Data Structures and Algorithms

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Session: Merge Sort

Divide and Conquer Approach to Sorting

- **Divide** the problem into a number of sub-problems.
- **Conquer** the sub-problems by solving them recursively. For small sizes, recursive call could be replaced by other straightforward alternatives.
- **Combine** the solutions to the sub-problems into the solution for the original problem.

Divide and Conquer Approach to Sorting

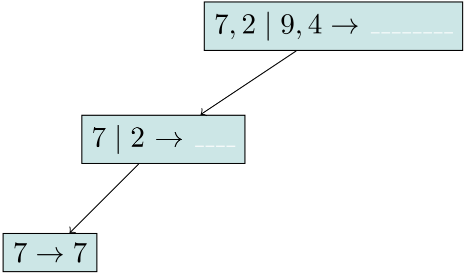
- **Divide** the problem into a number of sub-problems.
  - Divide the n-element sequence to be sorted into two sub-sequences of $\frac{n}{2}$ elements each
- **Conquer** the sub-problems by solving them recursively. For small sizes, recursive call could be replaced by other straightforward alternatives.
  - Sort the two subsequences recursively using merge sort.
- **Combine** the solutions to the sub-problems into the solution for the original problem.
  - Merge the two sorted subsequences to produce the sorted answer.

Merge-Sort Execution Tree

$$7, 2 \mid 9, 4 \rightarrow$$

$$7 \mid 2 \rightarrow$$

Merge-Sort Execution Tree



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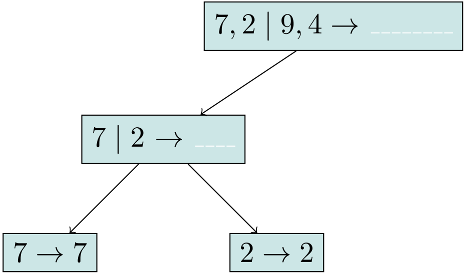

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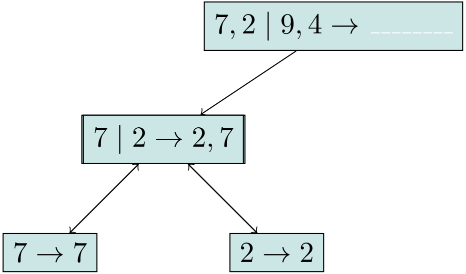

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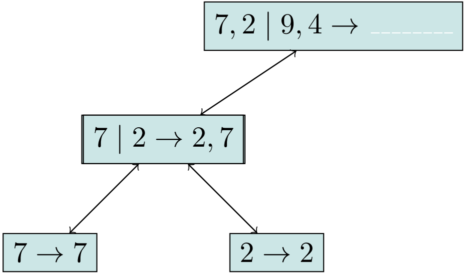

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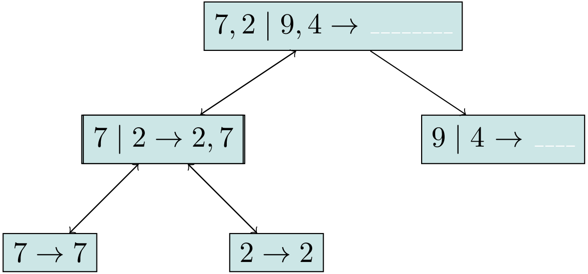

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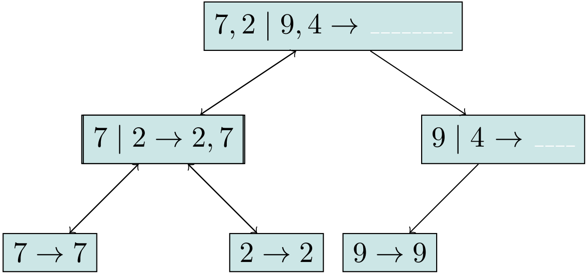

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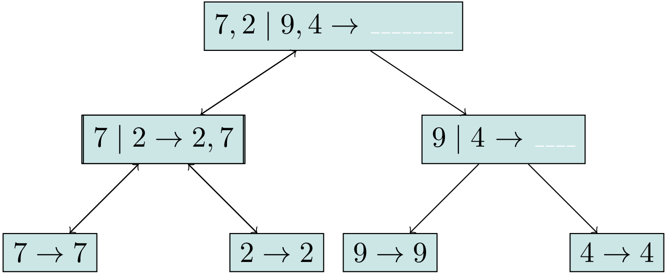

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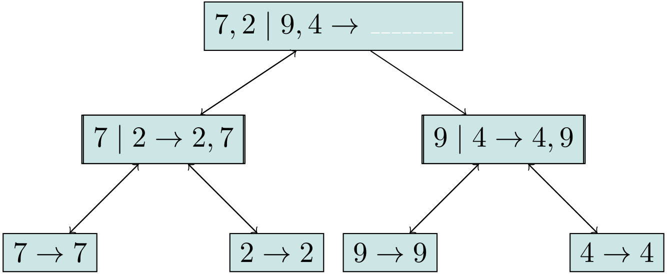

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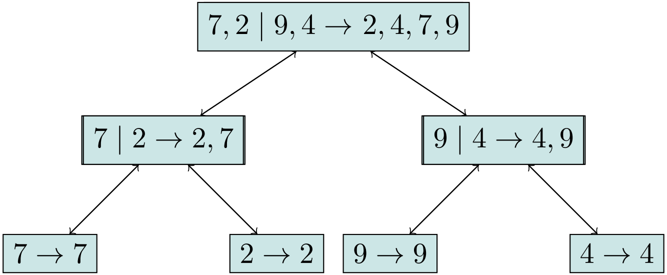


Merge Sort Algorithm

Algorithm MergeSort(S)
Input: Sequence S
Output: Sequence S sorted in increasing order
if $S.length > 1$ **then**

1. $(S_1, S_2) \leftarrow$ partition(S,$\frac{n}{2}$)
2. MergeSort(S_1)
3. MergeSort(S_2)
4. $S \leftarrow$ Merge(S_1,S_2)

end if

Figure: In-place Merge Sort

Merge Subroutine

Algorithm Merge(S)
Input: Sorted Sequence S_1 and S_2 with $\frac{n}{2}$ elements each
Output: Sorted Sequence S union of S_1 and S_2
$S \leftarrow$ empty sequence
while $not(S_1.isEmpty()) \wedge not(S_2.isEmpty())$ **do**
    if $S_1.firstElement() < S_2.firstElement()$ **then**
        $S.append(S_1.removeFirstElement())$
    else
        $S.append(S_2.removeFirstElement())$
    end if
end while

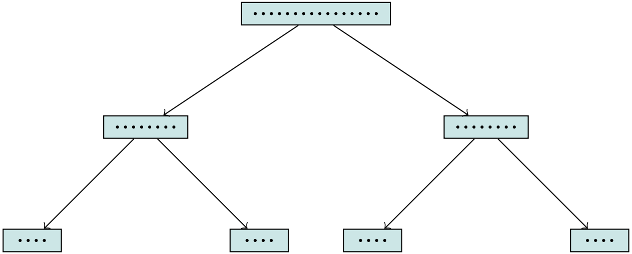
Figure: In-place Merge Sort

Merge Subroutine

Algorithm Merge(S) ...contd
while $not(S_1.isEmpty())$ **do**
  $S.append(S_1.removeFirstElement())$
end while
while $not(S_2.isEmpty())$ **do**
  $S.append(S_2.removeFirstElement())$
end while

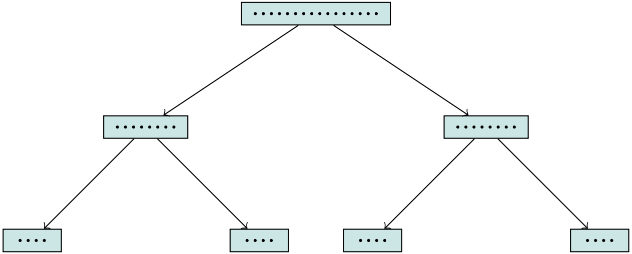
Figure: In-place Merge Sort

Analysis of Merge-Sort



Analysis of Merge-Sort

depth	# seqs	size



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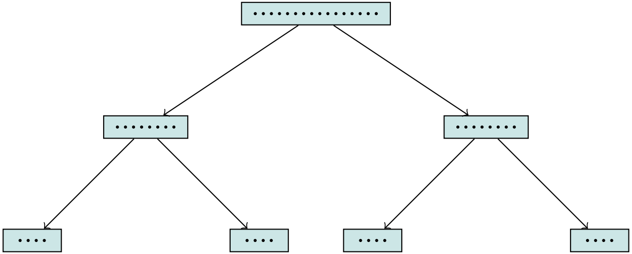

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0	1	n

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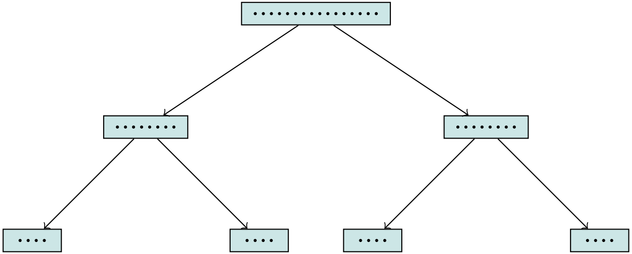

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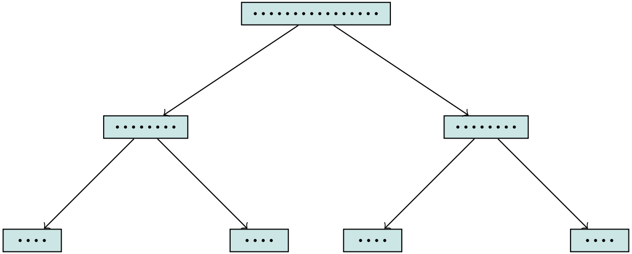
1	2	$n/2$

Analysis of Merge-Sort



depth	# seqs	size
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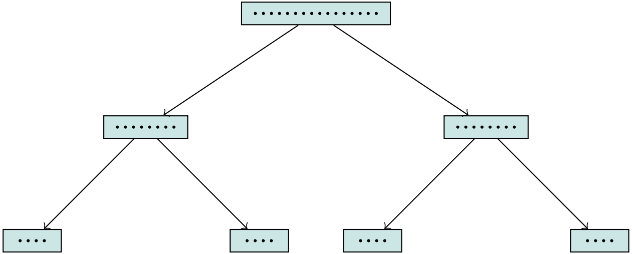
i	2^i	$n/(2^i)$

Analysis of Merge-Sort



- Each recursive call means division into two tasks of half original size $\Rightarrow$ Height h of tree $= O(\log n)$
- Amount of work done at any height $i = 2^i$ (number of sequences) $\times \frac{n}{2^i}$ (size of each sequence) $= O(n)$
- $\Longrightarrow$ total runtime $= O(n \log n)$

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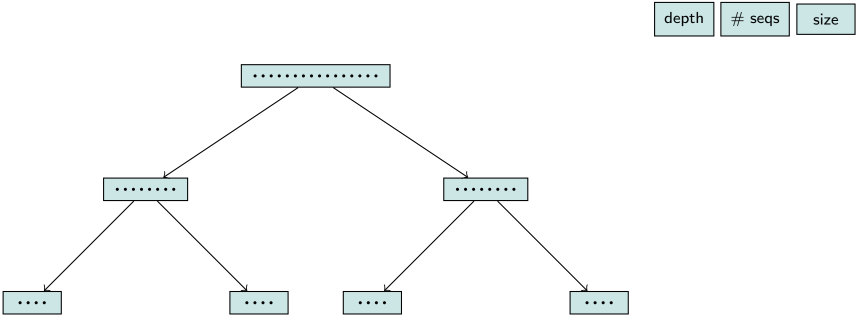

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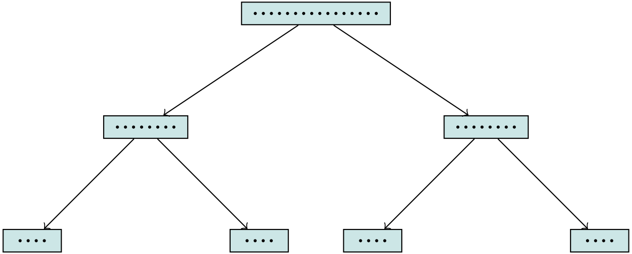

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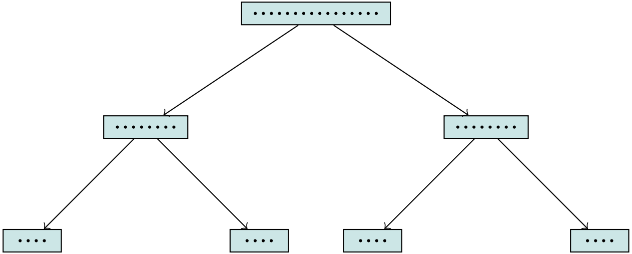

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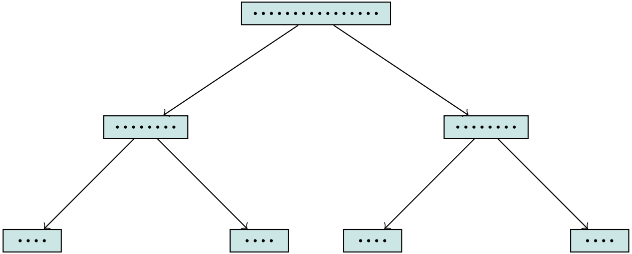

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- Next Session: Quick Sort

Thank you