PROGRESS REPORT: 2013-2014

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Research Description

My primary goal is to make Abelian functions and Riemann surfaces, two mathematical objects seen in a very wide variety of applications, as computationally accessible as trigonometric and hyperbolic functions. Doing so would provide to the scientific community an essential ingredient in finding large families of solutions to partial differential equations arising in a variety of fields including plasma physics, nonlinear optics, and water waves.

Research Update

At the start of this academic year I completed the first major feature of my software ABELFUNCTIONS: the calculation of period matrices of Riemann surfaces. Period matrices are important not only in applications to solutions to partial differential equations and optimization but have a large theoretical interest as well. Documentation for ABELFUNCTIONS can be found at `http://abelfunctions.cswiercz.info`.

The next step was to implement the Abel map, a function mapping a Riemann surface to its Jacobian. However, the initial implementation of my algorithms lacked the infrastructure needed to write this algorithm elegantly. I spent part of this year studying object-oriented programming design patterns and software architecture design [1, 2]. Most of ABELFUNCTIONS is now rewritten with these principles in place and the result has been not only code that's easier to work with and read but better performing code. All non-symbolic operations execute at least 80% faster. This sub-project is in line with my initial research goal of making an easy-to-extend platform for computing with Riemann surfaces.

Grady Williams has completed his work on developing a high-performance version of the Riemann theta function using GPUs. His implementation provides at least a x100 speedup to the algorithm. A paper on further improving Riemann theta performance via the Siegel transform is in the works. Grady will be attending the Ph.D. program in Robotics at Georgia Tech starting Autumn 2014.

I ended the winter quarter with the successful completion of my general examination. My committee includes my advisor Bernard Deconinck as well as Randy LeVeque and Robert O'Malley from the Applied Mathematics department and William Stein and Rekha Thomas from Mathematics. The major components of ABELFUNCTIONS that remain are the calculation of the Riemann constant vector and building a framework for defining and evaluating other types of differentials. With the software development portion of my work winding down I've begun a more extensive literature review of the background and applications of these computational tools.

Expected Academic Schedule

I expect to complete my degree at the end of Spring or Summer of 2015.

References

1. Gamma, Erich and Helm, Richard and Johnson, Ralph and Vlissides, John, *Design Patterns: Elements of Reusable Object-oriented Software*, Addison-Wesley Longman Publishing Co., Inc., Boston, MA, USA, 1995.
2. Martin, Robert *Design Principles and Design Patterns*, `www.objectmentor.com, 2000.`