The Multiresolution Analysis of Triangle Surface Meshes with Lifting Scheme

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Abstract. Nowadays, there are many applications that take advantage of the availability of three-dimensional (3D) data sets. These objects are represented as complex polygonal surfaces formed by hundreds of thousands of polygons, which causes a significant increase in the cost of storage, transmission and visualisation. Multiresolution modeling, which allows an object to be represented by set of approximations, each with a different number of polygons, has been successfully presented as a solution for the efficient manipulation of this type of objects. The main contribution of this work is the use of the complete lifting scheme for the multiresolution analysis of irregular meshes with proposition of new prediction block.

1 Introduction

3D objects have applications in computer graphics, medicine, games, simulators, scientific visualisation, CAD systems, GIS and virtual reality systems, etc. Better and more popular 3D geometry acquisition systems result in million-polygons models. Complex three-dimension objects must be processed, stored, transferred, animated and analysed which is very expensive. Multiresolution models (Fig. 1) arise a lot of interest because they make it possible to represent and process geometric data in different levels of detail (LOD) depending on application needs [7], [11].

Multiresolution analysis and wavelets have been very popular lately, after processing signals, sounds, images and video, wavelets have been applied for digital geometry processing. This new class of application needs a processing toolbox of fundamental algorithms such as denoising, compression, transmission, enhancement, detection, analysis and editing with suitable mathematical and computational representations. The basic framework for these tools is the multiresolution analysis that decomposes an initial data set into a sequence of approximations and details.

This paper presents a new wavelet-based multiresolution framework for decomposition and reconstruction of irregular, triangular meshes. The generalisation from the first generation of wavelets to the second one gives an opportunity to construct wavelets on irregular meshes. This solution is based on lifting scheme as a wavelet construction tool.

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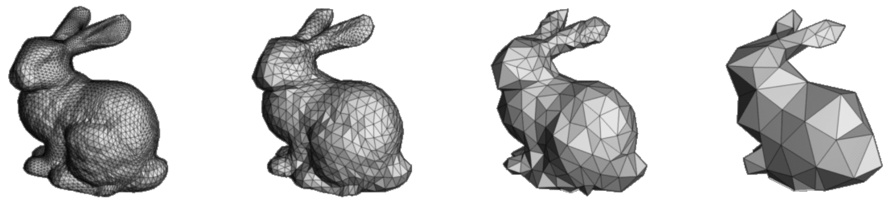

Fig. 1. Level of detail of model

2 Wavelet-Based Multiresolution Analysis of Surface Meshes

The triangle mesh is a pair $M = (P, T)$ where P is a set of n points $p_i = (x_i, y_i, z_i)$ with $1 \leq i \leq n$ and T is a simplicial complex which contains information about topology of a mesh. The complex is a set of three types subsets called simplices: vertices $v = \{i\} \epsilon V$, edges $e = \{i, j\} \epsilon E$, faces $v = \{i, j, k\} \epsilon F$. Two vertices $\{a\}$ and $\{b\}$ are neighbours if $\{a, b\} \epsilon E$. The 1-ring neighbourhood of vertex $\{a\}$ is the set $N(a) = \{b | \{a, b\} \epsilon E\}$. The valence of a vertex is the number of edges meeting at this vertex. There are three types of triangular meshes: regular (every vertex has valence equals 6), semi-regular (the most vertices have valence 6, few isolated vertices have any valence), irregular (vertices can have any valence).

The first connection between wavelets and subdivision (semi-uniform Loop subdivision) allows to define multiresolution surface representation for semi-regular mesh with subdivision connectivity, was presented in [10], [12]. In this case input mesh first must be rebuild with remeshing algorithms [12], [13]. The BLaC-wavelets [14], [15] define local decomposition which is generalisation of Haar wavelets for non-nested spaces. The connection of hierarchical Delaunay triangulation allows to build multiresolution model of planar or spherical, irregular surface mesh. The next proposition in literature is wavelet-based multiresolution analysis of irregular meshes using a new irregular subdivision scheme [16], [17]. This is expansion of the solution presented in [10], [12].

Another direction in research is the use of non-uniform subdivision schemes to build wavelet analysis. In [9], [8] the central ingredient of multiresolution analysis is a non-uniform relaxation operator which minimises divided differences. This operator together with the mesh simplification method (progressive mesh [20]) and pyramid algorithm (Burt-Adelson pyramid) allows to define signal processing tools for irregular connected triangle meshes. In [18] progressive meshes and a semi-uniform discrete Laplacian smoothing operator were used to perform multiresolution editing tool for irregular meshes. Lifting scheme is used for multiresolution analysis in [19] where a prediction block is non-uniform relaxation operator based on surface curvature minimisation. This transformation is not a complete lifting scheme because it does not have update block.

3 Second Generation Wavelets

The second generation wavelets [1], [3], [4] are generalisation of biorthogonal classic wavelets (called first generation wavelets) [5], [6]. The second generation wavelets are not necessarily translated and dilated of one function (mother function) so they cannot be constructed by Fourier transformation. The second generation wavelets may use many basic functions, but must be possible to define nested spaces necessary for describing multiresolution analysis.

The lifting scheme [1], [2], [3], [4] is a simple but powerful tool to construct the second generation wavelets. The main advantage of this solution is the possibility of building wavelet analysis on non-standard structures of data (irregular samples, bounded domains, curves, surfaces, manifolds) while keeping all powerful properties of the first generation wavelets such as speed and good ability of approximation. A general lifting scheme (Fig. 2) consists of three types of operation:

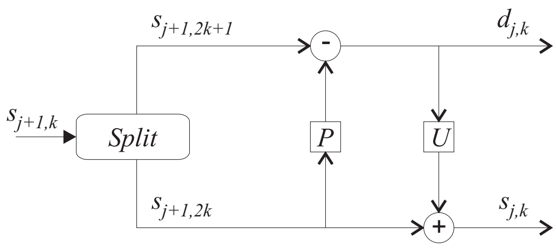

Fig. 2. The lifting scheme

- **Split**: splits input dataset into two disjoint sets of even and odd indexed samples (1). The definition of lifting scheme [2] doesn't impose any restriction on how the data should be split nor on the relative size of each subsets.

$$(s_{j+1,2k+1}, s_{j+1,2k}) = S(s_{j+1,k}) \tag{1}$$

- **Predict**: predicts odd indexed sample based on even samples. Next the odd indexed input value is replaced by the offset (difference) between the odd value and its prediction (2).

$$d_{j,k} = s_{j+1,2k+1} - P(s_{j+1,2k}) \tag{2}$$

- **Update**: updates the output, so that coarse-scale coefficients have the same average value as the input samples (3). This step is necessary for stable wavelet transform [3].

$$s_{j,k} = s_{j+1,2k} + U(d_{j,k}) \tag{3}$$

This calculations can be performed in-place. In all stages input samples can be overwritten by output of that step. Inverse transform (Fig. 3) is easy to find by reversing the order of operations and flipping the signs.

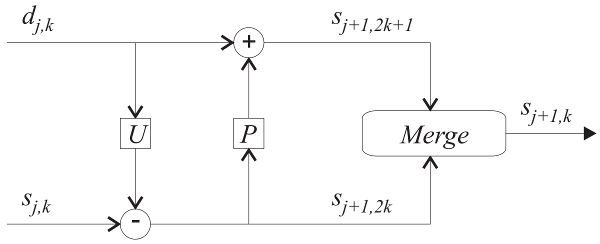

Fig. 3. The reverse lifting scheme

4 Analysis of Surface Mesh by Second Generation Wavelets

The most important advantage of the lifting scheme is generality which allows to construct the second generation wavelets on data with non-standard structures. This section presents proposition of transform based on the lifting scheme for multiresolution analysis of irregular, triangle closed mesh (2-manifold). Main operations can be singled out:

- Select from input mesh M^k (where k is the resolution level) the even and odd vertices (split block). The odd vertex will be removed from mesh. One step of lifting scheme removes only one vertex. It is possible to pick more odd vertices with disjoint neighbourhood and processes them in a parallel way.
- Predict odd vertex based on even ones.
- Compute detail vector as difference between predicted and original vertex.
- Remove the odd vertex from the mesh by an edge collapse [20]. The inverse operation is a vertex split.
- Update even vertices. The output is the coarser mesh M^{k-1} that is the approximation of initial mesh.

4.1 Split

The main task of this block is selecting vertex to remove (odd vertex) from input mesh. Different methods can be used to choose which vertex should be removed first. The criterion may be applied from mesh simplification methods [21], [22], [7], for example: quadric error metric [23], distance to local average plane [24], edge length.

The new suggested criterion is vertex distance to regression plane defined by 1-ring neighbourhood of this vertex (see Sect. 4.2).

4.2 Predict

The prediction operator estimates the odd vertex using even vertices. In this block relaxation operators from non-uniform subdivision schemes can be used,

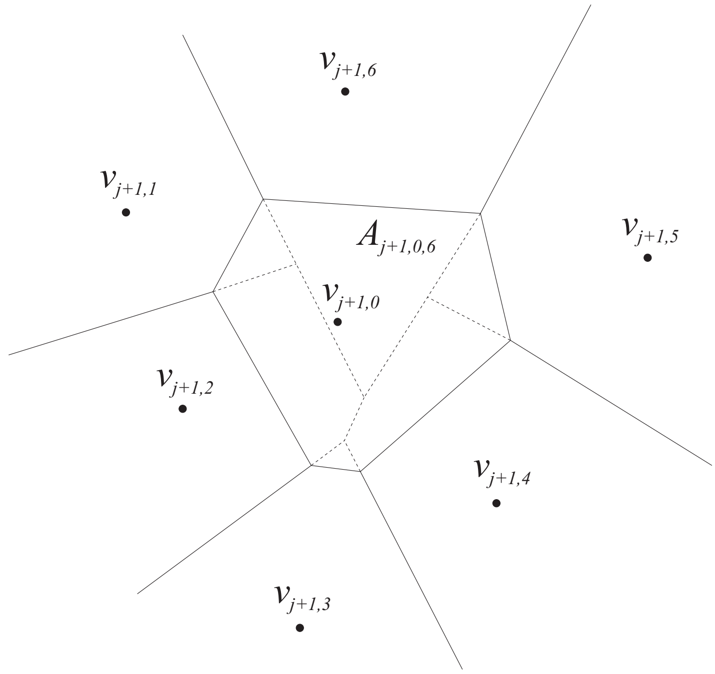

Fig. 4. Natural interpolation method

for example: surface curvature minimisation [19], minimisation of divided differences [9], [8], discrete Laplacian operator [18]. These prediction operators are based on the local surface geometry and allows to define the position of odd vertex as a combination of even vertices with suitable weight coefficients.

The new proposition of prediction block takes advantage of natural interpolation method [25]. The unsigned areas of the Voronoi cells of odd vertex (with index 0) and its 1-ring neighbours before and after removal (Fig. 4) are used for calculation in two dimensions. The prediction coefficients $\beta_{j,k}$ (where j is the resolution level, k is the index of neighbour of odd vertex) are simply the proportions of central cells areas, taken by neighbours (4).

$$\beta_{j,k} = \frac{A_{j+1,0,k}}{A_{j+1,0}} \tag{4}$$

In (4) $A_{j+1,0}$ is the area of central cell of odd vertex with index 0 and $A_{j+1,0,k}$ is the part of central cell area assigned to neighbour (even vertex) with index k. The prediction coefficients are positive and sum up to one, so the scheme has at least one vanishing moment and guarantee stable transform [3]. The value of prediction of odd vertex $P(v_{j+1,0})$ is linear combination of neighbouring values and its coefficients (5).

$$P(v_{j+1,0}) = \sum_{k \epsilon N(v_{j+1,0})} \beta_{j,k} v_{j+1,k} \tag{5}$$

Before calculating coefficients the local parameterization of odd vertex and its 1-ring neighbour should be determined. The searched parametric plane is total least squares plane [28]. This is the best-fitting plane to a given set of vertices in 1-ring neighbour by minimizing the sum of the squares of the perpendicular distances of the points from the plane. The best solution utilizes the Singular Value Decomposition (SVD) [27] which allows to find the point of plane (centroid of data) and normal vector. This is used as local parametric plane and after projection vertices on this plane, the parameter value (t, u) associated with odd and 1-ring neighbour could be computed (Fig. 5). After that we can treat three coordinates (x_i, y_i, z_i) as value of function in two dimension parameter space (t_i, u_i). This parameterization is needed for using natural interpolation.

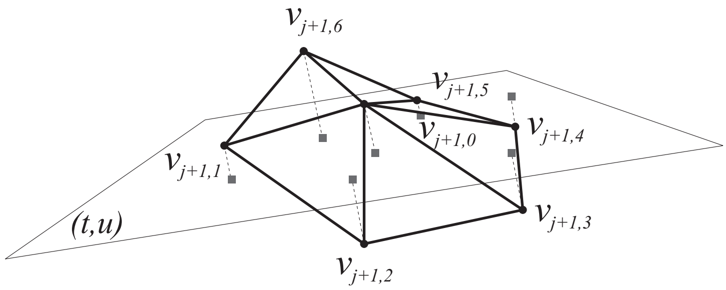

Fig. 5. Local parameterization on total least squares plane

4.3 Update

The suggested update block is a simple assurance that the values of the input and output (in coarser resolution) vertices have the same average. If this step is missing, the even indexed values proceed unchanged to the next level.

4.4 Results

The presented results concern the application of the lifting scheme into multiresolution analysis of irregular surface mesh. Initial mesh (Fig. 6.a) contains 2903 vertices and 5804 faces. In one step of transform one vertex is removed and the detail vector, which is the difference between the odd vertex and its prediction, is calculated. In this solution splitting block uses vertex distance to regression plane defined by its 1-ring neighbourhood criterion (see Sect. 4.1). Predict uses local parameterization on this plane. Coefficients are calculated by natural interpolation (see Sect. 4.2). Last step is update block (see Sect. 4.3). Fig. 6.b - 6.d show next approximations obtained from described transform. The average length of detail vector is 0.297280 and this value is between 0.088417 and 0.432680. One step of the lifting scheme transformation is presented on Fig.7.

The main advantage of proposed solution is that it has features of methods for irregular meshes [8], [9], [19], [20], [23], [24] such as progressive transmission, hierarchical representation and mesh approximation. Additionally the detail encode

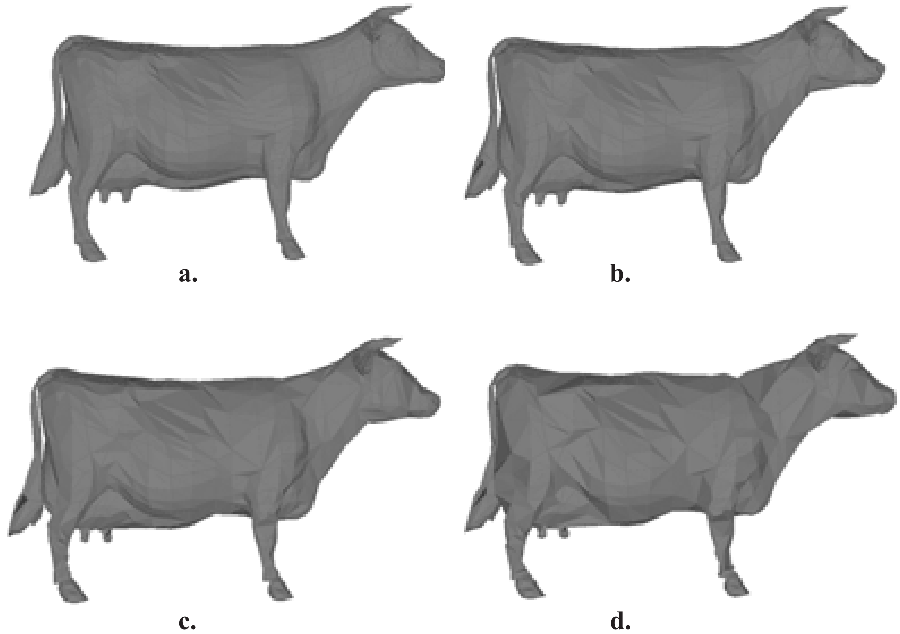

Fig. 6. Next approximations of the model. a. Original irregular mesh. b. After removal of 25% of vertices. c. After removal of 50% of vertices. d. After removal of 75% of vertices.

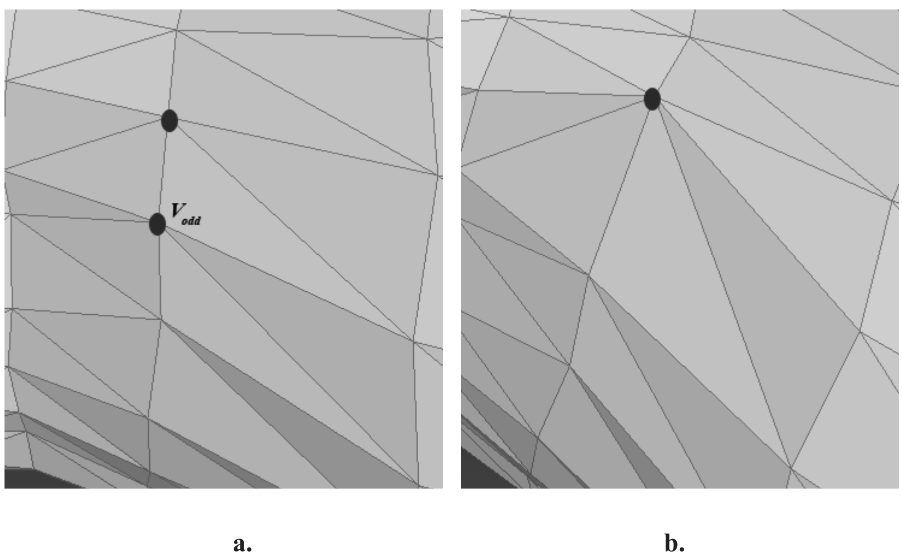

Fig. 7. The lifting scheme transformation for triangle mesh. a. The chosen odd vertex. b. The result mesh after the application of predict and update operation and removal of the odd vertex by the shortest edge collapse.

the data lost by the next approximations and can be seen as frequency spectrum. This make possible to build many applications such as filtering, smoothing, enhancement and denoising surface mesh. In hierarchical, multiresolution structure based on this scheme is no need to remember any additional coefficients only the value of detail vector. This method can be also used for processing vertex attributes.

5 Summary

This paper presents new proposal of connection criterions from mesh simplification methods, non-uniform subdivision and the lifting scheme. This solution allows to define the second generation wavelet transformation for multiresolution analysis of irregular, triangle, surface meshes. The paper also presents new prediction block which is built by taking advantage of natural interpolation. Further research will be focused on widening transform by processing different vertex attributes (such as color, temperature, texture coordinates) and examining properties of the proposed transform. The next step will be to build the multiresolution model with the support of level of detail (LOD) questions and variable resolution level of detail (region of interest, ROI) visualizations [7], [11], [26].

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