Human Standing Control Parameter Identification with Direct Collocation

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Parameter Identification

Find the parameters $\mathbf{p}$ such that the difference between the model simulation, $\mathbf{y}$, and measurements, $\mathbf{y}_m$ is minimized.

Dynamic system

- Equations of Motion: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{p})$
- Measurement variables: $\mathbf{y} = \mathbf{g}(\mathbf{x}, \mathbf{p})$

Objective

$$\min_{\mathbf{p}} J(\mathbf{p})$$

where

$$J(\mathbf{p}) = \int [\mathbf{y}_m - \mathbf{y}(\mathbf{p})]^2 dt$$

Parameter Identification By Shooting

- Repeated simulations are computationally costly
- Systems may be unstable and thus have an ill-defined objective
- Local minima are inevitable
    - May requires a superb guess
    - May need time intensive global optimization methods

Local Minima Example: Simple pendulum

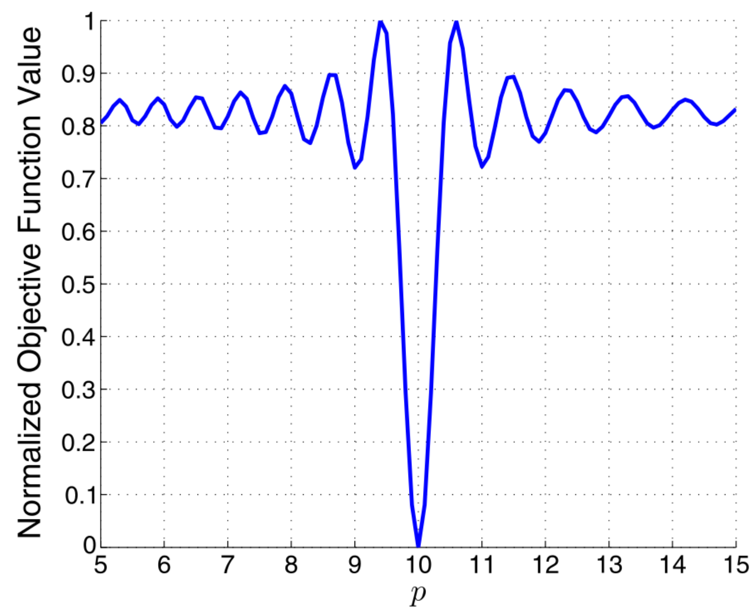
Vyasarayani, Chandrika P., Thomas Uchida, Ashwin Carvalho, and John McPhee. "Parameter Identification in Dynamic Systems Using the Homotopy Optimization Approach". Multibody System Dynamics 26, no. 4 (2011): 411-24.

1-DoF, 1 parameter pendulum equations of motion

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{\theta}(t) \\ \dot{\omega}(t) \end{bmatrix} = \begin{bmatrix} \omega(t) \\ -p \sin\theta(t) \end{bmatrix}$$

Objective: Minimize least squares

$$J(p) = \min_{p} \int_{t_0}^{t_f} [\theta_m(t) - \theta(\mathbf{x}, p, t)]^2 \, dt$$



Direct Collocation

Betts, J., and W. Huffman. "Large Scale Parameter Estimation Using Sparse Nonlinear Programming Methods." SIAM Journal on Optimization 14, no. 1 (January 1, 2003): 223–44. doi:10.1137/S1052623401399216.

Benefits

- Fast computation times
- Handles unstable systems with ease
- Less susceptible to local minima

Disadvantages

- Accurate solution requires large number of nodes
- Memory management for large sparse matrices and operations
- Tedious and error prone to form gradients, Jacobians, and Hessians

Our Direct Collocation Implementation

Implicit Continous Equations of Motion

No need to solve for $\dot{\mathbf{x}}$.

$$\mathbf{f}(\dot{\mathbf{x}}, \mathbf{x}, \mathbf{r}, \mathbf{p}, t) = 0$$

- $\mathbf{x}$, $\dot{\mathbf{x}}$: states and their derivatives
- $\mathbf{r}$: exogenous inputs
- $\mathbf{p}$: constant parameters

Discretization

First order discrete integration options:

Backward Euler:

$$\mathbf{f}(\tfrac{\mathbf{x}_i-\mathbf{x}_{i-1}}{h}, \mathbf{x}_i, \mathbf{r}_i, \mathbf{p}, t_i) = 0$$

Midpoint Rule:

$$\mathbf{f}(\tfrac{\mathbf{x}_{i+1}-\mathbf{x}_i}{h}, \tfrac{\mathbf{x}_i+\mathbf{x}_{i+1}}{2}, \tfrac{\mathbf{r}_i+\mathbf{r}_{i+1}}{2}, \mathbf{p}, t_i) = 0$$

Nonlinear Programming Formulation

$$\min_{\theta} J(\theta)$$
$$\text{where } \theta = [\mathbf{x}_i, \dots, \mathbf{x}_N, \mathbf{r}_i, \dots, \mathbf{r}_N, \mathbf{p}]$$
$$\text{subject to } \mathbf{f}(\mathbf{x}_i, \mathbf{x}_{i+1}, \mathbf{r}_i, \mathbf{r}_{i+1}, \mathbf{p}, t_i) = 0 \text{ and } \theta_L \leq \theta \leq \theta_U$$

Software Tool: opty

- User specifies continous symbolic:
  - objective
  - equations of motion (explicit or implicit)
  - additional constraints
  - bounds on free variables: $\mathbf{x}$, $\mathbf{r}$, $\mathbf{p}$
- EoMs can be generated with PyDy (http://pydy.org (http://pydy.org))
- Effficient just-in-time compiled C code is generated for functions that evaluate:
  - objective and its gradient
  - constraints and its Jacobian
- NLP problem automatically formed for IPOPT
- Open source: BSD license
- Written in Python
- http://github.com/csu-hmc/opty (http://github.com/csu-hmc/opty)

Example Code: 1 DoF, 1 Parameter Pendulum

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{\theta}(t) \\ \dot{\omega}(t) \end{bmatrix} = \begin{bmatrix} \omega(t) \\ -p\sin\theta(t) \end{bmatrix}$$

Paraphrased from https://github.com/csu-hmc/opty/blob/master/examples/vyasarayani2011.py (https://github.com/csu-hmc/opty/blob/master/examples/vyasarayani2011.py)

Symbolics

```
# Specify symbols for the parameters
p, t = symbols('p, t')

# Specify the functions of time
theta, omega, theta_m = symbols('theta, omega, theta_m', cls=Function)

# Specify the symbolic equations of motion
eom = (Matrix([theta(t), omega(t)]).diff(t) -
       Matrix([omega(t), -p * sin(theta(t))]))

# Specify the symbolic objective function
obj = Integral((theta_m(t) - theta(t))**2, t)
```

Example Code: 1 DoF, 1 Parameter Pendulum

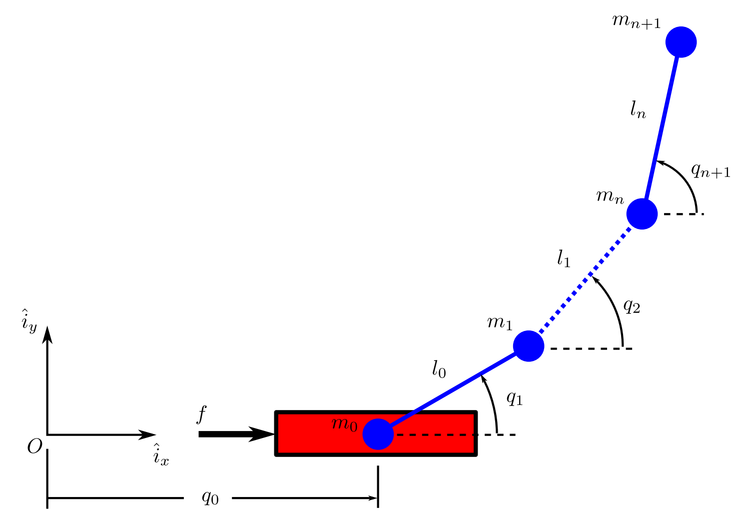
Numerics

```
# Choose discritzation values
num_nodes = 1000
interval = 0.01  # seconds

# Form the problem
prob = Problem(obj, eom, (theta(t), omega(t)), num_nodes, interval,
               known_trajectory_map={y1_m(t): measured_data},
               integration_method='midpoint')

# Set an initial guess
initial_guess = random(prob.num_free)

# Solve the system
solution, info = prob.solve(initial_guess)
```

Computational Speed

Example Larger System

10 link pendulum on sliding cart (not stiff)
11 DoF, 22 states, 22 parameters
12800 mathematical operations in constraint expressions
100 s sampled @ 100 hz



Computational Speed

Discretization Variables

- 10,000 collocation nodes
- 219,978 constraints
- 14,518,548 nonzero entries in the Jacobian
- 220,022 free variables

Timings

- Integrating with ODEPACK lsoda: **5.6 s**
- Constraint evaluation: **33 ms (0.033 s)**
- Jacobian evaluation: **128 ms (0.128 s)**

Case Study: Human Control Parameter Identification

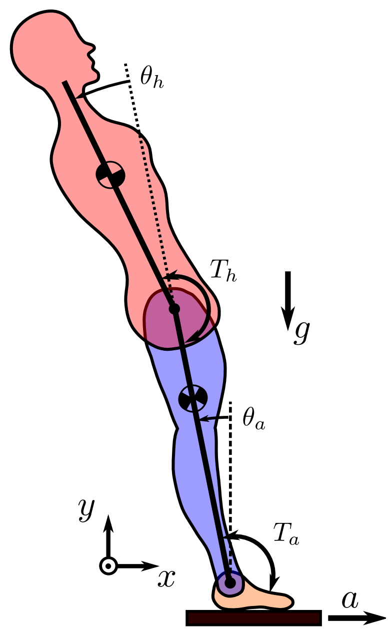
Plant

Torque driven two-link inverted pendulum with an accelerating base.
States: $\mathbf{x} = [\theta_a \quad \theta_h \quad \omega_a \quad \omega_h]^T$

Exogoneous inputs:

- Controlled: $\mathbf{r}_c = [T_a \quad T_h]^T$
- Specified: $\mathbf{r}_k = [a]$

Known parameters: $\mathbf{p}_k$



Open Loop Equations of Motion

$$\dot{\mathbf{x}} = \mathbf{f}_o(\mathbf{x}, \mathbf{r}_c, \mathbf{r}_k, \mathbf{p}_k, t)$$

Lumped Passive+Active Controller

- True human controller is practically impossible to isolate and identify
- Identify a controller for a similar system that causes the same behavior as the real system

Simple State Feedback

$$\mathbf{r}_c(t) = -\mathbf{K}\mathbf{x}(t)$$

Unknown Parameters

$$\mathbf{p}_u = \text{vec}(\mathbf{K})$$

Closed Loop Equations of Motion

$$\dot{\mathbf{x}} = \mathbf{f}_c(\mathbf{x}, \mathbf{r}_k, \mathbf{p}_k, \mathbf{p}_u, t)$$

Generate Data

Specify the psuedo-random platform acceleration

$$a(t) = \sum_{i=1}^{12} A_i \sin(\omega_i t) \quad \text{where,} \quad 0.15\text{rad/s} < \omega_i < 15.0\text{rad/s}$$

Choose a stable controller

$$\mathbf{K} = \begin{bmatrix} 950 & 175 & 185 & 50 \\ 45 & 290 & 60 & 26 \end{bmatrix}$$

Generate Data

Simulate closed loop system under the influence of perturbations for 60 seconds, sampled at 100 Hz

$$\dot{\mathbf{x}} = \mathbf{f}_c(\mathbf{x}, \mathbf{r}_k, \mathbf{p}_k, \mathbf{p}_u, t)$$

Add Gaussian measurement noise

$$\mathbf{x}_m(t) = \mathbf{x}(t) + \mathbf{v}_x(t)$$
$$a_m(t) = a(t) + v_a(t)$$

- σ_θ = 0.3 deg
- σ_ω = 4 deg/s
- σ_a = 0.42 ms^{-2}

Parameter Identification Problem Specification

Given noisy measurements of the states, $\mathbf{x}_m$, and the platform acceleration, a_m, can we identify the controller parameters $\mathbf{K}$?

$$\min_{\theta} J(\theta), \quad J(\theta) = \sum_{i=1}^{N} h[\mathbf{x}_{mi} - \mathbf{x}_i]^2$$

where

$$\theta = [\mathbf{x}_1, \ldots, \mathbf{x}_N, \mathbf{p}_u]$$

Subject to the constraints:

$$\mathbf{f}_c(\mathbf{x}_i, \mathbf{x}_{i+1}, a_{mi}, a_{mi+1}, \mathbf{p}_u) = 0, \quad i = 1 \ldots N$$

And the initial guess:

$$\theta_0 = [\mathbf{x}_{m1}, \ldots, \mathbf{x}_{mN}, \mathbf{0}]$$

For, N = 6000:

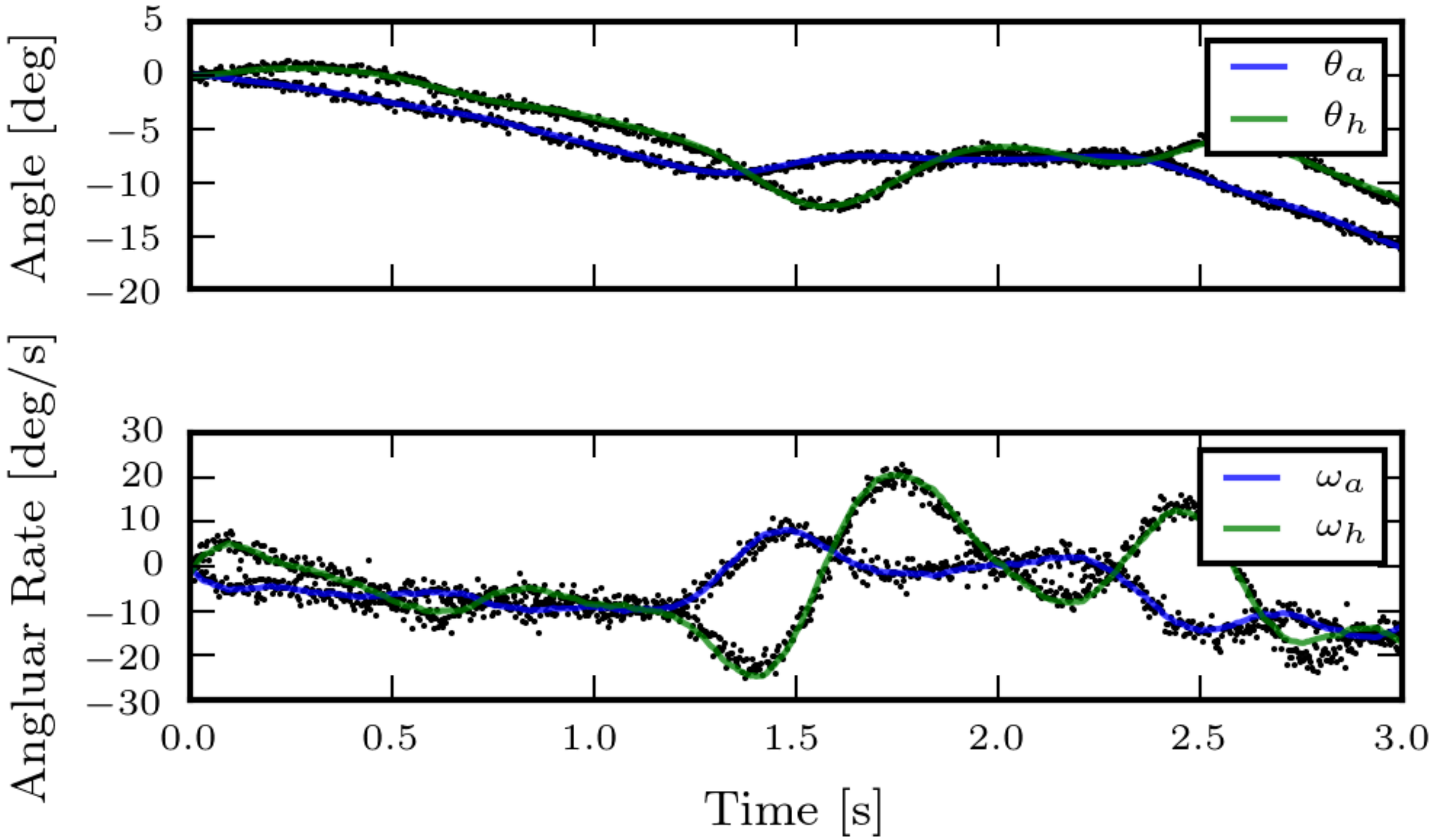
- 24008 free variables
- 23996 x 24008 Jacobian matrix with 384000 non-zero entries

Results

Converges in **11 iterations** in **2.8 seconds** of computation time.

	Known	**Identified**	**Error**
k_{00}	950	946	-0.4%
k_{01}	175	177	1.4%
k_{02}	185	185	-0.2%
k_{03}	50	55	9.4%
k_{10}	45	45	1.1%
k_{11}	290	289	-0.3%
k_{12}	60	59	-2.1%
k_{13}	26	27	4.2%

Identified State Trajectories



Conclusion

- Direct collocation is suitable for biomechanical parameter identification
- Computation speeds are orders of magnitude faster than shooting
- Parameter identification accuracy improves with # nodes
- Complex problems can be solved with few lines of code and high level mathematical abstractions