

# A Multiresolution Scheme for More Efficient Computations on Adaptive Mesh Refinement Blocks

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# Introduction

Many engineering applications depend on numerically solving systems of conservation laws of the form

$$\mathbf{U}_t + \mathbf{F}(\mathbf{U})_x = \mathbf{S}(\mathbf{U})$$

where  $\mathbf{U} = (\rho, \rho u, E)$  is a vector of conserved quantities,  $\mathbf{F}(\mathbf{U})$  is a flux vector, and  $\mathbf{S}(\mathbf{U})$  is a vector of source terms. The discretized solution are represented as averages over each cell

$$\mathbf{U}_i = \frac{1}{|V_i|} \int_{V_i} \mathbf{U} dV.$$

where the  $i$  denotes spatial index.

# Discretization

The semi-discretized form of system of PDEs is

$$\frac{\partial \mathbf{U}_i}{\partial t} = -\frac{1}{|V_i|} \left( \mathbf{F}_{i+\frac{1}{2}} - \mathbf{F}_{i-\frac{1}{2}} \right) + \mathbf{S}_i$$

where the source terms are also averaged over each cell

$$\mathbf{S}_i = \frac{1}{|V_i|} \int_{V_i} \mathbf{S} dV.$$

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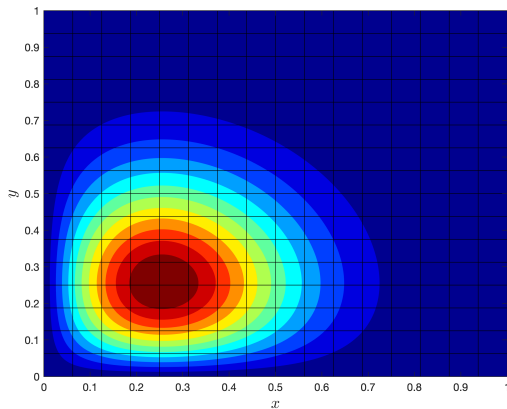
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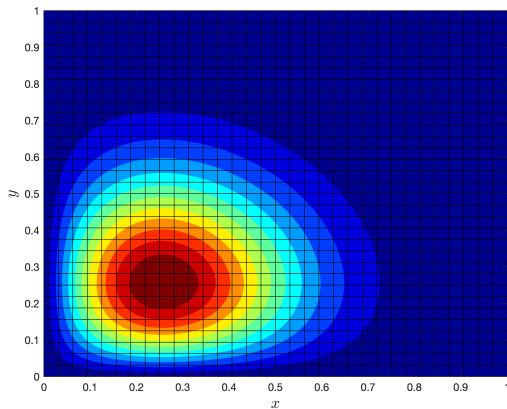
- ▶ the refinement needs to follow localized features
- ▶ some type of estimator of the local error is needed
- ▶ grid is typically refined in big groups (blocks) for efficiency
- ▶ blocks introduce inherent “overresolution” in some regions of the mesh



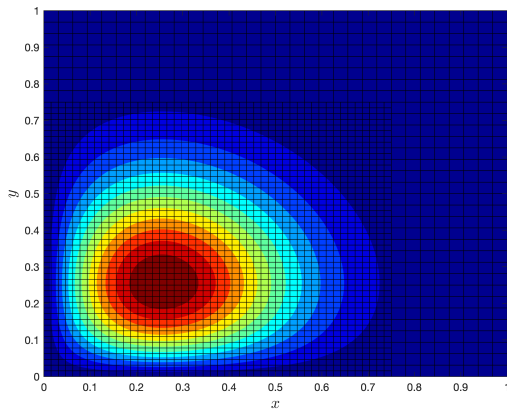
# Block-Structured AMR



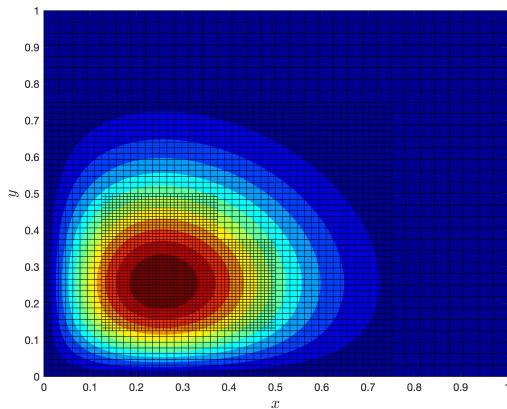
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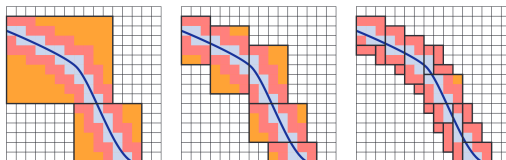
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# Filling Factor

The filling factor is the number of cells in a block which were flagged, divided by the total. As one reduces the block sizes...

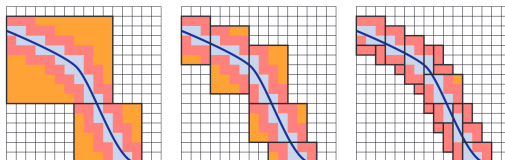
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- ▶ blocks with multiple parents becomes complicated
- ▶ communication between neighboring blocks becomes costly



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- ▶ “The goal of a multi-scale decomposition of a discrete set of data is a ”rearrangement” of its information content in such a way that the new discrete representation, exactly equivalent to the old one, is more ”manageable” in some respects.” - Arandiga, Donat

# Multiresolution

Define multiple, nested grids

$$\mathbf{G}^l = \left\{ x'_{i+\frac{1}{2}} \right\}_{i=0}^{N_l} = \left\{ x'^{l+1}_{i+\frac{1}{2}} \right\}_{i=0, \text{ i even}}^{N_{l+1}}.$$

Coarsening of average data in cell done via

$$\mathbf{u}'_i = \frac{1}{2} \left( \mathbf{u}^{l+1}_{2i} + \mathbf{u}^{l+1}_{2i+1} \right)$$

# Decomposition

The prediction from coarse to fine is done by

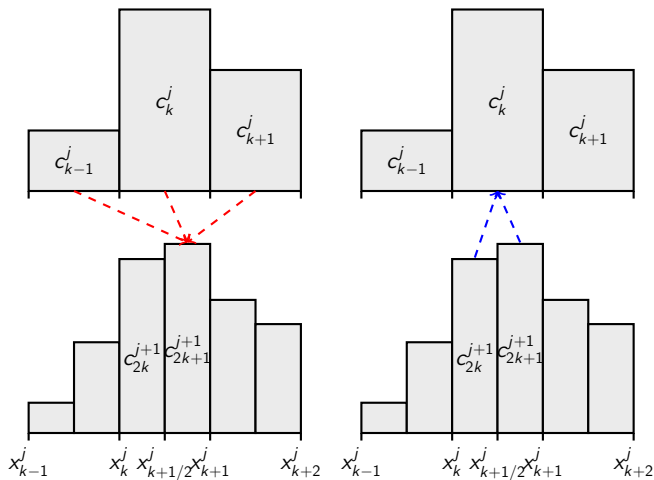
$$\hat{\mathbf{U}}_{2i+1}^{l+1} = \sum_{j=1-s}^{s-1} \gamma_j \mathbf{U}_{i+j}^l$$

The regularity information is assessed by computing detail coefficients as

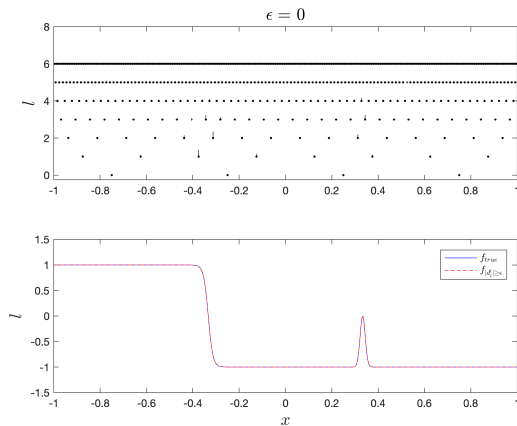
$$\mathbf{d}_i^l = \mathbf{U}_{2i+1}^{l+1} - \hat{\mathbf{U}}_{2i+1}^{l+1}.$$

A mask  $\{\mathbf{m}\}_i^{N'}$  is created for significant cells.

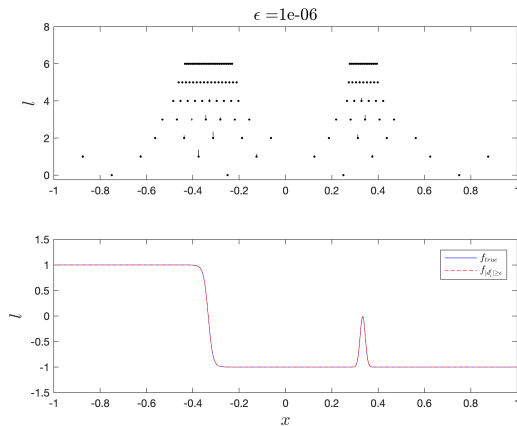
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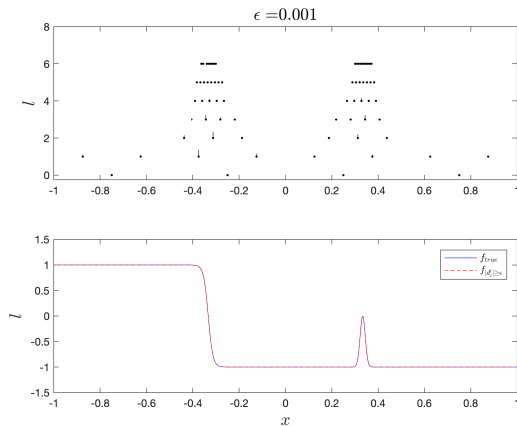
# Multiresolution Representation



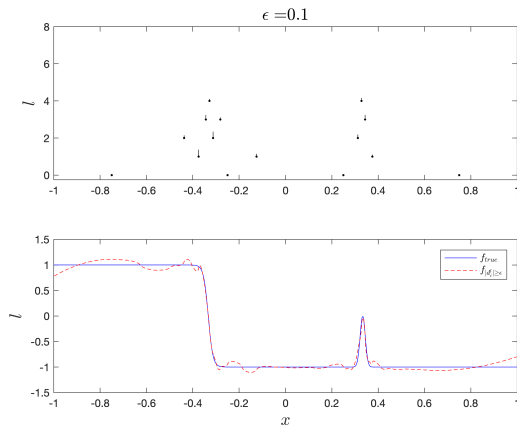
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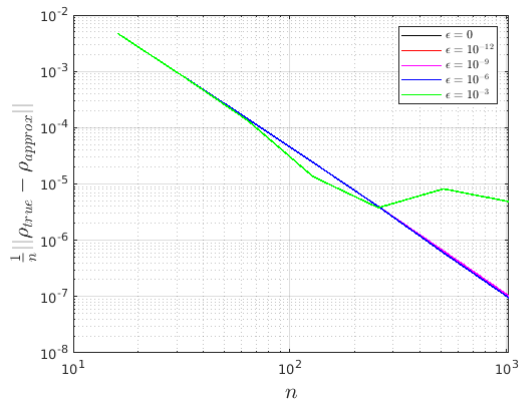
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- ▶ utilize this regularity information to identify sufficiently smooth regions in which to interpolate the flux.
- ▶ introduce sufficiently large buffer region (why?) around flagged cells
- ▶ perform inverse transform and compute or interpolate fluxes

# Convergence

Sine wave advection after one period:



# Outlook

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- ▶ weakly compressible turbulence (uniform mesh)
- ▶ compressible turbulence (potentially adaptive)
- ▶ turbulent combustion (adaptive)

# Turbulence

