Harten's Multiresolution Scheme on Adaptive Mesh Refinement Blocks for More Efficient Simulation of Reactive Flows

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March 25, 2019

Introduction

Many engineering applications depend on numerically solving systems of conservation laws of the form

$$\mathbf{U}_t + \mathbf{F}(\mathbf{U})_x = \mathbf{S}(\mathbf{U})$$

where $\mathbf{U} = (\rho, \rho u, E)$ is a vector of conserved quantities, $\mathbf{F}(\mathbf{U})$ is a flux vector, and $\mathbf{S}(\mathbf{U})$ is a vector of source terms. The discretized solution is represented as averages over each cell

$$\mathbf{U}_i = \frac{1}{|V_i|} \int_{V_i} \mathbf{U} dV.$$

where the i denotes spatial index.

Discretization

The semi-discretized form of the system of PDEs is

$$\frac{\partial \mathbf{U}_i}{\partial t} = -\frac{1}{|V_i|}\left(\mathbf{F}_{i+\frac{1}{2}} - \mathbf{F}_{i-\frac{1}{2}}\right) + \mathbf{S}_i$$

where the source terms are also averaged over each cell

$$\mathbf{S}_i = \frac{1}{|V_i|}\int_{V_i} \mathbf{S}\, dV.$$

Solution Approaches

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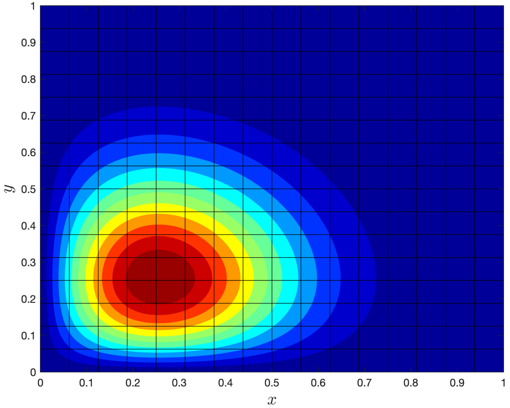
- the refinement is generally associated with localized features
- some type of estimator of the local error is needed
- typically a collection of cells (a block) is refined for efficiency
- blocks introduce inherent "overresolution" in some regions of the mesh

Block-Structured AMR



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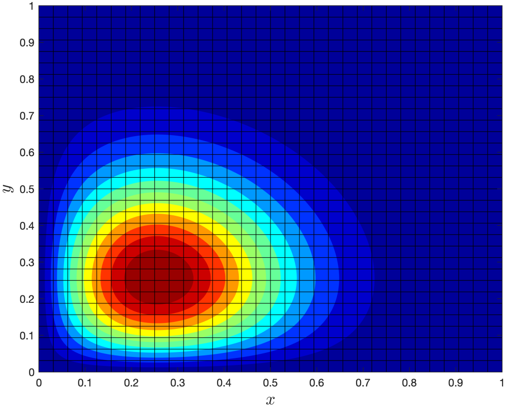

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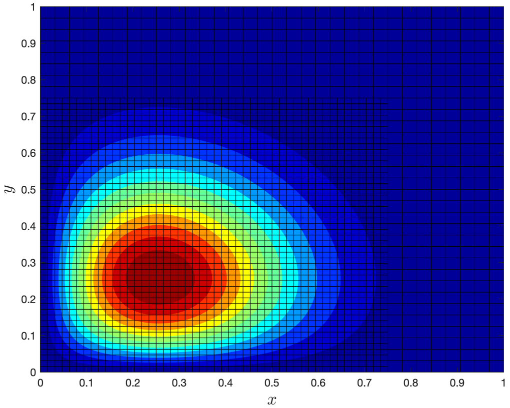

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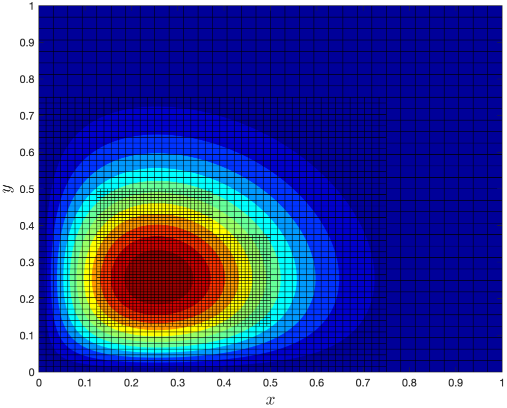
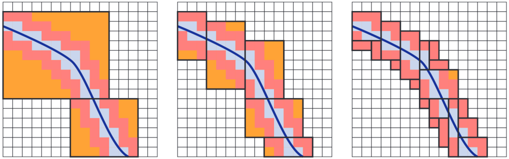


Filling Factor

The filling factor is the number of cells in a block which were flagged, divided by the total.



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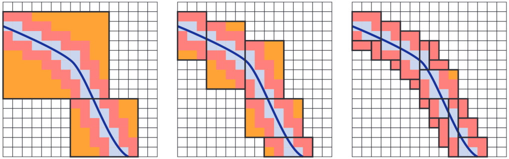

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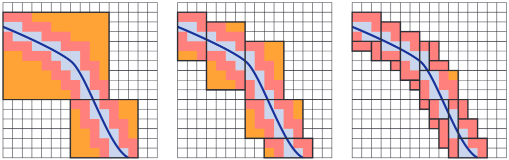
- blocks with multiple parents becomes complicated
- communication between neighboring blocks becomes costly



Multiresolution Representation

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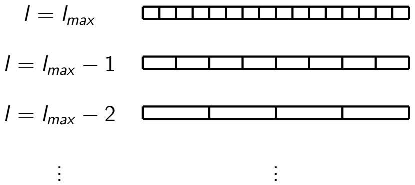
- The multiresolution scheme introduced by Harten does *not* adapt grid. Instead, a wavelet decomposition is performed on the uniform grid.

Multiresolution Representation

Define multiple levels of representation of the discrete data

$$\mathcal{G}^l = \left\{ x^l_{i+\frac{1}{2}} \right\}_{i=1}^{N_l} = \left\{ x^{l+1}_{i+\frac{1}{2}} \right\}_{i=1,\text{i even}}^{N_{l+1}}$$



Decomposition

Coarsening of avarage data in cell done via

$$\mathbf{U}_i^l = \frac{1}{2}\left(\mathbf{U}_{2i}^{l+1} + \mathbf{U}_{2i+1}^{l+1}\right),$$

and prediction from coarse to fine is done by

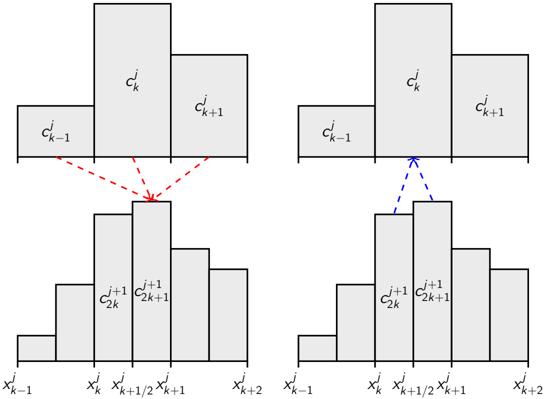
$$\hat{\mathbf{U}}_{2i+1}^{l+1} = \sum_{j=1-s}^{s-1} \gamma_j \mathbf{U}_{i+j}^l,$$

where γ_j are average-interpolation coefficients.

Detail Coefficients

The regularity information is assessed by computing detail coefficients (residuals) as

$$\mathbf{d}_i^l = \mathbf{U}_{2i+1}^{l+1} - \hat{\mathbf{U}}_{2i+1}^{l+1}.$$



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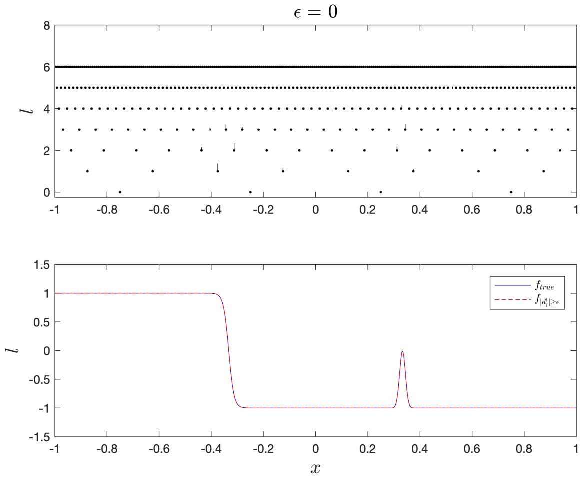

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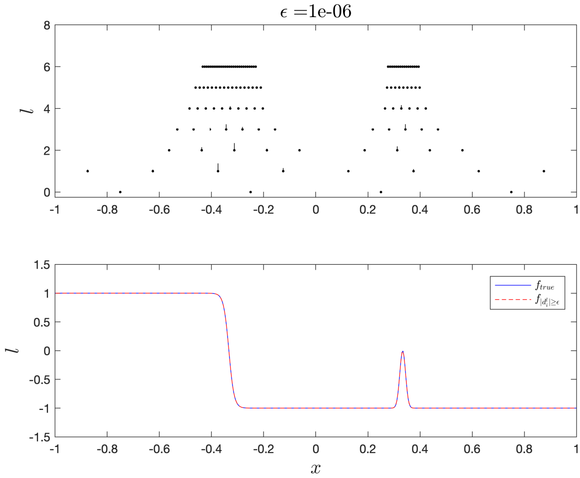

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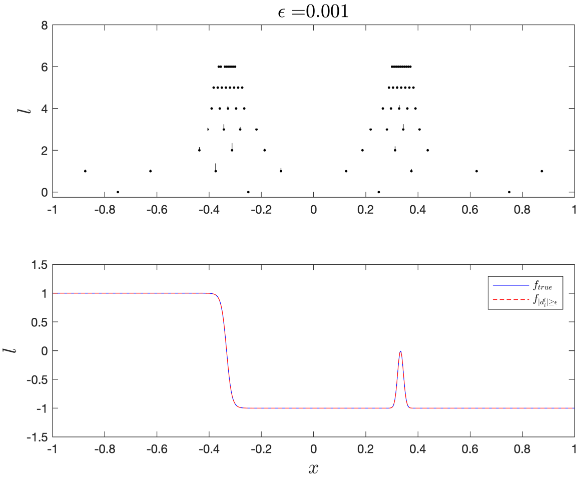

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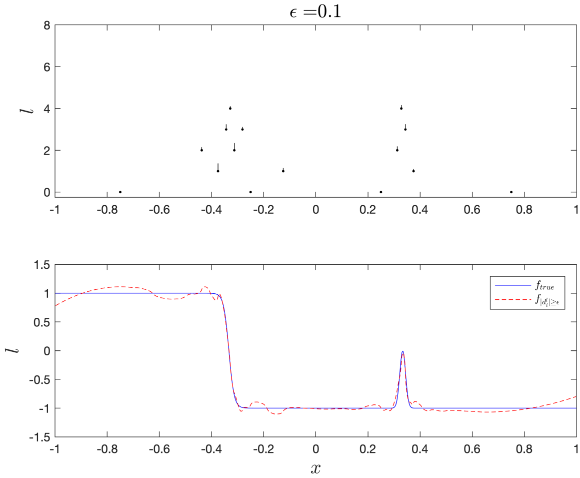

Harten's Approach

$$\frac{\partial \mathbf{U}_i}{\partial t} = -\frac{1}{|V_i|}\left(\mathbf{F}_{i+\frac{1}{2}} - \mathbf{F}_{i-\frac{1}{2}}\right) + \mathbf{S}_i$$

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- compute multiresolution decomposition on solution data
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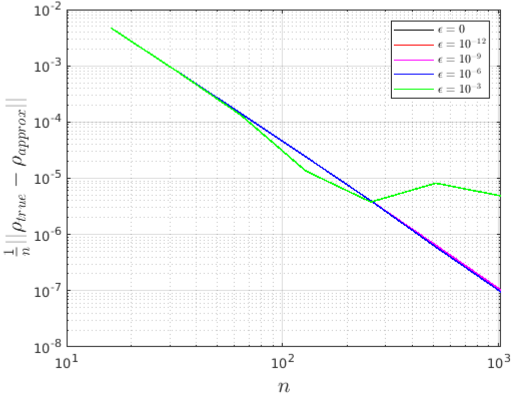
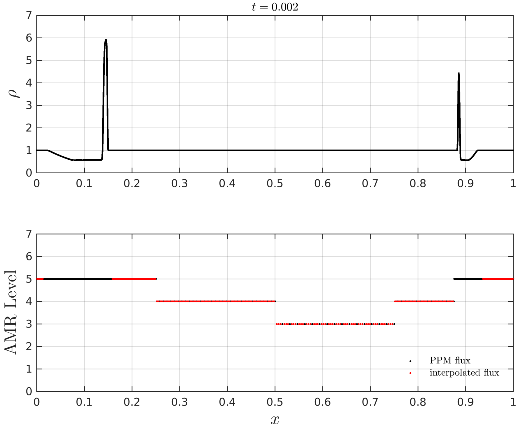
- move from coarse to fine, either compute or interpolate each $F^l_{i\pm\frac{1}{2}}$

Convergence

Sine wave advection after one period:



Two Interacting Blast Waves



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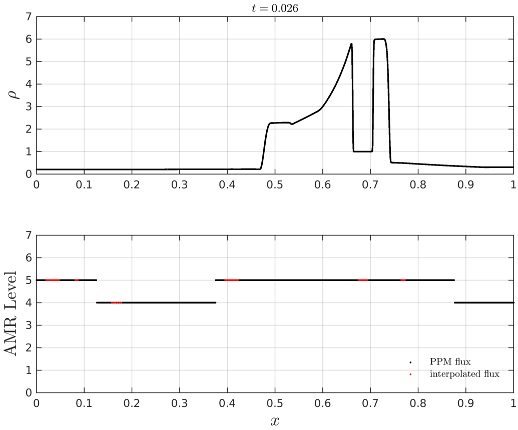

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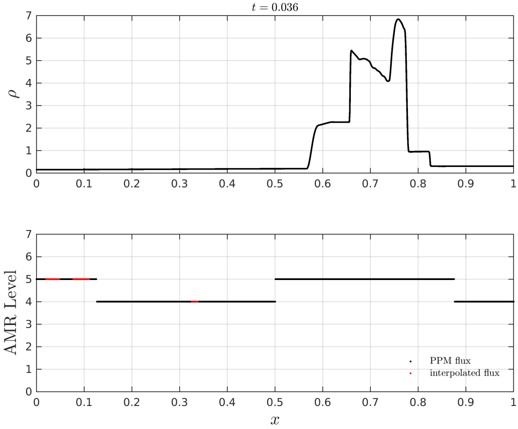

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- move to multi-dimensional problems