Quiet Standing Control Parameter Identification With Direct Collocation

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Introduction

It is hypothesized that a human operating during the quiet standing task uses feedback to remain upright in the face of perturbations. For various reasons, it is desirable to obtain mathematical models that predict a human's actuation patterns given measured estimates of the sensory information available to the human. Reasonably good models of the human's open loop musculoskeletal system exist but models of the human's control system and the system process noises are less than adequate. The control model can possibly be derived from first principles, but high level understanding of the human's sensory neurological feedback patterns are difficult to derive from the low level neurological first principles. These high level control descriptions may be more easily arrived at through optimal identification.

Here we present a numerical study demonstrating the merits of using a direct collocation formulation to identify the parameters of non-linear closed system, in this case a human quiet standing state feedback system. Parameter identification via direct collocation is exquisitely described in a paper by Betts and Huffman [1] but these methods have not, to the author's knowledge, been used for control parameter identification in biological systems. We aim to show that direct collocation is better suited to control parameter identification because it does not suffer from bias, computation times are extremely lower than competing optimization methods, and it is much less sensitive to initial guesses than shooting.

Methods

We make use of a common model of closed loop quiet standing taken from [2]. The plant is a two dimensional two body joint torque driven inverted pendulum with inertial characteristics of a human, see Fig. 1. The human's foot is attached rigidly to a moving base which can be accelerated, thus the ankle joint has a prescribed lateral motion. The open loop equations of motion of the system take this form

$$0 = \mathbf{M}(\mathbf{x}, t)\dot{\mathbf{x}} - \mathbf{F}(\mathbf{x}, \mathbf{u}, a, \mathbf{p}, t) \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{x}$ is the state, $\mathbf{F}$ is the forcing vector, $\mathbf{u}$ are the joint torques, a is the platform acceleration, $\mathbf{p}$ are the constant body segment parameters, and t is time.

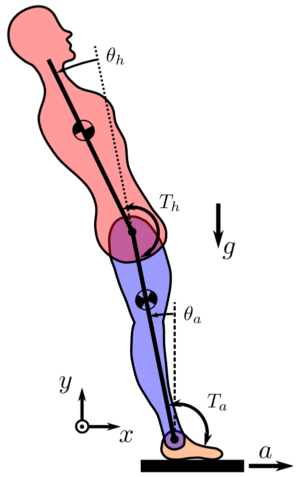

Figure 1: Free body diagram of plant model used in this study.

We then assume a controller that implements full state feedback based on a reference of 0. This controller operates to drive the state to zero. We model the human's inability to perfectly sense this error by injecting Gaussian noise onto the input to the controller which also can account for other system process noises. The controller take th form

$$\mathbf{u} = \mathbf{K}(\mathbf{x}_n - \mathbf{x}) \tag{2}$$

where $\mathbf{K}$ is the time invariant gain matrix and $\mathbf{x}_n$ is the reference noise.

We simulate the closed loop model with optimal gains take from [2] to generate measurement data. The simulation requires the exogenous input, a, to be specified for the time duration. We specify a to be a sum of sines to perturb the system. The measured data of interest includes the system state, the and the platform acceleration, all of which have measurement noise v applied before identification.

$$\mathbf{x}_m = [\theta, \omega]^T + [\mathbf{v}_\theta, \mathbf{v}_\omega]^T \tag{3}$$

$$a_m = a + v_a \tag{4}$$

We then construct an identical model, except without the process noise, for parameter identification purposes. We formulate the optimal control problem by direct collocation to minimize this model's trajectory with respect to the "measured" data from above. The cost function is simply

$$J(\mathbf{x}, \mathbf{K}) = \int_{t_0}^{t_f} (\mathbf{x}_m(t) - \mathbf{x}(t))^2 dt \tag{5}$$

Table 1: The percent relative error of the identified gains with respect to the known gains.

	θ_a	θ_h	ω_a	ω_h
T_a	1.822	-3.412	-1.097	-0.887
T_h	6.346	-0.617	-0.251	0.346

and subject to the constraints

$$\mathbf{c}(\mathbf{x}, \mathbf{K}) = \mathbf{M}\dot{\mathbf{x}} - \tilde{\mathbf{F}}(\mathbf{x}, a_m, \mathbf{p}, \mathbf{K}, t) \tag{6}$$

where $\tilde{\mathbf{F}}$ is the closed loop forcing vector.

We make use of first order midpoint integration to discretize the system at N nodes from t_0 to t_f. This constrained optimization problem has $4N+8$ unknowns and is subject to $4(N-1)$ constraint equations. Our initial guess for the state trajectories and the controller gains is zero and we use a large scale interior point optimization routine, IPOPT, to minimize the cost function.

All of the above is implemented with several open source software packages tied together with the Python programming language. The equations of motion are derived with SymPy and the model is simulated at 10x real time speeds with PyDy to generate the measurement data. The constraint equations and their sparse Jacobians are formulated symbolically with opty and efficient wrapped C code is generated to evaluate them numerically. Finally, IPOPT is used to solve the optimization problem.

Results

For an example result we simulate the closed loop model for a duration of 20 seconds under the influence of a platform acceleration with a standard deviation of 3.6 $\mathrm{m\,s^{-2}}$ and assume that the reference noise, x_n is zero. Gaussian normal measurement noise is then added to the state trajectories: $\sigma_\theta = 0.005$ rad and $\sigma_\omega = 0.07$ $\mathrm{rad\,s^{-1}}$ and acceleration $\sigma_a = 0.4$ $\mathrm{m\,s^{-2}}$ to produce the simulated measurements used for identification.

We then set up a direct collocation problem discretized with $N = 6001$ nodes, creating a problem with 24012 free variables. Given an initial guess of all zeros for the states and the unknown parameters, the solution is found after 27 iterations which only takes approximately 7 seconds on a four core Intel Core i5 processor.

Figure 2 shows a comparison of the measured state trajectories and the optimal trajectories and Table 1 provides the percent relative error in the identified gains with respect to the known gains.

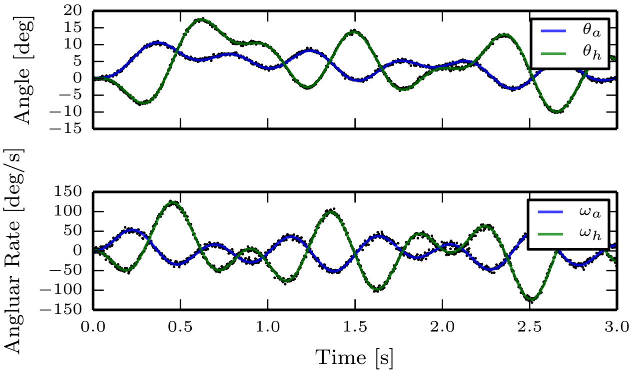

Figure 2: A comparison of the measured states, black dots, and the identified states, solid lines.

Discussion

The indirect control parameter identification via direct collocation has several advantages to more common nonlinear parameter identification methods. Firstly, it handles unstable systems with ease whereas single and multiple shooting often fail miserably with an unstable simulation. This makes the algorithm much less sensitive to poor initial guesses. Some evolutionary algorithms are able to home in on the optimal solution from a random initial guess but take many iterations and hours of parallel computation time. And lastly, computation speeds are a tiny fraction of various shooting algorithms. This method does not require the equations of motion to be in explicit first order form which saves computation time and the stiffness of the system does slow down the integration routine. Our formulation correctly identifies the parameters in approximately the time a single shooting iteration may take. The main limitations here are the requirement that the constraints are continuously differentiable and there could be memory issues if the number of nodes is extremely high, i.e. a long measurement duration.

The demonstration at the conference will include comparisons to other popular optimization algorithms, more details of how to deal with process noise, and several other parameter and trajectory optimization problems that can be handled by the open source software suite we have developed.

References

[1] J. Betts and W. Huffman. Large Scale Parameter Estimation Using Sparse Nonlinear Programming Methods. *SIAM Journal on Optimization*, 14(1):223–244, January 2003.

[2] Sukyung Park, Fay B. Horak, and Arthur D. Kuo. Postural feedback responses scale with biomechanical constraints in human standing. *Experimental Brain Research*, 154(4):417–427, February 2004.