Data Structures and Algorithms

Prof. Ganesh Ramakrishnan,
Prof. Ajit Diwan,
Prof. D.B. Phatak

Department of Computer Science and Engineering
IIT Bombay

Session: Properties of Line Segment

Introduction [1]

- branch of computer science that studies algorithms for solving geometric problems
- has applications in computer graphics, robotics, VLSI design, computer-aided design, molecular modeling, etc.
- input is a set of geometric objects, such as a set of points, a set of line segments, vertices of a polygon, etc
- output is response to a query about the objects, or a new geometric object

[1] Chapter 33, CLRS, Third Edition

Properties of Line Segment

- A convex combination of two distinct points $p_1 = (x_1, y_1)$ and $p_2 = (x_2, y_2)$ is any point $p_3 = (x_3, y_3)$ such that for some α in the range $0 \leq \alpha \leq 1$, we have $x_3 = \alpha x_1 + (1 - \alpha)x_2$ and $y_3 = \alpha y_1 + (1 - \alpha)y_2$ i.e. $p_3 = \alpha p_1 + (1 - \alpha)2_2$
- Consider 3 points p_0 at $(0, 0)$, p_1 at (x_1, y_1), and p_2 at (x_2, y_2)
  - Determine whether a segment $\overline{p_0 p_1}$ is clockwise or counter-clockwise from the other segment $\overline{p_0 p_2}$ with respect to the point p_0
  - Do the 2 line segments intersect?

Direction of p_1 from p_2 with respect to p_0

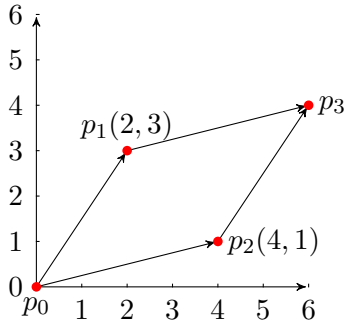
- Consider 3 points p_0 at $(0, 0)$, p_1 at (x_1, y_1), and p_2 at (x_2, y_2)
- To determine whether p_1 is clockwise or counter-clockwise from p_2 with respect to the origin $(0, 0)$, compute the determinant of a matrix as follows:

$$p_1 \times p_2 = \begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix}$$
$$= x_1 * y_2 - x_2 * y_1$$

- If this result is positive, then p_1 is clockwise from p_2, else it is counter-clockwise

Direction of p_1 from p_2 with respect to p_0

Cross Product of vectors p_1 and p_2



Computing Cross Product

$$\begin{aligned} p_3 &= p_1 + p_2 \\ &= (x_1 + x_2,\ y_1 + y_2) \\ &= (2 + 4,\ 3 + 1) \\ &= (6, 4) \end{aligned}$$

$$\begin{aligned} p_1 \times p_2 &= x_1 * y_2 - x_2 * y_1 \\ &= (2 * 1 - 4 * 3) \\ &= -10 \end{aligned}$$

Since, the result of $p_1 \times p_2$ is negative, p_1 is counter-clockwise from p_2, with respect to the origin p_0

Determine whether consecutive segments turn left or right

- Consider two line segments $\overline{p_0p_1}$ and $\overline{p_1p_2}$
- Do they turn left or right at point p_1
- We simply check whether directed segment $\overrightarrow{p_0p_2}$ is clockwise or counterclockwise with respect to directed segment $\overrightarrow{p_0p_1}$
- compute the cross product $(p_2 - p_0) \times (p_1 - p_0)$

Determine whether two line segments intersect

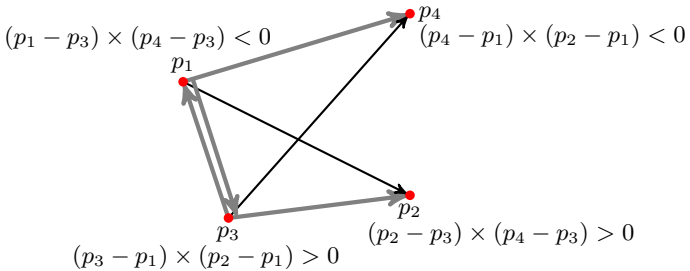


Figure: Case 1

Determine whether two line segments intersect

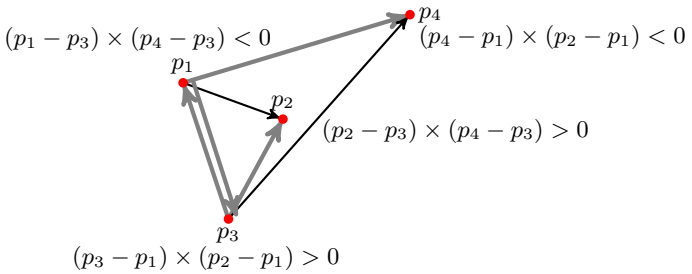


Figure: Case 2

Determine whether two line segments intersect

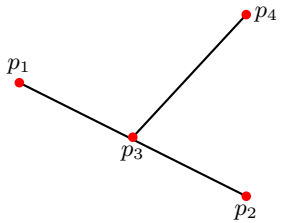


Figure: Case 3

Determine whether two line segments intersect

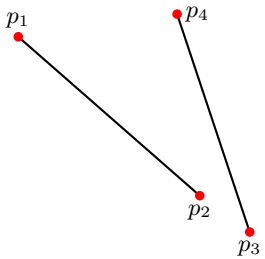


Figure: Case 4

Algorithm for detecting segments intersect

Algorithm SegmentsIntersect(p_1,p_2,p_3,p_4)
$d_1 = Direction(p_3, p_4, p_1)$
$d_2 = Direction(p_3, p_4, p_2)$
$d_3 = Direction(p_1, p_2, p_3)$
$d_4 = Direction(p_1, p_2, p_4)$
if (($d_1 > 0$ **and** $d_2 < 0$) **or** ($d_1 < 0$ **and** $d_2 > 0$)) **and** (($d_3 > 0$ **and** $d_4 < 0$) **or** ($d_3 < 0$ **and** $d_4 > 0$)) **then**
    return $TRUE$
else if $d_1 == 0$ **and** $ONSegment(p_3, p_4, p_1)$ **then**
    return $TRUE$
else if $d_2 == 0$ **and** $ONSegment(p_3, p_4, p_2)$ **then**
    return $TRUE$
else if $d_3 == 0$ **and** $ONSegment(p_1, p_2, p_3)$ **then**
    return $TRUE$
else if $d_4 == 0$ **and** $ONSegment(p_1, p_2, p_4)$ **then**
    return $TRUE$
else
    return $FALSE$
end if

Algorithm for detecting segments intersect

Algorithm Direction(p_i,p_j,p_k)
return $(p_k - p_i) * (p_j - p_i)$

Figure: Direction

Algorithm ONSegment(p_i,p_j,p_k)
if $min(x_i, x_j) \leq x_k \leq max(x_i, x_j)$ **and** $min(y_i, y_j) \leq y_k \leq max(y_i, y_j)$ **then**
    return $TRUE$
else
    return $FALSE$
end if

Figure: On-Segment

Thank you