Data Structures and Algorithms

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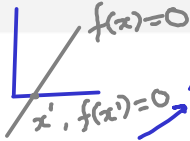
Session: Bisection Method

Root of $f(x)=0$

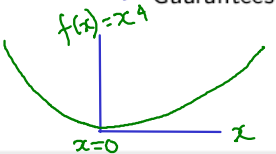
Introduction



Trivially for linear eqns: intercept on x-axis

- Finding the root of a nonlinear equation $f(x) = 0$
- Also called binary search method
- Given a function $f(x)$ continuous on the interval $[a, b]$
  - Let the initial guesses be a and b
  - The function has at least one root between a and b, if $f(a) * f(b) < 0$
  - If $f(a) * f(b) > 0$, there may or may not be any root
  - Guarantees only one root

Sufficient condition is $\text{sign}(f(a)) = -\text{sign}(f(b))$



Eg: sine/cosine functions

Algorithm for Bisection Method

Algorithm BisectionMethod($f(x)$)
Input Given a function $f(x)$ continuous on an interval $[a, b]$ and $f(a) * f(b) < 0$
$NMAX$ = max number of iterations → tolerance based on argument
TOL = tolerance limit → tolerance limit on $f(c) < TOL \Rightarrow$ stop
$N = 1$
$c = (a + b)/2$
while $N \leq NMAX$ & $f(c) < TOL$ **do**
  if $f(a) * f(c) < 0$ **then**
    $b = c$
  else
    $a = c$
  end if
  $N = N + 1$
  $c = (a + b)/2$
end while

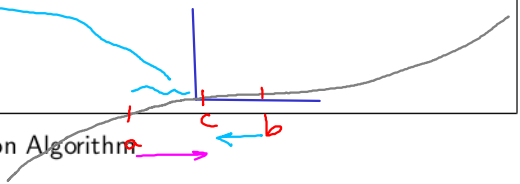


Figure: Bisection Algorithm

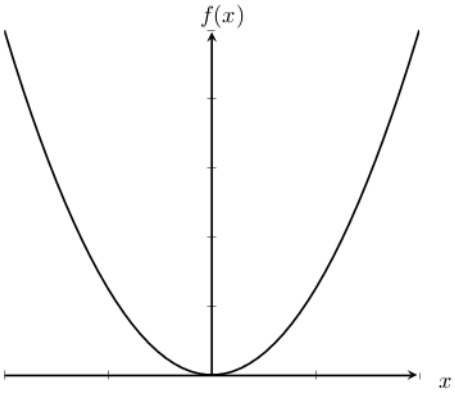
Advantages and Disadvantages

- Advantages
  - The method is guaranteed to converge
  - The search interval is halved for every iteration

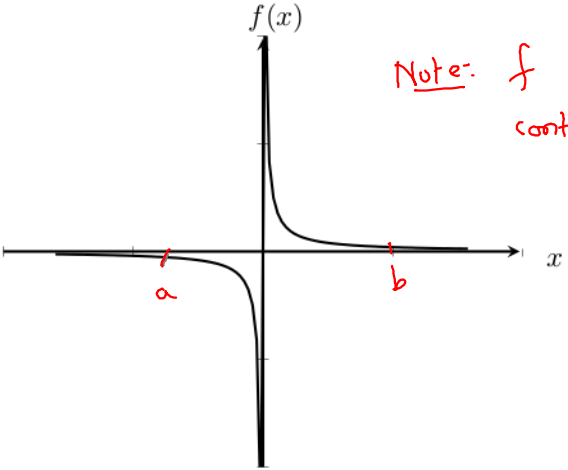
$\left(\frac{1}{2}\right)^k$.

- Disadvantages
  - Convergence is slow
  - If the initial guess is closer to the root, it will take more iterations to converge
  - It may not be possible to find the root for certain equations like $f(x) = x^2$, $f(x) = \frac{1}{x}$, etc.

Q: Can we reduce interval by factor $< \frac{1}{2}$? Can we adapt shrinkage?



The equation $f(x) = x^2 = 0$ cannot be solved using this method



Note: f was to be continuous in $[a, b]$

The equation $f(x) = \frac{1}{x} = 0$ has no root, but changes sign.

Thank you