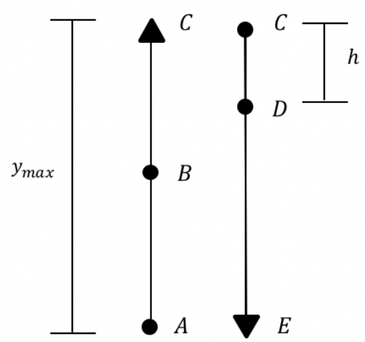
Description

One calm afternoon Calculus Cam decides to launch Hamster Huey into the air using a model rocket. The rocket is launched straight up off the ground, from rest. The rocket engine is designed to burn for specified time while producing non-constant net acceleration given by the equations below. After the engine stops the rocket continues upward in free-fall. A parachute opens after the rocket falls a specified vertical distance from its maximum height. When the parachute opens, assume the rocket instantly stops, and then increases speed to a terminal velocity given by the equation below. Assume the air resistance affects the rocket only during the parachute stage.

Diagram



Givens

$B\ coefficent = 0$
$C\ coefficent = 17$
$D\ coefficent = 8$
$t_{AB} = 3.9\ s$
$h = 80\ m$
$a_{BD} = -9.80\ m/s^2$
$v_T = -20\ m/s$

Equations

$a_{AB}[t] = -1.4t^2 + 17\ m/s^2$
$v_{DE}[t] = -20 + 20e^{\frac{-t}{8}}\ m/s$
$\Delta y_{CD} = \frac{1}{2}at^2 + v_{AB}t$
${v_E}^2 = {v_B}^2 + 2a\Delta y_{BC}$

Strategy

In order to simplify the problem, it's best to break it down into three stages. Stage AB is categorized by non-constant acceleration ($a_{AB}[t]$) caused by a rocket engine. Stage BD accounts for the motion when the engine shuts off and the rocket is accelerating at gravity ($-9.80\ m/s^2$) as a projectile. Lastly, stage DE accounts for the motion resulting from a parachute slowing down the rocket. The total time it takes for the rocket to complete this path can be calculated by summing up the times for each individual stage. To calculate the total time spent in stage BD, the final velocity and final position at stage AB is needed. This can be obtained by integrating the given acceleration function to produce the velocity and displacement functions. Because the rocket is moving as a projectile in Stage BD, the max height of its motion is calculated using EQ4. Then, EQ3 can be applied to calculate the total time spent in stage BD. The velocity at stage DE is given by the function $v_{DE}[t]$, which when integrated, gives the position as a function of time. Because the value for h is given, total time for this stage can be found by solving for t when $v_{DE}[t]$ equals Δy_{DE}. Lastly, all three times can be added up to find the total time the rocket is in the air.

Stage AB: Engine On

Integrate $a_{AB}[t]$ to find velocity as a function of time ($v_{AB}[t]$) and position as a function of time ($y_{AB}[t]$). Then, plug the given engine burn time into the position function and solve for the final height in stage AB. The constant C = 0 because the rocket starts from rest.

$$a_{AB}[t] = -1.4t^2 + 17\ m/s^2$$

$$\Delta v_{AB} = \int a_{AB}[t]\,dt$$

$$\Delta v_{AB}[t] = \int(-1.4t^2 + 17)\,dt, \text{ solver}$$
$$\Delta v_{AB}[t] = -\frac{7t^3}{15} + 17t + c$$

$$0 = -\frac{7(0)^3}{15} + 17(0) + c$$

$$0 = c$$

$$\Delta v_{AB}[t] = -\frac{7t^3}{15} + 17t$$

$$\Delta y_{AB} = \int v_{AB}[t]\,dt$$

$$\Delta y_{AB}[t] = \int(17t - \tfrac{7t^3}{15})\,dt, \text{ solver}$$

$$\Delta y_{AB}[t] = -\frac{7t^4}{60} + \frac{17t^2}{2}$$

$$\Delta v_{AB}[t_{AB}] = -\frac{7(3.9)^3}{15} + 17(3.9)$$

$$\boldsymbol{v_{AB}[t_{AB}] = 38.618\ m/s}$$

$$\Delta y_{AB}[t_{AB}] = -\frac{7(3.9)^4}{60} + \frac{17(3.9)^2}{2}$$

$$\boldsymbol{\Delta y_{AB}[t_{AB}] = \ 102.30\ \mathrm{m}}$$

Stage BD: Engine Off/Projectile

Use EQ4 to find the Δy from points BC. Add that value to the final height in stage AB (Δy_{AB}) to obtain the max height of the rocket. Lastly, use EQ3 to solve for time when given the Δy, acceleration, and velocity initial.

$$v_E{}^2 = v_B{}^2 + 2a\Delta y_{BC}$$

$$0 = (38.618)^2 + 2(-9.80)\Delta y_{BC}$$

$$-1491.35 = -19.60\Delta y_{BC}$$

$$\underline{76.089\ \mathrm{m} = \Delta y_{BC}}$$

$$\Delta y_{AC} = \Delta y_{AB} + \Delta y_{BC}$$

$$\Delta y_{AC} = 102.30 + 76.089$$

$$\underline{\Delta y_{AC} = 178.39\ m}$$

$$\Delta y_{CD} = \frac{1}{2}at^2 + v_{AB}t$$

$$(98.390 - 102.30) = \frac{1}{2}(-9.80)t^2 + (38.618)t$$

$$0 = -4.90t^2 + 38.618t + 3.91, \text{ solver}$$

$$t_{BD} = 7.9812\ s\ , \cancel{-0.09998}$$

Stage DE: Parachute

Integrate the terminal velocity function for the parachute to find position as a function of time. Then set the value of the position function equal to zero and solve for time.

$$v_{DE}[t] = -20 + 20e^{\frac{-t}{8}}\ m/s$$

$$\Delta y_{DE}[t] = \int v_{DE}[t]\,dt$$

$$\Delta y_{DE}[t] = \int \left(-20 + 20e^{\frac{-t}{8}}\right) dt$$

$$\Delta y_{DE}[t] = \ -20t - 160e^{\frac{-t}{8}} + c$$

$$98.390 = \ -20(0) - 160e^{\frac{-(0)}{8}} + c$$

$$258.39\ m = c$$

$$\Delta y_{DE}[t] = \ -20t - 160e^{\frac{-t}{8}} + 258.39$$

$$0 = \ -20t - 160e^{\frac{-t}{8}} + 258.39$$

$$t_{DE} = \cancel{-7.4941}, 10.861\ s$$

Final Step: Sum of Times

The total time the rocket spend in the air can be found by adding up the times of the individual stages

$$t_{total} = \ t_{AB} + \ t_{BD} + \ t_{DE}$$

$$t_{total} = 10.861 + 7.9812 + 3.9\ s$$

$$\boxed{\boxed{t_{total} = 22.742\ s}}$$

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Graphs: