Ohm's and Kirchoff's Laws

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1 Omh's Law

1.1 Introduction

Electrical potential measures the potential energy of a unit positive charge within a particular electric field. Electrical current is a flow of electric charge, that is current measures the amount of charge passing through a point per unit time. Electrical resistance measures the difficulty of an electrical current to pass though a conductor. Ohm's law predicts a relationship between these 3 quantities.

$$I = \frac{V}{R} \tag{1}$$

where I is the electrical current, V is the electrical potential, and R is resistance. In essence, Ohm's law says that the rate at which charge flows is proportional to the difference in electrical potential with some proportionality constant, or conductivity, $1/R$.

If this proportionality constant R is truly constant, one would expect to the relationship between the current I and the voltage V to be linear. If this is relationship holds, we call the conductor *ohmic*. If this resistance R is not truly constant we say that the resistor is *non-ohmic*.

1.2 Procedure

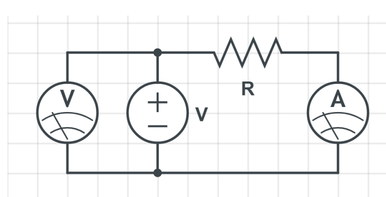


Figure 1: Depiction of Method of Measuring Current and Voltage

In order to verify that the relationship between voltage and current are indeed linear one may measure the current at through a resistor at various different voltages. A variable power supply was attached to a fixed resistor. The current through the resistor was measured with a digital ammeter connected in series with the resistor. The voltage drop across the power supply was also measured with a digital voltmeter. The variable power supply initially provided a potential difference of 0 V, this voltage was gradually increased from 0 V to 5 V over the course of about 50 seconds. The voltage was then decreased gradually from 5 V to 0 V over the course of another 50 s. Throughout the process, 3 data points (current and voltage) were collected every other second. A light bulb was used instead of a fixed resistance resistor for model a *non-ohmic* resistor. The bulb was positioned in the place where the resistor had been previously, and the voltage was slowly raised from 0 V. In order to avoid damaging the bulb, it was only raised to approximately 2.5 V.

1.3 Data

The procedure above was used to collect about 180 data points for each of the resistors and bulbs. Examining these shows that at least for 2 of the resistors the data seems extraordinarily linear. In fact, for the resistors with resistance 555 Ω and 2.63 kΩ the correlations for a linear fit are 0.998 and 0.997 respectively. Both also seem to be close to a proportional fit as well, that is, when the the voltage is zero the current seems to be zero as well. Examining the linear regression confirms this, the y-intercepts of the graphs are $-8 * 10^{-5} \pm 2 * 10^{-5}$ A and $-9 * 10^{-5} \pm 5 * 10^{-5}$ A respectively. This small deviation may be explained by a slight miscalibration in the current measuring probe.

The data for the other two schemes is not nearly as nice. For the largest resistor (4.68 kΩ) the current measured for voltages under 1.5 V fluctuated wildly. Perhaps because the current was so small (under 0.3 mA) the current probe was not able to accurately measure the current through the resistor. However, this range did not prove to be a major issue for the second resistor which also briefly included very small currents. After this point the data proved to be linear.

The bulb, as predicted was *non-ohmic*, the currents measured did not related proportionally with the voltage rather, the rate at which current increased decreased as voltage increase.

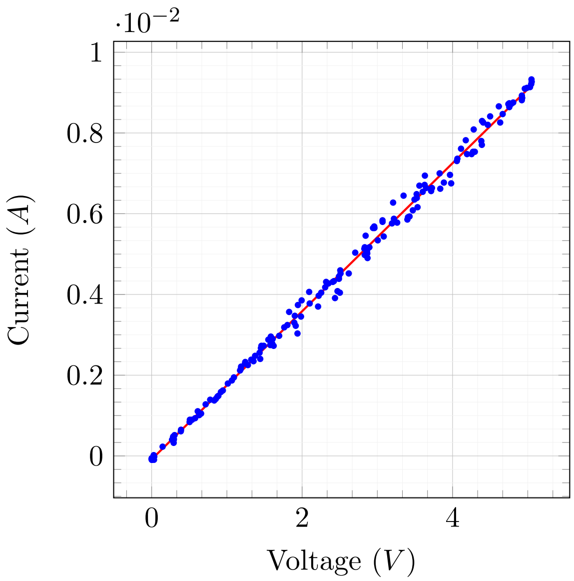
For the resistors, the slopes of the graphs and their inverses may be used to compute the overall resistance

Resistor	Slope (Ω^{-1})	Resistance (Ω)
555 $\pm 2\Omega$	$1.834 * 10^{-3} \pm 8 * 10^{-6}\ \Omega^{-1}$	$545 \pm 2\ \Omega$
2.63 $\pm 0.2k\Omega$	$3.76 * 10^{-4} \pm 2 * 10^{-6}\ \Omega^{-1}$	$2.66 \pm 0.01\ k\Omega$
4.68 $\pm 0.2k\Omega$	$1.94 * 10^{-4} \pm 7 * 10^{-6}\ \Omega^{-1}$	$5.1 \pm .2\ k\Omega$

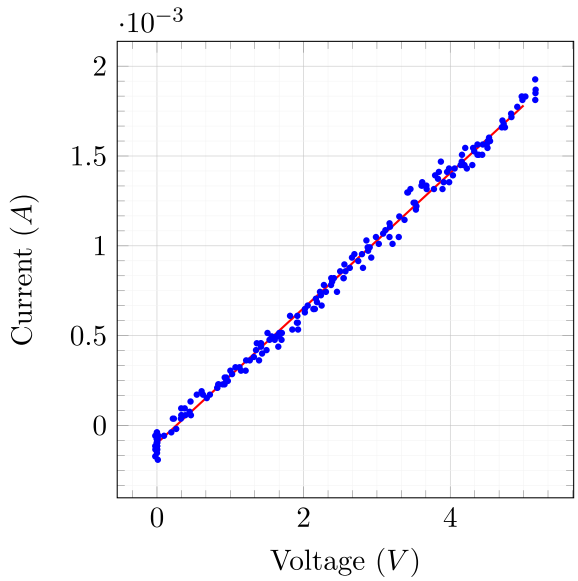
1.4 Analysis

Ohm's Law, $V = IR$ effectively models the current-voltage relationship for the purpose built resistors. However, being *non-ohmic*, the bulb is more difficult to model. Ultimately the non-linearity of the bulb's curve stems from its constant change in temperature. A change in

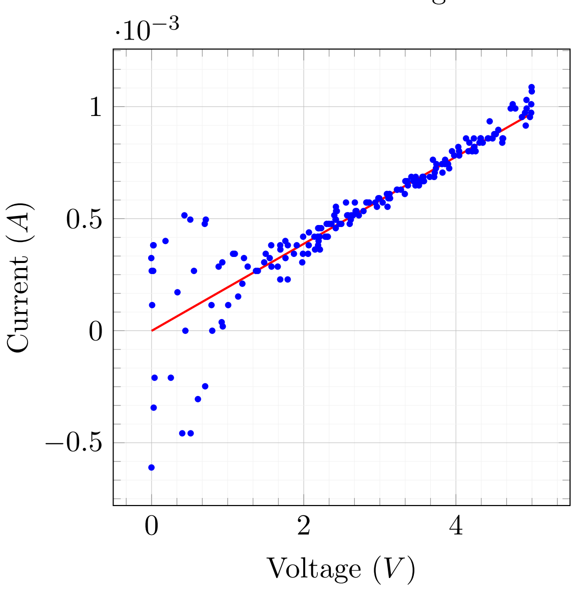
Current through Resistor (555 Ω)
at variable voltage



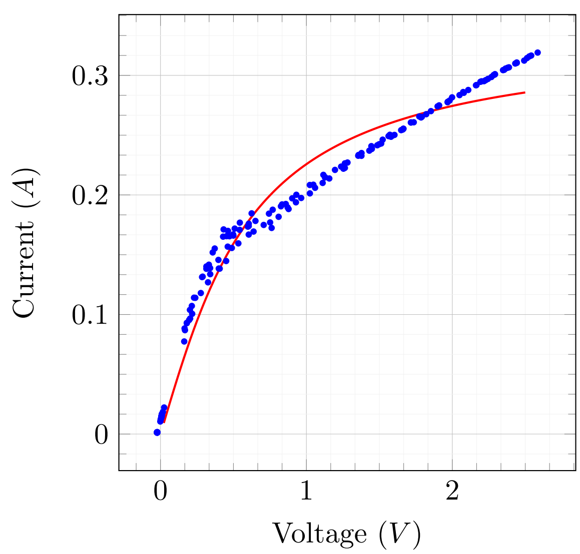
Current through Resistor (2.63 kΩ)
at variable voltage



Current through Resistor (4.68 kΩ)
at variable voltage



Current through Bulb
at variable voltage



temperature brings about a change in resistance. For most materials this means an increase in temperature will result in an increase in resistance. Typically this relationship is described as

$$R = R_0(1 + \alpha(T - T_0)) \tag{2}$$

where T is temperature and α is some proportionality constant called the temperature coefficient of resistance.

One might assume that the increase in temperature of the bulb is proportional to its power. The power dissipated by the bulb is

$$P \propto I^2 R \tag{3}$$

This means that the resistance should be expressible as

$$R = R_0(1 + \alpha\, I^2\, R) \tag{4}$$

It is convenient to solve for current in terms of voltage rather than resistance. Thus we may subsistent R for V/I.

$$\frac{V}{I} = R_0(1 + \alpha\, V\, I) \tag{5}$$

Solving for current yields the somewhat clunky expression

$$I = \frac{-R_0 + \sqrt{4\, R_0\, \alpha\, V^2 + R_0^2}}{2\, R_0\, \alpha\, V} \tag{6}$$

This should match the data collected for the bulb, however the fit seems quite disappointing. It generally captures the trend of graph but seems to curve more than the true data. The coefficients are found to be

$$R_0 = 2.42 \pm 0.6\Omega$$

$$\alpha = 3.6 \pm 0.2\; C^{-1}$$

1.5 Conclusions

The three resistors measured followed the trend of Ohm's law incredibly well. The current related to voltage in a linear relation. The resistances calculated using the linear fit to the data come relatively close to the measured resistances for each of the resistors. Resistor 1 was measured to be $555 \pm 2\Omega$ while the linear fit predicted it to be $545 \pm 2\Omega$. This constitutes an error of about 1.8 %. The second resistor had similarly good results. It was measured to be $2.63 \pm 0.02\; k\Omega$ while the data suggested a more appropriated value of $2.66 \pm 0.02\, k\Omega$, constituting an error of closer to 1.1%. While these values are quite close to their expected values, the third resistor is much farther off, measured at $4.68 \pm 0.02\; k\Omega$ while the data would suggest that the resistance is closer to $5.1 \pm 0.2\; k\Omega$ is error is a much larger 8.9%. Part of the increased error in resistor 3 may be explained by the issues with current measurement. While the other issues in measurement that account for the 1 - 2 % error in the others may be explained by some small internal resistance in the ammeter, some small amounts of current passing through the voltmeter and other issues in measurement.

The fit of the curve on the bulb is somewhat more troubling, however some of its short comings may be explained by the model. The model used for the fit assumes that the temperature of the bulb is directly proportional to the power dissipated. This is a naive interpretation as the bulb will of course have some sort of thermal inertia. That is, the bulb will take time to reach its stable temperature at any given voltage, however the voltage was changed rather rapidly and the bulb was not given sufficient time to adjust after each change in voltage.

The parameters for the fit have some meaning as well.R_0 is meant to represent the resistance of the bulb at room temperature. This would imply that the resistance of the bulb at this temperature is $2.42 \pm 0.6\Omega$. This value seems to agree visually with the inverse slope of the current voltage curve around zero which appears to be in the 2.5-3 Ω range. The calculated 2.42 value does not fall within this range bt has a very broad uncertainty which certainty pushes the value into this range. The α parameter was meant to represent the temperature coefficient of the material, however one might notice that $3.6 \pm 0.2\ C^{-1}$ is nowhere near the temperature coefficient of tungsten, $0.0044C^{-1}$ [1] However, this was based on the naive assumption that one Watt of power would result in a degree increase in temperature, this is simply not the case. Instead, alpha is the product of the temperature coefficient of tungsten and the ratio of watt power to degree Celsius. Dividing α by the temperature coefficient of tungsten should yield the amount of power required to increase the temperature of the resistor by one degree Celsius. This value is $810 \pm 50W/C$

2 Kirchoff's Laws

2.1 Introduction

Kirchoff's Laws are a series of equabilities that describe the distribution of current and electric potential within a circuit. Essentially they state that the voltage across any closed loop is 0, and that the current through any junction is 0. One may use these principles to determine the resistance, current, and voltage of any given circuit. For example, consider 3 resistors in series, R_1, R_2, R_3 and an ideal voltage source V. Because the current through a closed loop must be equal to 0, we know that the equality $V - IR_1 - IR_2 - IR_3 = 0$ must hold. This may be factored to show that $V - I(R_1 + R_2 + R_3)$. This shows that resistors effectively add in series. Similarly consider two resistors R_1, R_2 attached in parallel to an ideal voltage source V. Because currents must sum to zero in a junction

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}. \tag{7}$$

This shows that in parallel resistors are the inverse of the sum of the inverses.

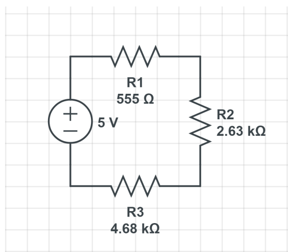
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}. \tag{8}$$

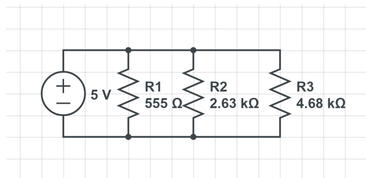
2.2 Procedure

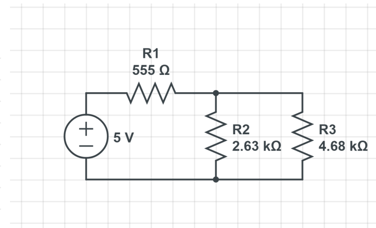
In order to demonstrate that Kirchoff's laws do in fact hold a series of circuits were arranged and the currents, voltages and resistances were measured at a variety of points. Three

[1] https://www.allaboutcircuits.com/textbook/direct-current/chpt-12/temperature-coefficient-resistance/

resistors, 555, 2630 and 4680 Ω were arranged in three separate schemes. First they were arranged in series, then all in parallel, and eventually in a combined scheme were resistors were both in series and in parallel. These were all connected to a 5 V ideal voltage supply. The three arrangements are pictured below.







2.3 Data

The voltage current and resistance were measured for each circuit.

Figure 2: Measurements of Series Circuit

Location	Resistance kΩ	Current (mA)	Voltage (V)
R_1	0.555	6.38	0.357
R_2	2.63	6.38	1.69
R_3	4.68	6.38	3.03
Total	7.92	6.38	5.07

Figure 3: Measurements of Parallel Circuit

Location	Resistance kΩ	Current (mA)	Voltage (V)
R_1	0.555	8.97	5.07
R_2	2.63	1.862	5.07
R_3	4.68	1.057	5.07
Total	0.419	11.85	5.07

The resistances under 2000 Ω are measured to the nearest 2 Ω while those above 2000 are measured to the nearest 20 Ω. Currents are accurate within 0.5 % of the stated value. Voltages under 2 V measure to the nearest millivolt while those above 2 V are accurate the the nearest 10 mV.

2.4 Analysis

2.4.1 Series Circuits

Kirchoff's Laws make several statements about properties that should hold in a series circuit. The most obvious of switch is simply the loop rule, that is, the voltages through each of the resistors should sum to the voltage drop across the power supply. Simply put

$$V = V_1 + V_2 + V_3. \tag{9}$$

Figure 4: Measurements of Combined Circuit

Location	Resistance kΩ	Current (mA)	Voltage (V)
R_1	0.555	2.24	1.254
R_2	2.63	1.140	3.81
R_3	4.68	0.795	3.81
Total	2.27	2.24	5.07

One may extend this notion to the resistances as was done in the introduction. Alternatively, the effective resistance across the circuit is equal to the sum of the sub resistors.

$$R = R_1 + R_2 + R_3. \tag{10}$$

Because there exist no junctions in the circuit the current through each of the components should remain constant.

2.4.2 Parallel Circuits

In a parallel circuit voltages should be constant as each of the components form their own loop with the voltages source. This may be expressed as

$$V = V_1 = V_2 = V_3. \tag{11}$$

Each of these voltages should be equal to the product of the current through the circuit and the resistance.

In parallel, resistances sum as the inverse of the sum of the inverses, this is simply a corollary of the loop rule and Ohm's law.

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \tag{12}$$

From these one may deduce the currents through each of the 3 resistors. By Ohm's law the current through a resistor with a resistance R and voltage V is a V/R.

$$I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3} \tag{13}$$

2.4.3 Combined Circuit

This circuit is a little bit more complicated than the previous two.

Being in parallel, the voltages through R_2 and R_3 should be equal and these should sum with V_1 to get the sum V.

$$V = V_1 + V_2 = V_1 + V_3 \tag{14}$$

The overall resistances are summed both as in series and as in parallel.

$$R = R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}} \tag{15}$$

The junction present in the circuit should obey Kirchoff's junction rule.

$$I = I_1 = I_2 + I_3 \tag{16}$$

2.5 Conclusions

All of the rules in the above section are obeyed by the three circuits.

2.5.1 Series Circuit

Resistance In a series circuit the resistances sum to give a single effective resistance. Summing the measured resistances yield a effective resistance of 7.87 ± 0.03kΩ. This summed value is just under the measured total resistance 7.92 ± 0.02kΩ. These values are just within each others uncertainty and have a total difference of 0.7 %.

Current Having computed the overall total resistance and measured the voltage though the power supply one may find the current through the whole system. Dividing the measured voltage by the calculated resistance yields a calculated current of 6.44 ± 3 mA. This is under measured current of 6.38 ± 0.03 mA and off by about 0.9%. This current should and did remain constant across the whole circuit.

Voltage The voltage drop through each resistor should be equal to the current though the resistor multiplied by its resistance. The calculated voltages are 0.355 ± 0.02 V, 1.69 ± 0.02 V, and 3.01 ± 0.02 V. These are remarkably close to the measured voltages. In fact, 2 of these values are the exact voltages measured. The only one that is off is the third voltage which is of by a mere 0.6 %. These voltages sum to 5.06 ± 0.03 V. This is an error of merely 0.2%. This confirms Kirchoff's law.

2.5.2 Parallel Circuit

Resistance In a parallel circuit, the resistances sum as the inverse of the sum of the inverses. The total effective resistance is computed to be $417 \pm 2\Omega$. This is within 0.5 % of the measured value.

Current One may use Ohm's law to compute the ideal currents across each of the resistors. This calculation shows that the currents through each of the resistors should be 9.14 ± 0.05 mA, 1.93 ± 0.02 mA, 1.083 ± 0.007 mA. These are generally good estimates but represent a overestimation of the measured currents by 1.9%, 3.6%, and 2.7%.

Voltage In parallel, voltage is equal, thus it makes sense that all measured voltages were the same as the total voltage measured.

2.5.3 Combined Circuit

Resistance In this combined circuit, the effective total resistance may be found by combining the methods for parallel and series circuits. Doing this will yield an calculated effective resistance of 2.24 ± 0.01 kΩ. This is 1.3 % below the measured value of 2.27 kΩ.

Current The current through the first resistor should be the same as the current through the whole system. Thus one can find this simply using Ohm's law. This calculation would conclude the current though the system to be 2.26 ± 0.01 mA, 0.9% higher than the measured value of 2.24mA. The other two currents should sum to get this previous current (Loop Rule) and should also be inversely proportional to the resistance through the resistors (Ohm's Law). This rules may be rearranged to say that

$$I_2 = \frac{IR_3}{R_2 + R_3},\ I_3 = \frac{IR_2}{R_2 + R_3} \tag{17}$$

These equations predict the currents through the resistors to be 1.447 ± 0.008 mA and 0.813 ± 0.006mA. The later of these values is an overestimation of 2.2%. But the first value is more troubling. Taken as written these predict an error of over 25%. This is troubling and doest fit well with the other data taken. It is possible and very likely that some kind of error in transcription occurred and the measured value was perhaps 1.410 mA or 1.440 mA which would better fit with the remaining data. However, the original manuscript is no longer in the possession of the author who can do no better than guess at the true value. These guessed values provide errors of 2.5% (for 1.410 mA) or 0.3% for (1.440 mA). The previous errors indicate that 1.410 is the most likely answer.

Voltage With calculated currents and measured resistances one need only take a product to find the voltages across each resistor. One should expect the second two voltages to be the same. The first voltage was found to be 1.254 ± 0.07 V, exactly the measured voltage. The second was found to be 3.81 ± 0.04 V, also exceptionally close to the measured voltage. The third and final was found to be 3.81 ± 0.05 V, like the other 3 accurate to within the significant figures of the measurement.

2.5.4 General Conclusions

The measurements gathered here agreed with both Kirchoff's laws and with Ohm's law. All values predicted were exceptionally close to their measured values, and the variability apparent is rather easily explained. Resistance was consistently underestimated, this means that there is some agent acting to increase the resistance of the system without our knowledge. This stems from the fact that the voltages source, resistance probe, and wires have some small unaccounted for resistance. The underestimation in resistance accounts for the constant over estimation in current. Voltage was consistently accurate.