Group Meeting
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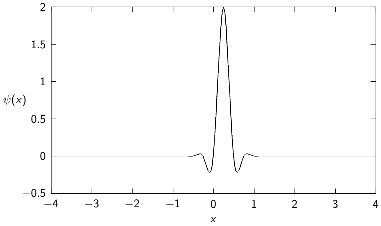
Hybrid Wavelet/AMR Scheme

The goal of this project is to marry the benefits of wavelet analysis with the more developed AMR strategies:

- can wavelet sensors augment LTE for refinement?
- can regularity information reduce computational expense on fine grids?



Harten's MR Scheme

The one-dimensional reference system of conservation laws is

$$\mathbf{U}_t + \mathbf{F}(\mathbf{U})_x = 0,$$

where $\mathbf{U} = (\rho, \rho u, E)$ is a vector of conserved quantities. In semi-discrete form,

$$\frac{\partial \mathbf{U}_i}{\partial t} = -\frac{1}{h}\left(\mathbf{F}_{i+\frac{1}{2}} - \mathbf{F}_{i-\frac{1}{2}}\right) = \mathbf{R}_i$$

where the i denotes spatial index, and $\mathbf{R}$ is residual.

Harten's MR Scheme

Define multiple, nested grids

$$\mathbf{G}^l = \left\{ x^l_{i+\frac{1}{2}} \right\}_{i=0}^{N_l} = \left\{ x^{l+1}_{i+\frac{1}{2}} \right\}_{i=0,\text{i even}}^{N^{l+1}} .$$

Coarsening of a cell done via

$$\mathbf{U}^l_i = \frac{1}{2}\left(\mathbf{U}^{l+1}_{2i} + \mathbf{U}^{l+1}_{2i+1} \right)$$

and the prediction from coarse to fine is

$$\hat{\mathbf{U}}^{l+1}_{2i+1} = \sum_{j=1-s}^{s-1} \gamma_l \mathbf{U}^l_{i+l}$$

Harten's MR Scheme

The regularity information is assessed by computing detail coefficients as

$$\mathbf{d}_i^l = \mathbf{U}_{2i+1}^{l+1} - \hat{\mathbf{U}}_{2i+1}^{l+1}.$$

A mask $\{\mathbf{m}\}_i^{N^l}$ is created for significant cells. The fluxes are initially computed on coarsest level, then via

$$\mathbf{F}_{2i-\frac{1}{2}}^{l+1} = \mathbf{F}_{i-\frac{1}{2}}^{l}$$

$$\text{if } m_i^l == \textit{true}, \text{then } \mathbf{F}_{2i+\frac{1}{2}}^{l+1} = \mathbf{F}(\mathbf{U})$$

else interpolate...

Multiresolution Representation

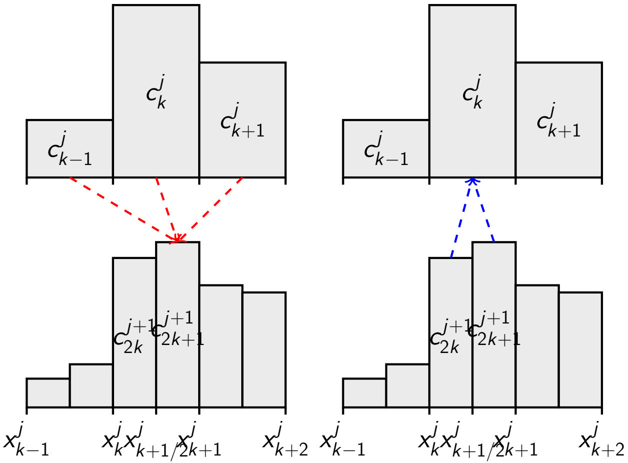
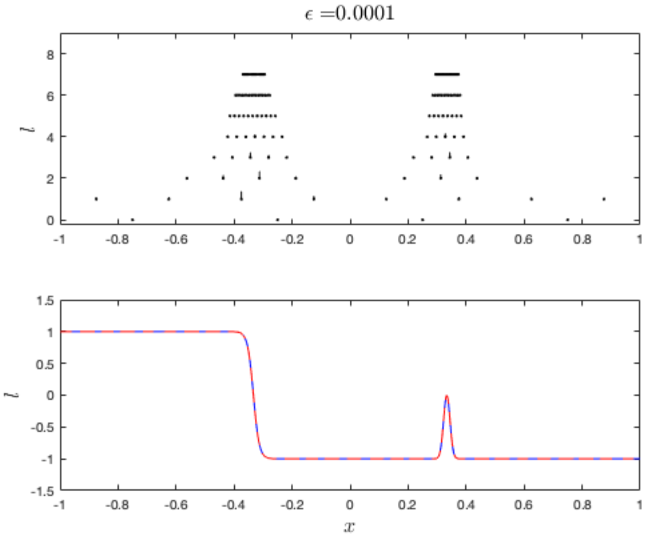


Figure: Left: quadratic prediction from coarse-scale j to fine-scale $j+1$, given cell-averages $\mathbf{c}^j$. Right: Fine-scale cell averages are coarsened.

Multiresolution Code - Examples



Multiresolution Scheme on AMR Patches

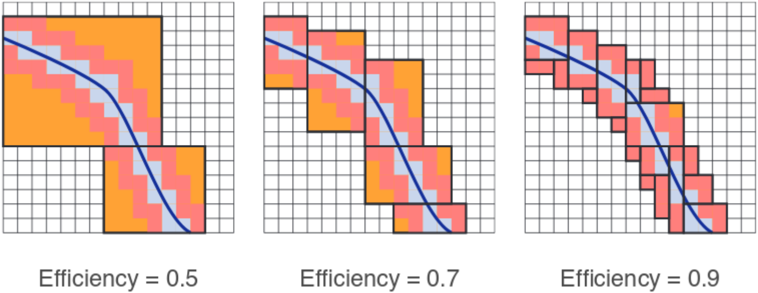


Figure: Increasing patch 'efficiency.'

Multiresolution Scheme on AMR Patches

The steps of the scheme for one patch are

1. take given (finest level) data on patch, and coarsen it to desired coarsest level L
2. compute the forward wavelet transform to obtain $\left\{\mathbf{d}^l\right\}_{l=L}^{l=1}$
3. on coarsest level L, compute the residuals $\left\{R_j^L\right\}_{j=0}^{N_L}$
4. loop through one finer level at a time, and according to detail coefficients, either interpolate or calculate remaining fluxes

Multiresolution Scheme on AMR Patches

The original (fine) data is coarsened by

$$w_j^l = \frac{1}{2}\left(w_{2j}^{l-1} + w_{2j+1}^{l-1}\right)$$

Then the residual R_j^l may be interpolated in smooth regions as

$$\begin{aligned} R_{2j+1}^{l-1} &= R_j^l - \frac{1}{8}\left(R_{j-1}^l - R_{j+1}^l\right) \\ R_{2j}^{l-1} &= 2R_j^l - R_{2j+1}^{l-1} \end{aligned}$$

Progress in FLASH Implementation

- created a new folder `source/flashUtilities/Wavelet/`
- writing a program `Wavelet_computeTransform.F90` which will be run in `Grid_computeUserVars.F90`

```
function imap( l, i, j, k, nx, ny, nz ) result(x)
  integer, intent(in) :: nx(:), ny(:), nz(:)
  integer, intent(in) :: l, i, j, k
  integer             :: x

  ! compute the map
  x = l + nx * ( i + ny * ( j + nz * k ) )

end function imap
```