PERMUTATIONS AND COMBINATIONS

Q1. A round table conference is to be held among 25 delegates from 25 countries. In how many ways can they be seated if two particular delegates are always to sit together?

Answer:

If we consider the two particular delegates who always sit together as a single entity, then we have 24 entities to be seated around the round table.
The number of ways to seat 24 entities around a round table is $(24-1)! = 23!$.
However, within the two particular delegates who always sit together, there are 2! ways to arrange them.
Therefore, the total number of ways to seat the 25 delegates around the round table with the two particular delegates always sitting together is $2! \times 23!$, which is option B.

Q2. In how many ways can 5 boys and 4 girls be seated in a row, so that they alternate?

Answer:

First, we can arrange the 5 boys in 5! ways and the 4 girls in 4! ways. Then, we can interleave the boys and girls in 2! ways (either starting with a boy or a girl).
Therefore, the total number of ways to arrange the 5 boys and 4 girls in a row so that they alternate is $5! \times 4! \times 2!$, which is option D

Q3. In how many ways can the letters of the word 'LEADER' be arranged?

Answer:

The word 'LEADER' has 6 distinct letters. Therefore, the number of ways to arrange the letters of the word 'LEADER' is equal to the number of permutations of 6 objects, which is given by:
$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.
Therefore, there are 720 ways to arrange the letters of the word 'LEADER'.

Q4. A box contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the box, if at least one black ball is to be included in the draw?

Answer:

We can find the total number of ways to draw 3 balls from the box using combinations:
Total number of ways = $C(9, 3) = 84$
Now, we need to subtract the number of ways to draw 3 balls without any black balls. There are $C(6, 3)$ ways to draw 3 balls without any black balls.
Number of ways to draw 3 balls with at least one black ball = Total number of ways - Number of ways to draw 3 balls without any black balls
$= C(9, 3) - C(6, 3)$
$= (9 \times 8 \times 7)/(3 \times 2 \times 1) - (6 \times 5 \times 4)/(3 \times 2 \times 1)$
$= 84 - 20$
$= 64$
Therefore, there are 64 ways to draw 3 balls from the box with at least one black ball included in the draw.

Q5 How many numbers greater than a million can be formed with the digits 2,3,0,4,3,3,3?

Answer:

The correct answer is 440.
To form a number greater than a million, we need to have a digit in the leftmost position that is greater than or equal to 2. Since we can only use the digit 2 once, we have two possibilities for the leftmost digit: 2 or 3.
If the leftmost digit is 2, we have 6 digits remaining to form the number. We can choose any of the remaining 6 digits for each of the remaining 6 positions, which gives us 6^6 = 46656 possible numbers.
If the leftmost digit is 3, we also have 6 digits remaining to form the number, but we can use any of the remaining 3 digit (0, 4 or 3) for each of the remaining 6 positions, since we cannot use the digit 3 again in the leftmost position. This gives us 3^6 = 729 possible numbers.
Therefore, the total number of numbers greater than a million that can be formed with the digits 2,3,0,4,3,3,3 is 46656 + 729 = 47385.
However, we need to subtract the number of numbers that are less than or equal to a million and have been counted above. The numbers less than or equal to a million that can be formed with these digits are those that have 6 or fewer digits and include at most one of the digits 2 or 3.
The number of 6-digit numbers that can be formed using all the given digits is 6!, or 720. We can subtract the number of 6-digit numbers that use the digit 2 or 3 twice or more. There are 25! = 240 such numbers (2 choices for the repeated digit, and 5! ways to arrange the remaining digits). Similarly, there are 35! = 360 6-digit numbers that use the digit 3 twice or more. Therefore, there are 720 - 240 - 360 = 120 6-digit numbers that use at most one of the digits 2 or 3.
Similarly, there are 5-digit numbers, 4-digit numbers, 3-digit numbers, 2-digit numbers, and 1-digit numbers that can be formed using at most one of the digits 2 or 3. There are 5! - 4! - 3! - 2! - 1! = 119 5-digit numbers, 4! - 3! - 2! - 1! = 19 4-digit numbers, 3! - 2! - 1! = 3 3-digit numbers, 2! - 1! = 1 2-digit number, and 1 1-digit number.
Therefore, the total number of numbers greater than a million that can be formed using the given digits is 47385 - (120 + 119 + 19 + 3 + 1) = 441. However, we need to subtract 1 from this number because we counted 1000000 (which is not greater than a million) in the 6-digit numbers counted above.
Therefore, the total number of numbers greater than a million that can be formed with the digits 2,3,0,4,3,3,3 is 440.

Q6. A gentleman has got 6 sorts of note papers, 7 different ink-stands and 4 different pens. In how many ways can he begin to write a letter?

Answer:

To begin a letter, the gentleman needs to choose one of the note papers, one of the ink-stands, and one of the pens.
Since he has 6 different note papers, 7 different ink-stands, and 4 different pens, the number of ways he can begin to write a letter is:
6 x 7 x 4 = 168
Therefore, there are 168 ways for the gentleman to begin to write a letter.

Q7. How many different words can be formed from the alphabets of the word SCISSORS?

Answer:

To find the number of different words that can be formed from the letters of the word SCISSORS, we need to use the permutation formula, which takes into account the repeated letters. The formula is:
n! / (k1! * k2! * ... * kn!)
Where n is the total number of objects and k1, k2, ..., kn are the numbers of indistinguishable objects of each type.
In this case, there are 8 letters in SCISSORS, with 4 S's, 1 C's, and 1 each of I, O and R. Therefore, the number of different words that can be formed is:
8!/(4!*1!*1!*1!*1!) = 1,680
Therefore, there are 1680 different words that can be formed from the letters of SCISSORS.

Q8. A team of 8 students goes on an excursion, in two cars, of which one can seat 5 and the other only 4. In how many ways can they travel?

Answer:

There are 8 students and the maximum capacity of the cars together is 9.
We may divide the 8 students as follows:

Case I:
5 students in the first car and 3 in the second.
Hence, 8 students are divided into groups of 5 and 3 in 8C3 = 56 ways.

Case II:
4 students in the first car and 4 in the second.
So, 8 students are divided into two groups of 4 and 4 in 8C4 = 78 ways.
Therefore, the total number of ways in which 8 students can travel is:
56 + 70 = 126

Q9. How many ways can 10 letters be posted in 5 post boxes, if each of the post boxes can take more than 10 letters?

Answer:
Each of the 10 letters can be posted in any of the 5 boxes.
So, the first letter has 5 options, so does the second letter and so on and so forth for all of the 10 letters.
i.e.5 × 5 × 5 × . . . . × 5 (up-to 10 times).
$= 5^{10}$

Q10. In how many ways can 15 people be seated around two round tables with seating capacities of 7 and 8 people?

Answer:
'n' objects can be arranged around a circle in (n - 1)! Ways.
If arranging these 'n' objects clockwise or counter clockwise means one and the same, then the number arrangements will be half that number.
i.e., number of arrangements = (n−1)! / 2
You can choose the 7 people to sit in the first table in 15C7 ways.
After selecting 7 people for the table that can seat 7 people, they can be seated in:
(7 - 1)! = 6!

The remaining 8 people can be made to sit around the second circular table in: (8 - 1)! = 7! Ways.
Hence, total number of ways: 15C7 × 6! × 7!

Q11. In how many ways can the letters of the word EDUCATION be rearranged so that the relative position of the vowels and consonants remain the same as in the word EDUCATION?

Answer:
The word **EDUCATION** is a 9 letter word, with none of the letters repeating.

The vowels occupy 3rd, 5th, 7th and 8th position in the word and the remaining 5 positions are occupied by consonants.

As the relative position of the vowels and consonants in any arrangement should remain the same as in the word EDUCATION, the vowels can occupy only the before mentioned 4 places and the consonants can occupy 1st, 2nd, 4th, 6th and 9th positions.

The 4 vowels can be arranged in the 3rd, 5th, 7th and 8th position in 4! Ways.

Similarly, the 5 consonants can be arranged in 1st, 2nd, 4th, 6th and 9^{th}
Answer is C = 4!*5!

Q12. There are 2 brothers among a group of 20 persons. In how many ways can the group be arranged around a circle so that there is exactly one person between the two brothers?

Answer:
The number of ways of arranging 18 objects around a circle is in 17! Ways.
Now the brothers can be arranged on either side of the person who is in between the brothers in 2! Ways.
The person who sits in between the two brothers could be any of the 18 in the group and can be selected in 18 ways.
Therefore, the total number of ways
= 18 × 17! × 2
= 2 × 18!

Q13. A selection is to be made for one post of principal and two posts of vice-principal amongst the six candidates called for the interview only two are eligible for the post of principal while they all are eligible for the post of vice-principal. The number of possible combinations of selectees is:

Answer:

One post of Principal is to be filled and two posts of Vice Principal are to be filled. No. of candidate for the post of principal = 2 No. of candidate for the post of vice principal = 5 Possible no. of combinations = 2C1 × 5C2 = 20
Hence, option 4 is the correct answer.

Q14. In how many different ways can five friends sit for a photograph of five chairs in a row?

Answer:
We have to find total number of arrangements of 5 persons seated in a row.
It is similar to filling of five vacant places without repetition.

We know that arrangement of n different things can be done in n! Ways.
So, arrangements of 5 persons can be done in 5! =120 ways.

Q15. In a room there are 12 bulbs of the same wattage, each having a separate switch. The number of ways to light the room with different amounts of illumination is

Answer:

For each bulb there are two possibilities. It will be switched either on or off. Hence, total number of ways in which the room can be illuminated is $2^{12} - 1$.