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1. $y=\frac{x}{ax+b} \Rightarrow \frac{1}{y}=a+\frac{b}{x} \Rightarrow \frac{1}{y}=b\cdot\frac{1}{x}+a$ 则

由表得 $E(\frac{1}{x})=0.45667,\ E(\frac{1}{y})=2.4568,\ n=5$

$$b=\frac{\sum_{i=1}^{n}\frac{1}{x_i}\frac{1}{y_i}-n\overline{\frac{1}{x}}\,\overline{\frac{1}{y}}}{\sum_{i=1}^{n}(\frac{1}{x_i})^2-n\times(\overline{\frac{1}{x_i}})^2}=1.0048,\quad a=\overline{\frac{1}{y}}-b\cdot\overline{\frac{1}{x}}=1.9979$$

$$\Rightarrow y=\frac{x}{1.9979x+1.0048}$$

2. $A=\begin{bmatrix}1&-1\\-1&1\\2&-2\end{bmatrix}\quad b=\begin{bmatrix}5\\-4\\10\end{bmatrix}$ 则 $A^TA=\begin{bmatrix}6&-6\\-6&6\end{bmatrix}\quad A^Tb=\begin{bmatrix}29\\-29\end{bmatrix}$

$\Rightarrow A^TAx=A^Tb \Rightarrow x_1=\frac{29}{6}+x_2$

取最小范数解 $\|\vec{x}\|_2=\sqrt{x_1^2+x_2^2}=\sqrt{2x_2^2+\frac{29}{3}x_2+\frac{841}{36}} \Rightarrow x_2=-\frac{29}{12},\ x_1=\frac{29}{12}$

则解为 $\vec{x}=\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}\frac{29}{12}\\-\frac{29}{12}\end{bmatrix}$

3. $\varphi_0(x)=\frac{1}{x}\quad \varphi_1(x)=x$ 则最佳平方逼近的形式为

$$\begin{bmatrix}(\varphi_0,\varphi_0)&(\varphi_0,\varphi_1)\\(\varphi_1,\varphi_0)&(\varphi_1,\varphi_1)\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix}=\begin{bmatrix}(\varphi_0,f)\\(\varphi_1,f)\end{bmatrix}\quad \text{取 } w=1$$

$(\varphi_0,\varphi_0)=\sum_{i=1}^{5}\varphi_0(x_i)\varphi_0(x_i)=1.3559\quad (\varphi_0,\varphi_1)=(\varphi_1,\varphi_0)=5$

$(\varphi_1,\varphi_1)=121,\ (\varphi_0,f)=16.065,\ (\varphi_1,f)=304.51$

则 $\begin{bmatrix}1.3559&5\\5&121\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix}=\begin{bmatrix}16.065\\304.51\end{bmatrix} \Rightarrow a=3.0296\ b=2.3914$

则为 $y=\frac{3.0296}{x}+2.3914x$

4. 前三阶 legendre 正交多项式为 $P_0(x)=1,\ P_1(x)=x,\ P_2(x)=\frac{3}{2}x\cdot x-\frac{1}{2}\cdot 1=\frac{3}{2}x^2-\frac{1}{2}$

则令 $\varphi(x)=\alpha_0P_0(x)+\alpha_1P_1(x)+\alpha_2P_2(x)$，$f(x)=\cos\pi x$ 则

$$\begin{bmatrix}(P_0,P_0)&(P_0,P_1)&(P_0,P_2)\\(P_1,P_0)&(P_1,P_1)&(P_1,P_2)\\(P_2,P_0)&(P_2,P_1)&(P_2,P_2)\end{bmatrix}\begin{bmatrix}\alpha_0\\\alpha_1\\\alpha_2\end{bmatrix}=\begin{bmatrix}(P_0,f)\\(P_1,f)\\(P_2,f)\end{bmatrix}$$

有 $(P_i,P_j)=\begin{cases}0, & i\neq j\\ 2/(2i+1), & i=j\end{cases}$

$(P_0,f)=\int_{-1}^{1}\cos\pi x\,dx=\frac{1}{\pi}\sin\pi x\Big|_{-1}^{1}=0$

$(P_1,f)=\int_{-1}^{1}x\cos\pi x\,dx=\frac{1}{\pi^2}(\pi x\sin\pi x+\cos\pi x)\Big|_{-1}^{1}=0$

$(P_2,f)=\int_{-1}^{1}(\frac{3}{2}x^2-\frac{1}{2})\cos\pi x\,dx=\left[\frac{3}{2\pi^3}((\pi x)^2\sin\pi x+2\pi x\cos\pi x-2\sin\pi x)-\frac{1}{2\pi}\sin\pi x\right]\Big|_{-1}^{1}=-\frac{6}{\pi^2}$

$$\Rightarrow \begin{bmatrix} 2 & & \\ & \frac{2}{3} & \\ & & \frac{2}{5} \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -\frac{6}{\pi^2} \end{bmatrix} \Rightarrow \alpha_0 = 0,\ \alpha_1 = 0,\ \alpha_2 = -\frac{15}{\pi^2}$$

$\Rightarrow$ 逼近为 $\varphi(x) = -\frac{15}{\pi^2}\left(\frac{3}{2}x^2 - \frac{1}{2}\right) = -\frac{45}{2\pi^2}x^2 + \frac{15}{2\pi^2}$