Exercises PHY981 Spring 2014

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The exercises are available at the beginning of the week and are to be handed in at the lecture the week thereafter on Wednesdays. This can be done electronically (as a pdf or postscript file) by email to hjensen@nscl.msu.edu or at the lecture. You can also send in a scanned version of your answer. The Friday lectures will be used to discuss the weekly exercises. The exercises will be graded and count 10% of the final mark.

Exercise 1: masses and binding energies

The data on binding energies can be found in the file aud11.dat, contained in the file at the webpage mass.zip. This file has also been sent as an email to you as well.

a) Write a small program which reads in the proton and neutron numbers and the binding energies and make a plot of all neutron separation energies up to $Z = 82$ (see figure 2.9 of Alex Brown's text), that is you need to plot

$$S_n = BE(N,Z) - BE(N-1,Z),$$

and similarly, the proton separation energies

$$S_p = BE(N,Z) - BE(N,Z-1).$$

See Fig. 2, upper two panels. The one neutron seperation energy shows obvious enhancements around $N = 28, N = 50, N = 82$ hence their definition as magic numbers. These lines are more difficult to see in the one proton seperation energy plot, but can just barely be made out.

b) In the same figures, you should also include the liquid drop model results of Eq. (2.17) of Alex Brown's text, namely

$$BE(N,Z) = \alpha_1 A - \alpha_2 A^{2/3} - \alpha_3 \frac{Z^2}{A^{1/3}} - \alpha_4 \frac{(N-Z)^2}{A},$$

with $\alpha_1 = 15.49$ MeV, $\alpha_2 = 17.23$ MeV, $\alpha_3 = 0.697$ MeV and $\alpha_4 = 22.6$ MeV. **See Fig. 2, middle two panels for the LD model estimates, and the bottom two panels for the difference in predicted one nucleon seperation energies. The plots from the included data are much more limited in scope than the liquid drop model, by looking at the difference plot, it appears that it does fairly well given it's simplicity. Whereas the proton magic numbers were difficult to see before, in the difference plots, they are very evident. Also, the effects of nuclear pairing are extremely visible in the difference plots; note how from one column/row to the next the seperation energy jumps color jumps from orange to red for neutrons/protons.**

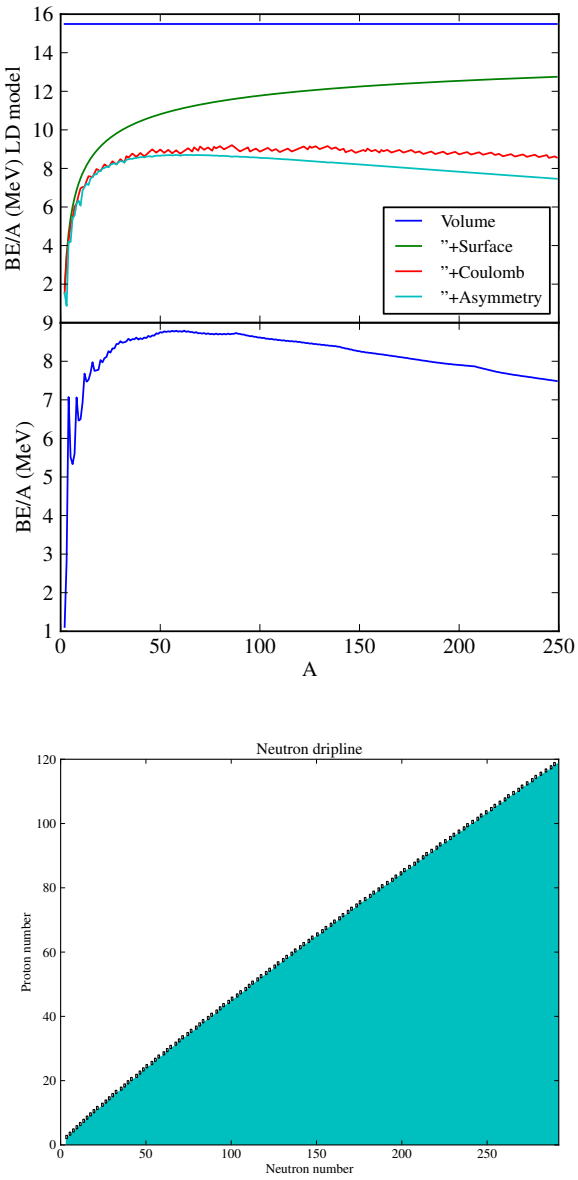
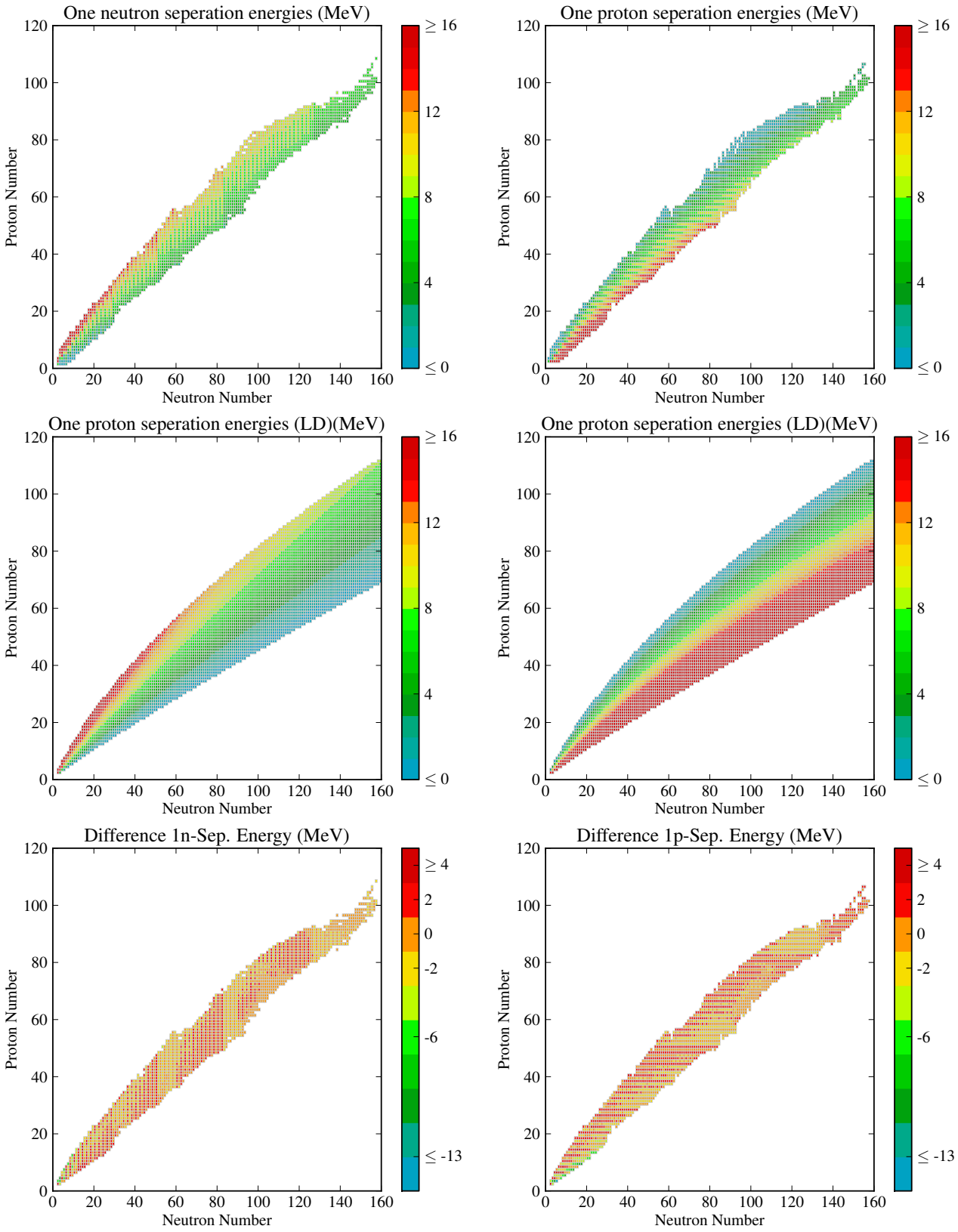


Figure 1: Upper panel details the binding energy per nucleon vs mass number from NNDC data stores, the top part of which details a running sum of the contributions to BE/A for each compnent of the Liquid-Drop model. The lower panel is the neutron dripline as predicted by the Liquid Drop model.

Figure 2: One nucleon seperation energies from NNDC data files and as predicted from the Liquid Drop model.



c) Make also a plot of the binding energies as function of A using the data in the file aud11.dat and the above liquid drop model. Make a figure similar to figure 2.5 where you set the various parameters $\alpha_i = 0$. **See Fig. 1, top panel. The liquid drop model does a fairly good job at reproducing the binding energy per nucleon, with the exception of the structure seen at low mass number nuclei. The surface term sets the overall shape of the curve, and at coulomb term becomes significant at large proton number due to it's Z^2 dependance. The asymmetry term doesn't affect the the binding energy very much for the most stable nuclei as $N - Z \approx 0$.**

d) Use the liquid drop model to find the neutron drip lines for Z values up to 120. Analyze then the oxygen and fluorine isotopes and find, where available the corresponding experimental data, and compare the liquid drop model predicition with experiment. **See Fig. 1, bottom panel. ^{23}O and ^{26}F are the last bound isotope of each element as predicted by the liquid drop model. According to experiment, the oxygen drip line is actually ^{24}O and the fluorine dripline is at ^{27}F or maybe even ^{29}F. As pairing has been the strongest contributer to differences in the rest of the above observations, I would postulate that the reason the dripline is actually quite a bit further out for these elements is because of their even number of neutrons, the pairing of which contributes to their binding energy.**

Exercise 2

Consider the Slater determinant

$$\Phi_\lambda^{AS}(x_1 x_2 \dots x_N; \alpha_1 \alpha_2 \dots \alpha_N) = \frac{1}{\sqrt{N!}} \sum_p (-)^p P \prod_{i=1}^{N} \psi_{\alpha_i}(x_i).$$

where P is an operator which permutes the coordinates of two particles. We have assumed here that the number of particles is the same as the number of available single-particle states, represented by the greek letters $\alpha_1 \alpha_2 \dots \alpha_N$.

a) Write out Φ^{AS} for $N = 3$.

$$\begin{aligned}
\Phi_H &= \psi_{\alpha_1}(x_1)\psi_{\alpha_2}(x_2)\psi_{\alpha_3}(x_3) \\
\Phi_\lambda^{AS} &= \frac{1}{\sqrt{3!}} \sum_p (-)^p P \Phi_H \\
&= \frac{1}{\sqrt{6}} (\sum_p (-)^p P) \psi_{\alpha_1}(x_1)\psi_{\alpha_2}(x_2)\psi_{\alpha_3}(x_3) \\
&= \frac{1}{\sqrt{6}} (1 - P_{12} - P_{23} - P_{31} + P_{12}P_{23} + P_{23}P_{12}) \Phi_H \\
&= \frac{1}{\sqrt{6}} (\psi_{\alpha_1}(x_1)\psi_{\alpha_2}(x_2)\psi_{\alpha_3}(x_3) \\
&- \psi_{\alpha_1}(x_2)\psi_{\alpha_1}(x_2)\psi_{\alpha_3}(x_3) \\
&- \psi_{\alpha_1}(x_1)\psi_{\alpha_2}(x_3)\psi_{\alpha_3}(x_2) \\
&- \psi_{\alpha_1}(x_3)\psi_{\alpha_2}(x_2)\psi_{\alpha_3}(x_1) \\
&+ \psi_{\alpha_1}(x_3)\psi_{\alpha_2}(x_1)\psi_{\alpha_3}(x_2) \\
&+ \psi_{\alpha_1}(x_2)\psi_{\alpha_2}(x_3)\psi_{\alpha_3}(x_1))
\end{aligned}$$

b) Show that

$$\int dx_1 dx_2 \dots dx_N \left|\Phi_\lambda^{AS}(x_1 x_2 \dots x_N; \alpha_1 \alpha_2 \dots \alpha_N)\right|^2 = 1.$$

$$\begin{aligned}
&\langle \Phi^{AS} | \Phi^{AS} \rangle \\
&= \frac{1}{N!} \int |((\sum_p (-)^p \hat{P}) \prod_{i=1}^{N} \psi_{\alpha_i}(x_i)|^2 d^N x \\
&= \frac{1}{N!} \sum_{p=0}^{\text{N-particle permutations}} (-)^p \int \hat{P} \prod_{i=1}^{N} \psi^*_{\alpha_i} \psi_{\alpha_i} d^N x \\
&= \frac{1}{N!} (N! + \sum_{i=1}^{N!} (-)^i) \int \hat{P} \prod_{i=1}^{N} \psi^*_{\alpha_i} \psi_{\alpha_i} d^N x \\
&= \frac{1}{N!} (N! * 1 + 0) \\
&= 1
\end{aligned}$$

Here the sum is over permutations of p particles, starting with one particle, then two particles, then three particles etc. on to permutations of an N-particle system. This could possibly be written out formly in terms of permutation matricies such as the levi-civita tensor and analogs for higher dimensions, but I am not sure it would be any more revealing.

c) Define a general onebody operator $\hat{F} = \sum_i^N \hat{f}(x_i)$ and a general twobody operator $\hat{G} = \sum_{i>j}^N \hat{g}(x_i, x_j)$ with g being invariant under the interchange of the coordinates of particles i and j. Calculate the matrix elements for a two-particle Slater determinant

$$\left\langle \Phi^{AS}_{\alpha_1\alpha_2} \right| \hat{F} \left| \Phi^{AS}_{\alpha_1\alpha_2} \right\rangle,$$

and

$$\left\langle \Phi^{AS}_{\alpha_1\alpha_2} \right| \hat{G} \left| \Phi^{AS}_{\alpha_1\alpha_2} \right\rangle.$$

Explain the short-hand notation for the Slater determinant.

The short-hand notation makes simple what would otherwise be arduous (even for latex!)

$$\begin{aligned}
&\langle \phi_\alpha(x_i), \phi_\beta(x_j) | f(x_i) | \phi_\alpha(x_i), \phi_\beta(x_j) \rangle \\
&= \langle \alpha = i, \beta = j | f_i | \alpha = i, \beta = j \rangle = \langle ij | f_i | ij \rangle
\end{aligned}$$

with the nice property that

$\langle ij | f_i | ij \rangle = \langle i | f_i | i \rangle$ and $\langle ij | f_i | ji \rangle = 0$

and $\langle ij | g_{ij} | ij \rangle = \langle ji | g_{ij} | ji \rangle$

For the onebody operator we have,

$$
\begin{aligned}
&\left\langle \Phi^{AS}_{\alpha_1\alpha_2} \right| \hat{F} \left| \Phi^{AS}_{\alpha_1\alpha_2} \right\rangle \\
&= \frac{1}{2} \left\langle 12 - 21 \right| f_1 + f_2 \left| 12 - 21 \right\rangle \\
&= \frac{1}{2} (\left\langle 12 \right| f_1 \left| 12 - 21 \right\rangle - \left\langle 21 \right| f_1 \left| 12 - 21 \right\rangle \\
&+ \left\langle 12 \right| f_2 \left| 12 - 21 \right\rangle - \left\langle 21 \right| f_2 \left| 12 - 21 \right\rangle) \\
&= \frac{1}{2} (\left\langle 1 \right| f_1 \left| 1 \right\rangle + \left\langle 1 \right| f_1 \left| 1 \right\rangle + \left\langle 2 \right| f_2 \left| 2 \right\rangle + \left\langle 2 \right| f_2 \left| 2 \right\rangle) \\
&= \left\langle 1 \right| f_1 \left| 1 \right\rangle + \left\langle 2 \right| f_2 \left| 2 \right\rangle .
\end{aligned}
$$

For the twobody operator we have,

$$
\begin{aligned}
&\left\langle \Phi^{AS}_{\alpha_1\alpha_2} \right| \hat{G} \left| \Phi^{AS}_{\alpha_1\alpha_2} \right\rangle \\
&= \frac{1}{2} \left\langle 12 - 21 \right| g_{12} \left| 12 - 21 \right\rangle \\
&= \frac{1}{2} (\left\langle 12 \right| g_{12} \left| 12 \right\rangle - \left\langle 21 \right| g_{12} \left| 12 \right\rangle - \left\langle 12 \right| g_{12} \left| 21 \right\rangle + \left\langle 21 \right| g_{12} \left| 21 \right\rangle) \\
&= \left\langle 12 \right| g_{12} \left| 12 \right\rangle - \left\langle 12 \right| g_{12} \left| 21 \right\rangle
\end{aligned}
$$

Which properties do you expect these operators to have in addition to an eventual permutation symmetry? **I would guess that they add linearly and that the onebody and two body operators would not commute.**