A Multiresolution Scheme for More Efficient Computations on Adaptive Mesh Refinement Blocks

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March 23, 2019

Introduction

Many engineering applications depend on numerically solving systems of conservation laws of the form

$$\mathbf{U}_t + \mathbf{F}(\mathbf{U})_x = \mathbf{S}(\mathbf{U})$$

where $\mathbf{U} = (\rho, \rho u, E)$ is a vector of conserved quantities, $\mathbf{F}(\mathbf{U})$ is a flux vector, and $\mathbf{S}(\mathbf{U})$ is a vector of source terms.

Discretization

The discretized solution are represented as averages over each cell

$$\mathbf{U}_i = \frac{1}{|V_i|} \int_{V_i} \mathbf{U} dV.$$

where the i denotes spatial index.

$$\frac{\partial \mathbf{U}_i}{\partial t} = -\frac{1}{h} \left(\mathbf{F}_{i+\frac{1}{2}} - \mathbf{F}_{i-\frac{1}{2}} \right)$$

Discretization

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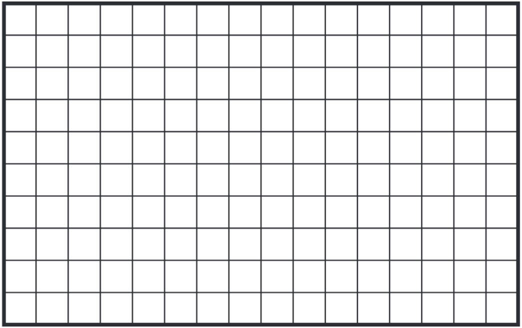
- computations typically done on blocks / patches for efficiency
- inherent "overresolution" in some regions of the mesh by using blocks
- can this be addressed by reducing block size? computational tradeoff?

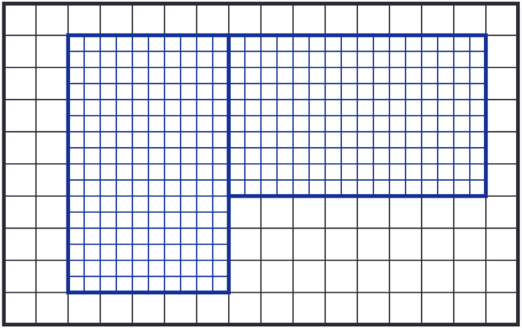
AMR



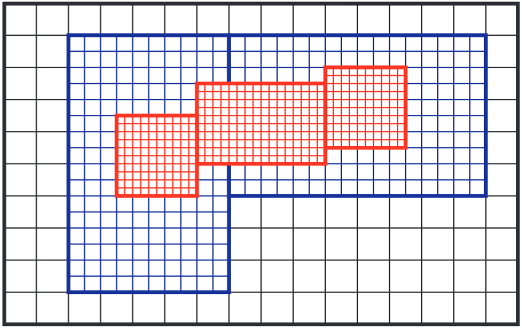
[1]

[1]Introduction to Block-Structured Adaptive Mesh Refinement (AMR), Ann S. Almgren

AMR

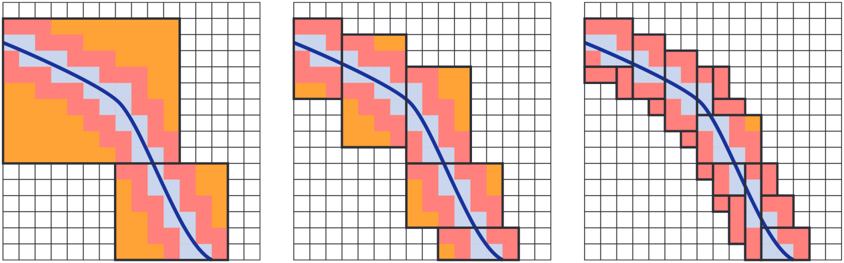


AMR



Filling

The filling factor is the number of cells in a block which were flagged, divided by the total.



a la Harten

- Besides the AMR concepts, a multiresolution approach was also introduced by Harten. Grid is *not* refined in space. Instead, a wavelet transform is performed on the uniform grid, and the fluxes are interpolated in smooth regions.

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- Besides the AMR concepts, a multiresolution approach was also introduced by Harten. Grid is *not* refined in space. Instead, a wavelet transform is performed on the uniform grid, and the fluxes are interpolated in smooth regions.
- “The goal of a multi-scale decomposition of a discrete set of data is a "rearrangement" of its information content in such a way that the new discrete representation, exactly equivalent to the old one, is more "manageable" in some respects.” - Arandiga, Donat

Multiresolution

Define multiple, nested grids

$$\mathbf{G}^l = \left\{ x^l_{i+\frac{1}{2}} \right\}_{i=0}^{N_l} = \left\{ x^{l+1}_{i+\frac{1}{2}} \right\}_{i=0,\text{i even}}^{N^{l+1}} .$$

Coarsening of avarage data in cell done via

$$\mathbf{U}^l_i = \frac{1}{2}\left(\mathbf{U}^{l+1}_{2i} + \mathbf{U}^{l+1}_{2i+1} \right)$$

Decomposition

The prediction from coarse to fine is done by

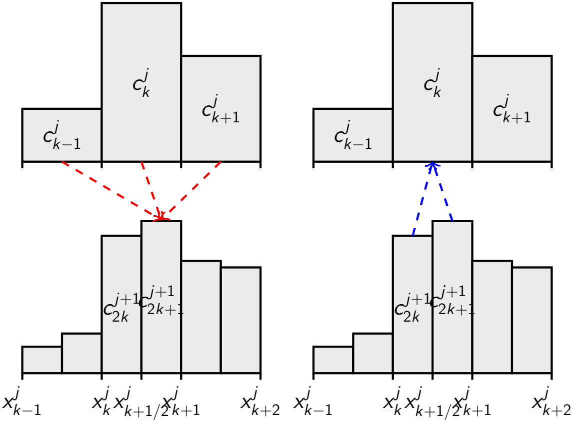
$$\hat{\mathbf{U}}_{2i+1}^{l+1} = \sum_{j=1-s}^{s-1} \gamma_j \mathbf{U}_{i+j}^{l}$$

The regularity information is assessed by computing detail coefficients as

$$\mathbf{d}_i^l = \mathbf{U}_{2i+1}^{l+1} - \hat{\mathbf{U}}_{2i+1}^{l+1}.$$

A mask $\{\mathbf{m}\}_i^{N^l}$ is created for significant cells.

Decomposition



Fluxes

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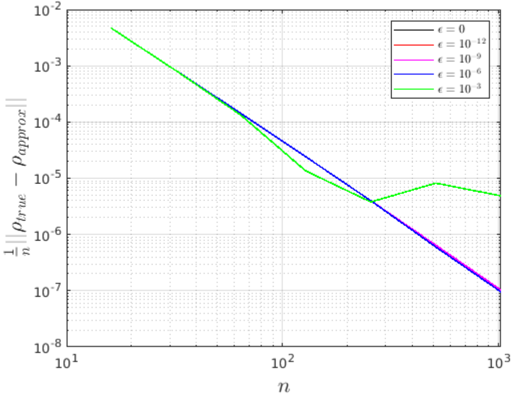
- utilize this regularity information to identify sufficiently smooth regions in which to interpolate the flux.
- introduce sufficiently large buffer region (why?) around flagged cells
- perform inverse transform and compute or interpolate fluxes

Convergence

Sine wave advection after one period:



Outlook

- develop performance study for effect of filling factor on time to solution, overheads... with and without wavelet scheme

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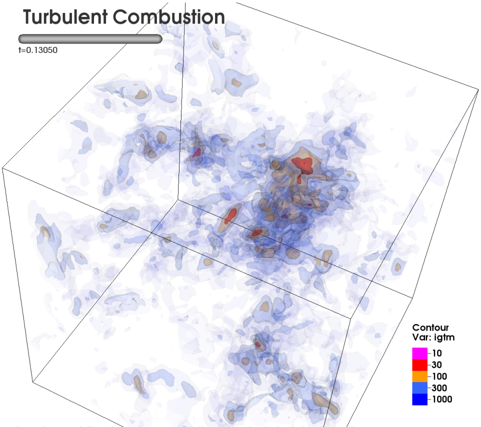
- weakly compressible turbulence (uniform mesh)
- compressible turbulence (potentially adaptive)
- turbulent combustion (adaptive)

Turbulence



when two white dwarfs merge, their material violently mixes
we model the evolution of small region containing such mixed material
fluctuations of burning time scale in plasma may cause detonation