

A Multiresolution Scheme for More Efficient Simulation on Adaptive Mesh Refinement Blocks?

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Introduction

The following project is concerned with the numerical solution of systems of conservation laws of the form

$$\mathbf{U}_t + \mathbf{F}(\mathbf{U})_x = 0$$

where $\mathbf{U} = (\rho, \rho u, E)$ is a vector of conserved quantities and $\mathbf{F}(\mathbf{U})$ is a flux vector. Consider a standard finite-volume discretization, with midpoint rule

$$\frac{\partial \mathbf{U}_i}{\partial t} = -\frac{1}{h} \left(\mathbf{F}_{i+\frac{1}{2}} - \mathbf{F}_{i-\frac{1}{2}} \right)$$

where the i denotes spatial index.

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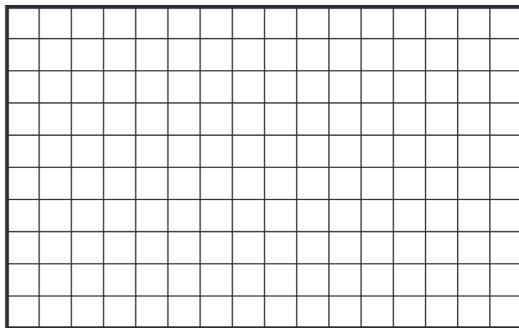
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- ▶ adaptive mesh refinement (AMR) based on local truncation error (or some other estimator)
- ▶ computations typically done on blocks / patches for efficiency
- ▶ inherent “overresolution” in some regions of the mesh by using blocks
- ▶ can this be addressed by reducing block size? computational tradeoff?

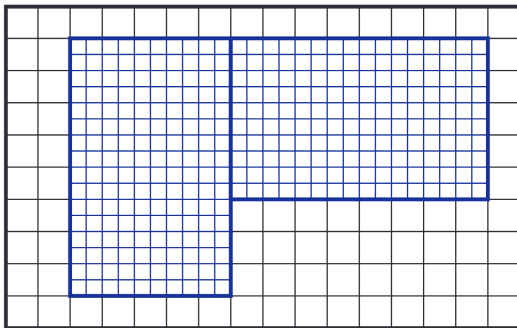
AMR



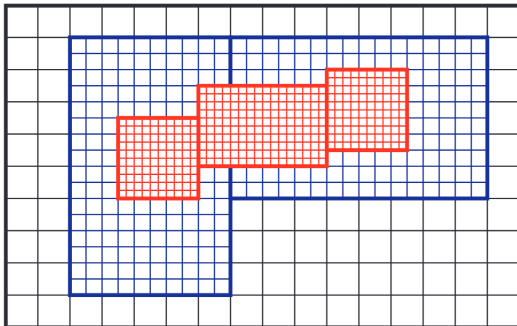
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¹Introduction to Block-Structured Adaptive Mesh Refinement (AMR), Ann S. Almgren

AMR

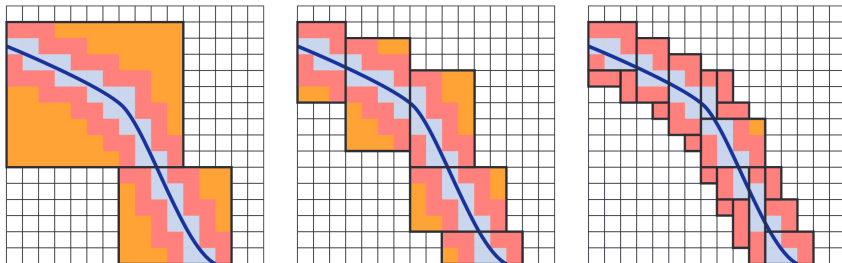


AMR



Filling

The filling factor is the number of cells in a block which were flagged, divided by the total.



a la Harten

- ▶ Besides the AMR concepts, adaptivity was also introduced by Harten in a multiresolution framework. Although in his scheme, the grid was *not* refined in space. Instead, a wavelet transform was performed on the uniform grid, and the fluxes were interpolated in smooth regions.

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- ▶ Multi-scale ideas behind Multigrid method, subdivision schemes for graphics / CAD

Multiresolution

Define multiple, nested grids

$$\mathbf{G}^l = \left\{ x_{i+\frac{1}{2}}^l \right\}_{i=0}^{N_l} = \left\{ x_{i+\frac{1}{2}}^{l+1} \right\}_{i=0, i \text{ even}}^{N^{l+1}}.$$

Coarsening of average data in cell done via

$$\mathbf{u}_i^l = \frac{1}{2} \left(\mathbf{u}_{2i}^{l+1} + \mathbf{u}_{2i+1}^{l+1} \right)$$

Decomposition

The prediction from coarse to fine is done by

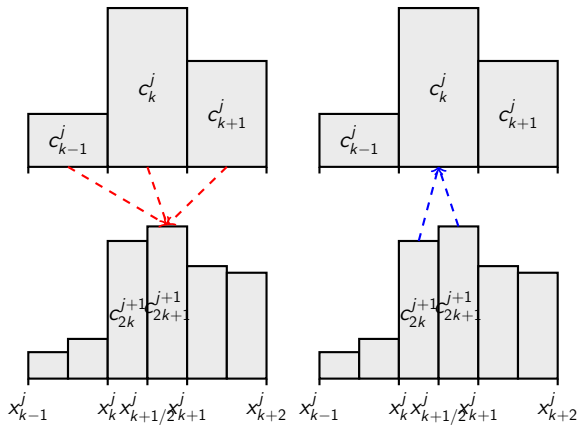
$$\hat{\mathbf{U}}_{2i+1}^{l+1} = \sum_{j=1-s}^{s-1} \gamma_j \mathbf{U}_{i+j}^l$$

The regularity information is assessed by computing detail coefficients as

$$\mathbf{d}_i^l = \mathbf{U}_{2i+1}^{l+1} - \hat{\mathbf{U}}_{2i+1}^{l+1}.$$

A mask $\{\mathbf{m}\}_i^{N'}$ is created for significant cells.

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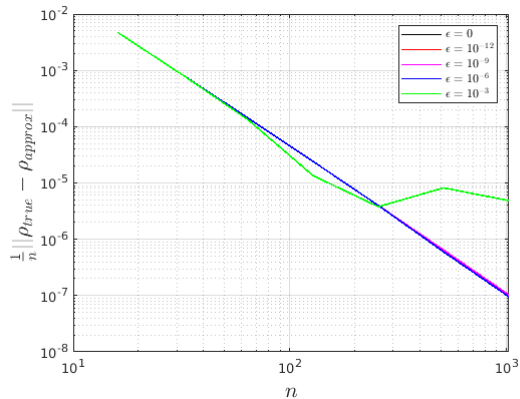
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- ▶ introduce sufficiently large buffer region (why?) around flagged cells

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- ▶ utilize this regularity information to identify sufficiently smooth regions in which to interpolate the flux.
- ▶ introduce sufficiently large buffer region (why?) around flagged cells
- ▶ perform inverse transform and compute or interpolate fluxes

Convergence



Outlook

- ▶ weakly compressible turbulence (uniform mesh)

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- ▶ compressible turbulence (potentially adaptive)

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- ▶ weakly compressible turbulence (uniform mesh)
- ▶ compressible turbulence (potentially adaptive)
- ▶ turbulent combustion (adaptive)