A multiresolution scheme for adaptive computations on block-structured AMR meshes: applications to reactive flows

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Abstract

We present a novel block-structured adaptive mesh refinement scheme featuring fully adaptive calculation of fluxes and source terms. The scheme addresses a major shortcoming of tree-based AMR codes: that the graded nature of the AMR mesh yields blocks whose cells are resolved beyond the desired error tolerance. To overcome this issue, we introduce a multiresolution representation of the solution not only for the purpose of grid adaptation but also to identify regions where fluxes and reaction-driven source terms can be interpolated from coarser levels. The error introduced by this approximation procedure is shown to be of the same order as the local truncation error of the reconstruction scheme. Thus the rate of convergence of the underlying spatial reconstruction scheme is preserved. Additionally with respect to parallel applications, the multiresolution transform and computation of fluxes and sources on adaptive blocks is asynchronous, requiring only one synchronization step which is equivalent to the filling of ghost cells for each block. The efficiency of the scheme is demonstrated for problems in compressible flow and reaction-driven combustion.

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[1]Since 1880.

Keywords: multiresolution, adaptive mesh refinement, reactive flows

2010 MSC: 00-01, 99-00

1. Introduction

Energetic, reacting, and turbulent flows are characterized by disparate spatial and temporal length scales. In the compressible regime, flows are capable of producing shock waves, resulting in steep gradients and an extremely thin discontinuity which propogates throughout the medium. Chemical reactions between fuels and oxidizers become a driving force for shock waves due to the large release of energy, and increase in static pressure. The burning fronts wherein these reactions take place are also highly spatially localized. These features require a level of mesh resolution that would make the problems intractable if applied over the entire domain. Therefore, efficient simulation of such flows requires a fully adaptive strategy, which we present in the following paper.

Accurately resolving regions of interest in fluid dynamics simulations for real-world applications is typically not feasible without introducing a non-uniform spatial mesh. Methods which introduce a hierarchy of nested grids are generally described as adaptive mesh refinement (AMR) methods. First introduced in (berger1984), AMR methods typically rely on estimates of the local truncation error (LTE) to determine regions where refinement is necessary for solution accuracy. Some more simple strategies may refine based on the magnitudes of the solution gradients, or concentration of a quantity of interest. While many strategies are possible, there is not yet a significant amount of mathematical theory available to quantify the solution accuracy for AMR simulations. For a full review of the LTE estimators and refinement criterion, readers are referred to BLANK.

Alternate approaches to dynamic grid adaptation based on wavelet theory became popular after the seminal papers by Harten [**?**], were introduced. In this work, a multiresolution representation of the discrete solution on a uniform grid was used for adaptively computing the divergence of the flux within a finite volume framework. Rather than adapt the grid directly, the idea was to accelerate the computation of fluxes using the multiresolution information. In this approach, eligible fluxes in sufficiently smooth regions are interpolated from fluxes obtained at interfaces corresponding to coarser grid levels. The original scheme was applied solely to hyperbolic conservation laws in one spatial dimension, but was then expanded by Bihari et. al. to two-dimensional simulations in, followed by the inclusion of viscous terms in, and then to source terms in the context of reactive flows in (bihari). These works retained the original spirit of Harten's scheme, which was to evolve the solution on a uniform grid, but use multiresolution information to identify regions where flux (and/or source term) computations may be avoided.

Regarding the implementation of AMR methods on large networks of parallel computers, certain engineering realities have neccessitated the reduction in granularity of the adaptive refinement. To use a single computational cell as the unit for refinement (i.e. cell-based refinement) introduces a number of costly compromises. Firstly, such an adapted grid requires the reconstruction method of choice to utilize nonuniform stencils, requiring increased computational resources. More significantly, the cell-based refinement requires costly data traversal. Traversing tree space requires on average $\mathcal{O}(n^d)$ operations, where n is the number of cells per dimension, d. Thus most AMR codes make use of some type of block-based approach. Tree-based block-structured codes, where each block consists of a fixed number of cells, are a very popular choice. These types of approaches are implemented in a number of AMR libraries including Paramesh, p4est, and others. This approach allows for simple mesh management procedures, and scales well for very large numbers of processors in parallel. One clear drawback however is the gradedness of the tree, which necessitates that no branch can have an incomplete set of children. This typically leads to refinement of many blocks which would not be otherwise flagged by refinement indicators. A further complication imposed by most finite volume solvers is that there can not be a jump in refinement greater than one level. Together these consequences of AMR represent a non-trivial decrease in performance due to the fact that the mesh is not optimal (in some sense).

Although Harten's original scheme was intended to be an alternative to spatially non-uniform grid adaptation, a series of papers have since reintroduced the concept of non-uniform grids within the MR framework. Thus the AMR approach was essentially redeveloped but with the refinement criterion defined by the MR representation rather than with the traditional metrics mentioned previously. The first fully adaptive scheme was presented by Cohen et. al. to study hyperbolic conservation laws in two dimensions in More recently, Rossinilli et. al. explored the use of wavelet-based refinement indicators for block-based adaptation.

In the following work, we present a novel block-based adaptive mesh refinement scheme using wavelet-based indicators, where Harten's scheme is used to essentially treat each block as a uniform grid. The indicators are used for two purposes: (1) refinement, and (2) adaptively calculating fluxes and source terms. We show how this scheme costs essentially nothing in additional computation (the MR information is recycled after adaptation), while not impacting the accuracy of the solver.

In the section 2 of this paper, we describe the conservations laws of interest as well as the underlying finite volume method and discretization. In section 3, we introduce the multiresolution decomposition of the numerical solution. In section 4, the block-based adaptive scheme is introduced, which combines grid adaptation with the interpolatory scheme of Harten.

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References