Group Meeting
Week 2, Spring 2019

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Finite Volume Approach

Here we consider the following one-dimensional refence PDE

$$w_t + f(w)_x = s(w)$$

where $w = (\rho, \rho u, E)$ represents the primitive solution variables, and initial and boundary conditions are supplied. The PDE in semi-discrete form (via finite volume w/ midpoint quadrature) is

$$(w_j)_t = -\frac{1}{\triangle x}\left(f_{j+1/2} - f_{j-1/2}\right) + s_j$$

where the j denotes spatial index. The solution is represented as cell averages

$$w_j^n \approx \frac{1}{\triangle x}\int_{x_{j-1/2}}^{x_{j+1/2}} w(x, t_n)dx$$

Finite Volume Approach

The numerical flux at cell interface is a function of $2k$ local cells

$$\hat{f}_{j+1/2} = f(w^n_{j-k+1}, \ldots, w^n_{j+k})$$

Computing the fluxes is usually the bulk of the computational effort (not considering overhead), requiring reconstructed states via ENO/WENO or TVD limiters, and Riemann solver.

Multiresolution Representation

Define multiple, nested grids

$$G^l = \left\{ x_j^l \right\}_{j=0}^{N_l}$$

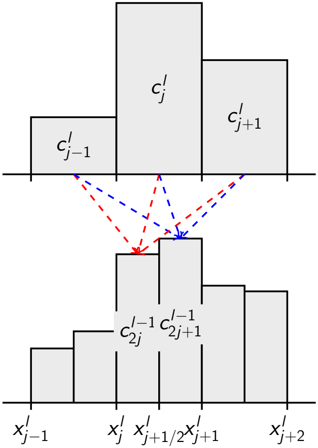
where the number of cells per level l is N_l and the cell width is

$$h_l = \frac{b-a}{N_l}$$

The aim is to decompose the field at the finest (given) level of resolution into a representation at a coarse level plus a series of differences at each finer level. The differences provide information about the function's regularity.

Multiresolution Representation

Consider some quantity of interest, $c(x)$. This will be represented as cell averages c_j^l at each level l. At some level l (coarse) the field at level $l-1$ (fine) is represented by a prediction from values at level l. A Lagrange interpolating polynomial is used to preserve the cell averages.



Multiresolution Representation

For a third order polynomial, the prediction is

$$\hat{c}_{2j}^{l-1} = c_j^l + \frac{1}{8}\left(c_{j-1}^l - c_{j+1}^l\right)$$
$$\hat{c}_{2j+1}^{l-1} = c_j^l - \frac{1}{8}\left(c_{j-1}^l - c_{j+1}^l\right)$$

and the detail coefficient for the cell corresponding to c_j^l is

$$d_j^l = c_{2j+1}^{l-1} - \hat{c}_{2j+1}^{l-1}$$

The value of this coefficient is indicitive of the smoothness of the function in that vicinity. Doing this at each level of resolution provides the desired information.

Multiresolution Code - Examples

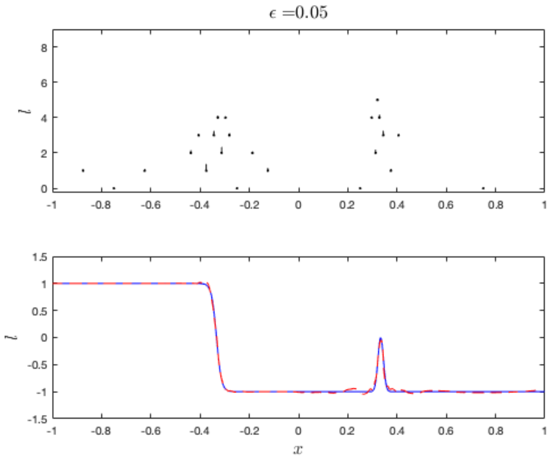


Figure : Coarse approximation (large threshold value).

Multiresolution Code - Examples

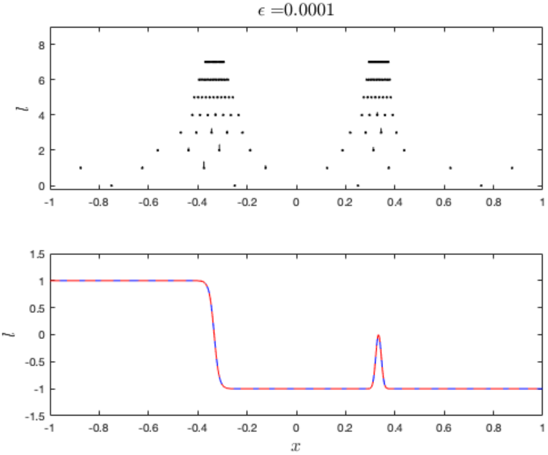


Figure : Better approximation (smaller threshold value).

Multiresolution Code - Examples

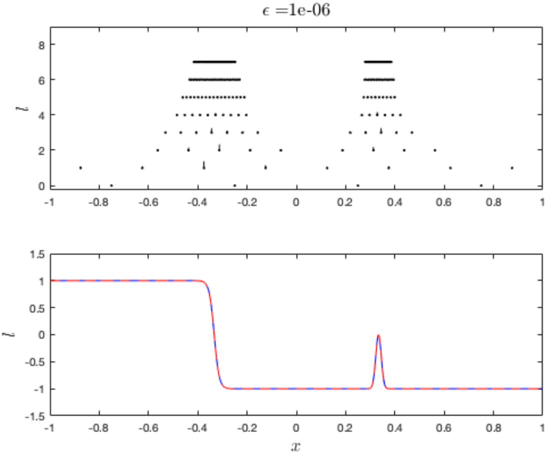
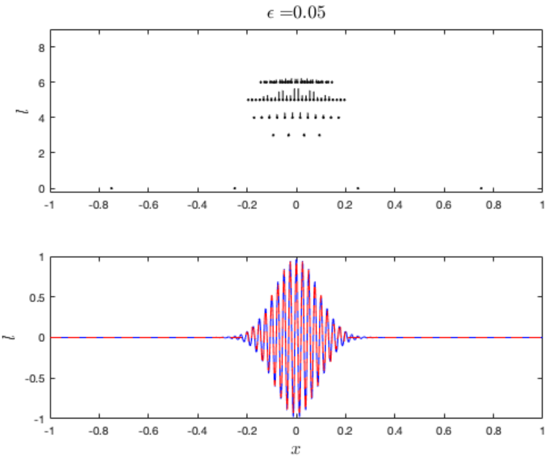


Figure : Even better approximation (even smaller threshold value).

Multiresolution Code - Examples



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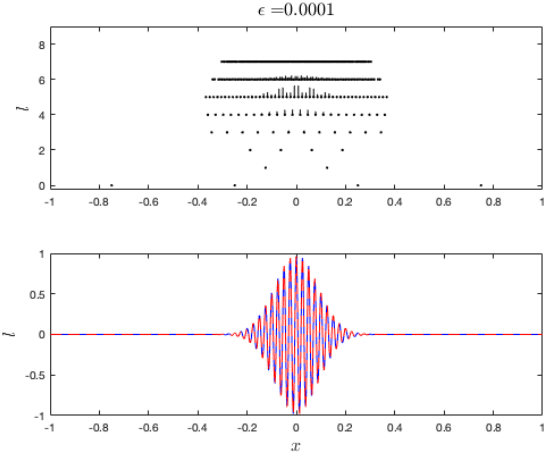

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Multiresolution Code - Examples

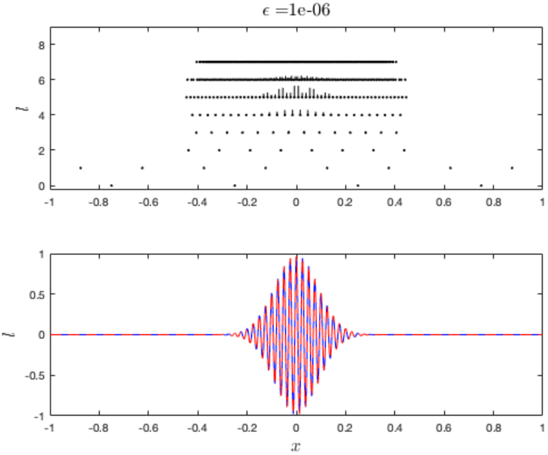

Figure : Even better approximation (even smaller threshold value).

Applications of Multiresolution to Finite Volume

What could we do with such information?

1. use regularity information to adapt the grid (just like AMR)
2. use regularity information to reduce number of costly flux evaluations

The first option is well established in the literature, and so is the second. But all production codes use AMR already... can we boost AMR with option 2?