Data Structures and Algorithms

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Session: Convex Hull

Introduction [1]

- The **convex hull** of a set Q of points, denoted $CH(Q)$, is the smallest convex polygon P for which each point in Q is either on the boundary of P or in its interior.
- Convex hull may be visualized as the shape enclosed by a rubber band stretched around all the points in Q
- Can be computed using a technique called rotational sweep

[1] Chapter 33, CLRS, Third Edition

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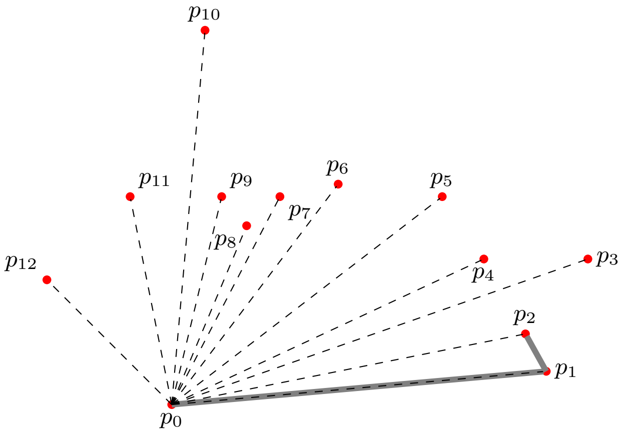
- Finally, the stack S contains all the points, from bottom to top, exactly the vertices of $CH(Q)$

Illustration



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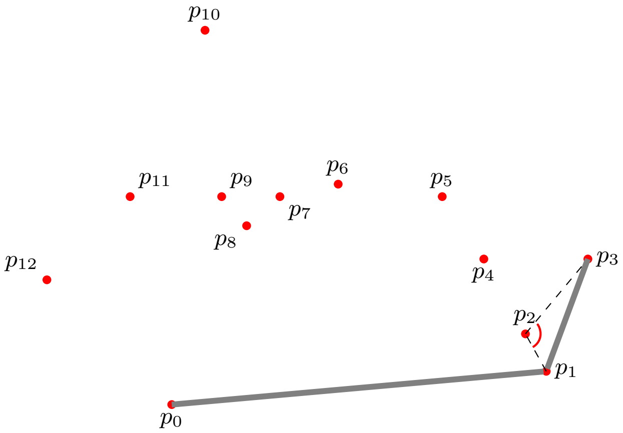

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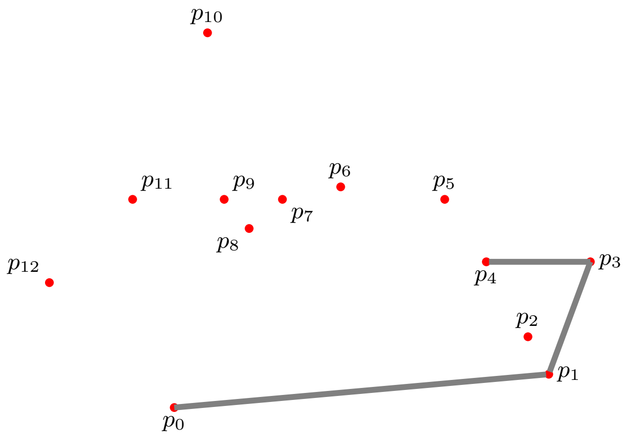

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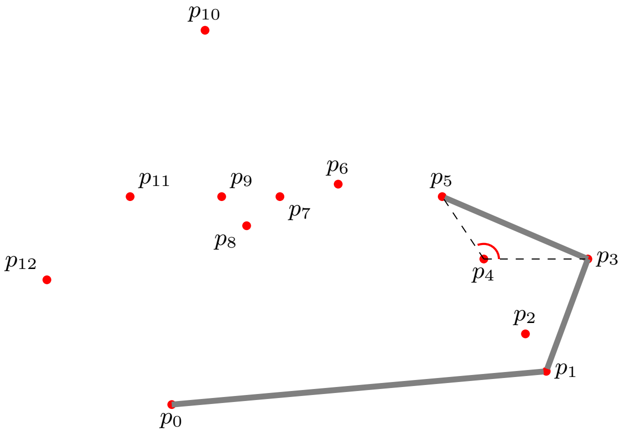

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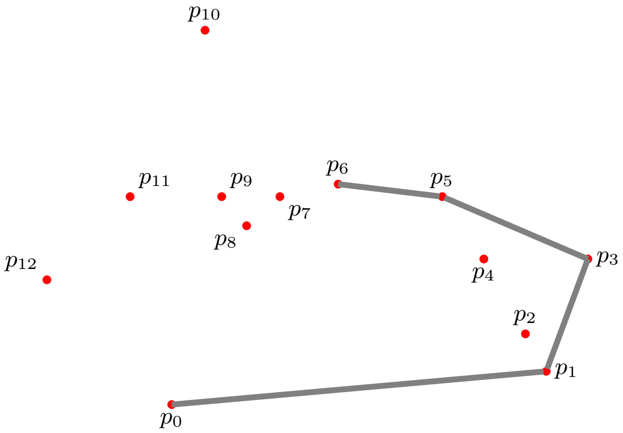

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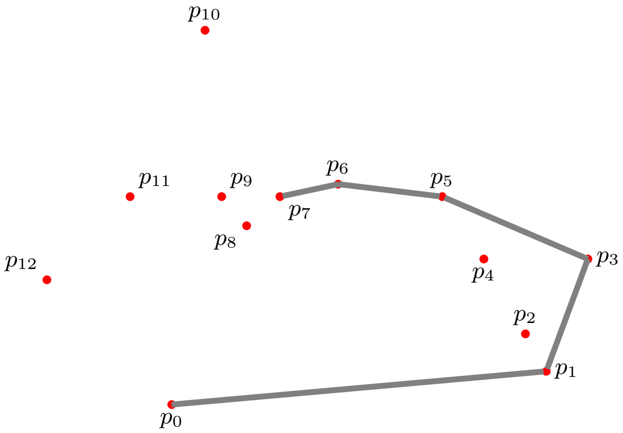

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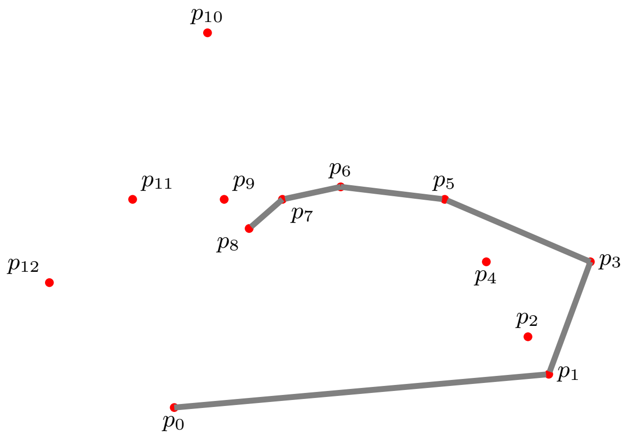

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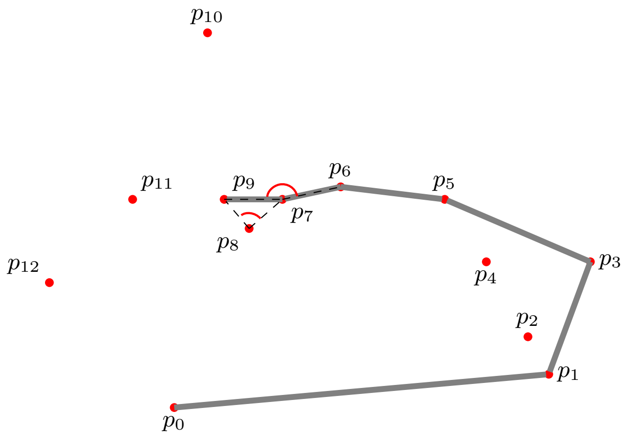

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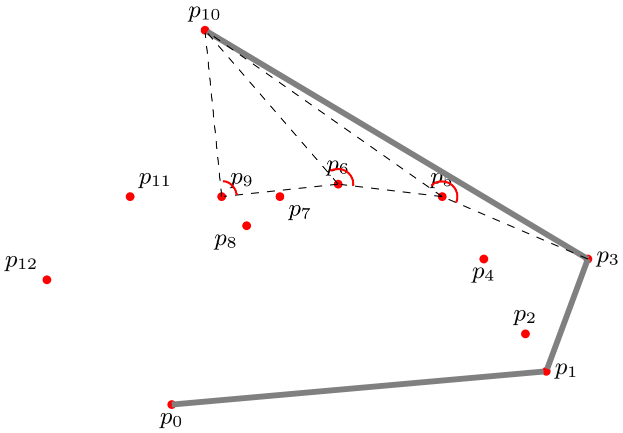

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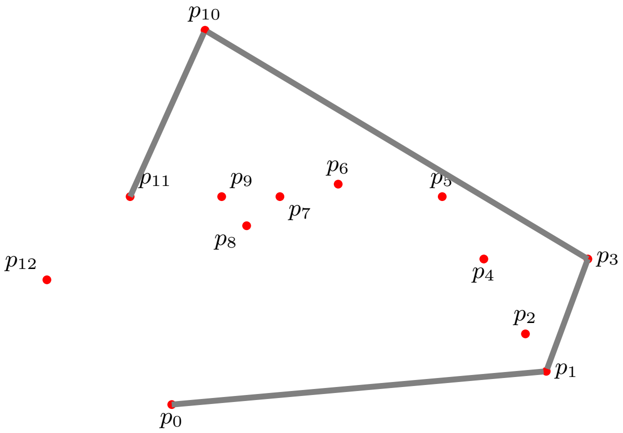

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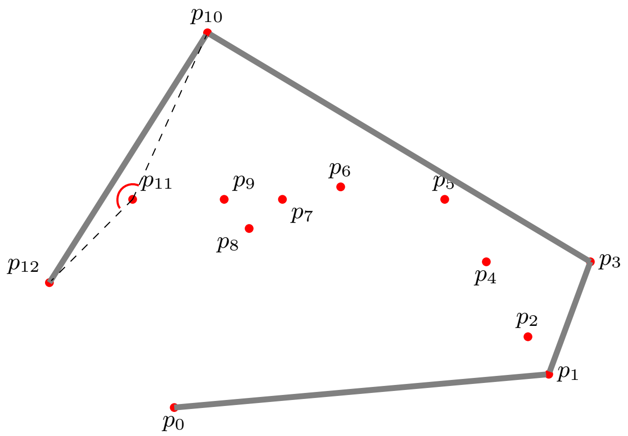

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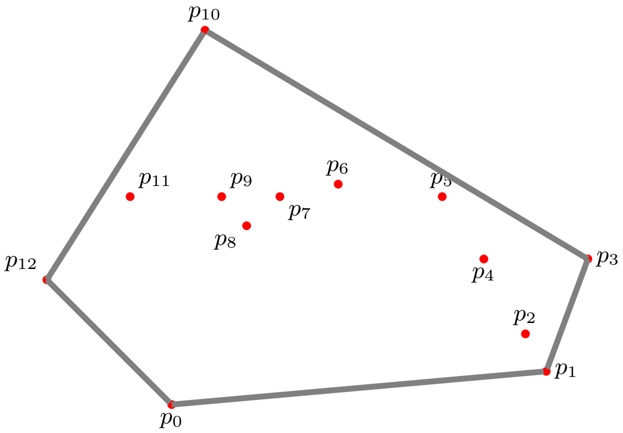

Algorithm for finding Convex Hull

```
Algorithm GrahamScanAlgorithm($Q$)
Let $p_0$ be the point in $Q$ with the minimum $y$-coordinate, or leftmost such point in case of tie
let $< p_1, p_2, ..., p_m >$ be the remaining points in $Q$, sorted by polar angle in counter-clockwise order around
$p_0$ (if more than one point has the same angle, remove all but the one that is farthest from $p_0$
let $S$ be an empty stack
PUSH($p_0, S$)
PUSH($p_1, S$)
PUSH($p_2, S$)
for $i \in (3...m)$ do
    while the angle formed by points $NextToTOP(S), TOP(S)$, and $p_i$ makes a non-left turn do
        $POP(S)$
    end while
    $PUSH(p_i, S)$
end for
return $S$
```

Figure: Graham-Scan-Algorithm

Thank you