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Chapter 3

THE MULTIRESOLUTION WAVELET-BASED TRANSFORMATION OF 3D OBJECT SURFACE MESHES

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Abstract

3D meshes are a piecewise-linear surface approximation and are the most popular representation of real scanned and modeled object used in computer graphics, medicine, games, simulators, scientific visualisation, CAD and virtual reality systems. Such meshes can be set of millions of polygons, vertices and edges. Complex three-dimension objects must be processed, stored, transferred, animated and analysed which is very expensive. Multiresolution models arise a lot of interest because they make it possible to represent and process geometric data in different levels of detail (LOD) depending on application needs. This chapter presents a survey of the different wavelet-based multiresolution methods proposed for triangle surface meshes. First the preliminary background on triangle meshes and multiresolution analysis is presented. Then the different proposed methods for regular, semi-regular and irregular meshes are described. This survey consists also description of Author solution based on second generation wavelets for decomposition and reconstruction of irregular, triangle mesh. The generalisation from the first generation of wavelets to the second one gives an opportunity to construct wavelets on irregular meshes. This solution is based on lifting scheme as a wavelet construction tool. The conclusion is the different application of result multiresolution representation in algorithms of digital geometry processing.

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Introduction

After processing signals, sounds, images and video, wavelets have been applied for digital geometry processing. Its goal is the research of mathematical and computational foundations of algorithms such as denoising, compression, transmission, enhancement, detection, analysis and editing. The basic framework for these tools is the multiresolution analysis that decomposes an initial data set into a sequence of approximations and details. Main advantages are reduction of data, quick access to data, hierarchical processing, LOD control and progressive transmission.

3D meshes are a piecewise-linear surface approximation and are the most popular representation of real scanned and modeled objects. Such meshes can be set of millions of polygons, vertices and edges. Multiresolution models arise a lot of interest because they make it possible to represent and process geometric data in different levels of detail (LOD) depending on application needs [1], [2], [3]. For given input original mesh the multiresolution representation is a decomposition to a sequence of topologically equivalent meshes with smaller geometric detail. Next approximations are next levels of detail of original mesh and consist of a lower number of vertices, edges and polygons. The wavelet-based multiresolution algorithms decompose an initial mesh into a sequence of mesh approximations and details - wavelet coefficients [4]. Result multiresolution representation can be base of different algorithms such as smoothing, enhancement, features detection, compression, progressive transmission, etc.

This chapter is a survey of different wavelet-based multiresolution methods proposed for triangle meshes (regular, semi-regular and irregular) [5]. The description of second generation wavelets for decomposition and reconstruction of irregular, triangle meshes is also presented. The conclusion is using of result multiresolution representation in the digital geometry processing applications.

Definition of Surface Mesh

There are three most popular representations of surfaces of the 3D objects: parametric surfaces (for example Bezier, B-spline surfaces), implicit surfaces and surface meshes. First two are continuous representation of surface and are commonly used for computer aided modeling (CAD). Mesh is discrete representation of surface. This is a piecewise-linear surface approximation and is the most popular representation of real scanned and modeled object used in computer graphics applications.

Because the triangle is the most popular graphics primitive so the triangles meshes are also most popular used. This chapter concerns only triangle surface meshes.

The triangle mesh is a pair $M = (P, K)$ where P is a set of n points $p_i = [x_i, y_i, z_i]$ with $1 \leq i \leq n$ and K is a simplicial complex which contains information about topology of a mesh. The complex is a set of three types of subsets called simplices: vertices $v_i = \{i\} \varepsilon V$, edges $e_{i,j} = \{i, j\} \varepsilon E$, faces $f_{i,j,k} = \{i, j, k\} \varepsilon F$. Two vertices $\{a\}$ and $\{b\}$ are neighbours if $\{a, b\} \varepsilon E$. The 1-ring neighbourhood of vertex $\{i\}$ is the set $N(v_i) = \{j | \{i, j\} \varepsilon E\}$. The valence of a vertex v_i is the number of edges meeting at this vertex $\#N(v_i)$. There are three types of triangular meshes shown on Figure 1:

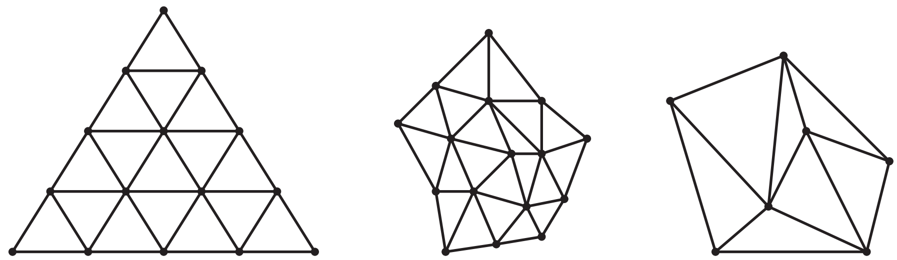


Figure 1. Types of triangle mesh: regular, semi-regular, irregular.

- regular where every vertex has valence equals 6;
- semi-regular where the most vertices have valence 6, few isolated vertices have any valence;
- irregular where vertices can have any valence.

Many models have surface properties beyond simple geometry. In computer graphics, the most common is RGB color values. Vertices of surface mesh, in addition to its position in space, have additional values which describe other properties. Each vertex is treated as a extended vector $v \varepsilon R^n$. The first 3 components of v are spatial coordinates, and the remaining components are attributes values. Example vertex with RGB color is $[x_i, y_i, z_i, r_i, b_i, g_i]$.

The triangle mesh which make surface must meet the assumptions of the topological 2-manifold. This means that every vertex of mesh has to have a neighbourhood for which there exists a homeomorphism mapping that neighbourhood to R^2 disc.

The topological 2-manifold mesh M define the domain of wavelet multiresolution representation. Functions are defined based on set of vertices V, edges E and triangles F with topological information about mutual proper connections.

Level of detail representation

For given input triangle mesh $M = M^n$ the multiresolution representation is a decomposition to a sequence of topologically equivalent meshes with smaller geometric detail: $M^n, M^{n-1}, ..., M^0$. Where M^0 is mesh in lowers resolution (base mesh). Next approximations are next levels of detail of original mesh and consist of a lower number of vertices V, edges E and polygons F.

Generally methods to generation multiresolution representation can be divided to two groups: mesh decimation and mesh refinement. Some elements and assumptions of these base methods are often used as building block in wavelets transformation framework. So the short introduction is needed.

Mesh decimation

Mesh decimation is coarsening of the mesh at full resolution through iterative application of local topological operator. These operators collapse vertex, edge, half-edge or triangle, the second group of operators remove vertex, edge or triangle and the result hole is triangulated.

This operator simplifies mesh region chosen by local or global criteria which depends of algorithm. Resolution levels are constructed until the approximation error reaches a user-defined maximum.

Most popular simplification algorithms are vertex decimation [6], Quadric Error Metric (*QEM*) [7], [8], mesh optimization [9], progressive mesh (*PM*) [10], [11]. Full survey of simplification methods can be fund in [12], [13], [14], [15], [16].

Mesh refinement

Mesh refinement is iterative algorithm which adds elements at each step to initial base mesh in coarser approximation. These algorithms need the input base mesh in lowest resolution. The main group of this algorithms are subdivision schemes.

Subdivision schemes were developed as generalizations of uniform B-spline knot insertion algorithms to settings with topologically irregular control meshes. Generally subdivision defines a smooth curve or surface as the limit of a sequence of successive refinements. A subdivision surface is created by iteratively refining a control mesh M^0 to produce a sequence on increasing detailed meshes that converge to a surface (1).

$$\sigma = \lim_{j \to \infty} M^j \tag{1}$$

The first step of algorithm is topological subdivision that splits each mesh face or edge. The second step, geometric positioning, computes positions for new vertices as a local affine combination of old vertices via subdivision relation. The subdivision process produces hierarchically sequence of meshes with increasing number of faces and vertices.

Example of semi-uniform subdivision schemes for triangle meshes are: approximating Loop [17] and $\sqrt{3}$ [18], interpolating Butterfly [19], [20]. To analysis of schemes and result surface the spectral analysis based on subdivision matrix can be used [21].

The non-uniform schemes (variational subdivision) can be used for irregular meshes [22], [23]. The subdivision operator computes positions of new vertex as a minimization of value of some function, for example: minimize of second order differences defined at every edge [24], [25], surface curvature minimization [22]. The disadvantage of that schemes are slower work, much more complex and not complete mathematical tool for analysis.

Wavelet-Based Multiresolution Transformation of Triangle Mesh

Background

Multiresolution analysis by using wavelets decomposes an initial mesh into a sequence of mesh approximations and details - wavelet coefficients (Figure 2). The approximations at different levels of resolution are computed by using scalar functions. Next approximations of original mesh consist of a lower number of vertices V, edges E and polygons F. The details are computed by using wavelet functions and represent the data lost between next approximations and consist information about vertices which were removed from mesh. The reconstruction process uses the dual scalar and wavelet functions.

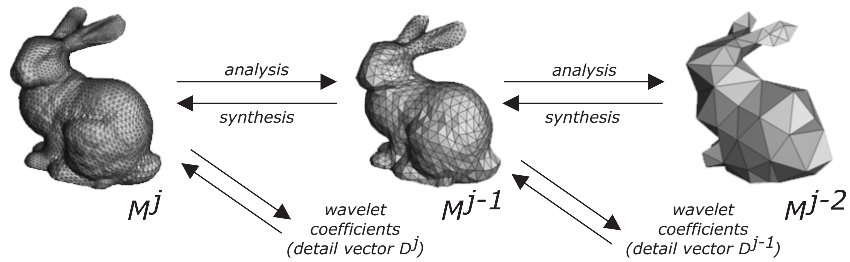


Figure 2. Wavelet based analysis of surface mesh.

The equation (2) is description of one step of wavelet transform. Wavelet coefficients D^j are set of difference vectors between meshes in $j+1$ and j resolution.

$$M^{j+1} \rightarrow M^j + D^j \tag{2}$$

The original mesh M^n is decomposed into base mesh in lowest resolution (M^0) and sequence of detail vectors. This is the result multiresolution representation (3).

$$M^n \rightarrow M^0 + \sum_{j=1}^{n} D^j \tag{3}$$

The general assumptions of wavelets methods for mesh transformation are:

- Meshes in lower resolutions should be a good approximation of the original surface mesh.
- The result of reconstruction process should be as close as possible to the original mesh. The calculations must be reversible.
- The detail vector should be small and capture local geometric details of represented surface.

Main problems in definition wavelet analysis and synthesis of surface mesh are no natural parametrization of mesh, irregularity of data and need for using non-separable wavelet bases.

Two groups of wavelets method can be singled out. First are methods for regular and semi-regular meshes with subdivision connectivity. These methods base on mathematical properties of uniform and semi-uniform subdivision methods. In this case input mesh first must be rebuild with remeshing algorithms [26], [27]. Another group of algorithms are methods to deal directly with irregular meshes with no need of remeshing process.

Lifting scheme and second generation wavelets

The second generation wavelets [28] are generalisation of biorthogonal classic wavelets, called first generation wavelets. The second generation wavelet is not necessarily constructed by translation and dilatation of one function (mother function). That wavelets

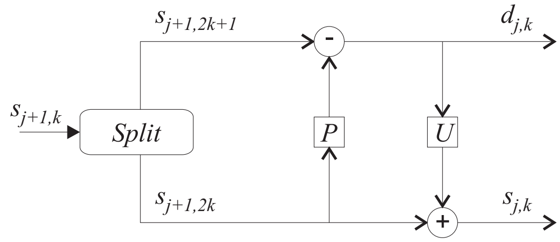


Figure 3. The lifting scheme

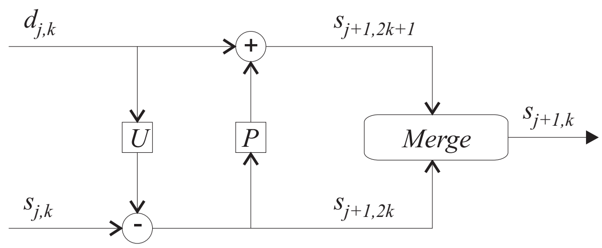


Figure 4. The reverse lifting scheme

can be built in more general settings where the Fourier can not be used as a construction tool. The second generation multiresolution representation may use many basic functions (wavelet and scaling functions), but must be possible to define nested spaces - must be meet the refinement relation of scaling functions.

The lifting scheme [29], [30], [31], [4] is a simple but powerful tool to construct the second generation wavelets. The main advantage of this solution is the possibility of building wavelet analysis on non-standard structures of data (irregular samples, bounded domains, curves, surfaces, manifolds) while keeping all powerful properties of the first generation wavelets such as speed and good ability of approximation.

A general lifting scheme (Figure 3) consists of three types of operation:

- **Split**: splits input dataset into two disjoint sets of even and odd indexed samples (4). The definition of lifting scheme doesn't impose any restriction on how the data should be split nor on the relative size of each subsets.

  $$(s_{j+1,2k+1}, s_{j+1,2k}) = S(s_{j+1,k}) \tag{4}$$

  Where first index denotes resolution level, higher index means more detailed resolution level. The second index denotes position of sample in data set.

- **Predict**: predicts odd indexed sample based on even samples. Next the odd indexed input value is replaced by the offset (difference) between the odd value and its prediction (5).

  $$d_{j,k} = s_{j+1,2k+1} - P(s_{j+1,2k}) \tag{5}$$

- **Update**: updates the output, so that coarse-scale coefficients have the same average value as the input samples (6). This step is necessary for stable wavelet transform [28].

$$s_{j,k} = s_{j+1,2k} + U(d_{j,k}) \tag{6}$$

This calculations can be performed in-place. In all stages input samples can be overwritten by output of that step. Inverse transform (Figure 4) is easy to find by reversing the order of operations and flipping the signs.

Method for regular and semi-regular meshes

The first connection between wavelets and subdivision (semi-uniform Loop subdivision) which allows to define multiresolution surface representation for semi-regular mesh with subdivision connectivity, was presented in [32], [26]. This is extension of multiresolution analysis based on tensor-product, which can be used only for functions parametrized on R^2, to arbitrary topological domains but with subdivision connectivity (semi-regular meshes). This solution is based on creating refinable scaling functions using recursive surface subdivision.

The recursive subdivision methods applied to surfaces meshes leads to a collection of refinable scaling functions and to a sequence of nested linear spaces required by multiresolution analysis. Only surfaces generated through subdivision can be hierarchically decomposed using that wavelets methods. The domain of scaling function is initial control mesh M^0. This mesh has the same topological type as the limit surface. A chain of nested linear space can be defined as:

$$V^0 \subset V^1 \subset \dots$$

where V^j is a set of mesh vertices in resolution j. The inner product is defined for functions which domain is M^0:

$$f, g \varepsilon V^j(M^0) :< f, g >= \sum_{\tau \varepsilon \Delta(M^0)} \frac{1}{area(\tau)} \int_{x \varepsilon \tau} f(x) g(x) dx \tag{7}$$

Where τ denote a triangle in set of triangles of M^0 mesh ($\Delta(M^0)$), $area(\tau)$ is area of triangle and x is the point of M^j corresponding to base mesh.

Result multiresolution transformation is computed by two analysis filters $\widetilde{H}^j$ and $\widetilde{G}^j$ for each resolution level j (8, 9).

$$V^j = \widetilde{H}^{j+1} V^{j+1} \tag{8}$$

$$W^j = \widetilde{G}^{j+1} V^{j+1} \tag{9}$$

The reconstruction is done with two synthesis filters H^j and G^j (10). Where W^j is the corresponding wavelet coefficients.

$$V^{j+1} = H^j V^j + G^j W^j \tag{10}$$

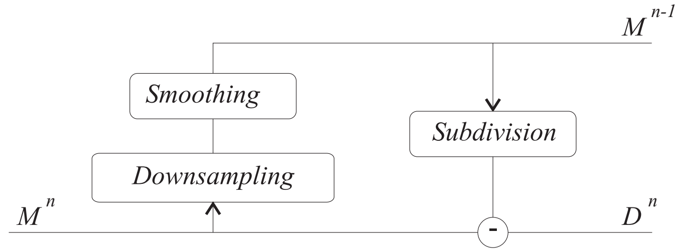


Figure 5. The Burt-Adelson piramid for decomposition of surface mesh

The analysis and synthesis filters make following assumption (11):

$$\begin{bmatrix} \widetilde{H}^j \\ \widetilde{G}^j \end{bmatrix} = \begin{bmatrix} H^j & G^j \end{bmatrix}^{-1} \tag{11}$$

In [33] this method was extended to processing meshes with additional attributes.

This biorthogonal wavelet basis can be build with lifting scheme notation. This *k-disc* wavelet construction begin with lazy wavelets and then is improved with use of lifting scheme [4].

Based on semi-uniform subdivision methods the prediction and update blocks of lifting scheme as a masks of subdivision schemes where also defined. In [34] the Loop scheme was used.

In [35], [36] the second generation wavelets on sphere were constructed as lifting scheme blocks. The Lazy, Linear, Quadratic, and Butterfly bases are lifted to assure that the wavelet has at least one vanishing moment.

The Loop wavelet transform with MAPS (Multiresolution Adaptive Parameterization of Surfaces, [27]) parametrisation was also introduced in [37]. The MAPS algorithm uses hierarchical simplification to efficiently induce a parameterization of the original mesh over a base domain consisting of a small number of triangles (base mesh M^0). In [38] a new biorthogonal wavelet decomposition using the $\sqrt{3}$ subdivision was described.

Another possibility, in construction of surface wavelets, is adaptation of Burt-Adelson pyramid scheme (Figure 5). This is combination of subdivision (upsampling), half-edge collapse topological operator (downsampling) and smoothing steps. Each step of this schema produces detail vector as a difference between meshes in two next resolution levels. In [39] the Loop subdivision and Taubin smoothing operator as elements of pyramid algorithm were used.

Method for irregular meshes

The BLaC-wavelets (Blending of Linear and Constant) [40], [41] define local decomposition which is generalisation of Haar wavelets for non-nested spaces. The connection of hierarchical Delaunay triangulation allows to build multiresolution model of planar or spherical, irregular surface mesh.

The next proposition in literature is wavelet-based multiresolution analysis of irregular meshes using a new irregular subdivision scheme [42], [43]. This is expansion of the solution presented in [32], [26]. The Authors proposed a new irregular subdivision scheme based on *subdivision cookbook* which defined different combinations of split and merge steps. Analysis and synthesis filters are defined in this same way as for semi-regular meshes but the new inner product was specified because the new subdivision scheme splits triangle on different count of new triangles (from 1 to 4) (12).

$$< f,g >= \sum_{\tau\varepsilon\Delta(M^j)} \frac{K^j(\tau)}{pow(\tau)} \int_{x\varepsilon\tau} f(x)g(x)dx \tag{12}$$

Where $K^j(\tau)$ is count of new triangles from τ triangle of M^j mesh.

Another direction in research is the use of non-uniform subdivision schemes to build wavelet analysis. In [24], [25] the central ingredient of multiresolution analysis is a non-uniform relaxation operator which minimises divided differences. This operator together with the mesh simplification method (progressive mesh [10]) and pyramid algorithm (Burt-Adelson pyramid) allows to define signal processing tools for irregular connected triangle meshes (Figure 5).

In [44] the progressive mesh algorithm is used to build the coarser resolution mesh and semi-uniform discrete Laplacian smoothing operator to estimate the higher resolution mesh. This method is not a wavelet transform but allows define multiresolution representation of surface mesh.

Lifting scheme is used for multiresolution analysis in [45] where a prediction block is non-uniform relaxation operator based on surface curvature minimisation (Meyer smoothing operator). This transformation is not a complete lifting scheme because it does not have update block. This framework also allows processing irregular meshes with attributes [46].

In [47], [48] the lifting scheme framework to processing irregular surface mesh was defined. The proposition of split, predict and update block with influence on result multiresolution representation (scaling and wavelet function) was described. In [49] the extension of lifting scheme for the multiresolution decomposition and reconstruction of irregular triangle surface meshes with additional attributes was presented.

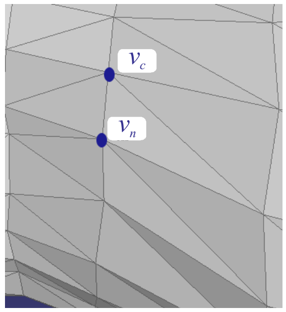
General lifting scheme framework to multiresolution transformation of irregular triangle mesh

This section summarises proposition of general transform based on the lifting scheme for multiresolution analysis of irregular, triangle closed mesh (the 2-manifold). Main operations of lifting scheme (Figure 7 and 6) can be singled out:

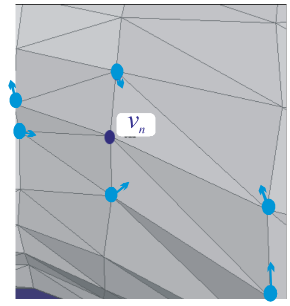
- Select from input mesh M^{j+1} (where j is the resolution level) the even and odd vertices (split block, S, definition in 13). The odd vertex (v_n) will be removed from mesh. The even vertices are 1-ring neighbourhood of odd vertex ($N(v_n)$).

  $$S(M^{j+1}) = (v_n^{j+1}, N(v_n^{j+1})) \tag{13}$$

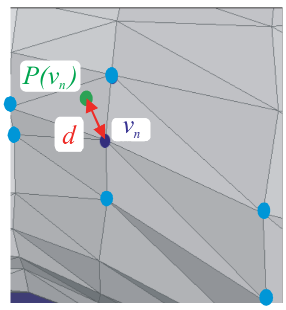
  One step of lifting scheme removes only one vertex (this is "one coefficient at a time" lifting scheme). One of proposition is vertex selection based on the value of criterion which is vertex distance to the regression plane defined by 1-ring neighbourhood of



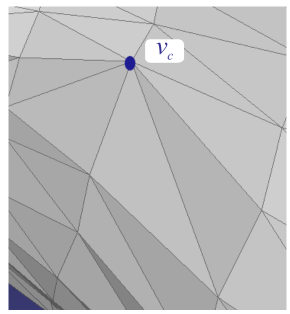
(a) split block - selecting odd vertex v_n and vertex to collapse v_c



(b) update block



(c) predict block and detail computation d



(d) odd vertex remove by half-edge collapse $(v_n, v_c) \rightarrow v_c$

Figure 6. Decomposition lifting scheme steps

the odd vertex ($S_d(v_n)$). This criterion reflected the curvature of surface very well. So the vertices from plain area of mesh are removed first.

The selected vertex is removed by half-edge collapse. Also the edge of vertex to collapse must be chosen. This is very important and has big influence on quality of result meshes. For every edge of selected vertex (v_n) the sum of differences between triangles normal before and after edge collapse is computed (14). Edge with minimal value β_n will be chose to collapse. This criteria minimize the changes in the curvature of surface in lower resolution.

$$\beta_n(v_n) = \min_{v_c \varepsilon N(v_n)} \sum_{m \varepsilon \Delta(v_n)} (1 - \cos(\alpha_{\Delta_m, \Delta'_m})) \tag{14}$$

Where $\alpha_{\Delta_m,\Delta'_m}$ is the angle between normal of triangle Δ_m and triangle Δ'_m after half-edge collapse $(v_n, v_c) \to v_c$.

- Update even vertices.

  Update block is a simple assurance that the values of the input and output vertices in coarser resolution have the same average. If this step is missing, the even indexed vertices proceed unchanged to the next level. This block smoothes mesh in lower resolution, which influences on better approximation by this mesh and in consequence smaller wavelet coefficients. The update value is added to every vertex in neighbourhood of processed vertex in this lifting scheme step (15).

$$\bigwedge_{v_k \varepsilon N(v_n^{j+1})} v_k^j = v_k^{j+1} + U(v_n^{j+1}, N(v_n^{j+1})) \tag{15}$$

  The suggested update block is: equal average value of single coordinates (AVV) before and after odd vertex removal. Odd and even vertices are used in computing so as to avoid allocation memory for additional coefficients in multiresolution model, update block is before prediction block ("update first lifting scheme", [50]).

- Prediction odd vertex based on even ones (predict block, *P*).

  The proposed prediction blocks take advantage of deterministic spatial interpolation methods such as natural neighbours (NNI) and thin plate splines interpolation based on first (TPS1) and second ring neighbourhoods (TPS2). To calculate prediction value the local parameterization of odd vertex and its neighbours should be determined, so the proposed prediction block consists of two parts: parameterization and interpolation. The searched parametric plane is the regression plane (total least squares plane) used also in split block. This is the best-fitting plane to a given set of vertices in 1-ring neighbourhood by minimizing the sum of the squares of the perpendicular distances of the points from the plane.

- Compute detail vector (wavelet coefficient, d) as difference between predicted and original vertex (16). Detail is stored in multiresolution model.

$$d^j = v_n^{j+1} - P(N(v_n^{j+1})) \tag{16}$$

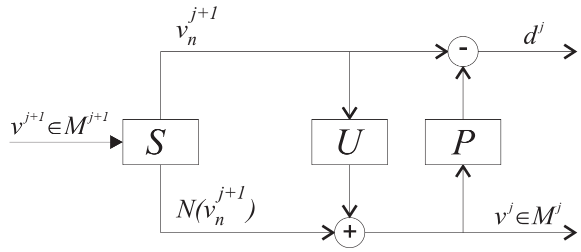


Figure 7. The lifting scheme for decomposition of surface mesh.

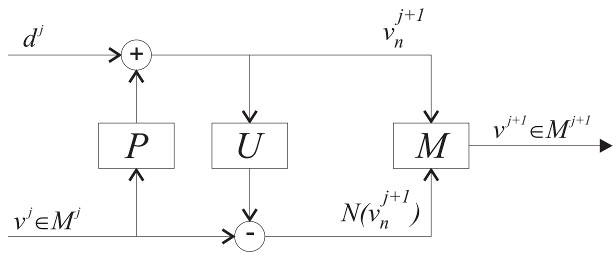


Figure 8. The reverse lifting scheme for reconstruction of surface mesh.

The detail vector is difference between odd vertex and its prediction. This encode the data lost by the next approximations and can be seen as local frequency spectrum of the mesh (output of high pass filter).

- Remove the odd vertex, selected in Split block, from the mesh by an half-edge collapse. The edge of odd vertex to collapse was also chosen in Split block. The output is the coarser mesh M^j that is the approximation of initial mesh.

The inverse lifting scheme consists of following operations (Figure 8):

- Prediction odd vertex based on even vertices from mesh M^j.
- Compute original value of odd vertex as sum of detail vector remembered in multiresolution model and prediction vertex.
- Update even vertices.
- Insertion computed vertex to mesh (merge block, *M*). Reconstruction of topological connections between odd vertex and its neighbors (even vertices). The output is the mesh M^{j+1}.

If functions in split, predict and update blocks are defined with respect to additional attributes in vertices this general scheme will be able to process meshes with additional attributes.

Wavelet and scaling function

The lifting scheme is a tool for constructing second generation wavelets, which are no longer dilates and translates of one single function. For irregular settings scaling and wavelets functions depend on local surroundings. For proposed scheme these functions depend on valence of odd vertex. Example scaling function for scheme with this same prediction block but for vertices with different valence are on Figure 9 and Figure 10. The basic idea behind lifting is to start with simple multiresolution analysis and modify it by prediction and update block. This starting analysis is called lazy wavelet, which split the input date into even and odd samples. In filter bank algorithm starting multiresolution analysis is implemented by analysis ($\widetilde{H}^{old}$ and $\widetilde{G}^{old}$) and synthesis filters (H^{old} and G^{old}). To find equations of scaling and wavelet functions, first the modification of input filters by update and prediction blocks of one step of reverse lifting scheme should be defined as in (18 and 18).

$$G^j = G^{j,old} - H^{j,old}U^j \tag{17}$$

$$H^j = G^{j,old}P^j + (1 - U^jP^j)H^{j,old} \tag{18}$$

Modification of scaling function of even vertices are defined in (20 and 20).

$$\Phi^j = \Phi^{j+1}H^j \tag{19}$$

$$\Phi^j_{N(v_n)} = \Phi^j_{N(v_n)} + (\phi^{j+1}_{v_n} - \Phi^j_{N(v_n)}U^j)P^j \tag{20}$$

After removal of odd vertex (v_n) from mesh the area taken by its scaling function $\phi^{j+1}_{v_n}$ is occupied by scaling functions of neighbours vertices $\Phi^j_{N(v_n)}$. Because proposed lifting scheme is update first lifting scheme so on scaling functions influence update and prediction blocks. This can be seen on Figure 9 and Figure 10 for scheme with prediction block NNI and on Figure 11 with $TPS1$ and $TPS2$. In classic scheme the scaling functions are modified only by prediction block.

Similarly the result wavelet functions can be analysed (22 and 22):

$$\Psi^j = \Phi^{j+1}G^j \tag{21}$$

$$\psi^j_{v_n} = \phi^{j+1}_{v_n} - \Phi^j_{N(v_n)}U^j \tag{22}$$

The update block modifies wavelet function to add vanishing moments. If scheme doesn't have update block wavelet function is this same as scaling function of next level (Figure 12).

To find shape of result scaling function the reverse lifting scheme can be used. This scaling function is defined as follows: set values of input data to Kronecker delta and wavelet coefficients equal to 0. After running reverse lifting scheme starting from level j ad infinitum we obtain scaling function $\phi^j_{v_n}$. For wavelet function the detail values are equal Kronecker delta and data set to 0.

Implementation

The input mesh is processing in main loop until archive requested level of resolution or no more verities can be removed from mesh. The main loop of lifting scheme is written in pseudocode on listing 1. First the split criteria for every vertex is computed. Next is the while loop with calling update, predict function, detail computing, removing one vertex and split criteria updating. The output is the continuous multiresolution model with base mesh and set of detail vectors. Such model supports LOD (for mesh in defined level of resolution) and ROI (for region of mesh in defined level of resolution) questions.

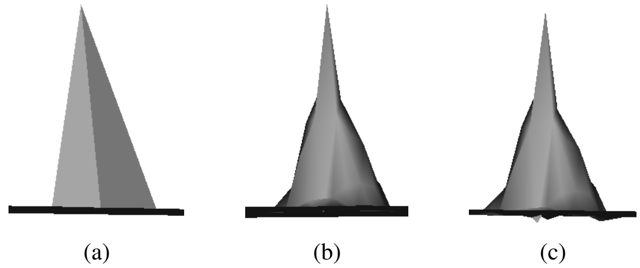


Figure 9. The scaling function of vertex with valence equals 6 after 268 steps of reverse scheme with prediction block *NNI*: (a) input scaling function, (b) result scaling function for scheme without update block, (c) result scaling function for scheme with update block.

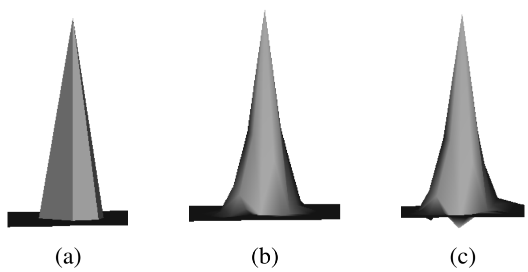


Figure 10. The scaling function of vertex with valence equals 4 after 268 steps of reverse scheme with prediction block *NNI*: (a) input scaling function, (b) result scaling function for scheme without update block, (c) result scaling function for scheme with update block.

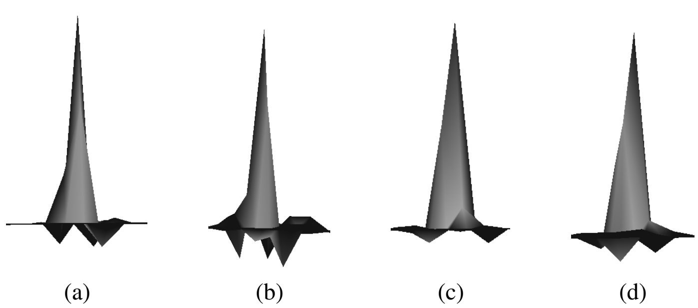
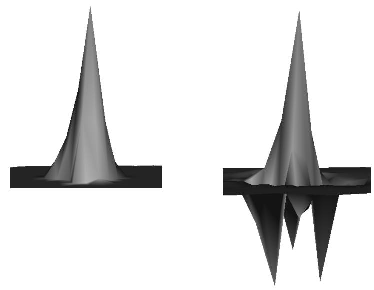


Figure 11. The scaling function of vertex with valence equals 4 after 268 steps of reverse scheme: (a) result scaling function for scheme with prediction block $TPS1$ without update block, (b) result scaling function for scheme with prediction block $TPS1$ with update block, (c) result scaling function for scheme with prediction block $TPS2$ without update block, (d) result scaling function for scheme with prediction block $TPS2$ with update block.



(a) (b)

Figure 12. The wavelet function for scheme with prediction block *NNI*: (a) result wavelet function for scheme without update block, (b) result wavelet function for scheme with update block.

Listing 1 Main loop of lifting scheme DOLIFTINGSCHEME

Input: M – triangle mesh
$MultiM$ – multiresolution model
$vCnt$ – count of vertices to remove
Output: mesh M after removing $vCnt$ vertices
and multiresolution model $MultiM$

function DOLIFTINGSCHEME(M, $vCnt$, $MultiM$):*void*
    for $v_i \in M$ **do** ▷ for every vertex from mesh M
        DOSPLIT(v_i) ▷ set the criteria value
    end for
    while ($vCnt > 0$) and (is possible to remove another vertex from M) **do**
        $v_n \leftarrow$ vertex from M with smallest value of split criteria
        DOUPDATE(v_n, $false$)
        $v_p \leftarrow$DOPREDICT(v_n)
        $v_n \leftarrow v_n - v_p$
        $MultiM \leftarrow MultiM \oplus v_n$ ▷ save detail vector in multiresolution model
        $M \leftarrow M \ominus v_n$ ▷ remove vertex v_n by half-edge collapse
        $v2Near \leftarrow$ second ring neighbourhoods of v_n
        for $v_i \in v2Near$ **do**
            DOSPLIT(v_i) ▷ update the split criteria value
        end for
        $vCnt \leftarrow vCnt - 1$
    end while
end function

The reverse lifting scheme reconstructs the mesh to defined level of resolution from multiresolution model. Loop is written on listing 2 and consists of calling predict, undo update function and inserting computing vertex to mesh by reconstruction the topological connections.

Listing 2 Loop of reverse lifting scheme DOREVERSELIFTINGSCHEME

Input: *MultiM* – multiresolution model
vCnt – count of vertices to process
Output: mesh *MultiM* after inserting *vCnt* verities (mesh in higher resolution)

function DOREVERSELIFTINGSCHEME(*vCnt*, *MultiM*):*void*
  while ($vCnt > 0$) **do**
    $d \leftarrow$ detail vector from $MultiM$
    $v_p \leftarrow$DOPREDICT(d)
    $d \leftarrow d + v_p$
    DOUPDATE(d, $true$)
    $MultiM \leftarrow MultiM \oplus d$ ▷ insert vertex to mesh
    $vCnt \leftarrow vCnt - 1$
  end while
end function

Application of multiresolution representation

In this section the most common applications of multiresolution representation are presented. That algorithms need multiresolution model which store base mesh and detail informations - wavelet coefficients.

Mesh compression

Generally meshes obtained by 3D scanners are very complex and big. They usually demand a huge amount of storage space and/or transmission bandwidth. So the most popular kind of application based of wavelets multiresolution representations is compression. In these methods wavelets coefficients of the analysed mesh are encoded using zerotree and entropy coding. More informations about multiresolution meshs compression can be found in [51], [52], [53], [37], [54], [55], [56].

Figure 13 is the example of mesh after the hard threshold, where 50% of smallest wavelets coefficients of lifting scheme multiresolution representation with *NNI* prediction block, were set to 0.

Feature extraction

Important features (like sharp edges) can be efficiently found by using detail coefficients through different scales or verities from base mash. This can be base of mesh segmentations

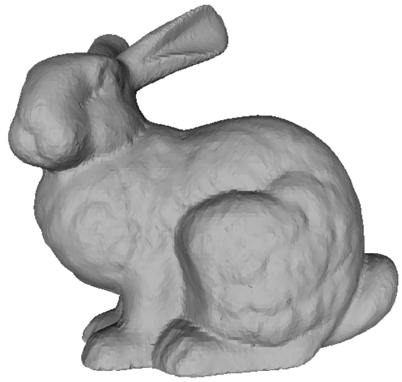


Figure 13. Mesh of Stanford Bunny after the hard threshold of 50% smallest wavelets coefficients.

algorithms [57] or base of classification algorithms to compare and recognise 3D objects [58], [59].

Filtering

Most of traditional multiresolution signal techniques can be defined for wavelet multresolution representation of surface mesh.

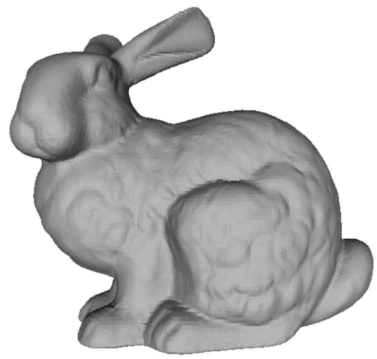
Example filter is a denosing algorithm using soft-thresholding of wavelet coefficients. First, the initial noisy mesh is decomposed using wavelet analysis. In this case the large coefficients are mainly the shape of surface and the smaller ones represent the noise. By the shrink the detail coefficients and next reconstruction the mesh, the noise can be removed [60].

Another examples are smoothing and enhancement filters. Here also the multiresolution representation is a base of these algorithms. The wavelet coefficients are the detail representations (high frequencies) and next meshes are the approximations of original surface (low frequencies). The smoothing filter sets to zero wavelets coefficients from determined levels of resolutions. Enhancement filter highlights some characteristics features in opposite to smoothing filter. This is available by scaling detail coefficients by multiply then by factor higher than 1. On the Figure 14 we can see the results of applying the smoothing and enhancement filter to multiresolution representation. This representation is the result of lifting scheme algorithm with *NNI* prediction block. This same filters can be applying to multiresolution representation of mesh with additional attributes in verities.

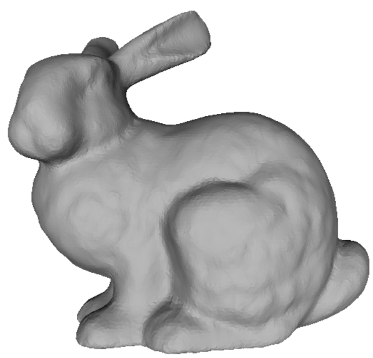
Level of detail visualization

When the visualizing complex object is far away from the object or is moving very fast, it is unnecessary and inefficient to draw a highly detailed representation [1], [61]. We can use level of detail (LOD) adaptive visualization where the information about the view determine the complexity of rendering model (Figure 15).

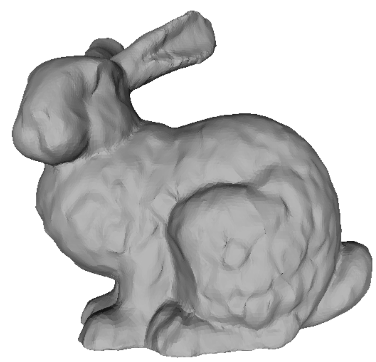
In LOD visualisation all object is in lower resolution. The second possibility is using the region of interest (ROI) visualisation where in higher resolution is only the part of object



(a) original mesh



(b) smoothing mesh



(c) enhancement mesh

Figure 14. Applying of smoothing and enhancement filters to mesh of the Stanford Bunny

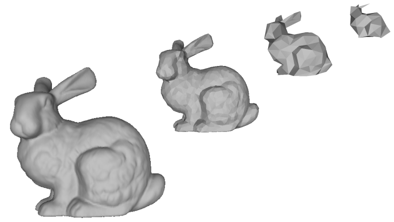
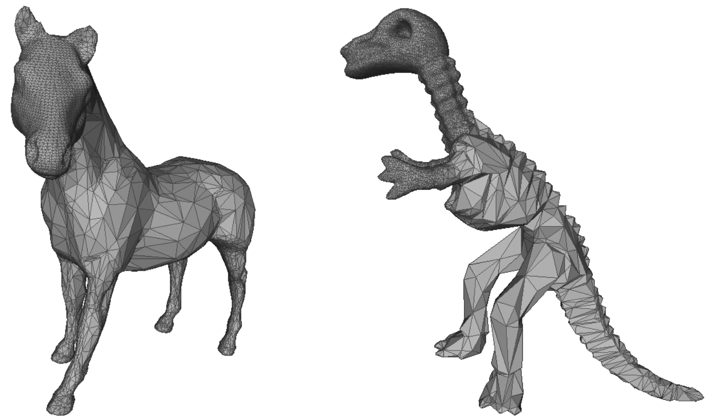


Figure 15. Level of detail (LOD) visualisation dependent of viewer distance



(a) mesh of the Horse model (b) mesh of the Dinozaur model

Figure 16. Region of interest visualisation (ROI)

- pick by user or actually in the viewport. Rest of mesh can be in lower resolution. Example is on Figure 16.

For this kind of visualization the results of wavelet decomposition must be stored in the continuous multiresolution model. This is a hierarchical structure with base mesh and details from all levels of resolution.

Multiresolution editing

The wavelet multiresolution representation can be used also to edit shapes at variety of resolutions. The idea is to edit the mesh at the lower resolution and use the reconstruction algorithm to obtain the high resolution mesh based on its edited version. The main advantage of these metod is the ability to edit very large meshes [39], [44].

Progressive transmission

The complex geometric models are becoming very common in the Internet. The distribution of these geometric object motivates the need for efficient transmissions methods of models across relatively low-bandwidth networks.

The progressive transmission allows gradually send complex model so the user does not have to wait until the entire model is received. First, the base mesh in lowers resolution is send. It is received and displayed very quickly. Next, the wavelets coefficients are transmitted in order of decreasing magnitude [51], [37], [54].

Watermarking

The digital watermarking allows copyright protection of various multimedia contents. This technique hides some secret information in the cover content [62]. Example algorithms for meshes can use wavelets coefficient vectors as authentication primitives [63], [64], [65].

Conclusions

The multiresolution methods for 3D surfaces meshes presented in this chapter do not always respect the strict definitions of the wavelet transform. In lots of examples there is lack of mathematical framework to define and proof of wavelets properties such as coverage, smoothness and stability. There are no mathematical utility to uniformly writing all this methods.

By the examples of using the result multiresolution representation we can see the ability of the multriresolution transformation to efficiently represent and process surface meshes data. This is base of efficients digital geometry processing algorithms and can by used in many 3D graphics applications such as compression, progressive transmission, filtering, feature extractions, LOD visualisations and watermarking.

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