The Lifting Scheme for Multiresolution Wavelet-Based Transformation of Surface Meshes with Additional Attributes

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Abstract. There are a variety of applications areas that take advantage of the availability of three-dimensional data sets. These objects are represented as complex polygonal surfaces formed by hundreds of thousands of polygons, which causes a significant increase in the cost of storage, transmission and visualisation. Such models are usually not only geometrically complex, but they may also have various surface properties such as colour, textures and temperature, etc. This paper presents a extension of lifting scheme for the multiresolution decomposition and reconstruction of irregular triangle surface meshes with additional attributes.

Keywords: lifting scheme, multiresolution, irregular surface mesh.

1 Introduction

Many applications in computer graphics and visualisation take advantage of 3D models. Better and more popular 3D geometry acquisition systems result in million-polygons models. Such models are usually not only geometrically complex, but they may also have various surface properties especially appearance attributes such as color values and textures coordinates.

Multiresolution analysis and wavelets have been very popular lately, after processing signals, sounds, images and video, wavelets have been applied for digital geometry processing. This new class of application needs a processing toolbox of fundamental algorithms such as denoising, compression, transmission, enhancement, detection, analysis and editing with suitable mathematical and computational representations. The basic framework for these tools is the multiresolution analysis that decomposes an initial data set into a sequence of approximations and details.

This paper presents a natural extension of lifting scheme framework presented in my previous papers [22], [23], [21] to new wavelet-based multiresolution framework for decomposition and reconstruction of irregular triangle surface meshes with additional attributes. This solution is based on complete lifting scheme as a second generation wavelet construction tool. The generalisation from the first generation of wavelets to the second one gives an opportunity to construct wavelets on irregular meshes.

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2 Definition of Surface Mesh

The triangle mesh is a pair $M = (P, K)$ where P is a set of n points $p_i = [x_i, y_i, z_i]$ with $1 \leq i \leq 1n$ and K is a simplicial complex which contains information about topology of a mesh. The complex is a set of three types of subsets called simplices: vertices $v_i = \{i\} \in V$, edges $e_{i,j} = \{i, j\} \in E$, faces $f_{i,j,k} = \{i, j, k\} \in F$. Two vertices $\{a\}$ and $\{b\}$ are neighbours if $\{a, b\} \in E$. The 1-ring neighbourhood of vertex $\{i\}$ is the set $N(v_i) = \{j|\{i, j\} \in E\}$. The valence of a vertex v_i is the number of edges meeting at this vertex $\#N(v_i)$. There are three types of triangular meshes: regular (every vertex has valence equals 6), semi-regular (the most vertices have valence 6, few isolated vertices have any valence), irregular (vertices can have any valence).

Many models have surface properties beyond simple geometry. In computer graphics, the most common is RGB color values. Vertices of surface mesh, in addition to its position in space, have additional values which describe other properties. Each vertex will be treated as a extended vector $v \in R^n$. The first 3 components of v will be spatial coordinates, and the remaining components will be attributes values.

3 Related Work

This paper concerns multiresolution analysis based on wavelet theory so classic simplification algorithms [19], [18], [6], [17] are not described.

The first connection between wavelets and subdivision (semi-uniform Loop subdivision) allows to define multiresolution surface representation for semiregular mesh with subdivision connectivity, was presented in [9], [10]. In this case input, irregular mesh must first be rebuild with remeshing algorithms [10]. The BLaC-wavelets [11], [12] define local decomposition which is generalisation of Haar wavelets for non-nested spaces. The connection of hierarchical Delaunay triangulation allows to build multiresolution model of planar or spherical, irregular surface mesh. The next proposition in literature is wavelet-based multiresolution analysis of irregular meshes using a new irregular subdivision scheme [13], [14]. This is expansion of the solution presented in [9], [10].

Another direction in research is the use of non-uniform subdivision schemes to build wavelet analysis. In [8], [7] the central ingredient of multiresolution analysis is a non-uniform relaxation operator which minimises divided differences. This operator together with the mesh simplification method (progressive mesh [17]) and pyramid algorithm (Burt-Adelson pyramid) allows to define signal processing tools for irregular connected triangle meshes. In [15] progressive meshes and a semi-uniform discrete Laplacian smoothing operator were used to perform multiresolution editing tool for irregular meshes. Lifting scheme is used for multiresolution analysis in [16] where a prediction block is non-uniform relaxation operator based on surface curvature minimisation. This transformation is not a complete lifting scheme because it does not have update block.

There has been comparatively less work done on processing meshes with additional attributes. There is few extensions of simplification algorithms, for example *Quadric Error Metrics* [20], *Progressive Meshes* [17], *Texture Deviation* [24], *DePreSS* [27] and algorithm with new iso-perceptual error metric [25]. The wavelet transformation of meshes with additional properties are discussed in [26] which is extension of [10]. The multirsolution framework based on not complete lifting scheme presented in [16] allows processing irregular meshes with attributes.

4 General Lifting Scheme for Decomposition and Reconstruction of Irregular Surface Mesh

The most important advantage of the lifting scheme is generality which allows to construct the second generation wavelets on data with non-standard structures [1], [3], [4] while keeping all powerful properties of the first generation wavelets (classic wavelets [5]) such as speed and good ability of approximation. This section summarises, developed in [22], [21], [23], proposition of transformation based on the lifting scheme for multiresolution analysis of irregular, triangle closed mesh (2-manifold). Main operations of lifting scheme (Fig. 1) can be singled out:

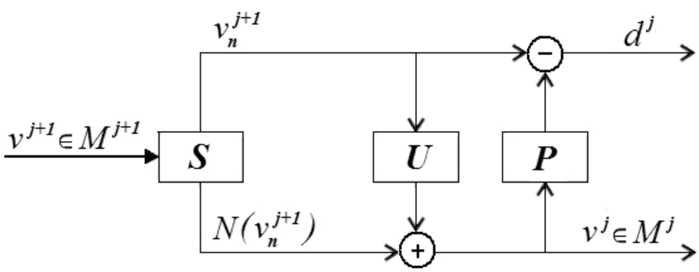


Fig. 1. The lifting scheme for decomposition of surface mesh

- Select from input mesh M^{j+1} (where j is the resolution level) the even and odd vertices (split block, S). The odd vertex (v_n) will be removed from mesh by half-edge collapse. The even vertices are 1-ring neighbourhood of odd vertex ($N(v_n)$). The vertex is selected based on the value of criterion which is vertex distance to the regression plane defined by 1-ring neighbourhood of the odd vertex ($S_d(v_n)$). This criterion reflected the curvature of surface very well. So the vertices from plain area of mesh are removed first.

$$S(M^{j+1}) = (v_n^{j+1}, N(v_n^{j+1}))$$

- Update even vertices (update block, U). This block smoothes mesh in lower resolution, which influences on better approximation by this mesh and in consequence smaller wavelet coefficients. The update value is added to every vertex in the neighbourhood of processed vertex in this lifting scheme step.

The suggested update block is: equal average value of single coordinates (AVV). Odd and even vertices are used in computing so as to avoid allocation memory for additional coefficients in multiresolution model, update block is before prediction block ("update first lifting scheme", [28]).

$$\bigwedge_{v_k \in N(v_n^{j+1})} v_k^j = v_k^{j+1} + U(v_n^{j+1}, N(v_n^{j+1}))$$

Prediction odd vertex based on even ones (predict block, P). The proposed prediction blocks take advantage of deterministic spatial interpolation methods such as natural neighbours (NNI) and thin plate splines interpolation based on first (TPS1) and second ring neighbourhoods (TPS2). To calculate prediction value the local parameterization of odd vertex and its neighbours should be determined, so the proposed prediction block consists of two parts: parameterization and interpolation. The searched parametric plane is the regression plane (total least squares plane) used also in split block. This is the best-fitting plane to a given set of vertices in 1-ring neighbourhood by minimizing the sum of the squares of the perpendicular distances of the points from the plane.

– Compute detail vector (wavelet coefficient, d) as difference between predicted and original vertex. Detail is stored in the multiresolution model [21].

$$d^j = v_n^{j+1} - P(N(v_n^{j+1}))$$

– Remove the odd vertex from the mesh by an half-edge collapse. The output is the coarser mesh M^j that is the approximation of initial mesh.

The inverse lifting scheme consists of following operations (Fig. 2):

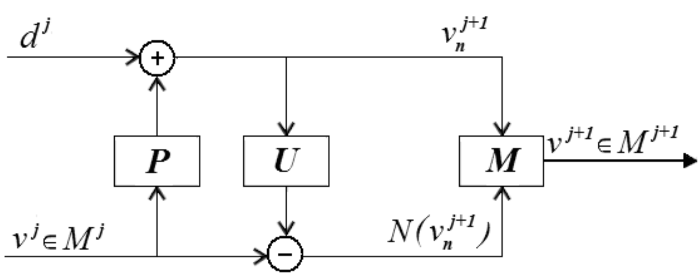


Fig. 2. The reverse lifting scheme for reconstruction of surface mesh

– Prediction odd vertex based on even vertices from mesh M^j.
– Compute original value of odd vertex as sum of detail vector remembered in multiresolution model and prediction vertex.
– Update even vertices.
– Insertion computed vertex to mesh (merge block, M). Reconstruction of topological connections between odd vertex and its neighbors (even vertices). The output is the mesh M^{j+1}.

We can write the equation of one step of wavelet transform:

$$M^{j+1} \rightarrow M^j + d^j$$

The original mesh (M) is decomposed into base mash in lowest resolution (M^0) and sequence of detail vectors:

$$M = M^0 + \sum_{j=1}^{m} d^j$$

This is the resulting multiresolution representation.

Application of second generation wavelets allows to build multiresolution processing directly on data from acquisitions systems without preprocessing step. In this step the irregular mesh is rebuilt to regular or semi-regular one.

5 Processing Meshes with Attributes

Vertices with additional attributes are treated as extended vector $v \in R^n$. For example vertex with RGB color values is vector $[x, y, z, r, g, b]$. The first 3 elements are coordinates values and next are color values. $A_k(v_i)$ is the k-th value of vertex vector vi.

5.1 Extension of Split Block

The vertex to remove criterion S_d is extended by additional criterion which minimises changes among next attributes of vertices in lower resolution mesh. From vertices with the smallest value of S_d criterion to remove is chosen this with the smallest value of criterion S_a. This is the minimal difference between average attribute value of neighbours vertices and attribute value of this vertex:

$$S_a(v_i) = \sum_{k=4}^{d} \alpha_k |A_k(v_i) - \overline{A}_k(N(v_i))|$$

Where $\overline{A}_k(N(v_i))$ is average attribute value of neighbours vertices of vertex v_i:

$$\overline{A}_k(N(v_i)) = \frac{\sum_{v_j \in N(v_i)} A_k(v_j)}{\#N(v_i)}$$

The coefficient α_i is weigh of attribute i in multiresolution processing of mesh. When all attributes are treated equally we can take $\alpha_i = \frac{1}{|A_{i,\max} - A_{i,\min}|}$ where values $A_{i,\max}$ and $A_{i,\min}$ are the minimal and maximal value of attribute i for all vertices of processing mesh.

5.2 Update Block

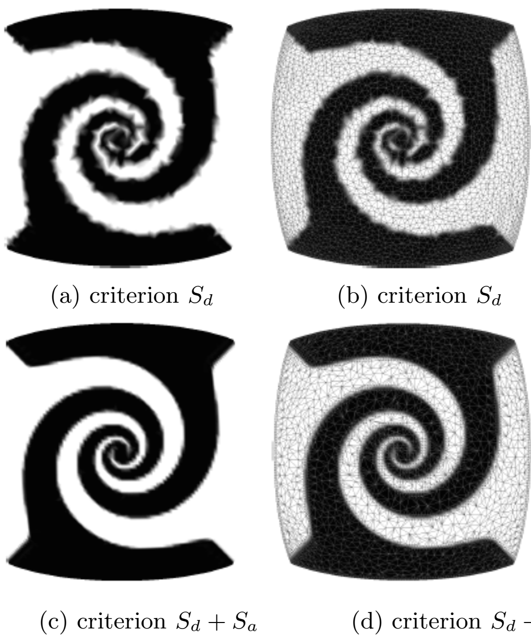
Additional attributes are treated in the same way as coordinates so this block assures equal average value of single coordinates and separate attributes.

5.3 Prediction Block

In prediction block three coordinates of vi are treated as value of function in two dimension parameter space (t_i, u_i). This is computed by the local parameterization of odd vertex and its neighbours on the regression plane. The prediction vertex is value of spatial interpolation in (t_0, u_0) which is the central point of its neighbours. In this same way can be computed prediction values of attributes. When the result of interpolation function is outside of range of property value (for example range of RGB color is form 0 to 1) it is clamped to nearest proper value.

6 Results

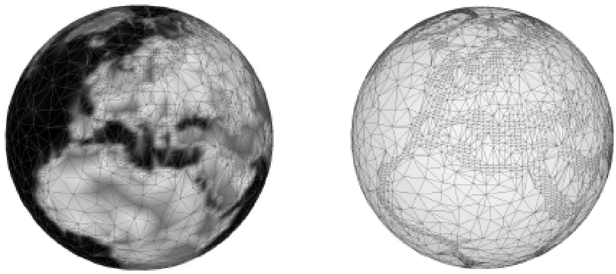
Presented results concern only meshes with RGB color attributes. The criterion $S_d + S_a$ in split block classifies vertices to remove first based on distance to the local regression plane next based on differences between attributes. Vertices which removing cause the smallest change in curvature of local surface (vertices from plain area of mesh) and in attributes (vertices with this same value of attributes) will be processed at the beginning. On Fig. 3 and Fig. 4 we can see that this criterion will retain vertices on the borderline between colors.



(a) criterion S_d (b) criterion S_d

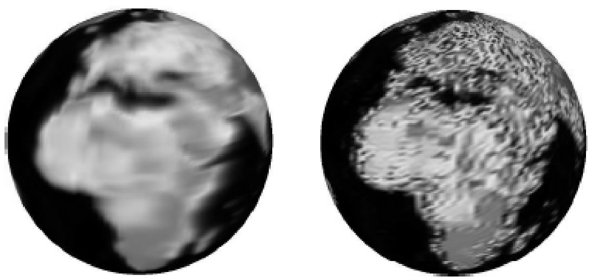
(c) criterion $S_d + S_a$ (d) criterion $S_d + S_a$

Fig. 3. Comparison meshes after removing of 60% of vertices by the lifting scheme with criterion S_d and extended criterion $S_d + S_a$



(a) (b)

Fig. 4. Mesh after removing 80% of vertices by the lifting scheme with criterion $S_d + S_a$.



(a) smoothing mesh (b) enhancement mesh

Fig. 5. Applying of smoothing and enhancement filter to multiresolution representation

7 Summary

This paper presents a extension of lifting scheme for the multiresolution decomposition and reconstruction of irregular triangle surface meshes with additional attributes. These attributes can be color values, texture coordinates, temperature and pressure values, etc. The result of processing is multiresolution representation which supports LOD and ROI questions [21]. The lifting scheme is second generation wavelets construction tool so this representation can be a base of digital geometry processing algorithms, such as smoothing or enhancement details of surface mesh (Fig. 5).

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