Data Structures and Algorithms

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Session: Graph Traversal Algorithm (BFS)

Techniques for Graph Traversal starting at source s

- Breadth First Search (BFS)
  - Discovers all vertices at distance d from s before discovering any vertices at distance $d+1$
- Depth First Search (DFS)
  - Search deeper in the graph whenever possible

Breadth-First Search (BFS)

- $UNEXP \equiv$ Unexplored
- $VIST \equiv$ Visited
- $DISC \equiv$ Discovery
- $SIB \equiv$ edge to a sibling or to a child shared with sibling.

Algorithm BFS(G)
Input: Graph G
Output: Labeling of the edges and partition of vertices of G
for each vertex $u \in V[G]$ **do**
    $setLabel(u, UNEXP)$
end for
for each edge $e \in E[G]$ **do**
    $setLabel(e, UNEXP)$
end for
for each vertex $v \in V[G]$ **do**
    if $getLabel(v) = UNEXP)$ **then**
        BFS(G,v)
    end if
end for

Figure: Breadth-First Search: BFS(G)

Breadth-First Search (BFS contd.)

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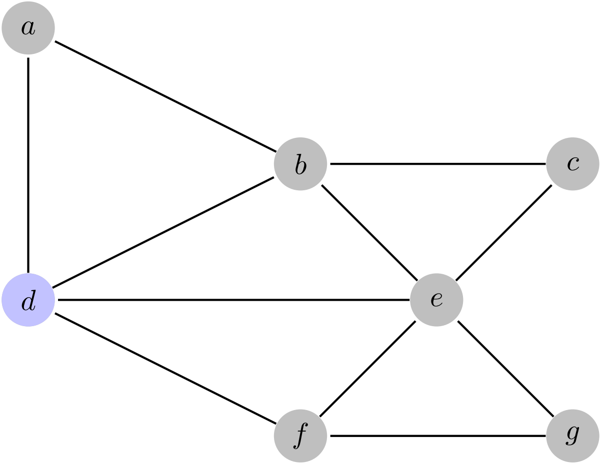
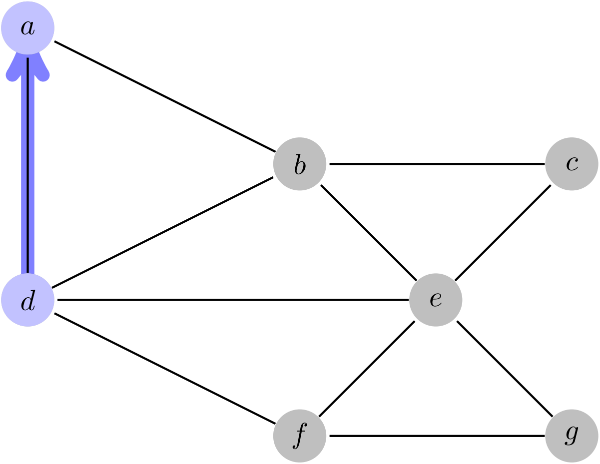
Algorithm BFS(G,v)
S_0 ← new empty sequence
S_0.apppend(v)
setLabel(v, VIS)
i ← 0
while ¬S_i.isEmpty() do
    S_{i+1} = newemptysequence
    for each u ∈ S_i.elements() do
        for each e ∈ G.incidentEdges(u) do
            if getLabel(e) == UNEXP then
                w ← e.getOtherVertex(u)
                if getLabel(w) = UNEXP then
                    setLabel(e, DISC)
                    setLabel(w, VIS)
                    S_{i+1}.apppend(w)
                else
                    setLabel(e, SIB)
                end if
            end if
        end for
    end for
    i = i + 1
end while
```

Figure: Breadth-First Search: BFS(G,s)

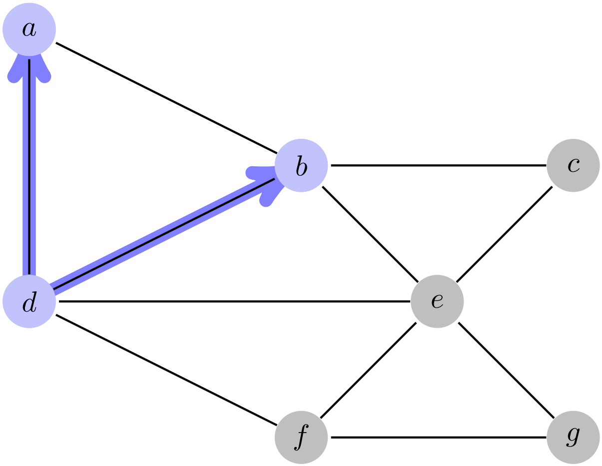
Breadth First Search



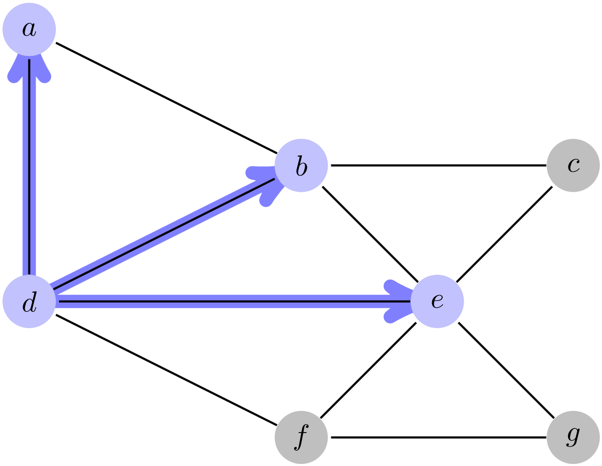
Breadth First Search



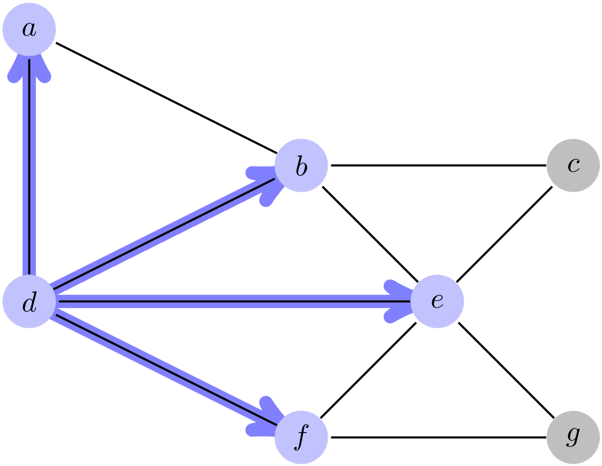
Breadth First Search



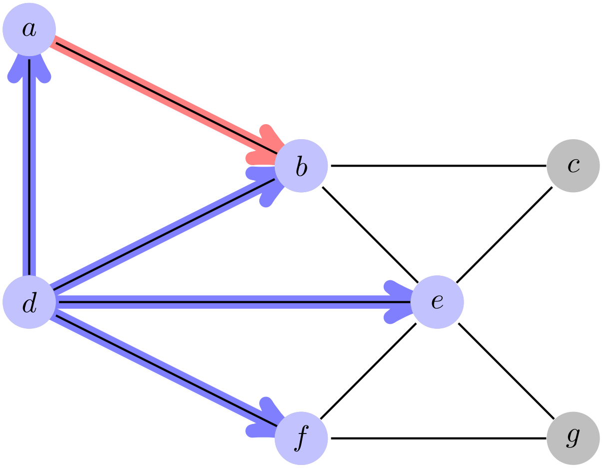
Breadth First Search



Breadth First Search



Breadth First Search



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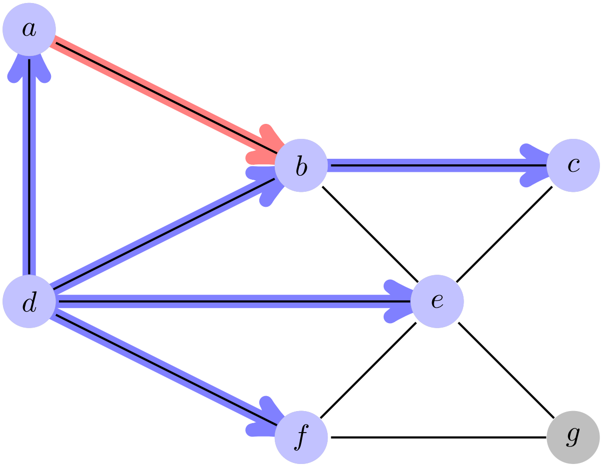

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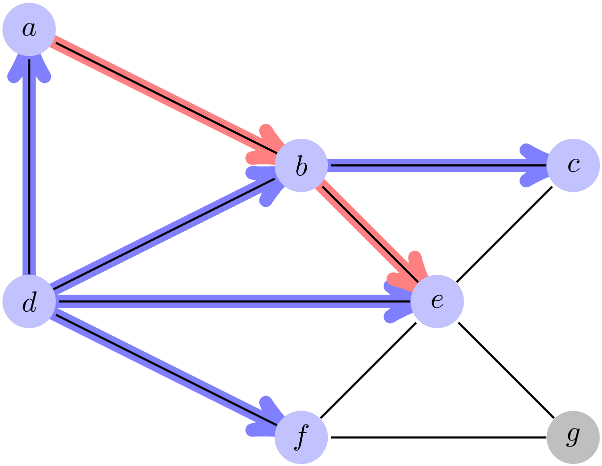
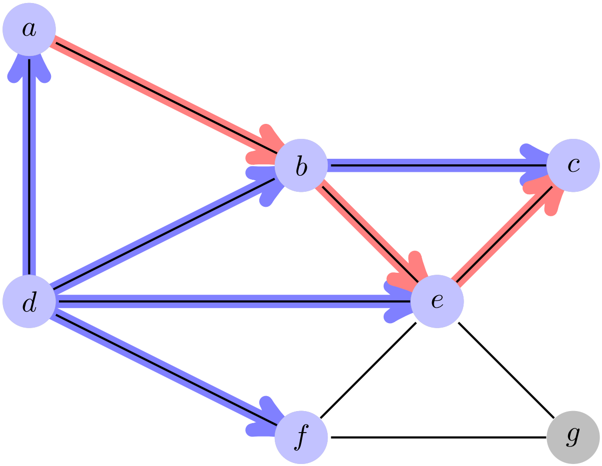
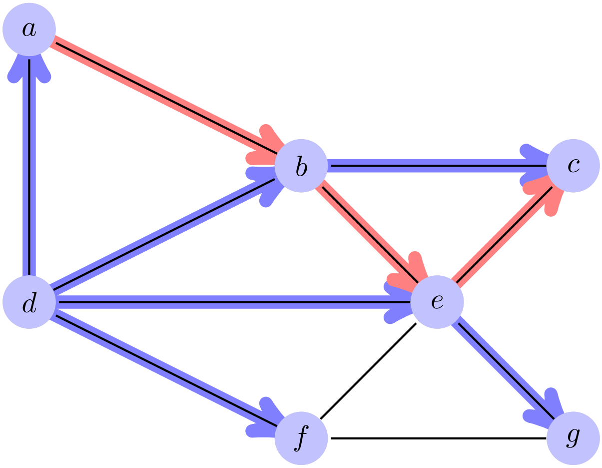


Breadth First Search



Breadth First Search



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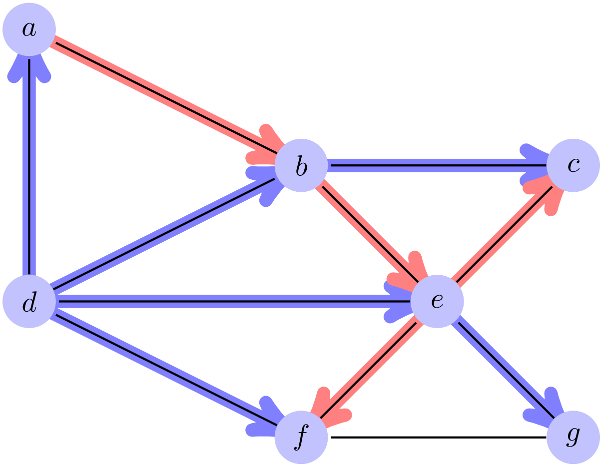
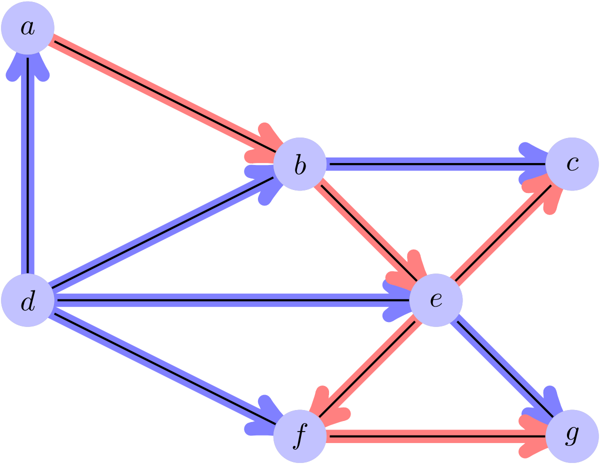
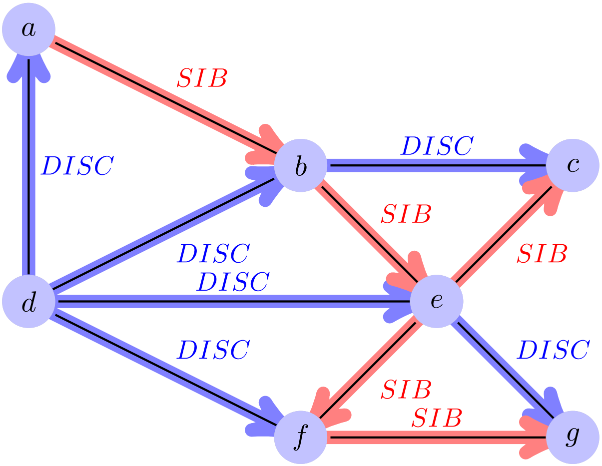


Breadth First Search



Breadth First Search



Properties of BFS

1. If G_v is a connected component of a graph G containing vertex v then $BFS(G, v)$ visits all the vertices and edges of G_v
2. The edges labeled $DISC$ by $BFS(G, v)$ form a spanning tree T_v of G_v
3. For each vertex $u \in S_i$,
   - The path from v to u in T_v has i edges
   - Every path from v to u in G_v has atleast i edges

Analysis of BFS

1. Each vertex is labeled twice
   - once as $UNEXP$ (Unexplored)
   - once as VIS (Visited)
2. Each edge is labeled twice
   - once as $UNEXP$ (Unexplored)
   - once as $DISC$ (Discovered) or as $INTR$ (an Intra edge)
3. Each vertex is inserted once into a S_i for some i
4. $G.incidentEdges(u)$ is called once for each vertex $u \implies O(E)$
5. $\implies$ BFS runs in $O(V+E)$ time

Applications of of BFS

BFS traversal of a graph G can be used to solve the following problems in $O(V+E)$ time

- Compute the connected components of G
- Compute a spanning forest of G
- Determine if G is a forest or find a simple cycle in G if there exists one
- Given two vertices of G, find a path (if there exists one) in G between them with the minimum number of edges
- BFS is the classic strategy for determining a solution to the rubix cube

Thank you