Data Structures and Algorithms

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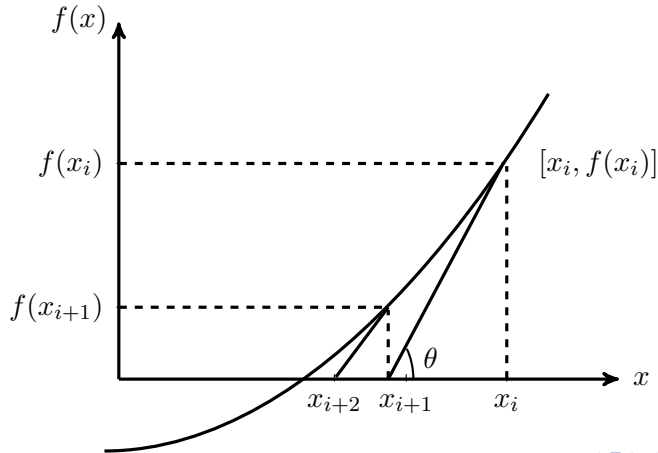
Session: Newton-Raphson Method

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Newton-Raphson Method

Introduction

■ As compared to bisection method, it requires only one initial guess of the root

■ If the method converges, it performs faster than bisection[1]

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[1]Content adapted from
http://mathforcollege.com/nm/mws/gen/03nle/mws_gen_nle_txt_newton.pdf and
https://en.wikipedia.org/wiki/Newton%27s_method

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Newton-Raphson Method

Newton-Raphson Method: Illustration

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Newton-Raphson Method

Principle of Newton-Raphson Method

- For a nonlinear function $f(x) = 0$, consider x_i, the initial guess of the root
- Find an improved estimate x_{i+1} as the $x-$intercept of the tangent to the curve at $f(x_i)$
- Using the slope definition

$$f'(x_i) = \tan \theta$$

$$= \frac{f(x_i)}{x_i - x_{i+1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

- Iteratively repeat the process till you get closer to the root with desirable tolerance

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Algorithm for Newton-Raphson Method

```
Algorithm Newton-RaphsonMethod(f(x))
Input Given a function f(x) = 0
Evaluate f'(x) symbolically
x_i = Initial guess of the root
NMAX = max number of iterations
TOL = tolerance limit
N = 1
while N < NMAX & |f'(x)| < TOL do
    x_{i+1} = x_i − f(x_i)/f'(x_i)
    x_i = x_{i+1}
    N = N + 1
end while
```

Figure: Newton-Raphson Algorithm

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Newton-Raphson Method

Problems with Newton-Raphson Method

■ Difficulty in calculating derivative of a function
■ Failure of the method to converge to the root in the following cases:
  ▶ overshooting
  ▶ division by zero
  ▶ wrong initial estimate
  ▶ oscillation

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Thank you

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