A Multiresolution Scheme for More Efficient Simulation on Adaptive Mesh Refinement Blocks?

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Introduction

The following project is concerned with the numerical solution of systems of conservation laws of the form

$$\mathbf{U}_t + \mathbf{F}(\mathbf{U})_x = 0$$

where $\mathbf{U} = (\rho, \rho u, E)$ is a vector of conserved quantities and $\mathbf{F}(\mathbf{U})$ is a flux vector. Consider a standard finite-volume discretization, with midpoint rule

$$\frac{\partial \mathbf{U}_i}{\partial t} = -\frac{1}{h}\left(\mathbf{F}_{i+\frac{1}{2}} - \mathbf{F}_{i-\frac{1}{2}}\right)$$

where the i denotes spatial index.

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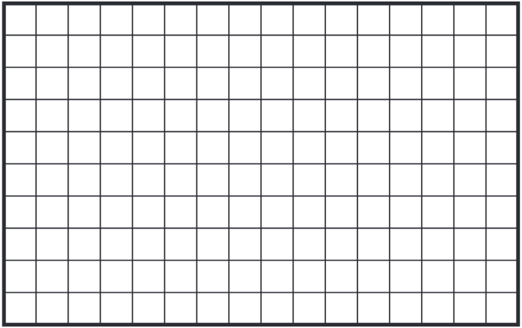
- computations typically done on blocks / patches for efficiency
- inherent "overresolution" in some regions of the mesh by using blocks
- can this be addressed by reducing block size? computational tradeoff?

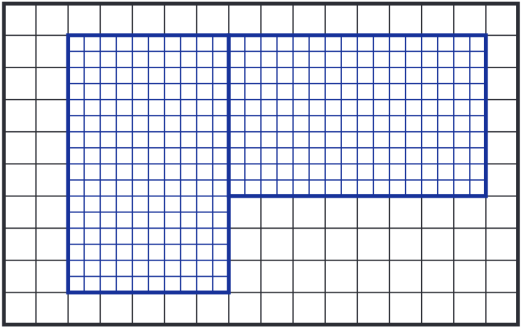
AMR



[1]

[1] Introduction to Block-Structured Adaptive Mesh Refinement (AMR), Ann S. Almgren

AMR



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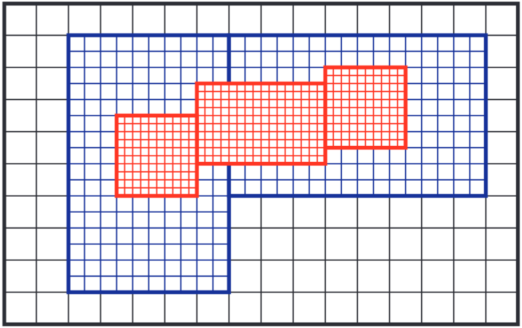
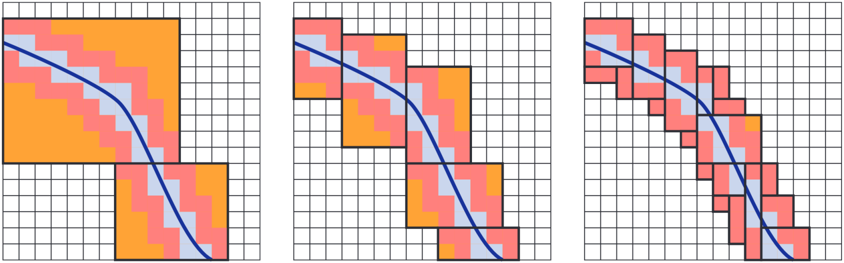


Filling

The filling factor is the number of cells in a block which were flagged, divided by the total.



a la Harten

- Besides the AMR concepts, adaptivity was also introduced by Harten in a multiresolution framework. Although in his scheme, the grid was *not* refined in space. Instead, a wavelet transform was performed on the uniform grid, and the fluxes were interpolated in smooth regions.

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- Multi-scale ideas behind Multigrid method, subdivision schemes for graphics / CAD

Multiresolution

Define multiple, nested grids

$$\mathbf{G}^l = \left\{ x^l_{i+\frac{1}{2}} \right\}_{i=0}^{N_l} = \left\{ x^{l+1}_{i+\frac{1}{2}} \right\}_{i=0,\text{i even}}^{N^{l+1}} .$$

Coarsening of avarage data in cell done via

$$\mathbf{U}^l_i = \frac{1}{2} \left(\mathbf{U}^{l+1}_{2i} + \mathbf{U}^{l+1}_{2i+1} \right)$$

Decomposition

The prediction from coarse to fine is done by

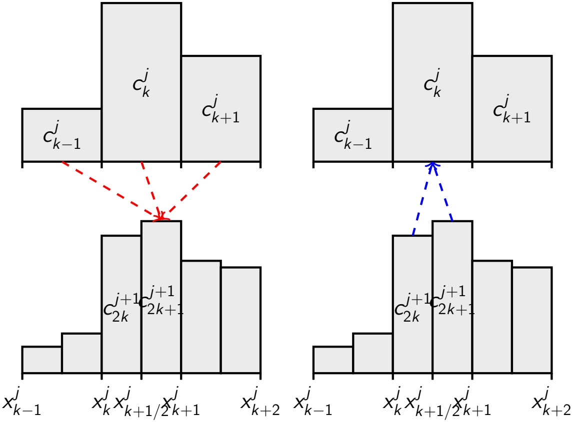
$$\hat{\mathbf{U}}^{l+1}_{2i+1} = \sum_{j=1-s}^{s-1} \gamma_j \mathbf{U}^{l}_{i+j}$$

The regularity information is assessed by computing detail coefficients as

$$\mathbf{d}^{l}_{i} = \mathbf{U}^{l+1}_{2i+1} - \hat{\mathbf{U}}^{l+1}_{2i+1}.$$

A mask $\{\mathbf{m}\}_i^{N^l}$ is created for significant cells.

Decomposition



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- introduce big buffer zone around cells that were flagged at each level
- perform inverse transform and compute or interpolate fluxes