Data Structures and Algorithms

Prof. Ganesh Ramakrishnan,
Prof. Ajit Diwan,
Prof. D.B. Phatak

Department of Computer Science and Engineering
IIT Bombay

Session: Running Time of Program: Empirical and Analytical Method

Running Time of Programs : Canonical example (searching e in $\underbrace{[\text{"e"}]}_{S}$)

- Consider algorithms for searching for an element e in a sequence S sorted
- Running time is function of ?
  - Input Size
  - Position of e
- Empirically finding running time?

Running Time of Programs

- Consider algorithms for searching for an element e in a sequence S
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  } different input parameters
- Empirically finding running time?
  - Run the program (with properly designed running time experiments) multiple times with a large number of different inputs (Sequence S, element e to search for)
  - Time each run and compute average times across runs

Empirical Analysis of Running Time

Need to normalize extraneous factors

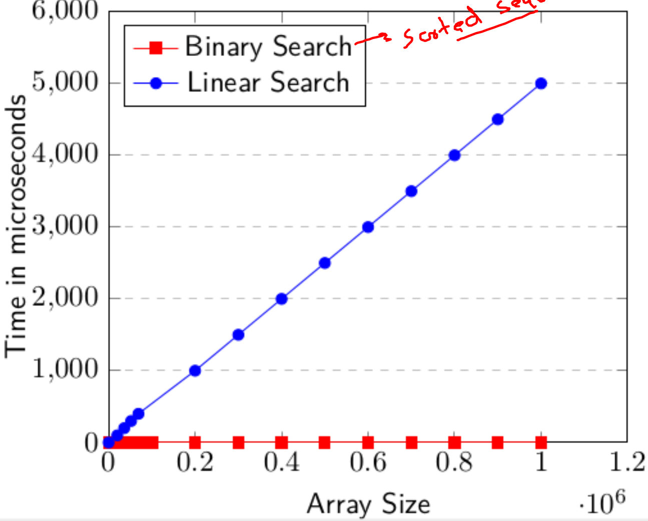
- No background processes
- Comparable data types (structures)
- Single threaded / no wasted steps
- To normalize w.r.t OS/Kernel related background processes
  - Average across multiple runs for each program
  - Use virtual machines with guaranteed resources

int. float

int[ ]

not comparable

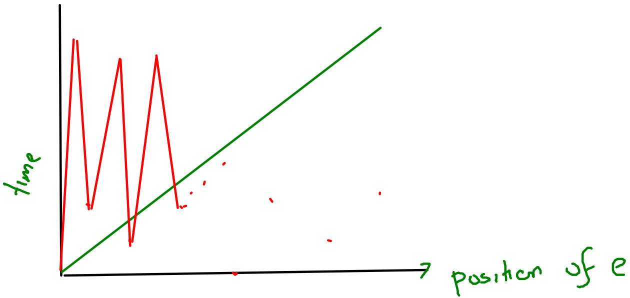
Results of Experiments (Execution time comparison)



→ sorted sequence

Question

How will plot(s) of time vs position of element e being searched for look like (for fixed size of S) ?



Issues with Empirical Analysis

(Post hoc)

- Primary Issue: Reasoning about the outcome should precede the experimentation
- Too time consuming
- Affected by several hidden factors (the hardware, the compiler, etc.)
- Might ignore the essential behaviour of the algorithm while focusing on unnecessary details
- Analysis on paper provides the foundational understanding of a system

Caveat: Such analysis always requires a model of the system under study

Model for Algorithm Analysis

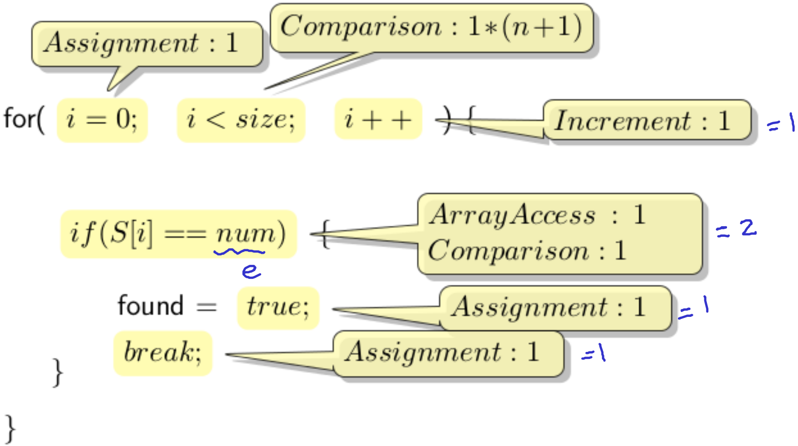
kx → shifting x to left by k positions

$2x$ → shifting x to left

- Sequential instructions and Infinite memory
- All basic instructions take one unit of time - addition, subtraction, multiplication, division, bit operations, comparison, assignment, etc.
- The above assumptions hold equally well for integer and floating point data types
- Ignores disk read times, memory hierarchies, paging, context switching, etc.

Analysis of Linear Search Algorithm A: Successful Search

LinearScan(e, S)



```
for( $i = 0$; $i < size$; $i++$ ){
    if$(S[i] == \underbrace{num}_{e})$ {
        found = $true$;
        $break$;
    }
}
```

$Assignment : 1$

$Comparison : 1*(n+1)$

$Increment : 1$ =1

$ArrayAccess : 1$
$Comparison : 1$ =2

$Assignment : 1$ =1

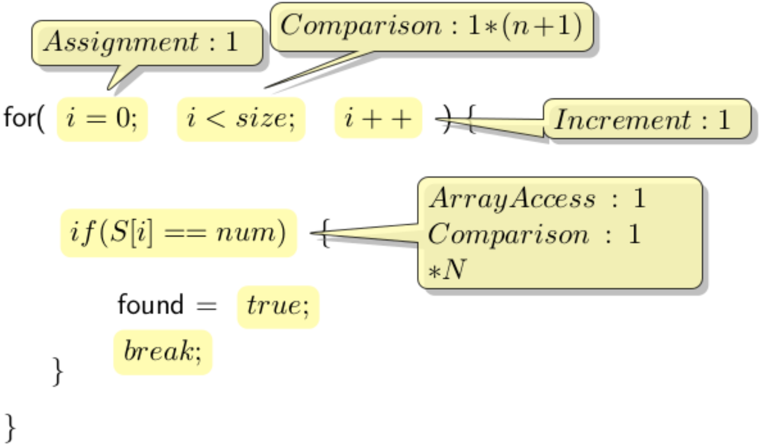
$Assignment : 1$ =1

Search Algorithm A: Time for Successful Search

- Total time $T_s(n)$, when element is at index $n = 0 \ldots N-1$ (successful search)
- $\underline{1} + \underline{2} * (n+1) + \underline{1} + \underline{1} * n + 1 * (n+1)$

  increment
- $T_s(n) = \underline{4n+5}$

Analysis of Search Algorithm A: Unsuccessful Search



```
for( i = 0;   i < size;   i++ ) {

    if(S[i] == num) {

        found = true;
        break;
    }

}
```

$Assignment : 1$

$Comparison : 1*(n+1)$

$Increment : 1$

$ArrayAccess\ : 1$
$Comparison\ : 1$
$*N$

Search Algorithm A: Time for Unsuccessful Search

- Total time for unsuccessful search
- $1 + 2 * N + 1 * N + 1 * (N + 1)$
- $T_u(N) = \underline{4N +} 2$

Question

How would you compute the average number of instructions executed by program A?

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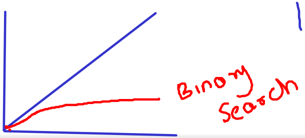
Here averaging is across all possible values of the position of ‘$\underbrace{\text{num}}_{(e)}$’ while holding the $\underbrace{\text{length of the list}}_{|s|}$ as a constant



Thank you