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EEL 5390 – Special Topics (FDTD)
HW #4

Notes:

P1 – I have broken up my documents into three documents, Maxwell Equations Derivation, 1D Derivation, and Sources. I believe that this will help me most in the future. So the Next major document should be 2D Derivation then 3D. I am still battling image floating formatting, but got I have nice section headers, Yay!!!.

P2 – I was battling "EXPLOSION!!!!" for the last couple of days. The cause was the manual calculation of dz, dt, and tau. Even though they all seem to fit the relevant conditions they did not work. I will have to explore further on next HW assignment. I used the parameters you gave and it worked just fine. Probably should have just used them in the first place, but I was trying to be all fancy.

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Maxwell Equations

Christopher Stricklan

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Abstract

This document describes the derivation of Maxwell's Equations from their Curl and Divergent forms to a final Finite Difference Form. A Yee Grid is used to derive the Finite Difference Equations.

Maxwell's Equations

$$\nabla \cdot \vec{D}(t) = \rho_v(t) \tag{1}$$

$$\nabla \cdot \vec{B}(t) = 0 \tag{2}$$

$$\nabla \times \vec{E}(t) = -\frac{\partial \vec{B}(t)}{\partial t} \tag{3}$$

$$\nabla \times \vec{H}(t) = \vec{J} + \frac{\partial \vec{D}(t)}{\partial t} \tag{4}$$

Constitutive relations

$$\vec{D} = [\epsilon(t)] * \vec{E}(t) \tag{5}$$

$$\vec{B} = [\mu(t)] * \vec{H}(t) \tag{6}$$

Eliminate Divergence

Our use of the Yee Grid Scheme alows us to elimate divergence via equations (7) and (8)

$$\nabla \cdot (\epsilon \vec{E}) = 0 \tag{7}$$

$$\nabla \cdot (\mu \vec{H}) = 0 \tag{8}$$

This also allows for the centering of the fields invoked by the curl equations Figure 1

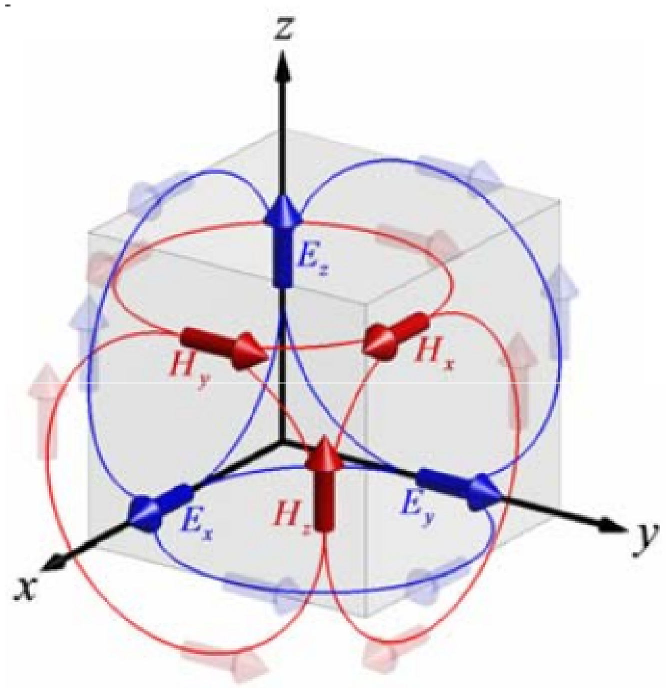

Figure 1: Yee Grid

Substitution

Because we have satisfied the divergence equations by using the Yee Grid Scheme, we are now able to focus on the curl equations with our constituive relation equations (5) and (6) subsituted in. We are also not injecting a Source.

Where $\vec{J} = 0$

$$\nabla \times \vec{E}(t) = -[\mu]\frac{\partial \vec{H}(t)}{\partial t} \tag{9}$$

$$\nabla \times \vec{H}(t) = [\epsilon]\frac{\partial \vec{H}(t)}{\partial t} \tag{10}$$

Normalize Magnetic Field

The Electric and Magnetic Fields are three orders of magnitude different. Rounding errors can propagate through our simulation, therefore we require that we normalize a Field. In this case we are normalizing the Magnetic Field.

$$\vec{\tilde{H}} = \sqrt{\frac{\mu_0}{\epsilon_0}}\vec{H} \Longrightarrow \nabla \times \vec{E} = -\frac{[\mu_r]}{c_0}\frac{\partial \vec{\tilde{H}}}{\partial t} \tag{11}$$

$$\nabla \times \vec{\tilde{H}} = \frac{[\epsilon_r]}{c_0}\frac{\partial \vec{E}}{\partial t}$$

Expand Curl Equations

$$\nabla \times \vec{E} = -\frac{[\mu_r]}{c_0}\frac{\partial \vec{\tilde{H}}}{\partial t} \implies \tag{12}$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{1}{c_0}\left(\mu_{xx}\frac{\partial \tilde{H}_x}{\partial t} + \mu_{xy}\frac{\partial \tilde{H}_y}{\partial t} + \mu_{xz}\frac{\partial \tilde{H}_z}{\partial t}\right)$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{1}{c_0}\left(\mu_{yx}\frac{\partial \tilde{H}_x}{\partial t} + \mu_{yy}\frac{\partial \tilde{H}_y}{\partial t} + \mu_{yz}\frac{\partial \tilde{H}_z}{\partial t}\right)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{1}{c_0}\left(\mu_{zx}\frac{\partial \tilde{H}_x}{\partial t} + \mu_{zy}\frac{\partial \tilde{H}_y}{\partial t} + \mu_{zz}\frac{\partial \tilde{H}_z}{\partial t}\right)$$

$$\nabla \times \vec{\tilde{H}} = \frac{[\epsilon_r]}{c_0}\frac{\partial \vec{E}}{\partial t} \implies \tag{13}$$

$$\frac{\partial \tilde{H}_z}{\partial y} - \frac{\partial \tilde{H}_y}{\partial z} = \frac{1}{c_0}\left(\epsilon_{xx}\frac{\partial E_x}{\partial t} + \epsilon_{xy}\frac{\partial E_y}{\partial t} + \epsilon_{xz}\frac{\partial E_z}{\partial t}\right)$$

$$\frac{\partial \tilde{H}_x}{\partial z} - \frac{\partial \tilde{H}_z}{\partial x} = \frac{1}{c_0}\left(\epsilon_{yx}\frac{\partial E_x}{\partial t} + \epsilon_{yy}\frac{\partial E_y}{\partial t} + \epsilon_{yz}\frac{\partial E_z}{\partial t}\right)$$

$$\frac{\partial \tilde{H}_y}{\partial x} - \frac{\partial \tilde{H}_x}{\partial y} = \frac{1}{c_0}\left(\epsilon_{zx}\frac{\partial E_x}{\partial t} + \epsilon_{zy}\frac{\partial E_y}{\partial t} + \epsilon_{zz}\frac{\partial E_z}{\partial t}\right)$$

Cross Out Diagonal Tensors

$$\nabla \times \vec{E} = -\frac{[\mu_r]}{c_0}\frac{\partial \vec{\tilde{H}}}{\partial t} \implies \tag{14}$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{1}{c_0}\left(\mu_{xx}\frac{\partial \tilde{H}_x}{\partial t} + \cancel{\mu_{xy}\frac{\partial \tilde{H}_y}{\partial t}} + \cancel{\mu_{xz}\frac{\partial \tilde{H}_z}{\partial t}}\right)$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{1}{c_0}\left(\cancel{\mu_{yx}\frac{\partial \tilde{H}_x}{\partial t}} + \mu_{yy}\frac{\partial \tilde{H}_y}{\partial t} + \cancel{\mu_{yz}\frac{\partial \tilde{H}_z}{\partial t}}\right)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{1}{c_0}\left(\cancel{\mu_{zx}\frac{\partial \tilde{H}_x}{\partial t}} + \cancel{\mu_{zy}\frac{\partial \tilde{H}_y}{\partial t}} + \mu_{zz}\frac{\partial \tilde{H}_z}{\partial t}\right)$$

$$\nabla \times \vec{\tilde{H}} = \frac{[\epsilon_r]}{c_0}\frac{\partial \vec{E}}{\partial t} \implies \tag{15}$$

$$\frac{\partial \tilde{H}_z}{\partial y} - \frac{\partial \tilde{H}_y}{\partial z} = \frac{1}{c_0}\left(\epsilon_{xx}\frac{\partial E_x}{\partial t} + \cancel{\epsilon_{xy}\frac{\partial E_y}{\partial t}} + \cancel{\epsilon_{xz}\frac{\partial E_z}{\partial t}}\right)$$

$$\frac{\partial \tilde{H}_x}{\partial z} - \frac{\partial \tilde{H}_z}{\partial x} = \frac{1}{c_0}\left(\cancel{\epsilon_{yx}\frac{\partial E_x}{\partial t}} + \epsilon_{yy}\frac{\partial E_y}{\partial t} + \cancel{\epsilon_{yz}\frac{\partial E_z}{\partial t}}\right)$$

$$\frac{\partial \tilde{H}_y}{\partial x} - \frac{\partial \tilde{H}_x}{\partial y} = \frac{1}{c_0}\left(\cancel{\epsilon_{zx}\frac{\partial E_x}{\partial t}} + \cancel{\epsilon_{zy}\frac{\partial E_y}{\partial t}} + \epsilon_{zz}\frac{\partial E_z}{\partial t}\right)$$

Final Analytical Equations

These equations are our final Maxwell Equations that will be used to derive the Finite-Difference Equations for all dimensions. We have one equation for each dimension for both the magentic and electric fields. This gives a total of six equations that will need to be tracked.

Magnetic Field Equations

$$\frac{\partial \tilde{H}_z}{\partial y} - \frac{\partial \tilde{H}_y}{\partial z} = \frac{\epsilon_{xx}}{c_0}\frac{\partial E_x}{\partial t} \tag{16}$$

$$\frac{\partial \tilde{H}_x}{\partial z} - \frac{\partial \tilde{H}_z}{\partial x} = \frac{\epsilon_{yy}}{c_0}\frac{\partial E_y}{\partial t} \tag{17}$$

$$\frac{\partial \tilde{H}_y}{\partial x} - \frac{\partial \tilde{H}_x}{\partial y} = \frac{\epsilon_{zz}}{c_0}\frac{\partial E_z}{\partial t} \tag{18}$$

Electric Field Equations

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{\mu_{xx}}{c_0}\frac{\partial \tilde{H}_x}{\partial t} \tag{19}$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{\mu_{yy}}{c_0}\frac{\partial \tilde{H}_y}{\partial t} \tag{20}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{\mu_{zz}}{c_0}\frac{\partial \tilde{H}_z}{\partial t} \tag{21}$$

Finite-Difference Derived Equations

Each equation needs to be derived based on the Yee Grid Model. Remember that adjacent cells will need to be used to perform the calcuation of the individual finite-difference equations for Magnetic and Electric fields in each direction x,y,z

Hx - Figure 2 Equation (22)

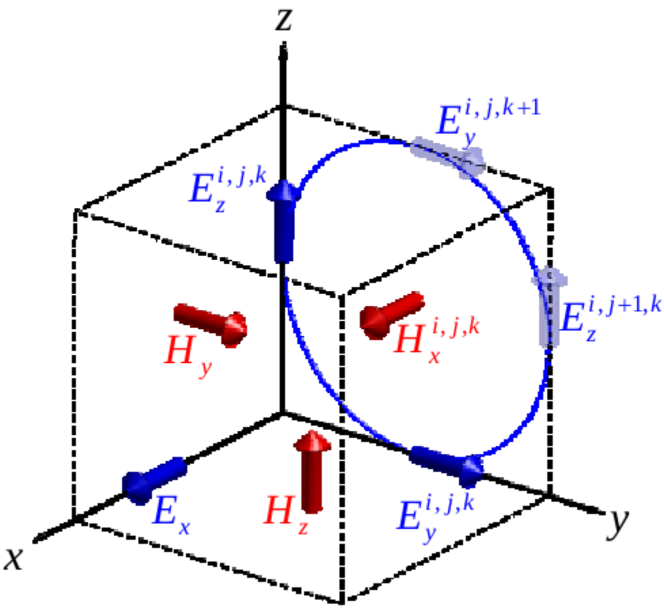

Figure 2: Yee Grid Hx

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{\mu_{xx}}{c_0}\frac{\partial \tilde{H}_x}{\partial t} \longrightarrow \tag{22}$$
$$\frac{E_z^{i,j+1,k}\mid_t - E_z^{i,j,k}\mid_t}{\Delta y} - \frac{E_y^{i,j,k+1}\mid_t - E_y^{i,j,k}\mid_t}{\Delta z} = -\frac{\mu_{xx}^{i,j,k}}{c_0}\frac{\tilde{H}_x^{i,j,k}\mid_{t+\frac{\Delta t}{2}} - \tilde{H}_x^{i,j,k}\mid_{t-\frac{\Delta t}{2}}}{\Delta t}$$

Hy - Figure 3 Equation (23)

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{\mu_{yy}}{c_0}\frac{\partial \tilde{H}_y}{\partial t} \longrightarrow \tag{23}$$
$$\frac{E_x^{i,j,k+1}\mid_t - E_x^{i,j,k}\mid_t}{\Delta z} - \frac{E_z^{i+1,j,k}\mid_t - E_z^{i,j,k}\mid_t}{\Delta x} = -\frac{\mu_{yy}^{i,j,k}}{c_0}\frac{\tilde{H}_y^{i,j,k}\mid_{t+\frac{\Delta t}{2}} - \tilde{H}_y^{i,j,k}\mid_{t-\frac{\Delta t}{2}}}{\Delta t}$$

Hz - Figure 4 Equation (24)

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{\mu_{zz}}{c_0}\frac{\partial \tilde{H}_z}{\partial t} \longrightarrow \tag{24}$$
$$\frac{E_y^{i+1,j,k}\mid_t - E_y^{i,j,k}\mid_t}{\Delta x} - \frac{E_x^{i,j+1,k}\mid_t - E_x^{i,j,k}\mid_t}{\Delta y} = -\frac{\mu_{zz}^{i,j,k}}{c_0}\frac{\tilde{H}_z^{i,j,k}\mid_{t+\frac{\Delta t}{2}} - \tilde{H}_z^{i,j,k}\mid_{t-\frac{\Delta t}{2}}}{\Delta t}$$

Ex - Figure 5 Equation (25)

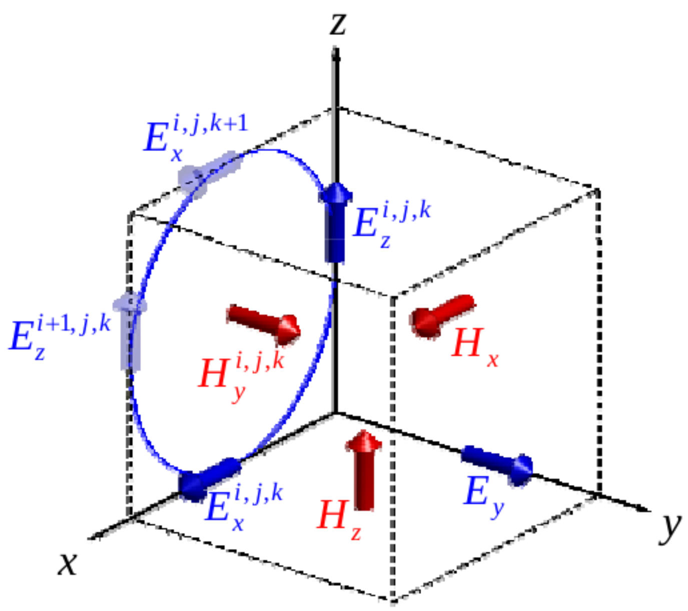

Figure 3: Yee Grid Hy

$$\frac{\partial \tilde{H}_z}{\partial y} - \frac{\partial \tilde{H}_y}{\partial z} = -\frac{\epsilon_{xx}}{c_0}\frac{\partial E_x}{\partial t} \longrightarrow \tag{25}$$
$$\frac{\tilde{H}_z^{i,j,k}\,|_{t+\frac{\Delta t}{2}} - \tilde{H}_z^{i,j-1,k}\,|_{t+\frac{\Delta t}{2}}}{\Delta y} - \frac{\tilde{H}_y^{i,j,k}\,|_{t+\frac{\Delta t}{2}} - \tilde{H}_y^{i,j,k-1}\,|_{t+\frac{\Delta t}{2}}}{\Delta z} = \frac{\epsilon_{xx}^{i,j,k}}{c_0}\frac{E_x^{i,j,k}\,|_{t+\Delta t} - E_x^{i,j,k}\,|_t}{\Delta t}$$

Ey - Figure 6 Equation (26)

$$\frac{\partial \tilde{H}_x}{\partial z} - \frac{\partial \tilde{H}_z}{\partial x} = -\frac{\epsilon_{yy}}{c_0}\frac{\partial E_y}{\partial t} \longrightarrow \tag{26}$$
$$\frac{\tilde{H}_x^{i,j,k}\,|_{t+\frac{\Delta t}{2}} - \tilde{H}_x^{i,j,k-1}\,|_{t+\frac{\Delta t}{2}}}{\Delta z} - \frac{\tilde{H}_z^{i,j,k}\,|_{t+\frac{\Delta t}{2}} - \tilde{H}_z^{i-1,j,k}\,|_{t+\frac{\Delta t}{2}}}{\Delta x} = \frac{\epsilon_{yy}^{i,j,k}}{c_0}\frac{E_y^{i,j,k}\,|_{t+\Delta t} - E_y^{i,j,k}\,|_t}{\Delta t}$$

Ez - Figure 7 Equation (27)

$$\frac{\partial \tilde{H}_y}{\partial x} - \frac{\partial \tilde{H}_x}{\partial y} = -\frac{\epsilon_{zz}}{c_0}\frac{\partial E_z}{\partial t} \longrightarrow \tag{27}$$
$$\frac{\tilde{H}_y^{i,j,k}\,|_{t+\frac{\Delta t}{2}} - \tilde{H}_y^{i-1,j,k}\,|_{t+\frac{\Delta t}{2}}}{\Delta x} - \frac{\tilde{H}_x^{i,j,k}\,|_{t+\frac{\Delta t}{2}} - \tilde{H}_x^{i,j-1,k}\,|_{t+\frac{\Delta t}{2}}}{\Delta y} = \frac{\epsilon_{zz}^{i,j,k}}{c_0}\frac{E_z^{i,j,k}\,|_{t+\Delta t} - E_z^{i,j,k}\,|_t}{\Delta t}$$

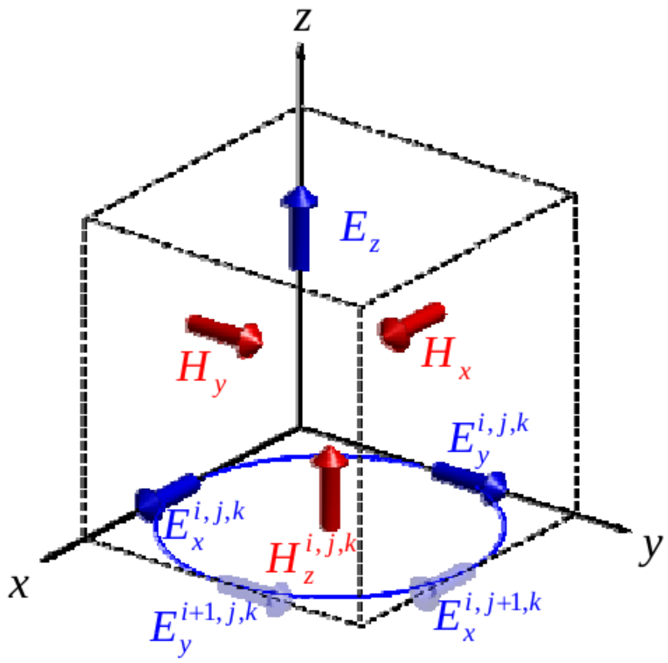

Figure 4: Yee Grid Hz

Summary of Finite-Difference Equations

The six sets equations below are our Finite-Difference equations derived from Maxwell's Equations. These equations we will use to derive equations for 1D, 2D, and 3D update equations for our FDTD algorithms.

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{\mu_{xx}}{c_0}\frac{\partial \tilde{H}_x}{\partial t}$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{\mu_{yy}}{c_0}\frac{\partial \tilde{H}_y}{\partial t}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{\mu_{zz}}{c_0}\frac{\partial \tilde{H}_z}{\partial t}$$

$$\frac{\partial \tilde{H}_z}{\partial y} - \frac{\partial \tilde{H}_y}{\partial z} = -\frac{\epsilon_{xx}}{c_0}\frac{\partial E_x}{\partial t}$$

$$\frac{\partial \tilde{H}_x}{\partial z} - \frac{\partial \tilde{H}_z}{\partial x} = -\frac{\epsilon_{yy}}{c_0}\frac{\partial E_y}{\partial t}$$

$$\frac{\partial \tilde{H}_y}{\partial x} - \frac{\partial \tilde{H}_x}{\partial y} = -\frac{\epsilon_{zz}}{c_0}\frac{\partial E_z}{\partial t}$$

$$\longrightarrow$$

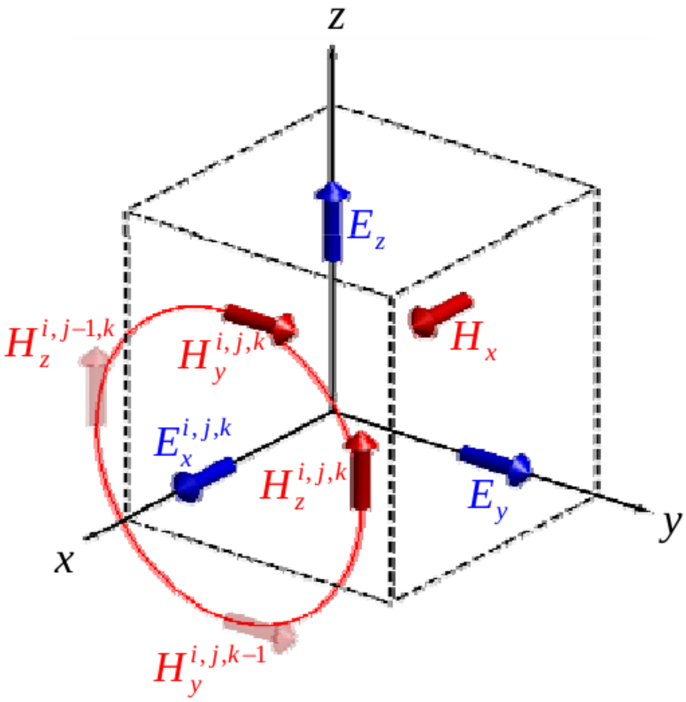

Figure 5: Yee Grid Ex

$$\frac{E_z^{i,j+1,k}|_t - E_z^{i,j,k}|_t}{\Delta y} - \frac{E_y^{i,j,k+1}|_t - E_y^{i,j,k}|_t}{\Delta z} = -\frac{\mu_{xx}^{i,j,k}}{c_0}\frac{\tilde{H}_x^{i,j,k}|_{t+\frac{\Delta t}{2}} - \tilde{H}_x^{i,j,k}|_{t-\frac{\Delta t}{2}}}{\Delta t}$$

$$\frac{E_x^{i,j,k+1}|_t - E_x^{i,j,k}|_t}{\Delta z} - \frac{E_z^{i+1,j,k}|_t - E_z^{i,j,k}|_t}{\Delta x} = -\frac{\mu_{yy}^{i,j,k}}{c_0}\frac{\tilde{H}_y^{i,j,k}|_{t+\frac{\Delta t}{2}} - \tilde{H}_y^{i,j,k}|_{t-\frac{\Delta t}{2}}}{\Delta t}$$

$$\frac{E_y^{i+1,j,k}|_t - E_y^{i,j,k}|_t}{\Delta x} - \frac{E_x^{i,j+1,k}|_t - E_x^{i,j,k}|_t}{\Delta y} = -\frac{\mu_{zz}^{i,j,k}}{c_0}\frac{\tilde{H}_z^{i,j,k}|_{t+\frac{\Delta t}{2}} - \tilde{H}_z^{i,j,k}|_{t-\frac{\Delta t}{2}}}{\Delta t}$$

$$\frac{\tilde{H}_z^{i,j,k}|_{t+\frac{\Delta t}{2}} - \tilde{H}_z^{i,j-1,k}|_{t+\frac{\Delta t}{2}}}{\Delta y} - \frac{\tilde{H}_y^{i,j,k}|_{t+\frac{\Delta t}{2}} - \tilde{H}_y^{i,j,k-1}|_{t+\frac{\Delta t}{2}}}{\Delta z} = \frac{\epsilon_{xx}^{i,j,k}}{c_0}\frac{E_x^{i,j,k}|_{t+\Delta t} - E_x^{i,j,k}|_t}{\Delta t}$$

$$\frac{\tilde{H}_x^{i,j,k}|_{t+\frac{\Delta t}{2}} - \tilde{H}_x^{i,j,k-1}|_{t+\frac{\Delta t}{2}}}{\Delta z} - \frac{\tilde{H}_z^{i,j,k}|_{t+\frac{\Delta t}{2}} - \tilde{H}_z^{i-1,j,k}|_{t+\frac{\Delta t}{2}}}{\Delta x} = \frac{\epsilon_{yy}^{i,j,k}}{c_0}\frac{E_y^{i,j,k}|_{t+\Delta t} - E_y^{i,j,k}|_t}{\Delta t}$$

$$\frac{\tilde{H}_y^{i,j,k}|_{t+\frac{\Delta t}{2}} - \tilde{H}_y^{i-1,j,k}|_{t+\frac{\Delta t}{2}}}{\Delta x} - \frac{\tilde{H}_x^{i,j,k}|_{t+\frac{\Delta t}{2}} - \tilde{H}_x^{i,j-1,k}|_{t+\frac{\Delta t}{2}}}{\Delta y} = \frac{\epsilon_{zz}^{i,j,k}}{c_0}\frac{E_z^{i,j,k}|_{t+\Delta t} - E_z^{i,j,k}|_t}{\Delta t}$$

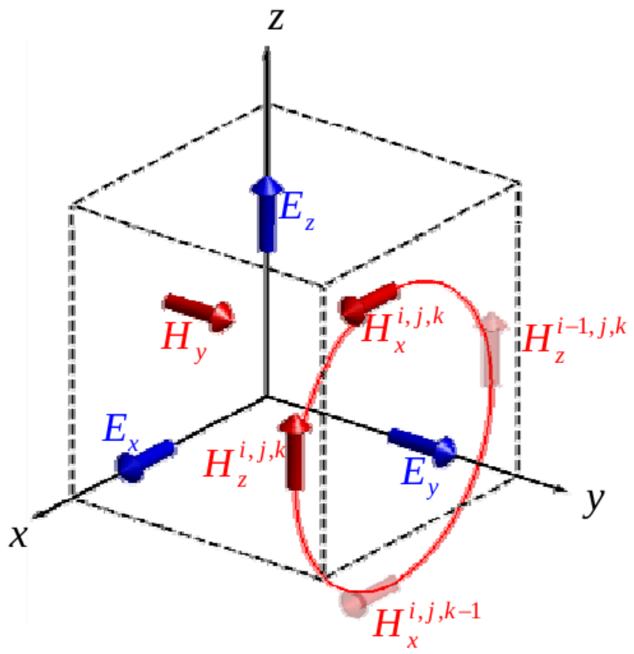

Figure 6: Yee Grid Ey

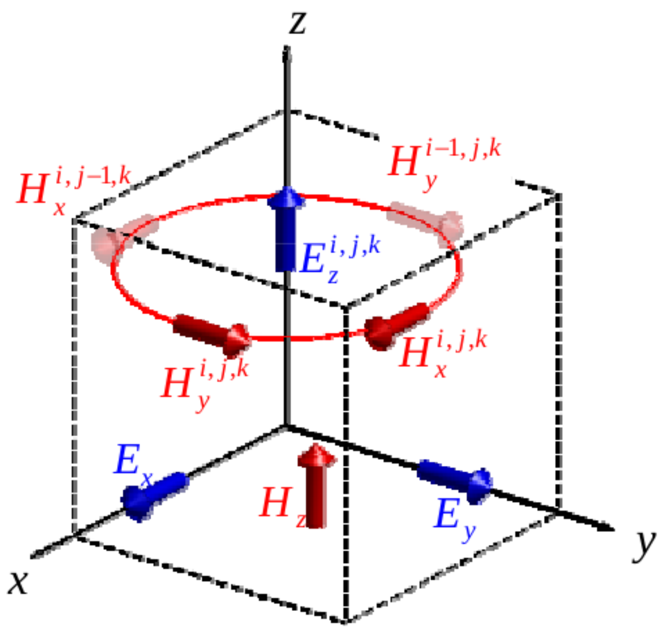

Figure 7: Yee Grid Ez

FDTD Sources

Christopher Stricklan

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Abstract

This document describes and formulates the various sources that can be used in FDTD models.

1D Sources

Gaussian Pulse

The Gaussian Pulse is an impulse function that allows us to excite the problem with a broad range of frequencies all at the same time.

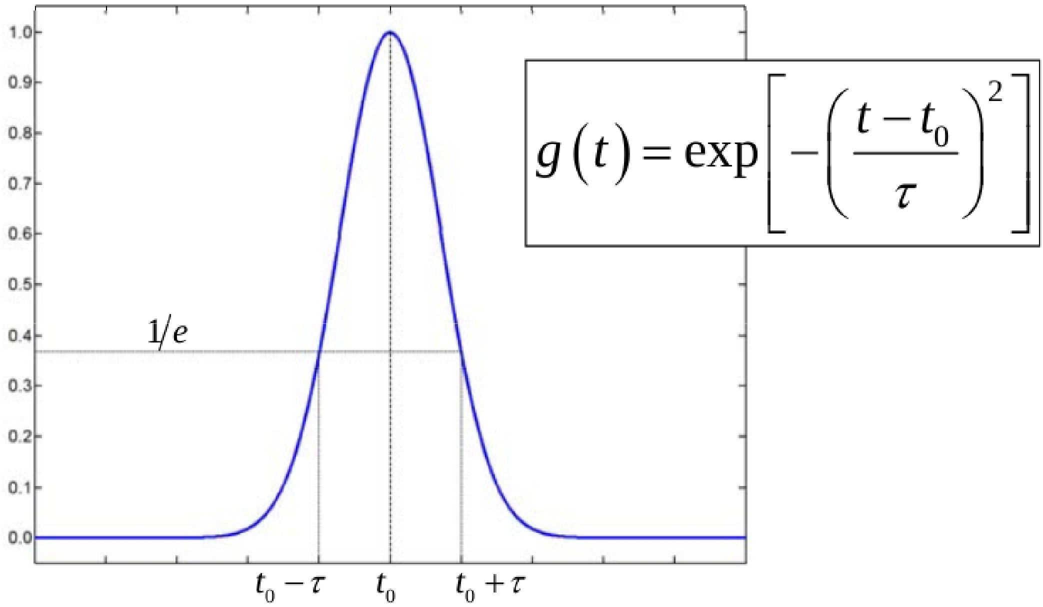

Figure 1: A Gaussian Pulse centered at t_0

To find the frequency content of the Pulse we must perform the Fourier Transform on the pulse.

$$g(t) = \exp\left(-\frac{t^2}{\tau^2}\right) \Longrightarrow G(f) = \frac{1}{\sqrt{\pi}B}\exp\left[-\frac{f^2}{B^2}\right]$$

This shows that the frequency content of the Gaussian Pulse extends from DC up to B where

$$B = \frac{1}{\pi\tau} \tag{1}$$

Pulse Design

To design the pulse for our simulations we must first decide on the maximum frequency we are interested in f_{max} then compute the pulse width with this upper frequency

$$B = f_{max} = \frac{1}{\pi\tau} \Longrightarrow \tau \leq \frac{1}{\pi f_{max}}$$

For simplification we can approximate τ as

$$\tau \cong \frac{0.5}{f_{max}} \tag{2}$$

In order to properly resolve our Guassian pulse, which should be completed by at least 10 to 20 time steps, we need to recalculate Δt. We now need to re-evaluate this value in conjuction with the Courant Stability Condition. We will determine Δt based on the maximum frequency and pick the smallest Δt.

$$\Delta t \leq \frac{\tau}{10} \tag{3}$$

Finally to properly inject our source without any adverse reactions within our model we must include delay 1.1. This will allow the pulse to ease into the problem space without producing large field gradients.

$$t_0 \geq 6\tau \tag{4}$$

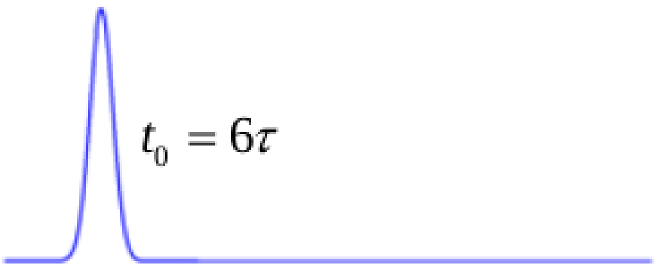

Figure 2: A Gaussian Pulse delayed by $t_0 = 6\tau$

Matlab Example

Program 1 The Gaussian Source Example

```
% Parameters
fmax = 1e9;                          %Max Frequency
dt = 0.5*dz/c0                       %Time Step
tau = 0.5/fmax;                      %Pulse Duration


% Compute Gaussian Source
nzc = round(Nz/2)                    %position of source
dt = 0.6*dz/c0
ta = [0:STEPS-1]*dt;                 %time axis
t0 = 6*tau                           %Pulse Position
s = dz/(2*c0) + dt/2                 %Total delay between E and H
Esrc = exp(-((ta-t0/tau).^2);        %E field source
A = - sqrt(ER(nzc)/UR(nzc));         %amplitude of H field
Hsrc = A * exp(-((ta-t0+s)/tau).^2); % H field source
```

Modeling 1D Problems with FDTD

Christopher Stricklan

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Abstract

This document describes the reduction from a 3-Dimensional Finite Difference set of Equations as derived in “Maxwell Equations” to a 1-Dimensional Form ready for use in numerical modeling algorithms.

Reduce to One-Dimension

We will reduce our Finite-Difference Equations to 1D. This means that we will have material that is uniform in two directions. This uniform material will cause the fields to be uniform as well. The change in material in our case we will define to be in the z-drection therefore the x anx y directions will be uniform. This means that the derivates in this direction will also equal zero.

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

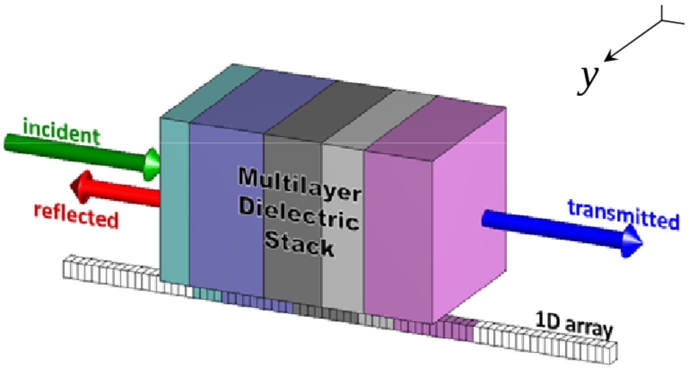


Figure 1: 1D Problem

From our Finite-Difference equations we need to cancel out the x and y derivates that are zero.

$$\cancel{\frac{\partial E_z}{\partial y}} - \frac{\partial E_y}{\partial z} = -\frac{\mu_{xx}}{c_0}\frac{\partial \tilde{H}_x}{\partial t}$$

$$\frac{\partial E_x}{\partial z} - \cancel{\frac{\partial E_z}{\partial x}} = -\frac{\mu_{yy}}{c_0}\frac{\partial \tilde{H}_y}{\partial t}$$

$$\cancel{\frac{\partial E_y}{\partial x}} - \cancel{\frac{\partial E_x}{\partial y}} = -\frac{\mu_{zz}}{c_0}\frac{\partial \tilde{H}_z}{\partial t}$$

$$\cancel{\frac{\partial \tilde{H}_z}{\partial y}} - \frac{\partial \tilde{H}_y}{\partial z} = \frac{\epsilon_{xx}}{c_0}\frac{\partial E_x}{\partial t}$$

$$\frac{\partial \tilde{H}_x}{\partial z} - \cancel{\frac{\partial \tilde{H}_z}{\partial x}} = \frac{\epsilon_{yy}}{c_0}\frac{\partial E_y}{\partial t}$$

$$\cancel{\frac{\partial \tilde{H}_y}{\partial x}} - \cancel{\frac{\partial \tilde{H}_x}{\partial y}} = \frac{\epsilon_{zz}}{c_0}\frac{\partial E_z}{\partial t}$$

$$\longrightarrow$$

$$\cancel{\frac{E_z^{i,j+1,k}\mid_t - E_z^{i,j,k}\mid_t}{\Delta y}} - \frac{E_y^{i,j,k+1}\mid_t - E_y^{i,j,k}\mid_t}{\Delta z} = -\frac{\mu_{xx}^{i,j,k}}{c_0}\frac{\tilde{H}_x^{i,j,k}\mid_{t+\frac{\Delta t}{2}} - \tilde{H}_x^{i,j,k}\mid_{t-\frac{\Delta t}{2}}}{\Delta t}$$

$$\frac{E_x^{i,j,k+1}\mid_t - E_x^{i,j,k}\mid_t}{\Delta z} - \cancel{\frac{E_z^{i+1,j,k}\mid_t - E_z^{i,j,k}\mid_t}{\Delta x}} = -\frac{\mu_{yy}^{i,j,k}}{c_0}\frac{\tilde{H}_y^{i,j,k}\mid_{t+\frac{\Delta t}{2}} - \tilde{H}_y^{i,j,k}\mid_{t-\frac{\Delta t}{2}}}{\Delta t}$$

$$\cancel{\frac{E_y^{i+1,j,k}\mid_t - E_y^{i,j,k}\mid_t}{\Delta x}} - \cancel{\frac{E_x^{i,j+1,k}\mid_t - E_x^{i,j,k}\mid_t}{\Delta y}} = -\frac{\mu_{zz}^{i,j,k}}{c_0}\frac{\tilde{H}_z^{i,j,k}\mid_{t+\frac{\Delta t}{2}} - \tilde{H}_z^{i,j,k}\mid_{t-\frac{\Delta t}{2}}}{\Delta t}$$

$$\cancel{\frac{\tilde{H}_z^{i,j,k}\mid_{t+\frac{\Delta t}{2}} - \tilde{H}_z^{i,j-1,k}\mid_{t+\frac{\Delta t}{2}}}{\Delta y}} - \frac{\tilde{H}_y^{i,j,k}\mid_{t+\frac{\Delta t}{2}} - \tilde{H}_y^{i,j,k-1}\mid_{t+\frac{\Delta t}{2}}}{\Delta z} = \frac{\epsilon_{xx}^{i,j,k}}{c_0}\frac{E_x^{i,j,k}\mid_{t+\Delta t} - E_x^{i,j,k}\mid_t}{\Delta t}$$

$$\frac{\tilde{H}_x^{i,j,k}\mid_{t+\frac{\Delta t}{2}} - \tilde{H}_x^{i,j,k-1}\mid_{t+\frac{\Delta t}{2}}}{\Delta z} - \cancel{\frac{\tilde{H}_z^{i,j,k}\mid_{t+\frac{\Delta t}{2}} - \tilde{H}_z^{i-1,j,k}\mid_{t+\frac{\Delta t}{2}}}{\Delta x}} = \frac{\epsilon_{yy}^{i,j,k}}{c_0}\frac{E_y^{i,j,k}\mid_{t+\Delta t} - E_y^{i,j,k}\mid_t}{\Delta t}$$

$$\cancel{\frac{\tilde{H}_y^{i,j,k}\mid_{t+\frac{\Delta t}{2}} - \tilde{H}_y^{i-1,j,k}\mid_{t+\frac{\Delta t}{2}}}{\Delta x}} - \cancel{\frac{\tilde{H}_x^{i,j,k}\mid_{t+\frac{\Delta t}{2}} - \tilde{H}_x^{i,j-1,k}\mid_{t+\frac{\Delta t}{2}}}{\Delta y}} = \frac{\epsilon_{zz}^{i,j,k}}{c_0}\frac{E_z^{i,j,k}\mid_{t+\Delta t} - E_z^{i,j,k}\mid_t}{\Delta t}$$

With these cross outs we come out with two observations. The first is that the longitudinal field components Ez and Hz are always zero. The second is that the Maxwell's equations have decoupled into two sets of two equations Ex/Hy Mode and Ey/Hx Mode.

$$-\frac{\partial E_y}{\partial z} = -\frac{\mu_{xx}}{c_0}\frac{\partial \tilde{H}_x}{\partial t}$$

$$\frac{\partial E_x}{\partial z} = -\frac{\mu_{yy}}{c_0}\frac{\partial \tilde{H}_y}{\partial t}$$

$$0 = -\frac{\mu_{zz}}{c_0}\frac{\partial \tilde{H}_z}{\partial t}$$

$$-\frac{\partial \tilde{H}_y}{\partial z} = \frac{\epsilon_{xx}}{c_0}\frac{\partial E_x}{\partial t}$$

$$\frac{\partial \tilde{H}_x}{\partial z} = \frac{\epsilon_{yy}}{c_0}\frac{\partial E_y}{\partial t}$$

$$0 = \frac{\epsilon_{zz}}{c_0}\frac{\partial E_z}{\partial t}$$

$$\longrightarrow$$

$$-\frac{E_y^{i,j,k+1}\mid_t - E_y^{i,j,k}\mid_t}{\Delta z} = -\frac{\mu_{xx}^{i,j,k}}{c_0}\frac{\tilde{H}_x^{i,j,k}\mid_{t+\frac{\Delta t}{2}} - \tilde{H}_x^{i,j,k}\mid_{t-\frac{\Delta t}{2}}}{\Delta t}$$

$$\frac{E_x^{i,j,k+1}\mid_t - E_x^{i,j,k}\mid_t}{\Delta z} = -\frac{\mu_{yy}^{i,j,k}}{c_0}\frac{\tilde{H}_y^{i,j,k}\mid_{t+\frac{\Delta t}{2}} - \tilde{H}_y^{i,j,k}\mid_{t-\frac{\Delta t}{2}}}{\Delta t}$$

$$\tilde{H}_z^{i,j,k} = 0$$

$$-\frac{\tilde{H}_y^{i,j,k}\mid_{t+\frac{\Delta t}{2}} - \tilde{H}_y^{i,j,k-1}\mid_{t+\frac{\Delta t}{2}}}{\Delta z} = \frac{\epsilon_{xx}^{i,j,k}}{c_0}\frac{E_x^{i,j,k}\mid_{t+\Delta t} - E_x^{i,j,k}\mid_t}{\Delta t}$$

$$\frac{\tilde{H}_x^{i,j,k}\mid_{t+\frac{\Delta t}{2}} - \tilde{H}_x^{i,j,k-1}\mid_{t+\frac{\Delta t}{2}}}{\Delta z} = \frac{\epsilon_{yy}^{i,j,k}}{c_0}\frac{E_y^{i,j,k}\mid_{t+\Delta t} - E_y^{i,j,k}\mid_t}{\Delta t}$$

$$E_z^{i,j,k} = 0$$

Summary of 1D Finite-Difference Equations

The equations are now broken apart into their respective modes. These modes are physical and propage independently from each other. In the end though they are numerically the same and will exhibit the same electromagnetic behavior. This allows us to only have to solve one set of two equations. We can also note that since we only changing materials along the z axis we can drop the i and j Array Indices.

Ex/Hy Mode

$$-\frac{\tilde{H}_y^{k}\mid_{t+\frac{\Delta t}{2}} - \tilde{H}_y^{k-1}\mid_{t+\frac{\Delta t}{2}}}{\Delta z} = \frac{\epsilon_{xx}^{k}}{c_0}\frac{E_x^{k}\mid_{t+\Delta t} - E_x^{k}\mid_t}{\Delta t} \tag{1}$$

$$\frac{E_x^{k+1}\mid_t - E_x^{k}\mid_t}{\Delta z} = -\frac{\mu_{yy}^{k}}{c_0}\frac{\tilde{H}_y^{k}\mid_{t+\frac{\Delta t}{2}} - \tilde{H}_y^{k}\mid_{t-\frac{\Delta t}{2}}}{\Delta t} \tag{2}$$

Ey/Hx Mode

$$\frac{\tilde{H}_x^{k}\mid_{t+\frac{\Delta t}{2}} - \tilde{H}_x^{k-1}\mid_{t+\frac{\Delta t}{2}}}{\Delta z} = \frac{\epsilon_{yy}^{k}}{c_0}\frac{E_y^{k}\mid_{t+\Delta t} - E_y^{k}\mid_t}{\Delta t} \tag{3}$$

$$\frac{E_y^{k+1}\mid_t - E_y^{k}\mid_t}{\Delta z} = \frac{\mu_{xx}^{k}}{c_0}\frac{\tilde{H}_x^{k}\mid_{t+\frac{\Delta t}{2}} - \tilde{H}_x^{k}\mid_{t-\frac{\Delta t}{2}}}{\Delta t} \tag{4}$$

Update Equations

We are now going to derive the update equations used during in FDTD algorithm. We have arbitrarily chosen to use Ey/Hx Mode found in Equations (3) and (4)

Update Equation for Ey

$$\frac{\tilde{H}_x^k \mid_{t+\frac{\Delta t}{2}} - \tilde{H}_x^{k-1} \mid_{t+\frac{\Delta t}{2}}}{\Delta z} = \frac{\epsilon_{yy}^k}{c_0} \frac{E_y^k \mid_{t+\Delta t} - E_y^k \mid_t}{\Delta t}$$

We want to solve for the E-Field at the future time value.

$$\frac{\epsilon_{yy}^k}{c_0} \frac{E_y^k \mid_{t+\Delta t} - E_y^k \mid_t}{\Delta t} = \frac{\tilde{H}_x^k \mid_{t+\frac{\Delta t}{2}} - \tilde{H}_x^{k-1} \mid_{t+\frac{\Delta t}{2}}}{\Delta z}$$

$$E_y^k \mid_{t+\Delta t} - E_y^k \mid_t = \frac{c_0 \Delta t}{\epsilon_{yy}^k} \left(\frac{\tilde{H}_x^k \mid_{t+\frac{\Delta t}{2}} - \tilde{H}_x^{k-1} \mid_{t+\frac{\Delta t}{2}}}{\Delta z} \right)$$

$$E_y^k \mid_{t+\Delta t} = E_y^k \mid_t + \left(\frac{c_0 \Delta t}{\epsilon_{yy}^k} \right) \left(\frac{\tilde{H}_x^k \mid_{t+\frac{\Delta t}{2}} - \tilde{H}_x^{k-1} \mid_{t+\frac{\Delta t}{2}}}{\Delta z} \right) \tag{5}$$

Update Equation for Hx

$$\frac{E_y^{k+1} \mid_t - E_y^k \mid_t}{\Delta z} = \frac{\mu_{xx}^k}{c_0} \frac{\tilde{H}_x^k \mid_{t+\frac{\Delta t}{2}} - \tilde{H}_x^k \mid_{t-\frac{\Delta t}{2}}}{\Delta t}$$

We want to sove for the H-Field at the future time value

$$\frac{\mu_{xx}^k}{c_0} \frac{\tilde{H}_x^k \mid_{t+\frac{\Delta t}{2}} - \tilde{H}_x^k \mid_{t-\frac{\Delta t}{2}}}{\Delta t} = \frac{E_y^{k+1} \mid_t - E_y^k \mid_t}{\Delta z}$$

$$\tilde{H}_x^k \mid_{t+\frac{\Delta t}{2}} - \tilde{H}_x^k \mid_{t-\frac{\Delta t}{2}} = \frac{c_0 \Delta t}{\mu_{xx}^k} \left(\frac{E_y^{k+1} \mid_t - E_y^k \mid_t}{\Delta z} \right)$$

$$\tilde{H}_x^k \mid_{t+\frac{\Delta t}{2}} = \tilde{H}_x^k \mid_{t-\frac{\Delta t}{2}} + \left(\frac{c_0 \Delta t}{\mu_{xx}^k} \right) \left(\frac{E_y^{k+1} \mid_t - E_y^k \mid_t}{\Delta z} \right) \tag{6}$$

Update Coefficients

Since the update coefficients don't change during the simulation. We can compute them once before our FDTD Algorithm is actually implemented in loop.

$$\tilde{H}_x^k \mid_{t+\frac{\Delta t}{2}} = \tilde{H}_x^k \mid_{t-\frac{\Delta t}{2}} + m_{H_x}^k \left(\frac{E_y^{k+1} \mid_t - E_y^k \mid_t}{\Delta z} \right) \tag{7}$$

$$E_y^k \mid_{t+\Delta t} = E_y^k \mid_t + m_{E_y}^k \left(\frac{\tilde{H}_x^k \mid_{t+\frac{\Delta t}{2}} - \tilde{H}_x^{k-1} \mid_{t+\frac{\Delta t}{2}}}{\Delta z} \right) \tag{8}$$

$$m_{E_y}^k = \frac{c_0 \Delta t}{\epsilon_{yy}^k} \tag{9}$$

$$m_{H_x}^k = \frac{c_0 \Delta t}{\mu_{yy}^k} \tag{10}$$

Boundary Conditions

We have two types of boundary conditions that can be incorporated into this 1-Dimensional Simulation. The Dirichlet and Perfectly Absorbing Boundary Conditions are methods that allow us to handle the fields that lie at the boundary of the model. Specifically these points lie at $k = 0$ and $k = N_z + 1$. At these points there is no grid to pull our Electric and Magnetic fields from, therefore to maintain the stability of the Finite Difference Maxwell's Equations we must handle these seperatly.

Dirichlet Boundary Condition

This boundary condition is the easiest to implement. We simply assume the fields that lie outside the grid are zero. This is the most crude method and will cause reflection back into our model as energy reaches the boundary.

$$\text{For } k < N_z \quad \tilde{H}_x^k \mid_{t+\frac{\Delta t}{2}} = \tilde{H}_x^k \mid_{t-\frac{\Delta t}{2}} + m_{H_x}^k \left(\frac{E_y^{k+1}|_t - E_y^k|_t}{\Delta z} \right)$$

$$\text{For } k = N_z \quad \tilde{H}_x^k \mid_{t+\frac{\Delta t}{2}} = \tilde{H}_x^k \mid_{t-\frac{\Delta t}{2}} + m_{H_x}^k \left(\frac{0 - E_y^k|_t}{\Delta z} \right)$$

$$\text{For } k > 1 \quad E_y^k \mid_{t+\Delta t} = E_y^k \mid_t + m_{E_y}^k \left(\frac{\tilde{H}_x^k|_{t+\frac{\Delta t}{2}} - \tilde{H}_x^{k-1}|_{t+\frac{\Delta t}{2}}}{\Delta z} \right)$$

$$\text{For } k = 1 \quad E_y^k \mid_{t+\Delta t} = E_y^k \mid_t + m_{E_y}^k \left(\frac{\tilde{H}_x^k|_{t+\frac{\Delta t}{2}} - 0}{\Delta z} \right)$$

Periodic Boundary Condition

This boundary condition assumes that the device being modeled is periodic along a particular direction. In our case the periodicity is along the z-axis.

$$\text{For } k < N_z \quad \tilde{H}_x^k \mid_{t+\frac{\Delta t}{2}} = \tilde{H}_x^k \mid_{t-\frac{\Delta t}{2}} + m_{H_x}^k \left(\frac{E_y^{k+1}|_t - E_y^k|_t}{\Delta z} \right)$$

$$\text{For } k = N_z \quad \tilde{H}_x^k \mid_{t+\frac{\Delta t}{2}} = \tilde{H}_x^k \mid_{t-\frac{\Delta t}{2}} + m_{H_x}^k \left(\frac{E_y^1|_t - E_y^{N_z}|_t}{\Delta z} \right)$$

$$\text{For } k > 1 \quad E_y^k \mid_{t+\Delta t} = E_y^k \mid_t + m_{E_y}^k \left(\frac{\tilde{H}_x^k|_{t+\frac{\Delta t}{2}} - \tilde{H}_x^{k-1}|_{t+\frac{\Delta t}{2}}}{\Delta z} \right)$$

$$\text{For } k = 1 \quad E_y^k \mid_{t+\Delta t} = E_y^k \mid_t + m_{E_y}^k \left(\frac{\tilde{H}_x^1|_{t+\frac{\Delta t}{2}} - \tilde{H}_x^{N_z}|_{t+\frac{\Delta t}{2}}}{\Delta z} \right)$$

Perfectly Absorbing Boundary Condition

The Perfectly Absorbing Boundary Condition provides a method to absorb almost all of the energy out of the model. There is approximately 10 orders of magnitude less energy reflected back into the problem space. This is very minor and should not affect our results. There are three conditions though that must be met in order to use this boundary conditionl.

- Waves at the boundaries are only traveling outward.
- Materials at the boundaries are linear, homogeneous, isotropic and non-dispersive.
- Time step is chosen so physical waves travel 1 cell in two time steps $\Delta t = \frac{\Delta z}{2c_o}$

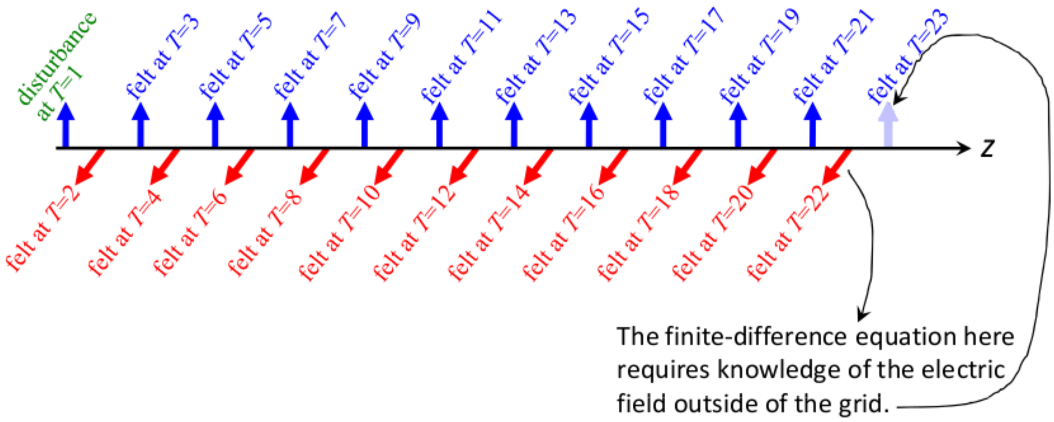


Figure 2: Perfeect Boundary Time Steps

As can be seen in 5.3 the last time step is magically off the grid. In order to implement the perfect boundary condition we must use a field that would have propagated past our last grid cell $2\Delta t$ time steps ago. In equations (11) and (12) the field value that exists outside our cells needs to be used when the wave reaches it at the $2\Delta t$ time frame

$$\tilde{H}_x^k \mid_{t+\frac{\Delta t}{2}} = \tilde{H}_x^k \mid_{t-\frac{\Delta t}{2}} + m_{H_x}^k \left(\frac{E_y^{N_z+1} \mid_t - E_y^{N_z} \mid_t}{\Delta z} \right) \tag{11}$$

$$E_y^k \mid_{t+\Delta t} = E_y^k \mid_t + m_{E_y}^k \left(\frac{\tilde{H}_x^1 \mid_{t+\frac{\Delta t}{2}} - \tilde{H}_x^0 \mid_{t+\frac{\Delta t}{2}}}{\Delta z} \right) \tag{12}$$

Since the field values did exist $2\Delta t$ ago in our boundary cell in the problem space. We can then infer the value is the same as the energy leaves the grid.

$$E_y^{N_z+2} \mid_t = E_y^{N_z} \mid_{t-2\Delta t}$$

$$\tilde{H}_x^0 \mid_t = \tilde{H}_x^1 \mid_{t-2\Delta t}$$

To implement the boundary condition we keep track of previous history in variables. At the z-low boundary we only modify the E-Field update equations. At the z-high boundary we only modify the H-Field update equations.

$$h_3 = h_2 \quad h_2 = h_1 \quad h1 = \tilde{H}_x^1$$

$$E_y^k \mid_{t+\Delta t} = E_y^1 \mid_t + m_{E_y}^1 \left(\frac{\tilde{H}_x^1 \mid_{t+\frac{\Delta t}{2}} - h_3}{\Delta z} \right)$$

$$e_3 = e_2 \quad e_2 = e_1 \quad e_1 = E_y^{N_z}$$

$$\tilde{H}_x^{N_z} \mid_{t+\frac{\Delta t}{2}} = \tilde{H}_x^{N_z} \mid_{t-\frac{\Delta t}{2}} + m_{H_x}^{N_z} \left(\frac{e_3 - E_y^{N_z} \mid_t}{\Delta z} \right)$$

Sources

There are two ways to incorporate a source into our models, Hard Source and Soft Source. The hard source is the simplest to implement. After we have performed our update equations over the entire grid we overwrite some fields with our source. This injects the source energy into our model. This does cause a problem because the point where the source was inject behaves like a perfect electric or magnetic conductor to scattered fields.

$$\tilde{H}_x^k \mid_{t+\frac{\Delta t}{2}} = g_H \mid_k \quad E_y^k \mid_{t+\Delta t} = g_E \mid_k$$

The soft source on the otherhand is the best way to inject energy into our model. After performing the udpate equations across the entire grid, the source is added to the current value of our source injection points. This injects energy into the model, but allows scattered waves to pass through the source region like it was transparent.

$$\tilde{H}_x^k \mid_{t+\frac{\Delta t}{2}} = \tilde{H}_x^k \mid_{t+\frac{\Delta t}{2}} + g_H \mid_k \quad E_y^k \mid_{t+\Delta t} = E_y^k \mid_{t+\Delta t} + g_E \mid_k$$

Model Parameters

Grid Resolution

There are multiple considerations that need to be addressed in order to properly calculate the grid resolution.

Wavelength

The grid resolution must be sufficient to resolve the shortest wavelength. First we must determine the smallest wavelength then we must devide this by a value that can be found in the chart. These values are used as a guideline depending on the material with the highest refractive index.

$$\lambda_{min} = \frac{c_0}{f_{max} nmax}$$

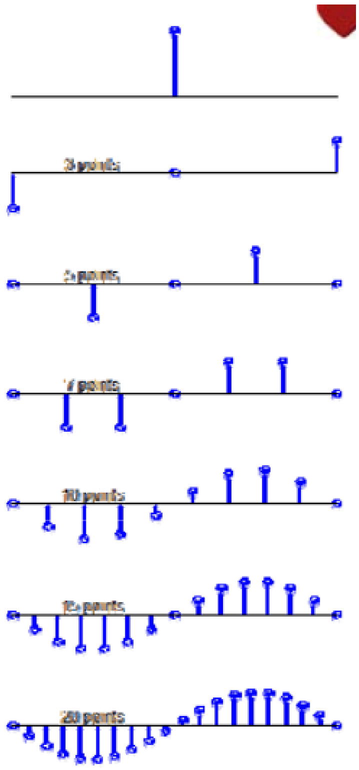


Figure 3: Building up wavelength resolution

$$\Delta_\lambda \approx \frac{\lambda_{min}}{N_\lambda} \quad N_\lambda \geq 10$$

Approximate N_λ Values		
N_λ	Refractive Index	Comment
10 to 20	$n = 1 - 10$	Low contrast dielectrics
10 to 30	$n = 10 - 30$	High contrast dielectrics
40 to 60	$n = 40 - 60$	Most metallic structures
100 to 200	$n \geq 60$	Plasmonic devices

Mechanical Features

The grid resolution mst be sufficient to resolve the smallest physical feature of the device. First, we must determine the smallest feature 7.1 d_{min}. then we must device this by 1 to 4 to accurately manage the width of our smallest feature.

$$\Delta_d \approx \frac{d_{min}}{N_d} \quad N_d \geq 1$$

Calculate Grid Resolution

We will first calculate an initial grid resolution by taking the minimum between the model feature Δ_d and wavelength Δ_λ.

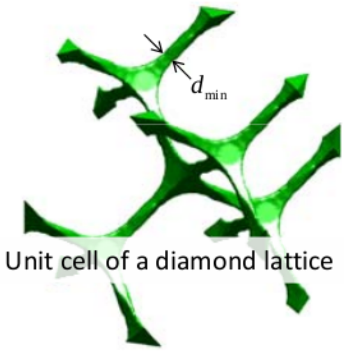


Figure 4:

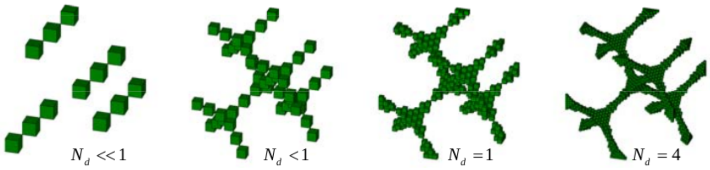


Figure 5:

$$\Delta_x = \Delta_y = min[\Delta_\lambda, \Delta_d]$$

Finally we need to "Snap" grid to critical dimensions. In 7.1 we have modified delta to not fall halfway through a grid cell. We can't fill half a cell in a discrete environment. First we Decide what dimensions along each axis are critical. d_x and d_y. We then compute how many grid cells comprise d_c and round up (13). Finally adjust grid resolution to fit this dimension in grid exactly (14).

$$\begin{aligned} M_x &= ceil(\frac{d_x}{\Delta_x}) \\ M_y &= ceil(\frac{d_y}{\Delta_y}) \end{aligned} \tag{13}$$

$$\begin{aligned} \Delta_x &= \frac{d_x}{M_x} \\ \Delta_y &= \frac{d_y}{M_y} \end{aligned} \tag{14}$$

Courant Stability Condition

For each iteration of our update equations a currently existing electric field will not immediatly affect adjacent fields. A disturbance in the electric field at one point will only be felt by the immediately adjacent magnetic fields. For an adjanent electric field to feel the disturbance it takes at least two time steps.

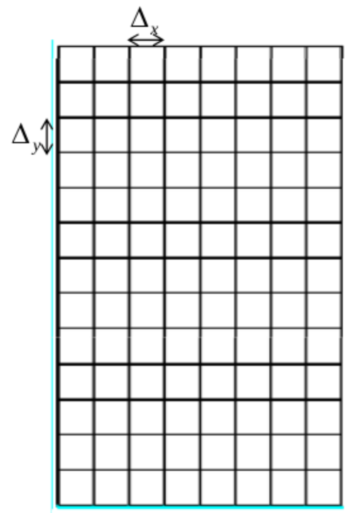


Figure 6: Grid X/Y

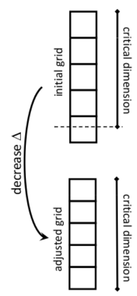


Figure 7: Critical Dimension Expansion

Keeping in mind that Electromagnetic waves propagate at the speed of light in a vacuum. Inside a material, a wave propagates at a reduce speed based on the refractive index (15)

$$v = \frac{c_0}{n} \tag{15}$$

Because we are dealing with a numerical algorithm it is not possible for a disturbance to travel farther than one unit cell in a single time step. So we need to force a condition that would force a wave not to propage farther than a single unit cell in one time step.

$$\frac{c_0 \Delta t}{n} < \Delta z$$

So now we can place an upper limit on a time steps

$$\Delta t < \frac{n \Delta z}{c_0}$$

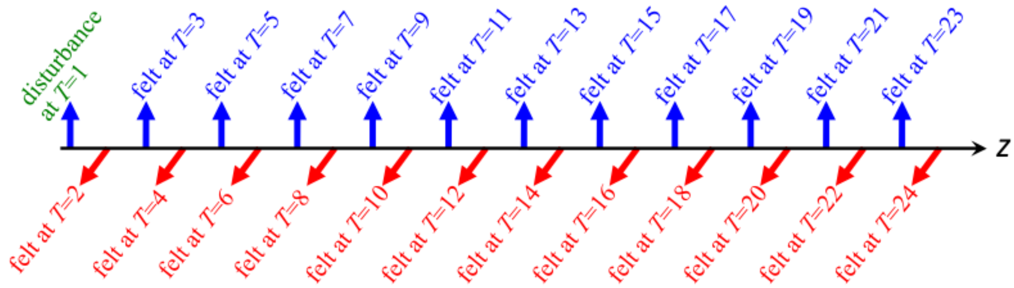


Figure 8: Timeline of electric field propation

To force our wave to travel the distance of one grid cell in exactly two time steps we finish up with (**??**)

$$\Delta t = \frac{n\Delta z}{2c_0} \tag{16}$$

FDTD Algorithm

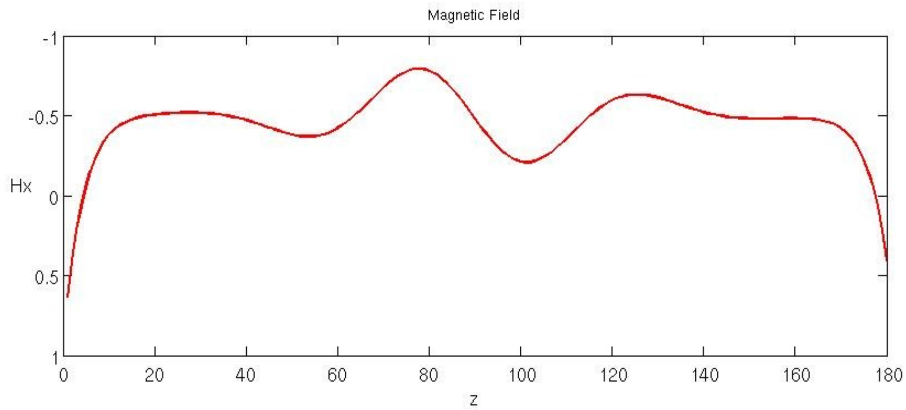
We first must do some initialization work

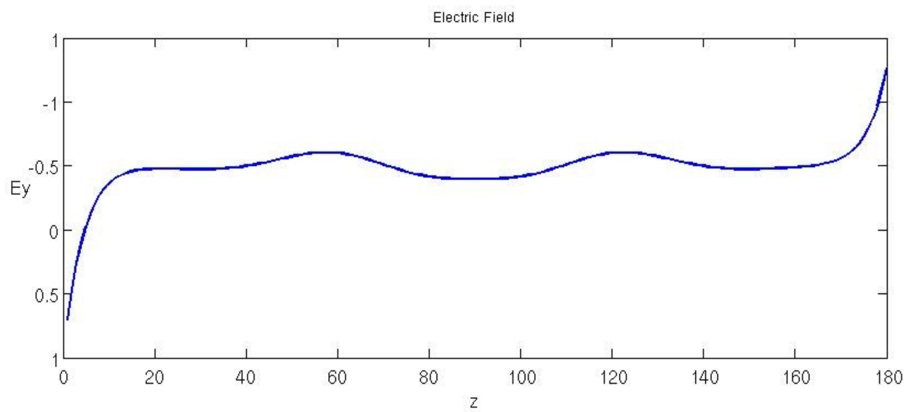
- Compute Grid Resolution
- Compute time step
- Compute Source
- Compute Update Coefficeints
- Intialize Fields

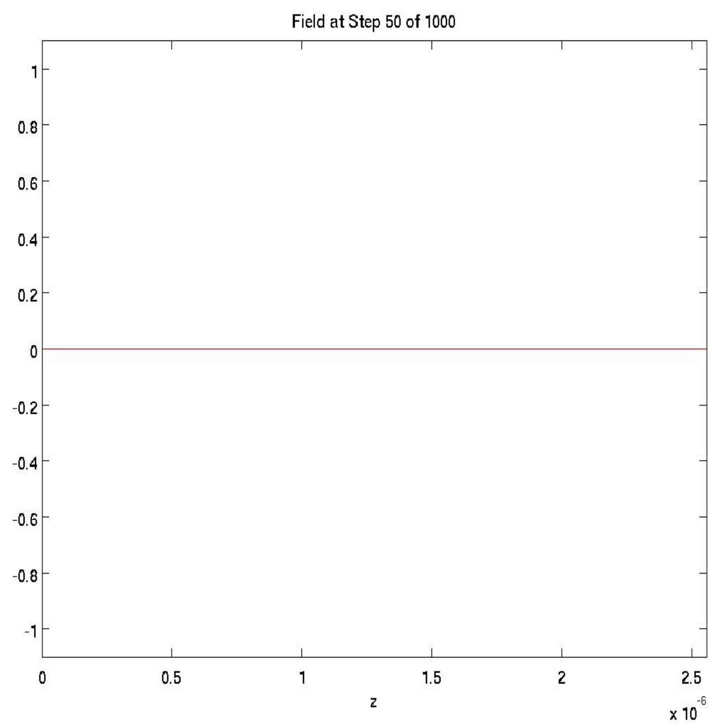
The meat of the algorithm is the loop

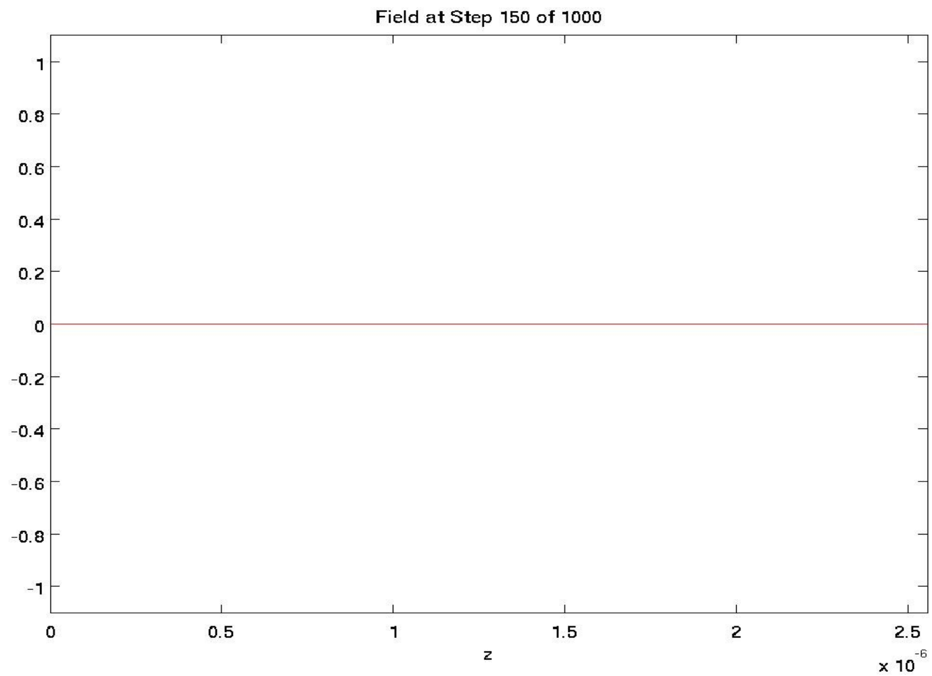
- Update H from E
- Update E from H
- Inject Source
- Visualize Fields

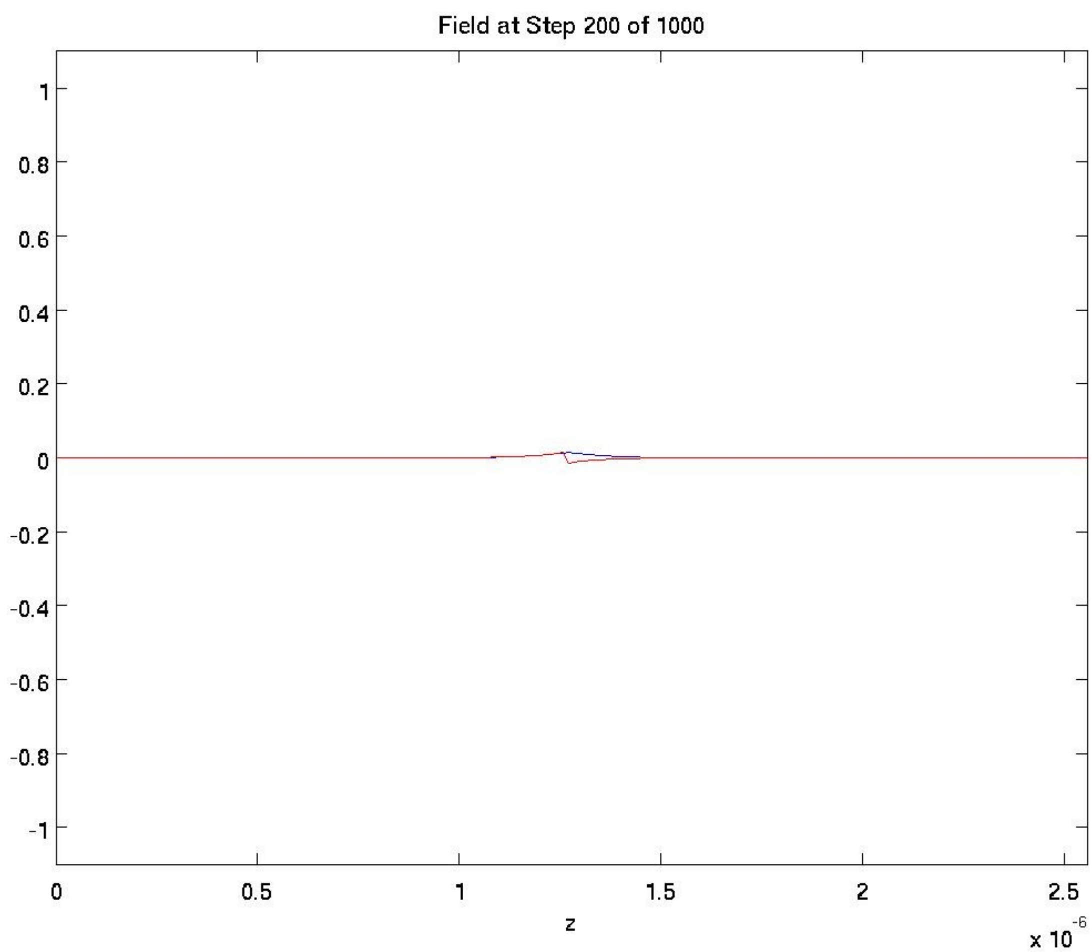
Problem 2

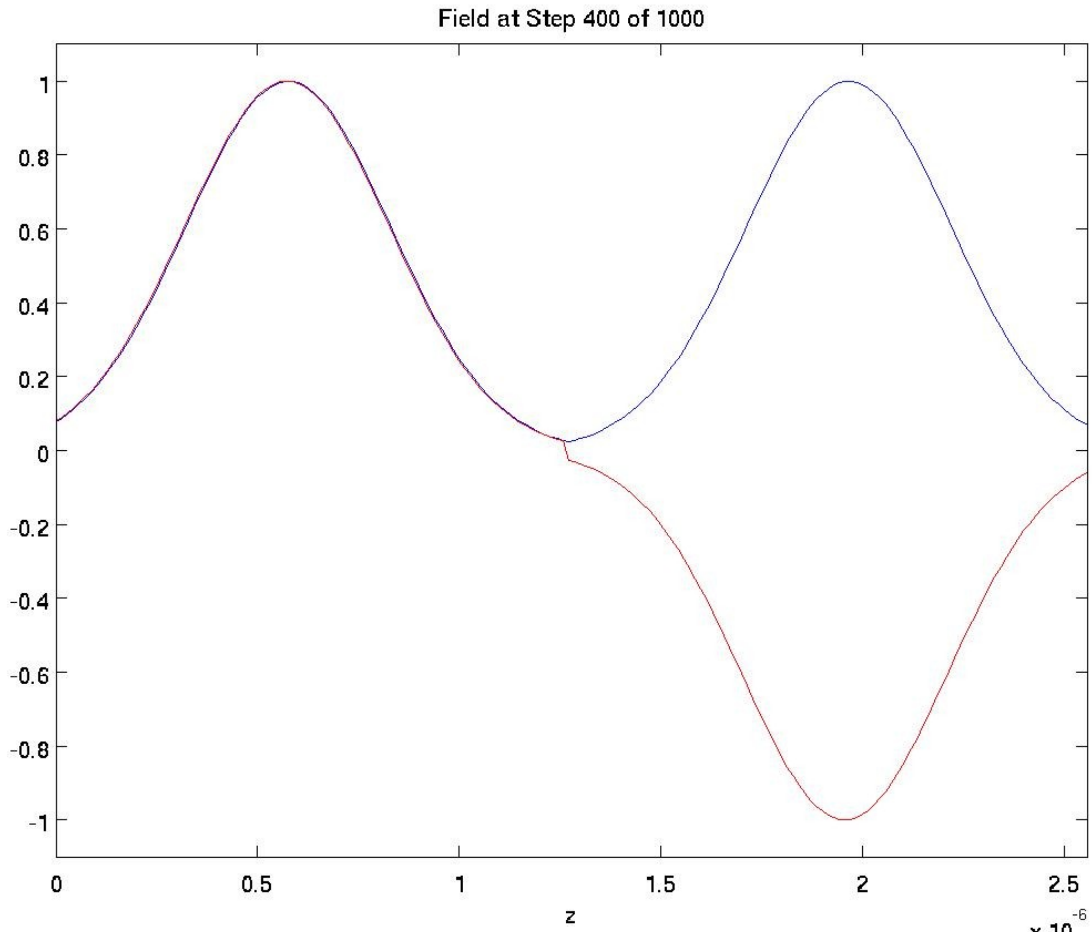

Magnetic Field
Hx
z

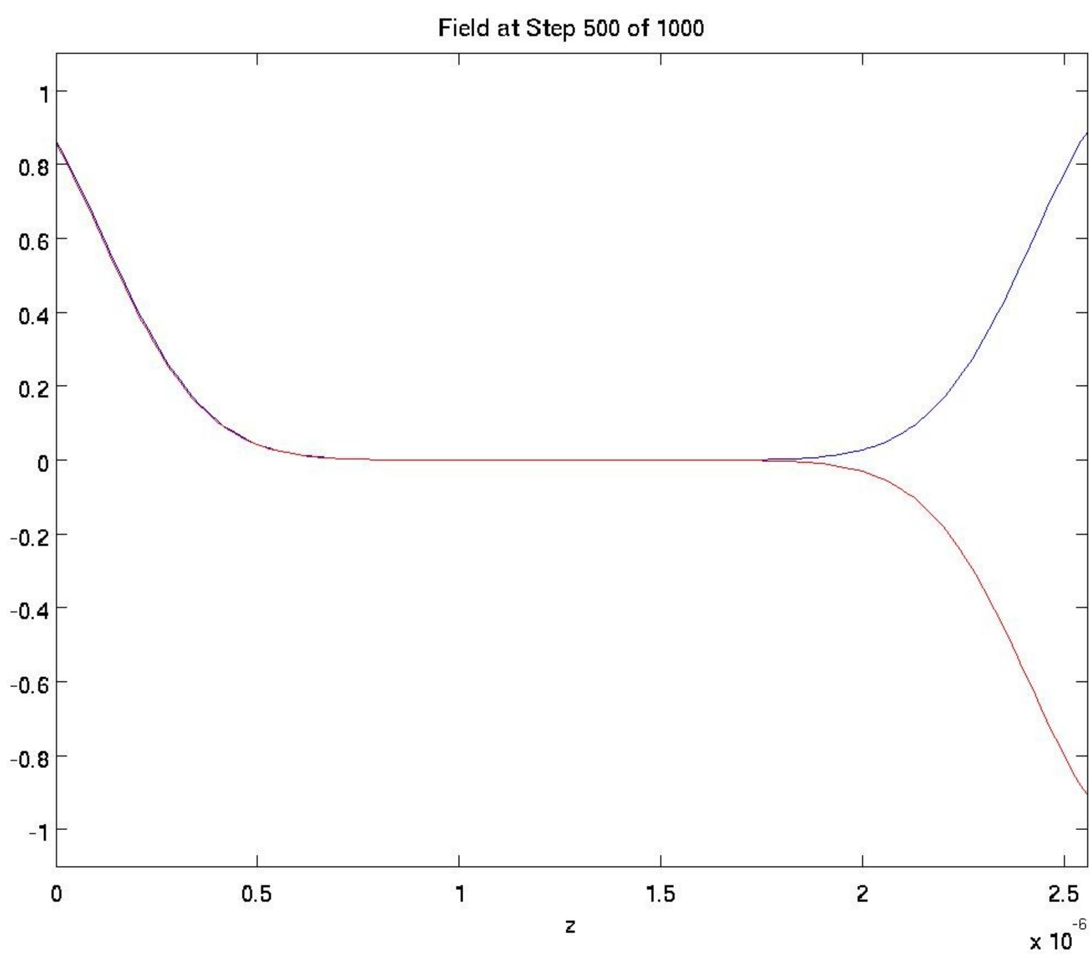

Electric Field
Ey
z

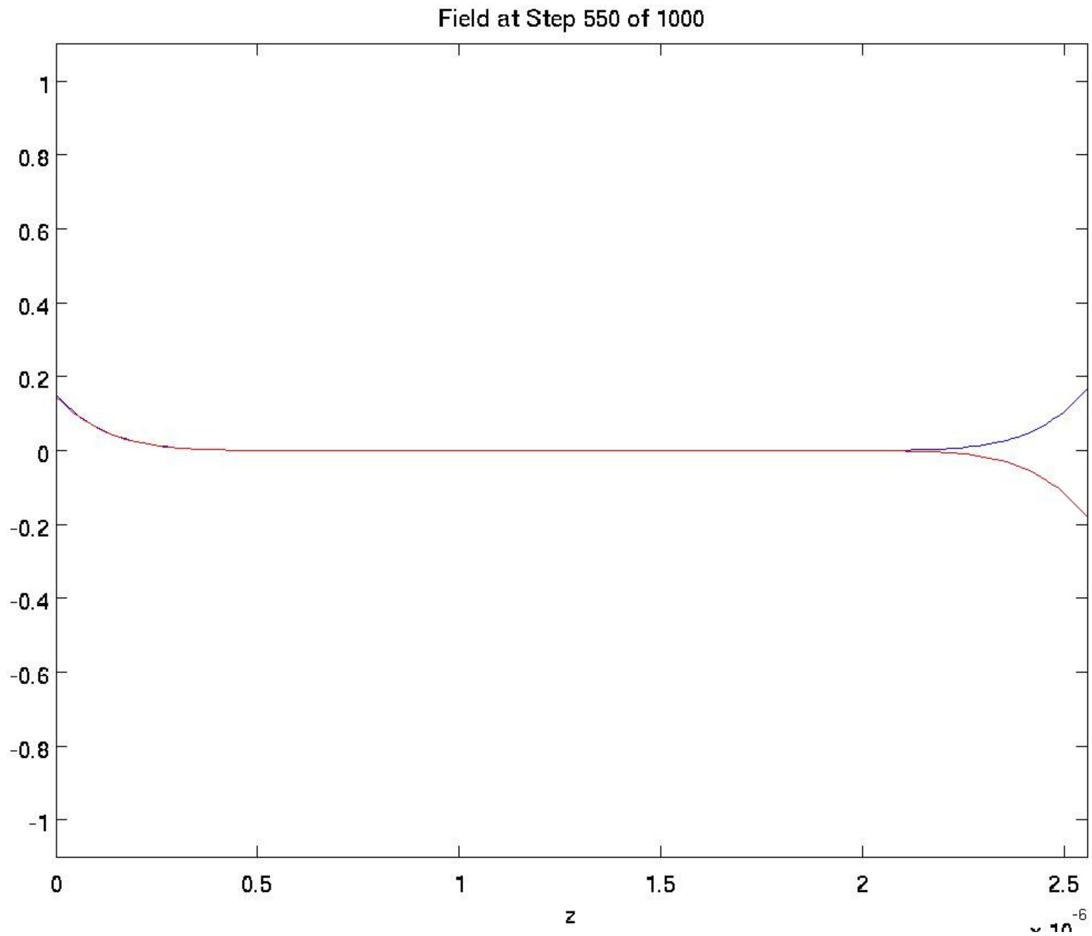

Field at Step 50 of 1000
z
x 10^-6

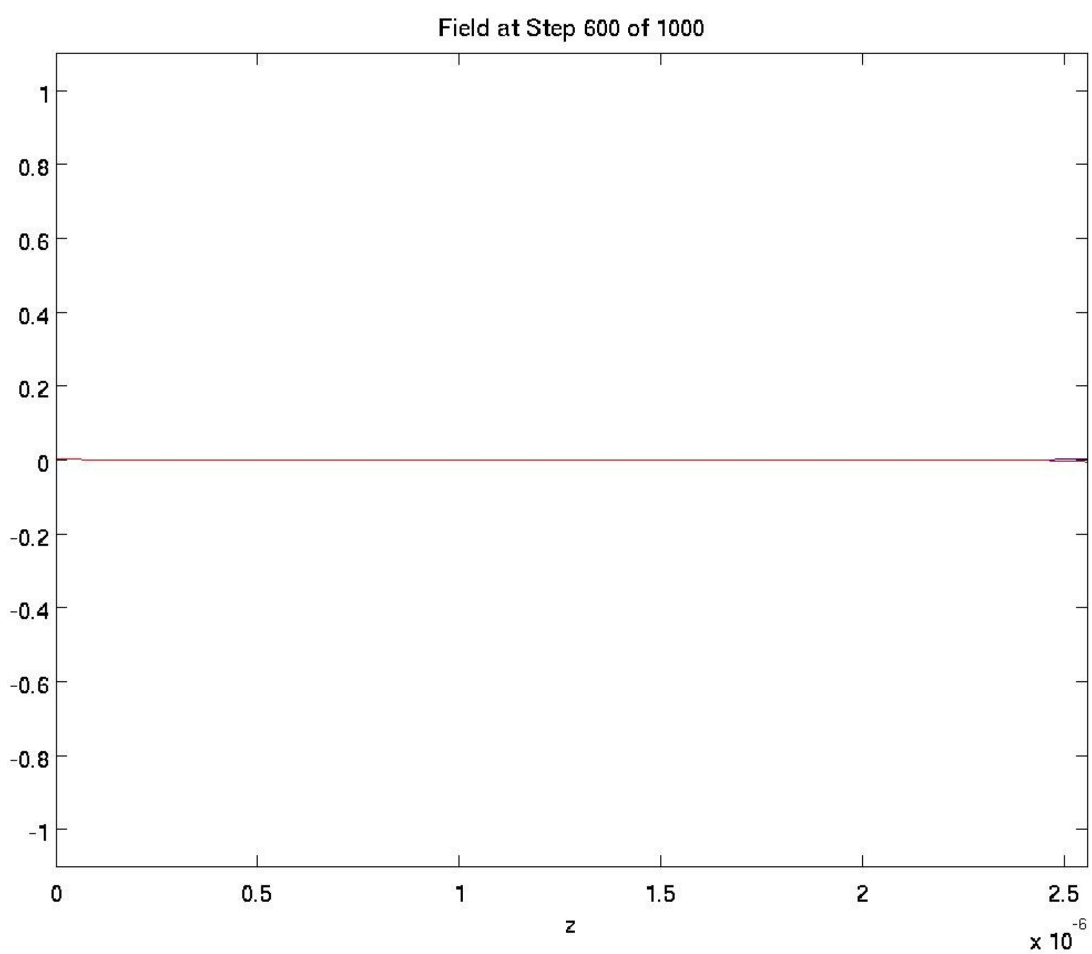

Field at Step 150 of 1000
z
x 10^-6

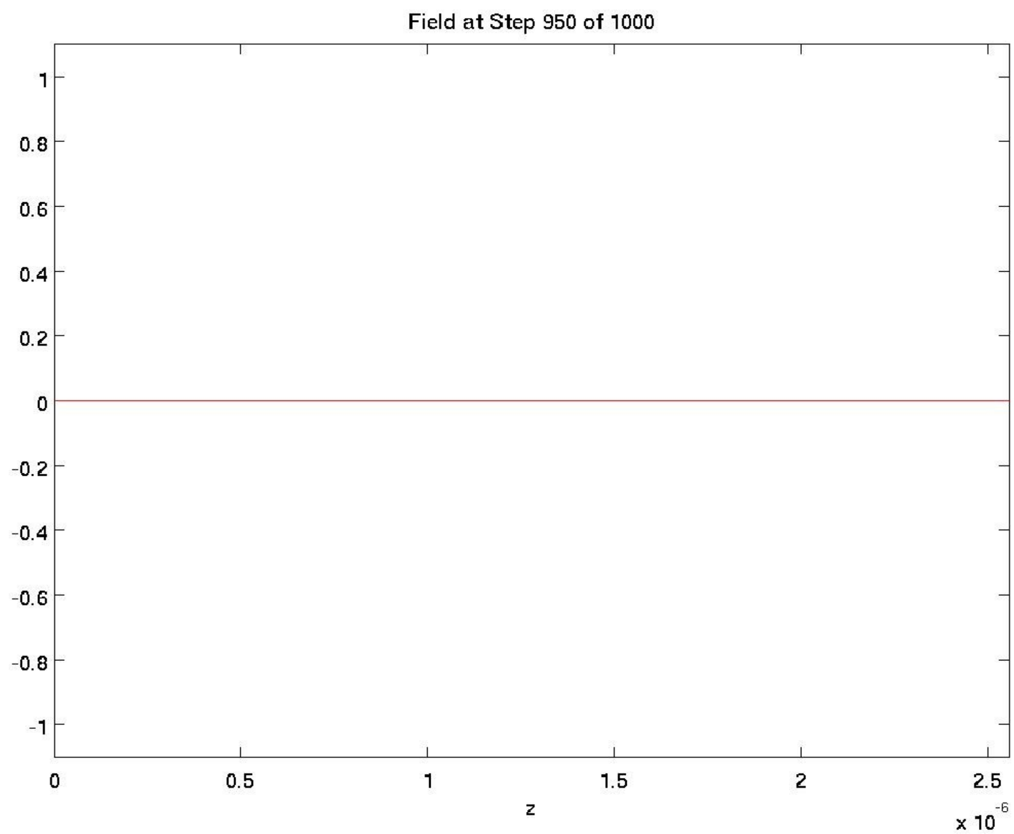

Field at Step 200 of 1000
z
x 10^-6

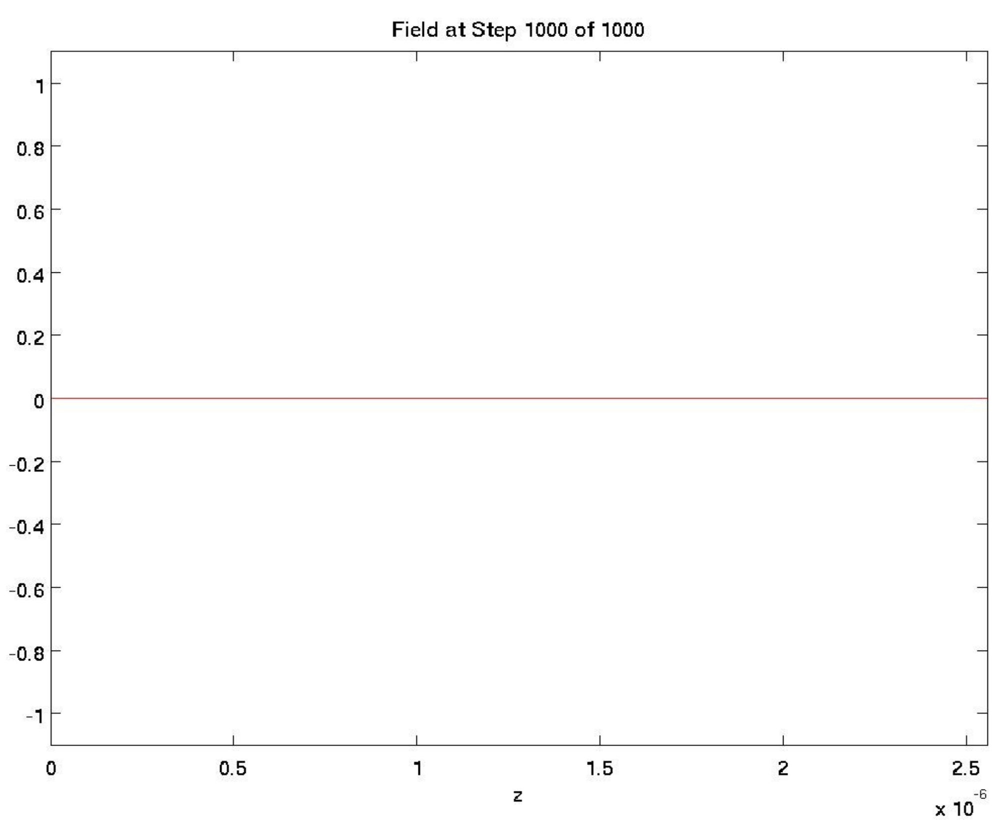

Field at Step 400 of 1000
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.5
1
1.5
2
2.5
z


Field at Step 500 of 1000
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.5
1
1.5
2
2.5
z
x 10^-6


Field at Step 550 of 1000
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.5
1
1.5
2
2.5
z


Field at Step 600 of 1000
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.5
1
1.5
2
2.5
z
x 10^-6


Field at Step 950 of 1000
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.5
1
1.5
2
2.5
z
x 10^-6


Field at Step 1000 of 1000
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.5
1
1.5
2
2.5
z
x 10^-6

Appendix – P2 Code

```
%FDTD1D

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Pre-Program Work
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Initialize MATLAB
close all; clc;
clear all;

%Constants
c0 = 299792458; %m/s
e0 = 8.854187817*10^-12; %F/m
u0 = 1.256637061*10^-6; %H/m

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Initialization
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Simulated Environment Settings
STEPS = 1000;
f_max = 1e9;  % 1Ghz
Nz = 180;
dz = 1.4286e-8;

%Compute Time Steps
dt = dz/(2*c0); %secs

%Grid Axis
za=[0:Nz-1]*dz;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Model
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Material Vectors
ER = ones([1 Nz]);
UR = ones([1 Nz]);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  Source
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
nzc = round (Nz/2);  %Position of Sources
tau = 1.2e-15        %normally 0.5/f_max *** Not working***
ta = [0:STEPS-1]*dt;     % Time Axis;
t0 = 6*tau;              % Delay
s = dz/(2*c0) + dt/2;    % Delay between E and H
Esrc = exp(-((ta-t0)/tau).^2); % E Source
A = -sqrt(ER(nzc)/UR(nzc));    % H Amplitude
Hsrc = A*exp(-((ta-t0+s)/tau).^2); % H Source

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%FDTD Initialization
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
% Compute Update Coefficients
mER = (c0*dt/dz)./ER;
mHR = (c0*dt/dz)./UR;

% Initialize Feilds
Ey = zeros([1 Nz]);
Hx = zeros([1 Nz]);


%PAB Parameters
h1 = 0; h2 = 0; h3 = 0;
e1 = 0; e2 = 0; e3 = 0;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Execute Simulation
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for t = 1:STEPS

  % Calculate H
  for nz = 1:Nz-1
    Hx(nz) = Hx(nz) + mHR(nz)*(Ey(nz+1)-Ey(nz));
  end

  Hx(Nz) = Hx(Nz) + mHR(Nz)*(e3 - Ey(Nz));

  h3 = h2; h2 = h1; h1 = Hx(1); % Boundary Params;

  % Calculate E
  Ey(1) = Ey(1) + mER(1)*(Hx(1) - h3);
  for nz = 2:Nz
    Ey(nz) = Ey(nz) + mER(nz)*(Hx(nz)-Hx(nz-1));
  end

  %Inject Source
  Ey(nzc) = Ey(nzc) + Esrc(t);

  e3=e2; e2=e1; e1=Ey(Nz); % Boundary Params;

  h = plot(za, Ey, '-b'); hold on;
  plot(za, Hx, '-r'); hold off;
  axis([za(1) za(Nz) -1.1 1.1]);
  xlabel('z');
  title(['Field at Step ' num2str(t) ' of ' num2str(STEPS)]);
  drawnow();

  if(mod(t,50) == 0)
    saveas(h, ['images/' num2str(t) '.jpg'], 'jpg');
  end
 end


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Plot Fields
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
fig = figure;
SetFigure(fig, 'HW#3-P2', [500 274 965 826]);
```

```
%Plot Magnetic Field
subplot(211)
h = plot(Hx, '-r', 'LineWidth', 2);
title('Magnetic Field');
h = get(h, 'Parent');
set(h, 'Fontsize', 14);
xlabel('z');
ylabel('Hx', 'Rotation', 0);
set(gca,'YTickLabel',{'1','0.5','0', '-0.5', '-1'})

%Plot Electric Field
subplot(212)
h = plot(Ey, '-b', 'LineWidth', 2);
title('Electric Field');
h = get(h, 'Parent');
set(h, 'Fontsize', 14);
xlabel('z');
ylabel('Ey', 'Rotation', 0);
set(gca,'YTickLabel',{'1','0.5','0', '-0.5', '-1'})
```