Easing Traffic

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Problem : The Lincoln Tunnel Authority is about to complete a tunnel under the city centre. Owing to financial cutbacks the tunnel is only one lane in either direction. The traffic Manager has realised that there will be holdups at both ends of the tunnel during the morning and evening rush hours. Bearing in mind aspects of safety and the desire to produce the maximum flow of traffic at peak times he wishes to put up signs indicating a maximum speed and the distance to be maintained between vehicles. What recommendations would you make to the Traffic Manager on this matter?

Givens

Step One: make a table with the known values:

```
In[∘]:= velocity = {30, 50, 70};

In[∘]:= thinkingDist = {30, 50, 70};

In[∘]:= brakingDist = {45, 125, 245};

In[∘]:= seperationDist = thinkingDist + brakingDist;

In[∘]:= XandYvalues = Transpose[{velocity, seperationDist}];

In[∘]:= Grid[Prepend[XandYvalues, {"Velocity", "Distance"}], Alignment → Center,
         Spacings → {2, 1}, Frame → All, ItemStyle → {"Text", {1 → {Bold, 14}}}]
```

Out[∘]=

Velocity	Distance
30	75
50	175
70	315

Step Two: graph a ListPlot to visualize the data

```
In[∘]:= givenListPlot = ListPlot[XandYvalues, PlotRange → {{0, 100}, {0, 350}},
           AxesLabel → {"Velocity", "Distance"}, AxesStyle → Arrowheads[0.025],
           PlotLabel → Style["Easing Traffic Given", Blue, 16],
           PlotStyle → {Red, PointSize[0.017]}, ImageSize → Large];
```

Step Three: find the curve of best fit

```
In[∘]:= Fit[XandYvalues, {x, x^2}, x]
```

Out[∘]= $1.\ x + 0.05\ x^2$

Step Four: create a function that gives us the total stopping distance when given the velocity

In[∘]:=
```
givenFunc[x_] := x + 0.05 x^2
```

In[∘]:=
```
givenFunc[{30, 50, 70}]
```

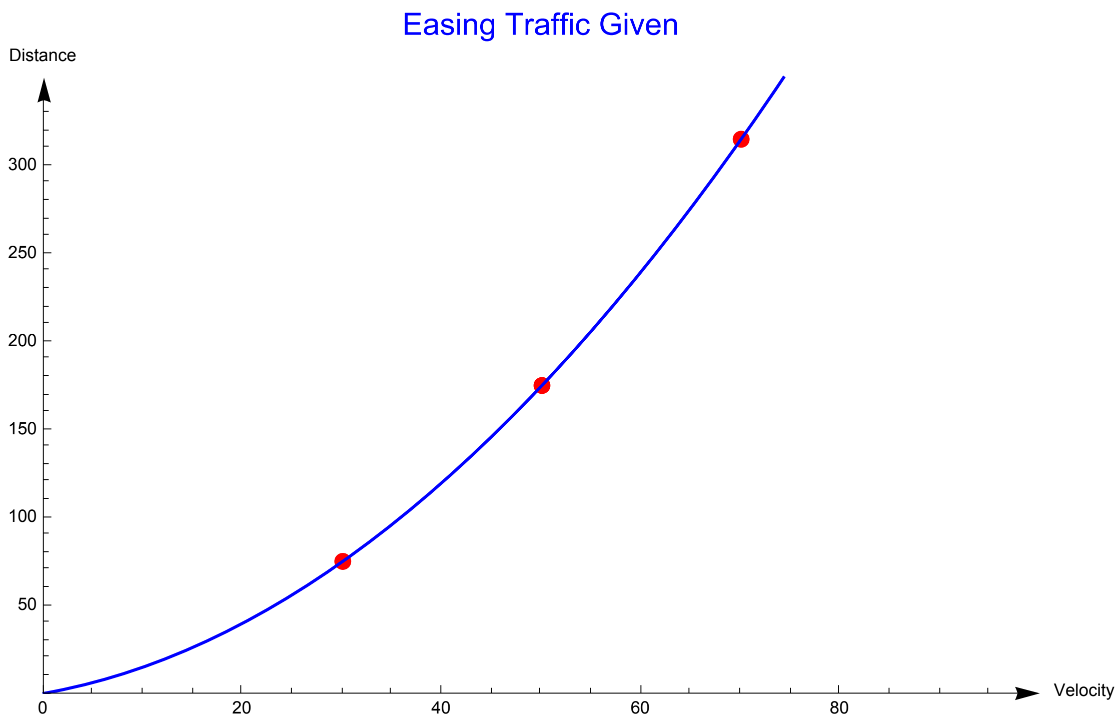
Out[∘]=
```
{75., 175., 315.}
```

Step Five: Graph the function

In[∘]:=
```
givenGraph = Plot[givenFunc[x], {x, 0, 100},
    PlotRange → {{0, 100}, {0, 350}}, AxesLabel → {"Velocity", "Distance"},
    PlotLabel → Style["Easing Traffic Given", Blue, 16], PlotStyle → Blue,
    ImageSize → Large, AxesStyle → Arrowheads[0.025], ImageSize → Large];
```

In[∘]:=
```
Show[givenListPlot, givenGraph]
```

Out[∘]=



Step Six: use the function to find more values to work with

In[∘]:=
```
moreVel = {5, 10, 20, 30, 40, 50, 60, 70}
```

Out[∘]=
```
{5, 10, 20, 30, 40, 50, 60, 70}
```

In[∘]:=
```
moreDist = givenFunc[moreVel]
```

Out[∘]=
```
{6.25, 15., 40., 75., 120., 175., 240., 315.}
```

In[∘]:=
```
moreXandYvalues = Transpose[{moreVel, moreDist}]
```

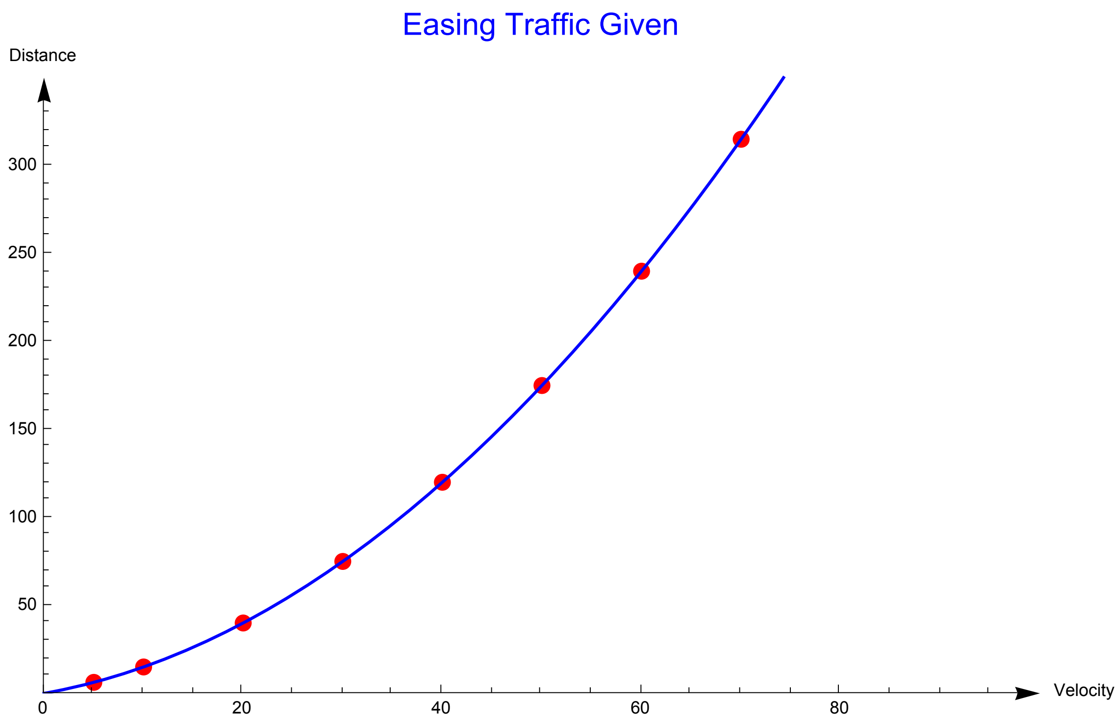
Out[∘]=
```
{{5, 6.25}, {10, 15.}, {20, 40.}, {30, 75.},
 {40, 120.}, {50, 175.}, {60, 240.}, {70, 315.}}
```

In[∘]:=
```
moreGivenListPlot = ListPlot[moreXandYvalues, PlotRange → {{0, 100}, {0, 350}},
    AxesLabel → {"Velocity", "Distance"}, AxesStyle → Arrowheads[0.025],
    PlotLabel → Style["Easing Traffic Given", Blue, 16],
    PlotStyle → {Red, PointSize[0.017]}, ImageSize → Large];
```

Step Six: visualize using a graph

In[∘]:=
```
Show[moreGivenListPlot, givenGraph]
```

Out[∘]=



The equation above, givenFunc[v] = v + 0.05 v^2, gives us the total stopping distance (time to think + actual braking distance) when given the velocity.

Model One

Finding the maximum number of cars in the tunnel given the velocity of each car

Step One: create the function to find the maximum number of cars in the tunnel given the velocity

In[∘]:=
```
numCarsPerMinFunc[moreVel_, moreDist_] := (88 * moreVel) / moreDist
```

In[∘]:=
```
numCarsPerMinFunc[15, 10]
```

Out[∘]= 132

In[∘]:=
```
numCarsPerMin = Round[numCarsPerMinFunc[moreVel, moreDist]]
```

Out[∘]= {70, 59, 44, 35, 29, 25, 22, 20}

In[∘]:=
```
numCarsXandY = Transpose[{moreVel, numCarsPerMin}]
```

Out[∘]= {{5, 70}, {10, 59}, {20, 44}, {30, 35}, {40, 29}, {50, 25}, {60, 22}, {70, 20}}

In[∘]:=
```
Grid[Prepend[numCarsXandY, {"Velocity", "Num Cars"}], Alignment → Center,
 Spacings → {2, 1}, Frame → All, ItemStyle → {"Text", {1 → {Bold, 14}}}]
```

Out[∘]=

Velocity	**Num Cars**
5	70
10	59
20	44
30	35
40	29
50	25
60	22
70	20

Step Two: visualize the data and find the curve of best fit.

In[∘]:=
```
modelOneLP = ListPlot[numCarsXandY, PlotRange → {{0, 100}, {0, 100}},
   AxesLabel → {"Velocity", "Num Cars"}, AxesStyle → Arrowheads[0.025],
   PlotLabel → Style["Easing Traffic Model One", Blue, 16],
   PlotStyle → {Red, PointSize[0.017]}, ImageSize → Large];
```

In[∘]:=
```
modelOneShell = a Exp[-k t]
```

Out[∘]= $a\,e^{-k\,t}$

In[∘]:=
```
fit = FindFit[numCarsXandY, modelOneShell, {a, k}, t]
```

Out[∘]= {a → 74.1303, k → 0.0221302}

In[∘]:=
```
modelOneFunc[x_] := 74.13 * Exp[-0.022 x]
```

In[∘]:=
```
modelOneFunc[1]
```

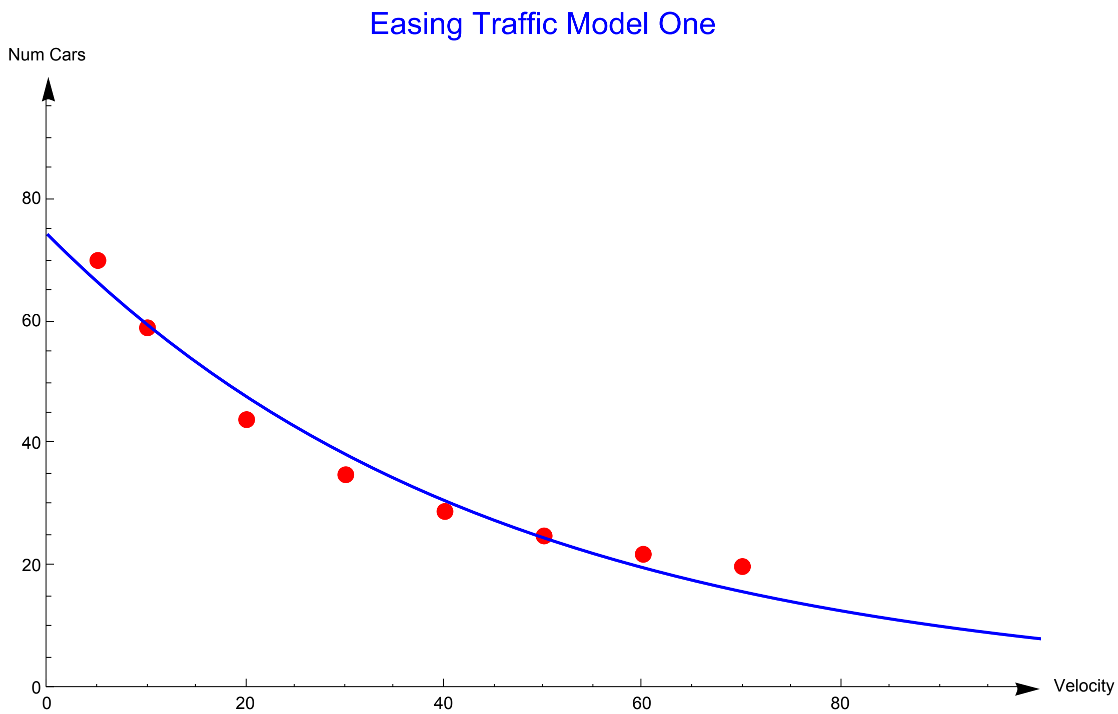
Out[∘]= 72.5169

Step Three: transpose the curve of best fit with the given data

In[∘]:=
```
modelOneGraph = Plot[modelOneFunc[x], {x, 0, 100},
   PlotRange → {{0, 100}, {0, 100}}, AxesLabel → {"Velocity", "Num Cars"},
   PlotLabel → Style["Easing Traffic Model 1", Blue, 16], PlotStyle → Blue,
   ImageSize → Large, AxesStyle → Arrowheads[0.025], ImageSize → Large];
```

In[∘]:=
```
Show[modelOneLP, modelOneGraph]
```

Out[∘]=



Step Four: obtain more values from the curve of best fit

In[∘]:=
```
expFitData = Round[modelOneFunc[moreVel]]
```
Out[∘]= {66, 59, 48, 38, 31, 25, 20, 16}

In[∘]:=
```
expFitXandY = Transpose[{moreVel, expFitData}]
```
Out[∘]= {{5, 66}, {10, 59}, {20, 48}, {30, 38}, {40, 31}, {50, 25}, {60, 20}, {70, 16}}

In[∘]:=
```
Grid[Prepend[expFitXandY, {"Velocity", "Num Cars"}], Alignment → Center,
 Spacings → {2, 1}, Frame → All, ItemStyle → {"Text", {1 → {Bold, 14}}}]
```
Out[∘]=

Velocity	Num Cars
5	66
10	59
20	48
30	38
40	31
50	25
60	20
70	16

From the data above, it seems that the maximum number of cars that can be in the tunnel is 88 cars with a velocity of 0 mph for each car. This result does not make sense, so a different model is needed.

Model Two

Assumes that the total braking distance includes the average car length of 13 feet

Step One: create a table of values

In[•]:=
```
modelTwoVelAndDist = Transpose[{moreVel, moreDist, moreDist + 13}]
```

Out[•]=
```
{{5, 6.25, 19.25}, {10, 15., 28.}, {20, 40., 53.}, {30, 75., 88.},
 {40, 120., 133.}, {50, 175., 188.}, {60, 240., 253.}, {70, 315., 328.}}
```

In[•]:=
```
Grid[Prepend[modelTwoVelAndDist,
  {"Velocity", "Braking Dist", "Braking Dist with car length"}],
 Alignment → Center, Spacings → {2, 1}, Frame → All,
 ItemStyle → {"Text", {1 → {Bold, 14}}}]
```

Out[•]=

Velocity	Braking Dist	Braking Dist with car length
5	6.25	19.25
10	15.	28.
20	40.	53.
30	75.	88.
40	120.	133.
50	175.	188.
60	240.	253.
70	315.	328.

Step Two: gather more data

In[•]:=
```
numCarsModelTwo = Round[numCarsPerMinFunc[moreVel, moreDist + 13]]
```

Out[•]=
```
{23, 31, 33, 30, 26, 23, 21, 19}
```

In[•]:=
```
modelTwoData = Transpose[{moreVel, moreDist, moreDist + 13, numCarsModelTwo}]
```

Out[•]=
```
{{5, 6.25, 19.25, 23}, {10, 15., 28., 31}, {20, 40., 53., 33}, {30, 75., 88., 30},
 {40, 120., 133., 26}, {50, 175., 188., 23}, {60, 240., 253., 21}, {70, 315., 328., 19}}
```

In[•]:=
```
ModelTwoXandY = Transpose[{moreVel, numCarsModelTwo}]
```

Out[•]=
```
{{5, 23}, {10, 31}, {20, 33}, {30, 30}, {40, 26}, {50, 23}, {60, 21}, {70, 19}}
```

In[•]:=
```
Grid[Prepend[modelTwoData,
  {"Velocity", "Braking Dist", "Braking Dist with car length", "Num Cars"}],
 Alignment → Center, Spacings → {2, 1}, Frame → All,
 ItemStyle → {"Text", {1 → {Bold, 14}}}]
```

Out[∘]=

Velocity	Braking Dist	Braking Dist with car length	Num Cars
5	6.25	19.25	23
10	15.	28.	31
20	40.	53.	33
30	75.	88.	30
40	120.	133.	26
50	175.	188.	23
60	240.	253.	21
70	315.	328.	19

Step Three: visualize the data

In[∘]:=
```
modelTwoLP = ListPlot[ModelTwoXandY, PlotRange → {{0, 80}, {0, 40}},
  AxesLabel → {"Velocity", "Num Cars"}, AxesStyle → Arrowheads[0.025],
  PlotLabel → Style["Easing Traffic Model Two", Blue, 16],
  PlotStyle → {Red, PointSize[0.017]}, Joined → True,
  ImageSize → Large, PlotMarkers → {Automatic, 11}]
```

Out[∘]=

In[∘]:= **From the list plot above, it is clear that the maximum number of cars that can go through tunnel per minute is approximately 34 at a speed of about 15 mph.**

In[∘]:=
```
givenFunc[15]
```

Out[∘]= 26.25

The 15 mph and the 34 cars per minute constraints translates to a separation distance of about

26.25 feet between each car.

Model Three

Assumes that the total separation distance includes the thinking distance only.

Note: I use the variable moreVel in the second and third positions of the table however, this value is actually meant to be the thinking distance. I simply used the moreVel variable because I thought it would be a waste to create another moreThinkingDist variable

Step One: obtain more values and construct the data table

```
In[∘]:= modelThreeDist = Transpose[{moreVel, moreVel, moreVel + 13}]
Out[∘]= {{5, 5, 18}, {10, 10, 23}, {20, 20, 33}, {30, 30, 43},
  {40, 40, 53}, {50, 50, 63}, {60, 60, 73}, {70, 70, 83}}
```

```
In[∘]:= numCarsModelThree = Round[numCarsPerMinFunc[moreVel, moreVel + 13]]
Out[∘]= {24, 38, 53, 61, 66, 70, 72, 74}
```

```
In[∘]:= modelThreeData = Transpose[{moreVel, moreVel, moreVel + 13, numCarsModelThree}]
Out[∘]= {{5, 5, 18, 24}, {10, 10, 23, 38}, {20, 20, 33, 53}, {30, 30, 43, 61},
  {40, 40, 53, 66}, {50, 50, 63, 70}, {60, 60, 73, 72}, {70, 70, 83, 74}}
```

```
In[∘]:= modelThreeXandY = Transpose[{moreVel, numCarsModelThree}]
Out[∘]= {{5, 24}, {10, 38}, {20, 53}, {30, 61}, {40, 66}, {50, 70}, {60, 72}, {70, 74}}
```

```
In[∘]:= Grid[Prepend[modelThreeData, {"Velocity", "Braking Dist (thinking only)",
    "Braking Dist (thinking only) with car length", "Num Cars"}],
  Alignment → Center, Spacings → {2, 1}, Frame → All,
  ItemStyle → {"Text", {1 → {Bold, 14}}}]
```

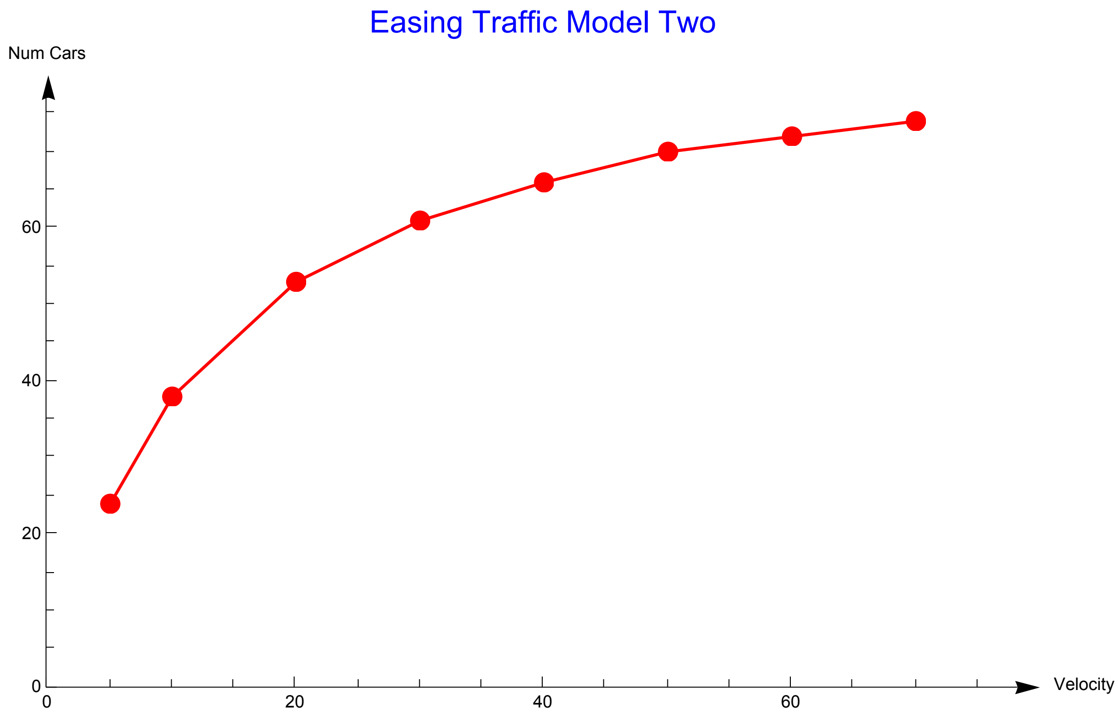
Out[∘]=

Velocity	Braking Dist (thinking only)	Braking Dist (thinking only) with car length	Num Cars
5	5	18	24
10	10	23	38
20	20	33	53
30	30	43	61
40	40	53	66
50	50	63	70
60	60	73	72
70	70	83	74

Step Two: visualize the data

In[∘]:=
```
modelThreeLP = ListPlot[modelThreeXandY, PlotRange → {{0, 80}, {0, 80}},
  AxesLabel → {"Velocity", "Num Cars"}, AxesStyle → Arrowheads[0.025],
  PlotLabel → Style["Easing Traffic Model Two", Blue, 16],
  PlotStyle → {Red, PointSize[0.017]}, Joined → True,
  ImageSize → Large, PlotMarkers → {Automatic, 11}]
```

Out[∘]=



From the plot above, it' s clear that the maximum cars per minute when their velocity is 70 mph is 70 feet, or about 5 car lengths. It seems extremely dangerous to go at such high speeds in a tunnel, so a new model is needed.

Model Four

Assumes that drivers do not keep the full stopping distance between themselves and the car. This means that the total separation is the thinking distance plus half the braking distance

Step One: create a function to find the braking distance based on velocity

In[∘]:=
```
brakingDistVel = Transpose[{velocity, brakingDist}]
```

Out[∘]=
```
{{30, 45}, {50, 125}, {70, 245}}
```

In[∘]:=
```
Grid[Prepend[brakingDistVel, {"Velocity", "Braking Only"}], Alignment → Center,
 Spacings → {2, 1}, Frame → All, ItemStyle → {"Text", {1 → {Bold, 14}}}]
```

Out[∘]=

Velocity	Braking Only
30	45
50	125
70	245

In[∘]:=
```
brakingOnlyLP = ListPlot[brakingDistVel, PlotRange → {{0, 100}, {0, 350}},
   AxesLabel → {"Velocity", "Distance"}, AxesStyle → Arrowheads[0.025],
   PlotLabel → Style["Easing Traffic Given", Blue, 16],
   PlotStyle → {Red, PointSize[0.017]}, ImageSize → Large];
```

In[∘]:=
```
Fit[brakingDistVel, {x, x^2}, x]
```

Out[∘]= $-1.24788 \times 10^{-15}\, x + 0.05\, x^2$

In[∘]:=
```
brakingDistFunc[x_] := 0.05 x^2
```

In[∘]:=
```
brakingDistFunc[50]
```

Out[∘]= 125.

In[∘]:=
```
moreBrakingDistOnly = brakingDistFunc[moreVel]
```

Out[∘]= {1.25, 5., 20., 45., 80., 125., 180., 245.}

In[∘]:=
```
modelFourNumCars =
 Round[numCarsPerMinFunc[moreVel, (moreVel + (0.5 * moreBrakingDistOnly)) + 13]]
```

Out[∘]= {24, 35, 41, 40, 38, 35, 32, 30}

Step Two: make a new data table using values for the newly created function

In[∘]:=
```
modelFourXandY = Transpose[{moreVel, modelFourNumCars}]
```

Out[∘]= {{5, 24}, {10, 35}, {20, 41}, {30, 40}, {40, 38}, {50, 35}, {60, 32}, {70, 30}}

In[∘]:=
```
modelFourData = Transpose[{moreVel, moreVel + (0.5 * moreBrakingDistOnly),
    (moreVel + (0.5 * moreBrakingDistOnly)) + 13, modelFourNumCars}]
```

Out[∘]=
```
{{5, 5.625, 18.625, 24}, {10, 12.5, 25.5, 35},
 {20, 30., 43., 41}, {30, 52.5, 65.5, 40}, {40, 80., 93., 38},
 {50, 112.5, 125.5, 35}, {60, 150., 163., 32}, {70, 192.5, 205.5, 30}}
```

```
In[∘]:= Grid[Prepend[modelFourData, {"Velocity", "Seperation (thinking - 0.5(braking))",
    "seperation with car length", "Num Cars"}], Alignment → Center,
  Spacings → {2, 1}, Frame → All, ItemStyle → {"Text", {1 → {Bold, 14}}}]
```

Out[∘]=

Velocity	**Seperation (thinking - 0.5(braking))**	**seperation with car length**	**Num Cars**
5	5.625	18.625	24
10	12.5	25.5	35
20	30.	43.	41
30	52.5	65.5	40
40	80.	93.	38
50	112.5	125.5	35
60	150.	163.	32
70	192.5	205.5	30

Step Three: visualize the data

```
modelFourLP = ListPlot[modelFourXandY, PlotRange → {{0, 70}, {0, 50}},
  AxesLabel → {"Velocity", "Num Cars"}, AxesStyle → Arrowheads[0.025],
  PlotLabel → Style["Easing Traffic Model Four", Blue, 16],
  PlotStyle → {Red, PointSize[0.017]}, Joined → True,
  ImageSize → Large, PlotMarkers → {Automatic, 11}]
```

Out[∘]=

This model gives a more reasonable speed of about 25 mph for the maximum flow. The

separation would be about 40 feet or roughly 3 car lengths, which is also reasonable.

Recommendation

Using model 4, a reasonable solution to the problem can be obtained. In order to ease the traffic that could occur within the narrow tunnel, the Traffic Manager should establish a maximum speed limit of 20 mph with a separation of 4 car lengths in front of each car.