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1. 取 $\lambda_0 = 9.6$, $B = A - \lambda_0 I = \begin{bmatrix} -8.6 & 2 & 3 \\ 2 & -6.6 & 4 \\ 3 & 4 & -4.6 \end{bmatrix}$ 取初始向量 $x^{(1)} = [1, 1, 1]$

则 $Bx^{(1)} = x^{(2)} \Rightarrow \begin{cases} -8.6x_1 + 2x_2 + 3x_3 = 1 \\ 2x_1 - 6.6x_2 + 4x_3 = 1 \\ 3x_1 + 4x_2 - 4.6x_3 = 1 \end{cases} \Rightarrow x^{(1)} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 27.5 \\ 40 \\ 52.5 \end{bmatrix}$

$\Rightarrow$ 特征值 $\lambda^* = \lambda_0 + 1/\max(x) = \lambda_0 + \frac{1}{52.5} = 9.619$

特征向量 $u = \frac{x}{\|x\|} = \begin{bmatrix} 0.3846 \\ 0.5594 \\ 0.7342 \end{bmatrix}$

2. ① a_{12} $Q_1 = \begin{bmatrix} \cos\theta & -\sin\theta & \\ \sin\theta & \cos\theta & \\ & & 1 \end{bmatrix} \Rightarrow \frac{1}{2}(5-4)\sin 2\theta + 2\cos 2\theta = 0 \Rightarrow \tan 2\theta = -4$

$\Rightarrow A^{(1)} = \begin{bmatrix} 6.56 & 0 & 2 \\ 0 & 2.44 & -0.96 \\ 2 & -0.96 & 6 \end{bmatrix}$

② a_{13} $Q_2 = \begin{bmatrix} \cos\theta & & -\sin\theta \\ & 1 & \\ \sin\theta & & \cos\theta \end{bmatrix} \Rightarrow \tan 2\theta = 7.14$

$\Rightarrow A^{(2)} = \begin{bmatrix} 8.32 & 0 & 0 \\ 0 & 2.44 & -0.72 \\ 0 & -0.72 & 4.24 \end{bmatrix}$

则 $\lambda = 8.32, 2.44, 4.24$

3. $x_1 = \begin{bmatrix} -4 \\ 2 \\ 4 \end{bmatrix} \Rightarrow u_1 = x_1 + \sigma e_1 = \begin{bmatrix} -4 \\ 2 \\ 4 \end{bmatrix} + (-6)\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -10 \\ 2 \\ 4 \end{bmatrix}$

$w_1 = \frac{u_1}{\|u_1\|} = \begin{bmatrix} -\frac{5}{\sqrt{30}} \\ \frac{1}{\sqrt{30}} \\ \frac{2}{\sqrt{30}} \end{bmatrix}$ $H_1 = I - 2w_1 w_1^T = \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{14}{15} & -\frac{2}{15} \\ \frac{2}{3} & -\frac{2}{15} & \frac{11}{15} \end{bmatrix}$

$R_1 = H_1 A = \begin{bmatrix} 6 & \frac{13}{3} & 10 \\ 0 & \frac{23}{15} & -\frac{7}{5} \\ 0 & -\frac{14}{15} & \frac{1}{5} \end{bmatrix}$

$x_2 = \begin{bmatrix} \frac{23}{15} \\ -\frac{14}{15} \end{bmatrix} \Rightarrow u_2 = x_2 + \sigma e_1 = \begin{bmatrix} \frac{23}{15} \\ -\frac{14}{15} \end{bmatrix} + \frac{\sqrt{29}}{3}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{23+5\sqrt{29}}{15} \\ -\frac{14}{15} \end{bmatrix}$

$w_2 = \frac{u_2}{\|u_2\|} = \begin{bmatrix} 0.9629 \\ -0.2703 \end{bmatrix}$ $H_2 = I - 2w_2 w_2^T = \begin{bmatrix} -0.8542 & 0.5199 \\ 0.5199 & 0.8541 \end{bmatrix}$

$$R_2 = \begin{bmatrix} 1 & & \\ & H_2 & \\ & & \end{bmatrix} \quad R_1 = \begin{bmatrix} -6 & \frac{13}{5} & 10 \\ 0 & -\frac{\sqrt{89}}{5} & 1.2958 \\ 0 & 0 & -0.5571 \end{bmatrix}$$