Measuring the Charge-to-Mass Ratio of an Electron

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March 22, 2018

Date Performed:	February 9, 2018
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1 Introduction

An electron is a subatomic particle. It has a charge, called the elemental charge. It also has mass. Both of these are very small and thus not particularly easy to measure with standard measuring apparatus. That is, one might find it rather difficult to measure the mass of an electron on a standard kitchen scale. Rather, it is more natural to find some more easily measurable occurrence which depends on the properties of electrons and from them infer these properties. Chief among the properties of an electron that one might want to measure are the aforementioned charge and mass.

Historically, the first property of an electron to be measured was actually neither charge nor mass. In 1897, J. J. Thomson, discoverer of the electron, first characterized the charge-to-mass of this newly discovered particle. About 10 years later, American physicist Robert Millikan calculated the charge, and thus, using Thomson's charge-to-mass ratio, the mass, of an electron. Later experiments yielded similar values for other electrons.

Thomson's experiment used a cathode ray tube, a vacuum tube containing an electron gun. This cathode ray tube produces a stream quickly moving electrons. Being charged particles under motion these electrons are liable to be acted upon by a magnetic field. Actually, this is a method by which one may show that electrons have negative charge. If one is careful, and one chooses to apply a magnetic field which is perpendicular to the direction of the electrons' motion, then one may assume a constant force due to magnetism acting on the electrons perpendicular to the direction of both the electron beam and the magnetic field. This will bend the electron beam into a circular arc whose radius is dependent on several properties, one of which is the charge-to-mass ratio of the electron.

2 Procedure

An electron beam was placed within a glass tube of low pressure, in side of which was helium gas which ionized and thus became visible when passed through by the electron beam. This glass tube had a spherical bulb in which the electron beam was to rotate. The magnetic field was generated by a pair of Helmholtz coils with 130 loops and a current passed through them. This bent the electron beam into a circle of diameter between 5 and 11 cm. A small glass ruler was inside of the glass bulb to measure the diameter of the electron beam radius. The diameter of the loop was measured at various currents voltages and orientations. Currents ranged from approximately 1 to 2.5 A, while the experiment was done with voltages of 200, 250, and 300 V. These were done in easterly, westerly, northerly, and southerly directions to account for variation in the magnetic field of the earth.

Figure 1: The magnetic field from a pair of Helmholtz coils causes the electron beam to move in a circle

Figure 2: Fitting Parameter for Inverse Relationship

Trial	Orientation 1 (A m)	Orientation 2 (A m)
(200 V E-W)	0.1255 ± 0.005	0.1233 ± 0.008
(200 V N-S)	0.1224 ± 0.008	0.1282 ± 0.004
(300 V N-S)	0.1558 ± 0.003	0.1611 ± 0.004
(150 V N-S)	0.1058 ± 0.005	0.1094 ± 0.004

3 Data

Initially the current through the Helmholtz coils was varied such that the diameter of the electron circle fell on each half inch interval. After this, the apparatus was rotated 180 degrees and the diameter was measured at the currents used in the previous orientation.

These points seem to signal an inverse relationship between the diameter of the loop of current and the current through the coil. One might try to fit an inverse curve of the sort A/x to the data.

These might be used to find the charge-to-mass ratio as demonstrated later on.

The radius and separation between the Helmholtz coils was measured to be 15.3 ± 0.3 cm.

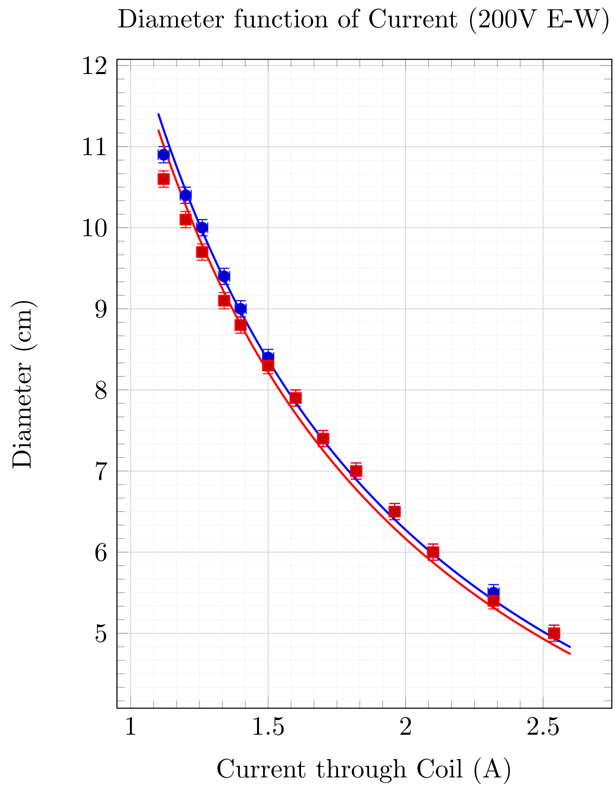
4 Analysis

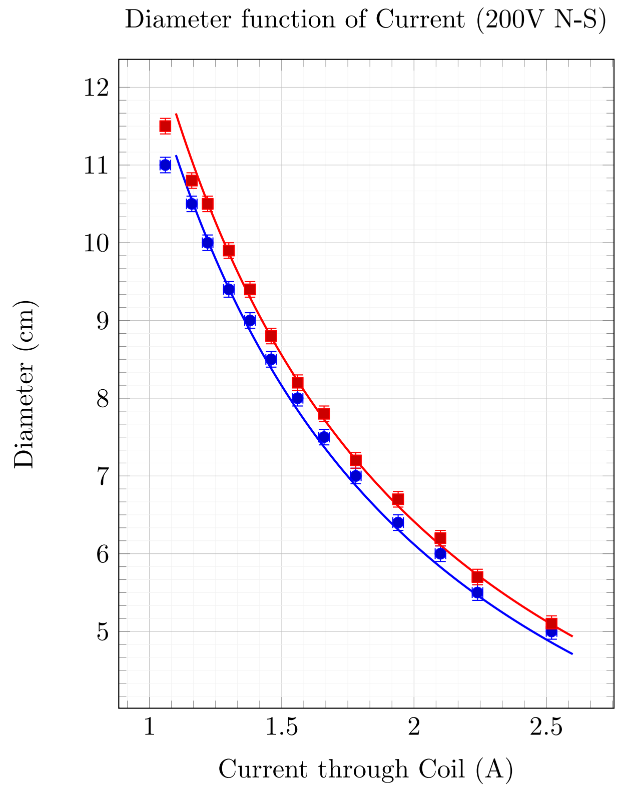
One should be able to find the charge-to-mass ratio function of the diameters and currents measured above. Consider a beam of electrons at velocity $\mathbf{v}$ moving perpendicular to a magnetic field $\mathbf{B}$. Each of these electrons should have some unknown charge q. The force from the magnetic field acting on these electrons should hence be

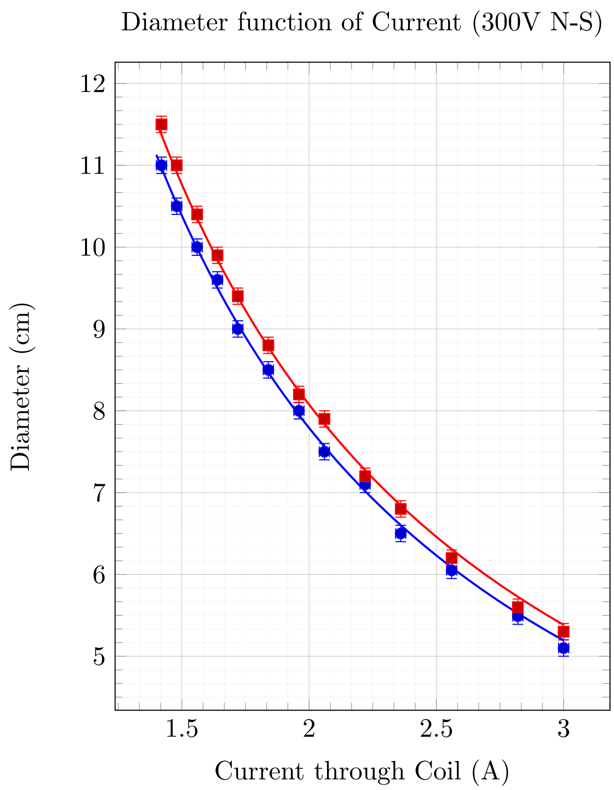
$$\mathbf{F} = q\,\mathbf{v} \times \mathbf{B} \tag{1}$$

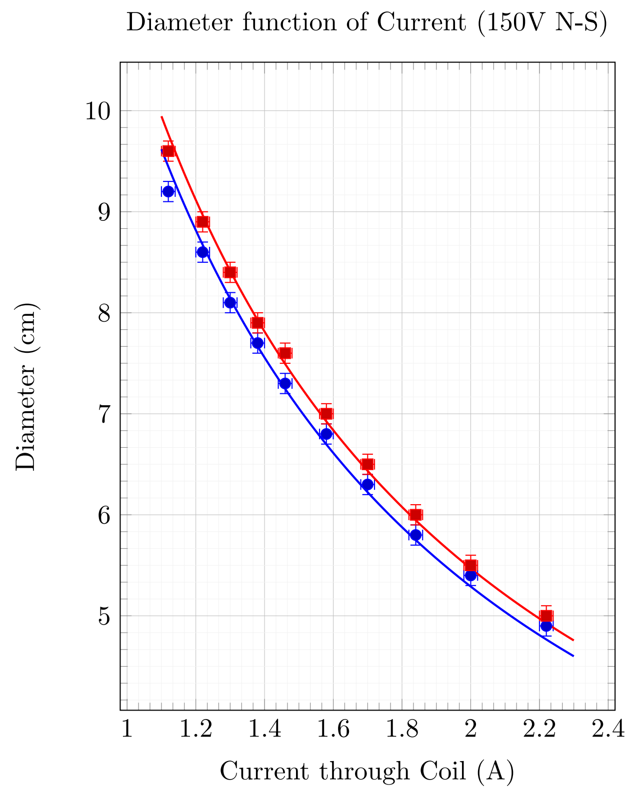
Notice that these electrons are moving in a circular path. Recall that the force required perpendicular to the direction of motion is described as

$$\mathbf{F} = \frac{m\,\mathbf{v}^2}{r}\hat{\mathbf{r}} \tag{2}$$


Diameter function of Current (200V E-W)
Diameter (cm)
Current through Coil (A)
12
11
10
9
8
7
6
5
1
1.5
2
2.5


Diameter function of Current (200V N-S)
Diameter (cm)
Current through Coil (A)
12
11
10
9
8
7
6
5
1
1.5
2
2.5


Diameter function of Current (300V N-S)
Diameter (cm)
Current through Coil (A)
12
11
10
9
8
7
6
5
1.5
2
2.5
3


Diameter function of Current (150V N-S)
Diameter (cm)
Current through Coil (A)
10
9
8
7
6
5
1
1.2
1.4
1.6
1.8
2
2.2
2.4

One might set these forces equal to each other to conclude that

$$q\,\mathbf{v}\times\mathbf{B} = \frac{m\,\mathbf{v}^2}{r}. \tag{3}$$

And from there one is left to solve for the charge-to-mass ratio.

$$\frac{q}{m} = \frac{v}{r\,\mathbf{B}} \tag{4}$$

This is a wonderful little expression. The trouble is that it isn't particularly useful, for one the speed at which the electrons move is unknown. However it may be rearranged to be more useful. Consider the energy of the electrons. The gain of energy from an electron is simply the product of the potential difference V and the charge q. While the kinetic energy is $m\,v^2/2$ setting these equal allows one to solve for the velocity.

$$V\,q = \frac{m\,v^2}{2} \tag{5}$$

$$v = \sqrt{\frac{2V q}{m}} \tag{6}$$

substituting this into the previous expression yields the somewhat nasty expression.

$$\frac{q}{m} = \frac{1}{r\,\mathbf{B}}\sqrt{\frac{2V q}{m}} \tag{7}$$

This expression is unfortunately in terms of q/m the very ratio we wish to find. Fortunately we may square this equality and simplify.

$$\frac{q^2}{m^2} = \frac{2\,V\,q}{r^2\,\mathbf{B^2}\,m} \tag{8}$$

$$\frac{q}{m} = \frac{2\,V}{r^2\,\mathbf{B^2}} \tag{9}$$

This is a nice, convenient expression, but it relies on knowledge of $\mathbf{B}$. This magnetic field comes from a pair of Helmholtz coils, luckily there is a expression for the magnetic field from this configuration.

$$\mathbf{B} = \left(\frac{4}{5}\right)^{3/2}\frac{\mu_0\,n\,I}{R} \tag{10}$$

Where μ_0 is the permeability of free space, n is the number of loops in the Helmholtz coil, I is the current passing through the coils, and R is both the radius of the coils and the separation between the two. Substituting this into the expression for the charge-to-mass ratio yields .

$$\frac{q}{m} = \frac{125\,V\,R^2}{32\,r^2\,\mu_0^2\,n^2\,I^2} \tag{11}$$

Now, one might try look at the function of current to radius, as these were the quantities directly measured in the experiment. Solving for radius yields

$$r = \frac{R}{\mu_0\,n\,I}\sqrt{\frac{125\,V\,m}{32\,q}} \tag{12}$$

And hence the diameter would be

$$d = \frac{R}{\mu_0\,n\,I}\sqrt{\frac{125\,V\,m}{8\,q}} \tag{13}$$

This would mean that the current through the Helmholtz coils and the diameter of circle measured will be related by an inverse relationship. One could attempt to fit the data on a current diameter graph with a simple relationship

$$d = \frac{A}{I} \tag{14}$$

where A is some constant whose value should be equal to

$$A = \frac{R}{\mu_0\, n}\sqrt{\frac{125\, V\, m}{8\, q}} \tag{15}$$

solving for the the ratio q/m

$$\frac{q}{m} = \frac{125\, R^2\, V}{8A^2\, \mu_0^2\, n^2} \tag{16}$$

4.1 Uncertainty

One must also attempt to quantify the uncertainty in the above expression. The natural way to do this is to look at the contribution each of the variable parameters makes to the above expressions. This is done by taking the partial derivative of q/v with respect to the variable parameters A,R, and V.

$$\frac{\partial q/m}{\partial A} = \frac{-2\, q}{A\, m} \tag{17}$$

$$\frac{\partial q/m}{\partial R} = \frac{2\, q}{R\, m} \tag{18}$$

$$\frac{\partial q/m}{\partial V} = \frac{q}{V\, m} \tag{19}$$

These may be combined with and the square root of the sum of their squares may be used to quantify the uncertainty in the overall expression

$$\delta_{q/m} = \frac{q}{m}\sqrt{\frac{4\delta_A^2}{A^2} + \frac{4\delta_R^2}{R^2} + \frac{\delta_V^2}{V^2}} \tag{20}$$

5 Conclusion

The above expression as well as the fitting parameters calculated for the graphs in section 3 may be used to quickly find the charge-to-mass ratio for each of the orientations and voltages. These figures

Figure 3: Charge-to-mass ratio

Trial	Orientation 1 (C/kg $\cdot 10^{11}$)	Orientation 2 (C/kg $\cdot 10^{11}$)
(200 V E-W)	1.74 ± 0.15	1.80 ± 0.24
(200 V N-S)	1.83 ± 0.25	1.67 ± 0.12
(300 V N-S)	1.69 ± 0.09	1.59 ± 0.1
(150 V N-S)	1.84 ± 0.19	1.72 ± 0.14

seem relatively close to the $1.76 \cdot 10^{11}$C/kg[1] accepted value for the charge to mass ratio of the electron. However, it is useful to be able to point to a single number for the calculated charge-to-mass ratio of the electron. To do this, one may take a simple average of the above estimates. However, it is useful to take a weighted average and to give larger weights to estimates with smaller variance. In this case,

[1]2014 CODATA

a weight proportional to the inverse variance was assigned to each of the estimates. This process yields an estimate of $1.69 \pm 0.05 \cdot 10^{11}$C/kg, an error of 3.4%. One might notice that the estimates for the charge-to-mass ratio done while facing in a northerly direction are consistently larger than those measured when facing a southerly direction. This is consistent with the wildly held belief that compasses tend to point north. That is, the earth has a magnetic field which points northward. This magnetic field contributes to that of the Helmholtz coil, and when facing north results in a larger magnetic field. This will result in an overestimation of the charge to mass ratio when facing north, and an underestimate when facing south. Interestingly this isn't held when comparing the east-west orientations. There actually is a small easterly component to the magnetic field in Chapel Hill, however this isn't represented in the data. However, it is worth noting that these values are very close to each other and each have a very large uncertainty.

The relatively small error present in the estimate, however it may be explained by some of the qualities if the experiment and apparatus. For example, the coils used to create the magnetic field were not quite circular and were actually rather squished. This will have an effect of the estimate for the charge-to-mass ratio. Also, the electron beam was not actually a thin beam, and thus measuring the point at which the beam hits the glass rod is not easy to accurately measure. It is also possible that the viewer didn't look at the rod and thus had some parallax distortion. Also, it is possible that the moving electron beam, a current, created its own small magnetic field which had a very small effect on the path of them.