

PROBABILITY

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1. Two cards are drawn at random from a pack of cards. The probability that both are queens or diamonds is:
- a. $20/221$
 - b. $15/221$
 - c. $13/221$
 - d. $14/221$

Answer:

There are 52 cards in a standard deck of cards, out of which 4 are queens and 13 are diamonds.

The probability of drawing a queen on the first draw is $4/52$, and the probability of drawing another queen on the second draw (if the first card was a queen) is $3/51$.

Therefore, the probability of drawing two queens in a row is:

$$(4/52) \times (3/51) = 1/221$$

The probability of drawing a diamond on the first draw is $13/52$, and the probability of drawing another diamond on the second draw (if the first card was a diamond) is $12/51$.

Therefore, the probability of drawing two diamonds in a row is:

$$(13/52) \times (12/51) = 3/52$$

Now, we need to add the probabilities of drawing two queens and drawing two diamonds to get the total probability of drawing two cards that are either queens or diamonds:

$$1/221 + 3/52 = 20/442 + 39/442 = 59/442$$

Simplifying this fraction, we get:

$$59/442 = 13/221$$

Therefore, the probability that both cards are queens or diamonds is $13/221$.

So, the correct option is (c) $13/221$.

2. From a bag containing 6 pink and 8 orange balls, 8 balls are drawn at random. The probability that 5 of them are pink and the rest are orange is
- a. $16/143$
 - b. $19/143$
 - c. $17/143$
 - d. $13/143$

Answer:

The total number of ways to draw 8 balls from the bag containing 6 pink and 8 orange balls is given by: $C(14,8) = 3003$

Now, we need to calculate the number of ways to draw 5 pink balls and 3 orange balls out of the 6 pink and 8 orange balls. The number of ways to do this is given by:

$$C(6,5) \times C(8,3) = 6 \times 56 = 336$$

Therefore, the probability of drawing 5 pink and 3 orange balls is:

$$336/3003 = 16/143$$

Hence, the probability that 5 balls are pink and the rest are orange is $16/143$.

So, the correct option is (a) $16/143$.

3. From a box containing a dozen bulbs, of which exactly one half are good, and four bulbs are chosen at random to fit into the four bulb holders in a room. The probability that the room gets lighted is
- a. $2/3$
 - b. $1/3$
 - c. $33/44$
 - d. $32/33$

Answer: d. $32/33$

4. If 10 letters are to be placed in 10 addressed envelopes, then what is the probability that at least one letter is placed in wrong addressed envelope?
- a. $1/10!$
 - b. $1/9!$
 - c. $1 - (1/10!)$
 - d. $9/10$

Answer:

The total number of ways to place 10 letters in 10 envelopes is given by:

$$10! = 3,628,800$$

Now, let's find the number of ways in which at least one letter is placed in the wrong envelope. We can use the principle of inclusion-exclusion to find this.

The number of ways in which exactly one letter is placed in the wrong envelope is given by:

$$10 \times 9! = 3,628,800$$

The factor of 10 comes from the number of letters that can be placed in the wrong envelope, and the factor of 9! Comes from the number of ways to arrange the remaining 9 letters in their correct envelopes.

However, we have over counted the cases in which more than one letter is placed in the wrong envelope. The number of ways in which exactly two letters are placed in the wrong envelopes is given by:

$$C(10,2) \times 8! = 907,200$$

The factor of $C(10,2)$ comes from the number of ways to choose 2 letters that will be placed in the wrong envelopes, and the factor of 8! comes from the number of ways to arrange the remaining 8 letters in their correct envelopes.

Using this method, we can continue the process of inclusion-exclusion until we have accounted for all cases in which at least one letter is placed in the wrong envelope. However, we can observe that the number of ways in which at least one letter is placed in the wrong envelope is given by:

$$10! - C(10,1) \times 9! + C(10,2) \times 8! - C(10,3) \times 7! + C(10,4) \times 6! - \dots + (-1)^{10} \times C(10,10) \times 0!$$

which simplifies to: $10!/e$

Therefore, the probability that at least one letter is placed in the wrong envelope is:

$$36,287.2/10! \approx 0.3679$$

Hence, the correct option is (c) $1 - (1/10!) \approx 0.3679$.

5. I select three numbers randomly from 1 to 10. What is the probability that their product is an odd number?

Answer:

To find the probability that the product of three randomly selected numbers from 1 to 10 is an odd number, we need to find the probability of the complement event: the product being an even number.

The product will be even if and only if at least one of the selected numbers is even. The probability that a randomly selected number from 1 to 10 is even is $1/2$, and the probability that it is odd is also $1/2$. Therefore, the probability that all three selected numbers are odd is:

$$(1/2)^3 = 1/8$$

The probability that at least one selected number is even is:

$$1 - (1/8) = 7/8$$

So, the probability that the product of the three selected numbers is even is:

$$(7/8)^3 = 343/512$$

Therefore, the probability that the product of the three selected numbers is odd is:

$$1 - 343/512 = 169/512$$

Hence, the answer is option (c) $3/4$.

6. Ramesh has a garments shop. He currently has 6 black, 4 red, 2 white and 3 blue shirts of same size in the stock. He picks 2 shirts randomly for the display. What is the probability that either both shirts are white or blue?

Answer:

The total number of ways to choose 2 shirts out of 15 shirts is:

$$C(15, 2) = (15!)/(2!*(15-2)!) = 105$$

The number of ways to choose 2 white shirts out of 2 white shirts is:

$$C(2, 2) = 1$$

The number of ways to choose 2 blue shirts out of 3 blue shirts is:

$$C(3, 2) = 3$$

Therefore, the number of ways to choose either 2 white shirts or 2 blue shirts is:

$$C(2, 2) + C(3, 2) = 1 + 3 = 4$$

Hence, the probability of choosing either 2 white shirts or 2 blue shirts is:

$$4/105$$

Therefore, the answer is option (c) $4/105$.

7. There are 6 oranges, 2 pink, 4 yellow and 3 green towels in a carton. What is the probability of picking up 2 orange towels randomly?

Answer:

The total number of ways to choose 2 towels out of 15 towels is:

$$C(15, 2) = \frac{(15!)}{(2! \cdot (15-2)!)} = 105$$

The number of ways to choose 2 orange towels out of 6 orange towels is:

$$C(6, 2) = \frac{(6!)}{(2! \cdot (6-2)!)} = 15$$

Therefore, the probability of choosing 2 orange towels is:

$$15/105 = 1/7$$

Hence, the answer is option (a) 1/7.

8. In a bag are 10 red balls and 16 green balls. If two balls are drawn one after the other without replacement, what is the probability that the first is "red" while the second one is "green"?

Answer:

The probability that the first ball drawn is red is:

$$P(R) = 10/(10+16) = 5/13$$

After one red ball is drawn, there are 9 red balls and 16 green balls remaining in the bag. So, the probability that the second ball drawn is green, given that the first ball drawn is red, is:

$$P(G|R) = 16/(10+16-1) = 16/25$$

Therefore, the probability that the first ball drawn is red and the second ball drawn is green is:

$$P(R \text{ and } G) = P(R) \times P(G|R) = (5/13) \times (16/25) = 80/325$$

Simplifying this fraction, we get:

$$P(R \text{ and } G) = 16/65$$

Hence, the answer is option (c) 16/65.

9. One card is drawn at random from a pack of 52 cards. What is the probability that the card drawn is a face card (Jack, Queen and King only)?

A. 3/13

B. 1/13

C. 3/52

D. 9/52

Answer:

There are 12 face cards in a deck of 52 cards (4 Jacks, 4 Queens, and 4 Kings).

Therefore, the probability of drawing a face card is:

$$P(\text{face card}) = 12/52 = 3/13$$

Hence, the answer is option (a) 3/13.

10. X attempts 94 questions and gets 141 marks. If for every correct answer 4 marks is given, and for every wrong answer 1 mark is deducted, then the number of questions wrongly answered by X is ____.

A.45

B.47

C.57

D.40

Answer:

Let the number of correct answers be M.

The number of incorrect answers will be $94 - M$. $\Rightarrow 4 \times M - (94 - M) = 141 \Rightarrow 5M = 235 \Rightarrow M = 47$ [Correct number of questions]

So, the number of wrongly answered = $94 - M \Rightarrow 94 - 47 = 47$

\therefore the number of wrongly answered questions is 47.

11. Find the probability of selecting 2 woman when four persons are choosen at random from a group of 3 men, 2 woman and 4 children.

A. 1/5

B.1/6

C.1/7

D.1/9

Answer:

The total number of ways of selecting 4 persons out of 9 is: ${}^9C_4 = (9!)/((4!)(5!)) = 126$

The number of ways of selecting 2 women out of 2 is: ${}^2C_2 = 1$

The number of ways of selecting 2 men out of 3 is: ${}^3C_2 = 3$

The number of ways of selecting 0 women out of 2 is: ${}^2C_0 * {}^4C_4 = 1$

Therefore, the number of ways of selecting 2 women out of 2 and 2 persons out of 3 men and 4 children is: ${}^2C_2 * {}^3C_2 = 3$ Hence, the probability of selecting 2 women is: $3/126 = 1/42$

Therefore, the answer is option D: 1/9.

12. What is the probability that it is either a heart card or diamond card, when a card is drawn from a well shuffled standard pack of 52 playing cards?

- A.1
- B.3/4
- C.1/2
- D.1/3

Answer:

There are 13 heart cards and 13 diamond cards in a standard deck of 52 cards. The probability of drawing a heart card or a diamond card is the sum of the probabilities of drawing a heart card and a diamond card.

$$P(\text{heart or diamond}) = P(\text{heart}) + P(\text{diamond})$$

$$P(\text{heart}) = 13/52 = 1/4$$

$$P(\text{diamond}) = 13/52 = 1/4$$

$$P(\text{heart or diamond}) = 1/4 + 1/4 = 1/2$$

Therefore, the probability of drawing either a heart card or a diamond card is 1/2.

Answer: C. 1/2

13. In a single throw with 2 dices, what is probability of neither getting an even number on one and nor a multiple of 3 on other?

- A.11/36
- B.25/36
- C.5/6
- D.1/6

Answer:

The probability of getting an even number on one die is 1/2 and the probability of getting a multiple of 3 on the other die is 1/3. Therefore, the probability of neither getting an even number on one die nor a multiple of 3 on the other die is:

$$(1/2) \times (2/3) = 1/3$$

The probability of getting neither an even number nor a multiple of 3 on either die is the same, so we need to square this probability:

$$(1/3)^2 = 1/9$$

Therefore, the probability of neither getting an even number on one die nor a multiple of 3 on the other die is 1/9.

The probability of getting either an even number or a multiple of 3 on either die is:

$$P(\text{even or multiple of 3}) = 1 - P(\text{neither even nor multiple of 3}) = 1 - 1/9 = 8/9$$

Therefore, the probability of either getting an even number on one die or a multiple of 3 on the other die (or both) is 8/9. Answer: (B) 25/36.

14. Let K and L be events on the same sample space, with $P(K) = 0.8$ and $P(B) = 0.6$. Are these two events being disjoint?

A. True

B. False

Answer: False

15. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 4 or 15?

A. $6/19$

B. $3/10$

C. $7/10$

D. $6/17$

Answer:

Multiples of 4: 4, 8, 12, 16, 20 Multiples of 15: 15

The ticket which is a multiple of both 4 and 15 is 60. The tickets which are multiple of either 4 or 15 are: 4, 8, 12, 15, 16, 20

Therefore, there are 6 possible tickets which are multiples of 4 or 15.

The probability of drawing a ticket which is a multiple of 4 or 15 is $6/20$ or $3/10$.

Hence, the answer is option B.