Data Structures and Algorithms

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$f(x) = 0$

Session: Newton-Raphson Method → starts with a point estimate.

Bisection: $[a, b]$

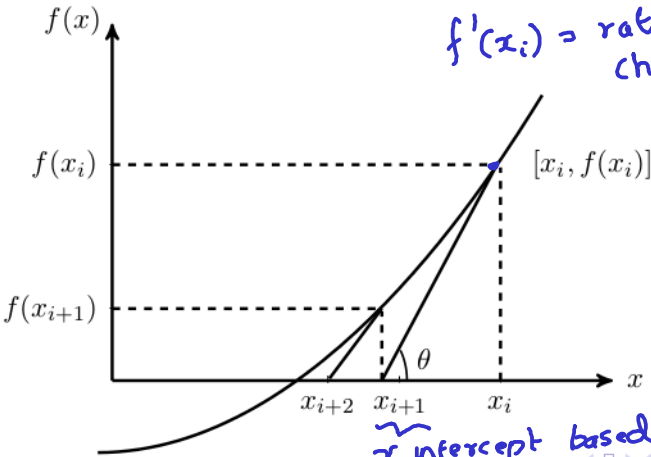
Introduction

- As compared to bisection method, it requires only one initial guess of the root
- If the method converges, it performs faster than bisection[1]

rate depends on curvature of f.

[1] Content adapted from
http://mathforcollege.com/nm/mws/gen/03nle/mws_gen_nle_txt_newton.pdf and
https://en.wikipedia.org/wiki/Newton%27s_method

Newton-Raphson Method: Illustration



Principle of Newton-Raphson Method

- For a nonlinear function $f(x) = 0$, consider x_i, the initial guess of the root
- Find an improved estimate x_{i+1} as the $x-$intercept of the tangent to the curve at $f(x_i)$
- Using the slope definition

small rate of change $\Rightarrow f'$ small $\Rightarrow$ large steps

$$f'(x_i) = \tan\theta = \frac{f(x_i) - 0}{x_i - x_{i+1}}$$

large rate of change $\Rightarrow f'$ large $\Rightarrow$ small steps

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

- Iteratively repeat the process till you get closer to the root with desirable tolerance

Algorithm for Newton-Raphson Method

Algorithm Newton-RaphsonMethod($f(x)$)
Input Given a function $f(x) = 0$
Evaluate $f'(x)$ symbolically
x_i = Initial guess of the root
$NMAX$ = max number of iterations
TOL = tolerance limit
$N = 1$
while $N < NMAX$ & $|f'(x)| < TOL$ **do**
  $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$
  $x_i = x_{i+1}$
  $N = N + 1$
end while

$f'(x) = 0$

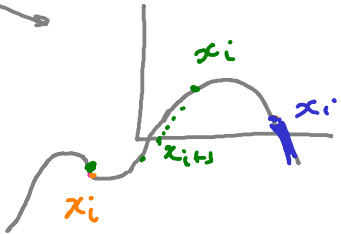
Figure: Newton-Raphson Algorithm

Problems with Newton-Raphson Method

Numerical errors! → Programmatically computing derivative is challenging.

- Difficulty in calculating derivative of a function
- Failure of the method to converge to the root in the following cases:
  - overshooting
  - division by zero
  - wrong initial estimate
  - oscillation

$f'(x) \approx 0$



Thank you