Group Meeting
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Finite Volume Approach

Here we consider the following one-dimensional refence PDE

$$w_t + f(w)_x = s(w)$$

where $w = (\rho, \rho u, E)$ represents the primitive solution variables, and initial and boundary conditions are supplied. The PDE in semi-discrete form (via finite volume w/ midpoint quadrature) is

$$(w_j)_t = -\frac{1}{\triangle x}\left(f_{j+\frac{1}{2}} - f_{j-\frac{1}{2}}\right) + s_j = R_j(w)$$

where the j denotes spatial index. The solution is represented as cell averages

$$w_j^n \approx \frac{1}{\triangle x}\int_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}} w(x, t_n)dx$$

Finite Volume Approach

The numerical flux at cell interface is a function of $2k$ local cells

$$\hat{f}_{j+\frac{1}{2}} = f(w^n_{j-k+1}, \ldots, w^n_{j+k})$$

Computing the fluxes is bulk of computational effort.

Multiresolution Representation

Define multiple, nested grids

$$G^l = \left\{ x_j^l \right\}_{j=0}^{N_l}$$

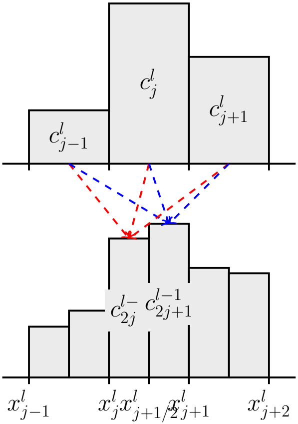
where the number of cells per level l is N_l and the cell width is

$$h_l = \frac{b-a}{N_l}$$

The aim is to decompose the field at the finest (given) level of resolution into a representation at the coarsest level plus a series of differences at each finer level. The differences provide information about the function's regularity.

Multiresolution Representation

Consider some quantity $c(x)$ represented as cell averages c_j^l at each level l. At some level l (coarse) the field at level $l-1$ (fine) is represented by a prediction from values at level l. An interpolating polynomial is used to preserve the cell averages.



Multiresolution Representation

Using a standard third-order polynomial, the prediction in one-dimension is

$$\hat{c}_{2j}^{l-1} = c_j^l + \frac{1}{8}\left(c_{j-1}^l - c_{j+1}^l\right)$$
$$\hat{c}_{2j+1}^{l-1} = c_j^l - \frac{1}{8}\left(c_{j-1}^l - c_{j+1}^l\right)$$

and the detail coefficient for the cell corresponding to c_j^l is

$$d_j^l = c_{2j+1}^{l-1} - \hat{c}_{2j+1}^{l-1}$$

The value of this coefficient is indicitive of the smoothness of the function in that vicinity. Doing this at each level of resolution provides the desired information.

Multiresolution Code - Examples

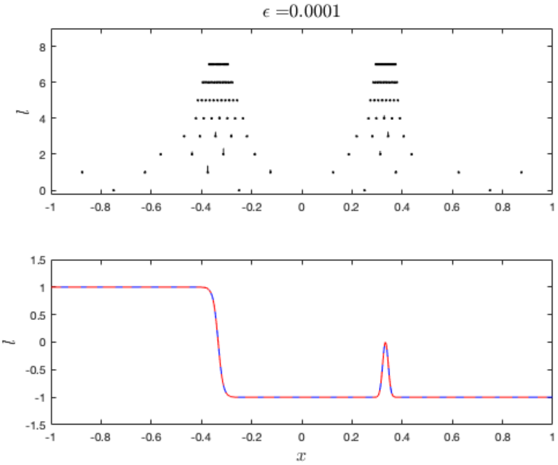


Figure: Better approximation (smaller threshold value).

Multiresolution Scheme on AMR Patches

Apply MR scheme on an AMR patch, use regularity information (detail coefficients) to reduce number of direct flux calculations. Compute the residual R_L at coarsest level of resolution. For smooth regions, the residual at next finer level will be interpolated

$$R_{2j+1}^{l-1} = R_j^l - \frac{1}{8}\left(R_{j-1}^l - R_{j+1}^l\right)$$
$$R_{2j}^{l-1} = 2R_j^l - R_{2j+1}^{l-1}$$

For significant cells, compute flux at finest (given) level as usual.