YSP Introductory Material

July 4, 2019

1 Data Averages

Let's suppose we are looking at data on an interval $x \in [0, 1]$, where x is the coordinate along this interval. Suppose there is a function $u(x)$ defined on this interval. This function is discretized by cutting the interval into a number of cells, N.

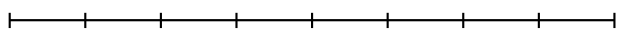

Figure 1: Interval with $N = 8$.

We denote the centers of each cell as x_i, and the left and right boundaries as $x_{i-1/2}$ and $x_{i+1/2}$, respectively. We define the width of each cell as $\triangle x = \frac{1}{N} = x_{i+1/2} - x_{i-1/2}$. For each cell we calculate the average amount of u in each interval, and we denote these averages as $\overline{u}_i$. This is done by perfoming integration of $u(x)$ within the cell as

$$\overline{u}_i = \frac{1}{\triangle x} \int_{x_{i-1/2}}^{x_{i+1/2}} u(x)dx. \tag{1}$$

2 Wavelet Transform

Now let's imagine that we have a second grid measuring the same function, but with half the number of intervals ($N = 4$).

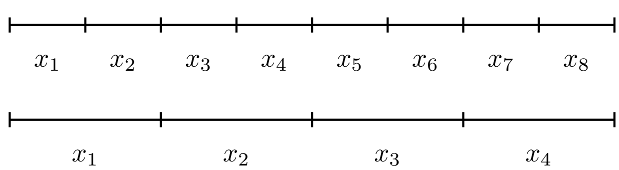

Figure 2: Fine grid with $N = 8$ and coarse grid with $N = 4$.

Note that each cell on the coarse level has two child cells which live on the fine grid. We can *predict* the child values on the finer grid by creating an interpolating polynomial based on coarse grid values. Denote the coarse cells by u_i^c and the fine cells by u_i^f. The prediction is made with the following interpolation

$$\tilde{u}_{2i}^f = u_i^c - \frac{1}{8}\left(u_{i-1}^c - u_{i+1}^c\right) \tag{2}$$

Then we want to compute the difference between this polynomial approximation, and the true value that we know. We compute the difference for each coarse cell,

$$d_i^c = u_{2i}^f - \tilde{u}_{2i}^f. \tag{3}$$

Note that near the boundaries we need to use biased interpolation. On the left boundary compute

$$\tilde{u}_{2i}^f = \frac{5}{8}u_i^c + \frac{1}{2}u_{i+1}^c - \frac{1}{8}u_{i+2}^c. \tag{4}$$

Near the right boundary compute

$$\tilde{u}_{2i}^f = \frac{11}{8}u_i^c - \frac{1}{2}u_{i-1}^c + \frac{1}{8}u_{i-2}^c. \tag{5}$$