Group Meeting
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Finite Volume Approach

Here we consider the following one-dimensional reference PDE

$$w_t + f(w)_x = s(w)$$

where $w = (\rho, \rho u, E)$ represents the primitive solution variables, and initial and boundary conditions are supplied. The PDE in semi-discrete form (via finite volume w/ midpoint quadrature) is

$$(w_j)_t = -\frac{1}{h}\left(f_{j+\frac{1}{2}} - f_{j-\frac{1}{2}}\right) + s_j = R_j(w)$$

where the j denotes spatial index.

Finite Volume Approach

The numerical flux at cell interface is a function of $2k$ local cells

$$\hat{f}_{j+\frac{1}{2}} = f(w^n_{j-k+1}, \ldots, w^n_{j+k})$$

The solution is represented as cell averages

$$w^n_j \approx \frac{1}{\triangle x} \int_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}} w(x, t_n) dx$$

Multiresolution Representation

Define multiple, nested grids

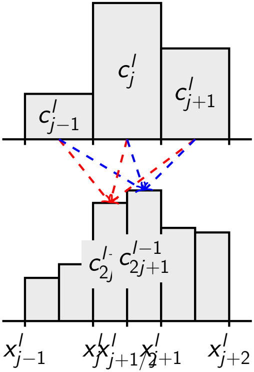
$$G^l = \left\{ x_j^l \right\}_{j=0}^{N_l}$$

such that $x_j^l = x_{2j}^{l-1}$ for any j, l. The number of cells per level l is N_l and the cell width is

$$h_l = \frac{b-a}{N_l}$$

Multiresolution Representation

Consider some quantity $c(x)$ represented as cell averages c_j^l at each level l. At some level l (coarse) the field at level $l-1$ (fine) is represented by a prediction from values at level l.



Multiresolution Representation

Using a standard third-order polynomial, the prediction in one-dimension is

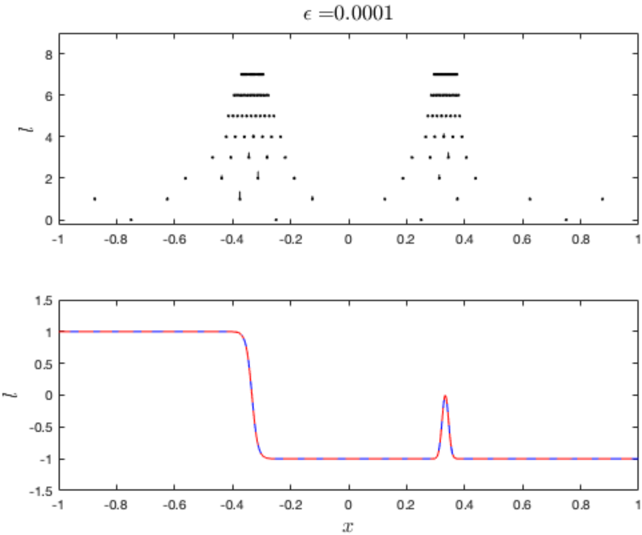
$$
\begin{aligned}
\hat{c}_{2j}^{l-1} &= c_j^l + \frac{1}{8}\left(c_{j-1}^l - c_{j+1}^l\right) \\
\hat{c}_{2j+1}^{l-1} &= c_j^l - \frac{1}{8}\left(c_{j-1}^l - c_{j+1}^l\right)
\end{aligned}
$$

and the detail coefficient for the cell corresponding to c_j^l is

$$
d_j^l = c_{2j+1}^{l-1} - \hat{c}_{2j+1}^{l-1}
$$

The value of this coefficient is indicitive of the smoothness of the function in that vicinity.

Multiresolution Code - Examples



Multiresolution Scheme on AMR Patches

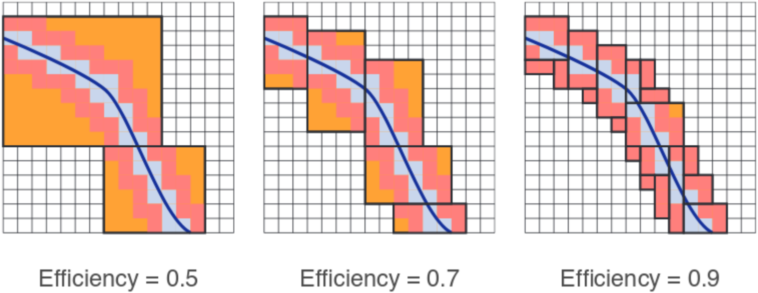


Figure : Increasing patch 'efficiency.'

Multiresolution Scheme on AMR Patches

The steps of the scheme for one patch are

1. take given (finest level) data on patch, and coarsen it to desired coarsest level L
2. compute the forward wavelet transform to obtain $\{\mathbf{d}^l\}_{l=L}^{l=1}$
3. on coarsest level L, compute the residuals $\left\{R_j^L\right\}_{j=0}^{N_L}$
4. loop through one finer level at a time, and according to detail coefficients, either interpolate or calculate remaining fluxes

Multiresolution Scheme on AMR Patches

The original (fine) data is coarsened by

$$w_j^l = \frac{1}{2}\left(w_{2j}^{l-1} + w_{2j+1}^{l-1}\right)$$

Then the residual R_j^l may be interpolated in smooth regions as

$$R_{2j+1}^{l-1} = R_j^l - \frac{1}{8}\left(R_{j-1}^l - R_{j+1}^l\right)$$
$$R_{2j}^{l-1} = 2R_j^l - R_{2j+1}^{l-1}$$

Progress in FLASH Implementation

- created a new folder `source/flashUtilities/Wavelet/`
- writing a program `Wavelet_computeTransform.F90` which will be run in `Grid_computeUserVars.F90`

```
function imap( l, i, j, k, nx, ny, nz ) result(x)
  integer, intent(in) :: nx(:), ny(:), nz(:)
  integer, intent(in) :: l, i, j, k
  integer             :: x

  ! compute the map
  x = l + nx * ( i + ny * ( j + nz * k ) )

end function imap
```