Name ______________________

18.05 Exam 1

No books or calculators. You may have one 4×6 notecard with any information you like on it.
6 problems, 8 pages

Use the back side of each page if you need more space.

Simplifying expressions: You don't need to simplify complicated expressions. For example, you can leave $\frac{1}{4}\cdot\frac{2}{3}+\frac{1}{3}\cdot\frac{2}{5}$ exactly as is. Likewise for expressions like $\frac{20!}{18!2!}$.

Table of normal probabilities: The last page of the exam contains a table of standard normal cdf values.

Problem 0. (5 pts)
Turn in notecard. Note: Students were allowed to bring in a 4×6 notecard with anything they wanted on it. For 5 points it had to be their own work and contain the basic details from the unit including pdf, cdf, expected value, variance of the important distributions.

Problem 1. (20 pts: 4,4,4,8)
We asked 6 students how many times they rebooted their computers last week.
There were 4 Mac users and 2 PC users.
The PC users rebooted 2 and 3 times.
The Mac users rebooted 1, 2, 2 and 8 times.
Let C be a Bernoulli random variable representing the type of computer of a randomly chosen student (Mac = 0, PC = 1).
Let R be the number of times a randomly chosen student rebooted (so R takes values 1,2,3,8).

(a) Create a joint probability table for C and R. Be sure to include the marginal probability mass functions.

(b) Compute $E(C)$ and $E(R)$.

(c) Determine the covariance of C and R and explain its significance for how C and R are related. (*A one sentence explanation is all that's called for.*)
Are R and C independent?

(d) Independently choose a random Mac user and a random PC user.
Let M be the number of reboots for the Mac user
and W the number of reboots for the PC user.

(i) Create a table of the joint probability distribution of M and W, including the marginal probability mass functions.

(ii) Calculate $P(W > M)$.

(iii) What is the correlation between W and M?

Scores	
0.	______
1.	______
2.	______
3.	______
4.	______
5.	______
6.	______
	______

Put your answer to problem 1 on this page.

Problem 2. (8 pts)
Recall the relation between degrees Fahrenheit and degrees Celsius

$$\text{degrees Celsius} = \frac{5}{9} \cdot \text{degrees Fahrenheit} - \frac{160}{9}.$$

Let X and Y be the daily high temperature in degrees Fahrenheit for the summer in Los Angeles and San Diego. Let T and S be the same temperatures in degrees Celsius.

Suppose that $\text{Cov}(X, Y) = 4$ and $\rho(X, Y) = 0.8$. Compute $\text{Cov}(T, S)$ and $\rho(T, S)$ ($\rho(T, S)$ = correlation)

Problem 3. (16 pts: 8,8)
I have a bag with 3 coins in it. One of them is a fair coin, but the others are biased trick coins. When flipped, the three coins come up heads with probability 0.5, 0.6, 0.1 respectively.

Suppose that I pick one of these three coins uniformly at random and flip it three times.

(a) What is $P(HTT)$? (That is, it comes up heads on the first flip and tails on the second and third flips.)

(b) Assumimg that the three flips, in order, are HTT, what is the probability that the coin that I picked was the fair coin?

(Remember, there is no need to simplify fractions.)

Problem 4. (16 pts: 4,4,4,4)
You are taking a multiple choice test for which you have mastered 70% of the material. Assume this means that you have a 0.7 chance of knowing the answer to a random test question, and that if you don't know the answer to a question then you randomly select among the four answer choices. Finally, assume that this holds for each question, independent of the others.

(a) What is your expected score (as a percent) on the exam?

Let p be the probability of getting a random question correct. You likely found p in part (a), but in any case you should assume $0.7 < p < 0.9$.

In parts (b), (c) and (d) you can just use the letter p for this probability.

(b) If the test has 10 questions, what is the probability you score 90% or higher? *(Do not simplify the expression you get.)*

(c) What is the probability you get the first 6 questions on the exam correct? *(Again, do not simplify the expression you get.)*

(d) Suppose you need a 90% to keep your scholarship. Would you rather have a test with 10 or 100 questions? Why? (*Your answer only needs to be one sentence long.*)

Problem 5. (20 pts: 4,4,4,4,4)
Let X have range [0,3] and density $f_X(x) = kx^2$. Let $Y = X^3$.

(a) Find k and the cumulative distribution function of X.

(b) Find the 30$^{\text{th}}$ percentile of X.

(c) Compute $E(Y)$.

(d) Write down an explicit formula, involving an integral, for $\text{Var}(Y)$.
(Do not compute the value of the integral.)

(e) Find the probability density function $f_Y(y)$ for Y.

if Fx known, might be easier to calculate Fy

Problem 6. (15 pts: 10,5)
J & M have their child in daycare twice a week. Being busy people they are often a few minutes late to pick her up. The daycare has a strict policy that parents need to be on time. They enforce this by charging $1 per minute for tardiness.

Suppose that each day the amount of time in minutes that they are late follows an exponential distribution with mean 6.

(a) Their child will be in daycare for 100 days this year. Estimate the probability that they will pay more than $630 in late fees?

(b) The late fees were not effective in getting J & M to arrive on time, so the daycare changed the rate to $t^2 + t$ dollars for t minutes of tardiness. On average, how much will J & M pay in late fees each day?

For part (a) we want a numerical answer. For part (b) leave your answer as an integral. Do not compute it out.

Table of Standard Normal Cumulative Probabilities

z	-3.0									
$\Phi(z)$	0.001									
z	-2.9	-2.8	-2.7	-2.6	-2.5	-2.4	-2.3	-2.2	-2.1	-2.0
$\Phi(z)$	0.002	0.003	0.003	0.005	0.006	0.008	0.011	0.014	0.018	0.023
z	-1.9	-1.8	-1.7	-1.6	-1.5	-1.4	-1.3	-1.2	-1.1	-1.0
$\Phi(z)$	0.029	0.036	0.045	0.055	0.067	0.081	0.097	0.115	0.136	0.159
z	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0.0
$\Phi(z)$	0.184	0.212	0.242	0.274	0.309	0.345	0.382	0.421	0.460	0.500
z	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\Phi(z)$	0.500	0.540	0.579	0.618	0.655	0.691	0.726	0.758	0.788	0.816
z	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
$\Phi(z)$	0.841	0.864	0.885	0.903	0.919	0.933	0.945	0.955	0.964	0.971
z	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9
$\Phi(z)$	0.977	0.982	0.986	0.989	0.992	0.994	0.995	0.997	0.997	0.998
z	3.0									
$\Phi(z)$	0.999									