Maxwell's Equations

$$\nabla \cdot \vec{D}(t) = \rho_v(t) \tag{1}$$

$$\nabla \cdot \vec{B}(t) = 0 \tag{2}$$

$$\nabla \times \vec{E}(t) = -\frac{\partial \vec{B}(t)}{\partial t} \tag{3}$$

$$\nabla \times \vec{H}(t) = \vec{J} + \frac{\partial \vec{D}(t)}{\partial t} \tag{4}$$

Constitutive relations

$$\vec{D} = [\epsilon(t)] * \vec{E}(t) \tag{5}$$

$$\vec{B} = [\mu(t)] * \vec{H}(t) \tag{6}$$

Eliminate Divergence

Our use of the Yee Grid Scheme allows us to elimate divergence via equations (7) and (8)

$$\nabla \cdot (\epsilon \vec{E}) = 0 \tag{7}$$

$$\nabla \cdot (\mu \vec{H}) = 0 \tag{8}$$

This also allows for the centering of the fields invoked by the curl equations Figure 1

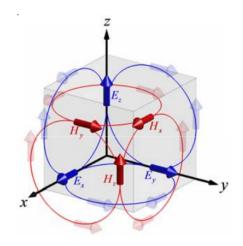


Figure 1: Yee Grid

Substitution

Because we have satisfied the divergence equations by using the Yee Grid Scheme, we are now able to focus on the curl equations with our constituive relation equations (5) and (6) substituted in. We are also not injecting a Source.

Where
$$\vec{J} = 0$$

$$\nabla \times \vec{E}(t) = -[\mu] \frac{\partial \vec{H}(t)}{\partial t} \tag{9}$$

$$\nabla \times \vec{H}(t) = [\epsilon] \frac{\partial \vec{H}(t)}{\partial t} \tag{10}$$

Normalize Magnetic Field

The Electric and Magnetic Fields are three orders of magnitude different. Rounding errors can propagate through our simulation, therefore we require that we normalize a Field. In this case we are normalizing the Magnetic Field.

$$\vec{\tilde{H}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \vec{H} \tag{11}$$