Average-Intepolating Wavelets with Lifting Scheme

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Predict Stage

We are interested in obtaining the difference between approximation spaces at varying levels of resolution. We are given cell-averaged values as input data to our wavelet transform. This data is considered at some maximum resolution level J, and the wavelet transform will produce details coefficients down to the coarsest level $j = 0$. We consider an interpolating polynomial $p(x)$ such that

$$c_{k-1}^j = \int_{x_{k-1}^j}^{x_k^j} p(x)dx \tag{1}$$

$$c_k^j = \int_{x_k^j}^{x_{k+1}^j} p(x)dx \tag{2}$$

$$c_{k+1}^j = \int_{x_{k+1}^j}^{x_{k+2}^j} p(x)dx. \tag{3}$$

However this can be cast in a more suitable form for interpolation algorithms by introducting $P(x) = \int_0^x p(y)dy$. The problem is then to interpolate the following data

$$0 = P(x_{k-1}^j) \tag{4}$$

$$c_k^j = P(x_k^j) \tag{5}$$

$$c_k^j + c_{k+1}^j = P(x_{k+1}^j) \tag{6}$$

$$c_k^j + c_{k+1}^j + c_{k+2}^j = P(x_{k+2}^j) \tag{7}$$

$$\tag{8}$$

Lifting Scheme

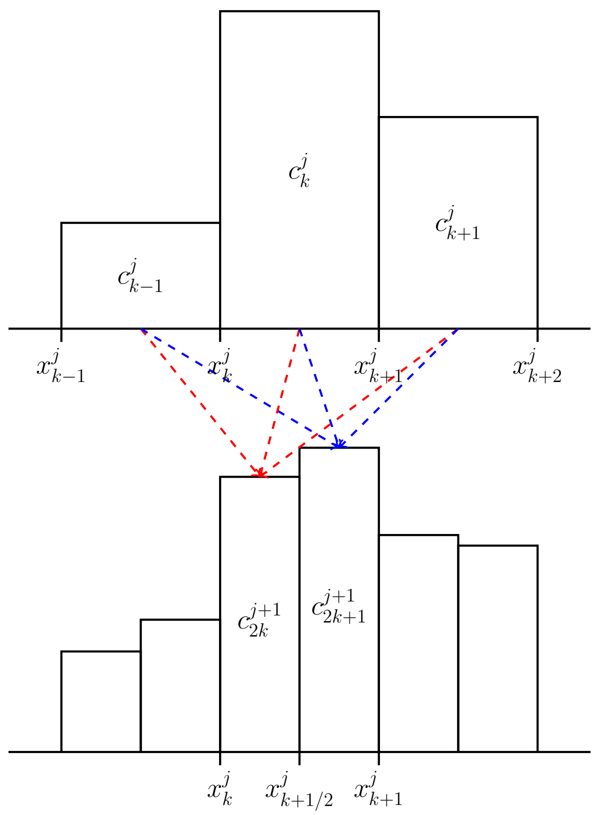

Figure 1: Prediction operator from coarse-scale j to fine-scale $j+1$, given cell-averaged data $\mathbf{c}^j$. Red and Blue arrows indicate interpolation dependency for each cell at level $j+1$.