A Sparse Signal Reconstruction Perspective for Direction-of-Arrival (DoA) Estimation

Xiang Zhang

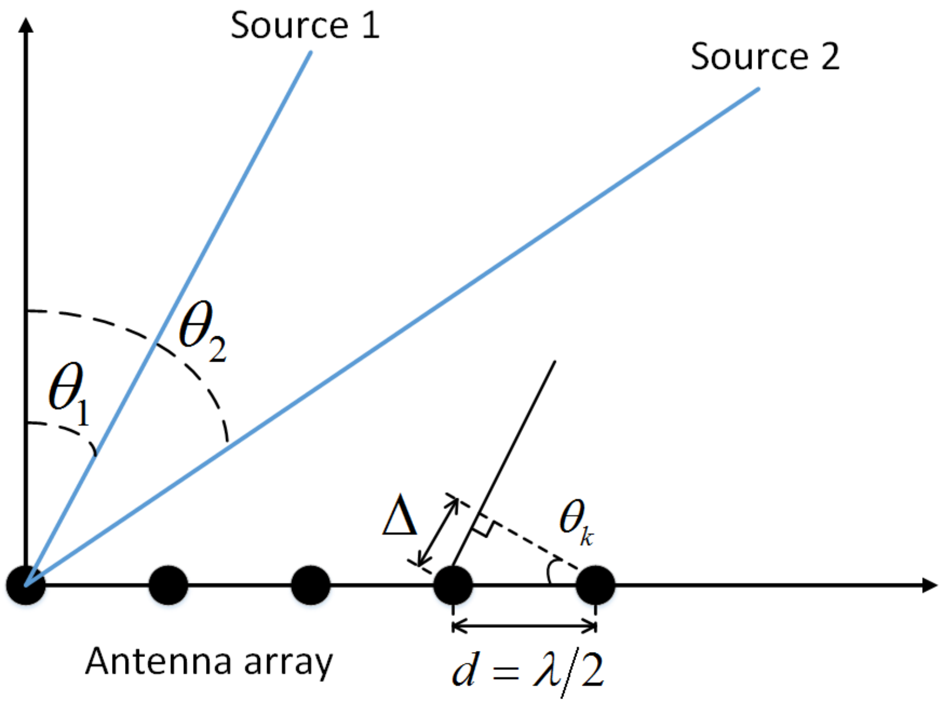
Outline

- Introduction to DoA Estimation
- Sparse Signal Reconstruction -based Approach
- Simulation results
- Conclusion

Reference

Malioutov D, Cetin M, Willsky A S. ***A sparse signal reconstruction perspective for source localization with sensor arrays****[J]. IEEE transactions on signal processing, 2005, 53(8): 3010-3022.*

Direction of Arrival (DoA) Estimation



- Assumption: far-field narrowband signals
- Distance difference: $\Delta = d \sin\theta_k$
- Goal: estimate all possible DoAs

Signal Model

Source signal: $s_k(t) = f_k(t)\,\mathrm{e}^{j\omega_k t}, \quad k = 1, 2, \ldots, K$

Delayed signal at *m-th* antenna

$$
\begin{aligned}
s_k\big(t-(m-1)\Delta t\big) &= \underline{f_k\big(t-(m-1)\Delta t\big)}\,\mathrm{e}^{j\omega_k (t-(m-1)\Delta t)} \\
&\approx \underline{f_k(t)}\,\mathrm{e}^{j\omega_k (t-(m-1)\Delta t)} \\
&= s_k(t) e^{-j\omega_k (m-1)\Delta t} && \Delta t = d \sin\theta_k / c \\
&= s_k(t) e^{-j(m-1)\frac{2\pi c}{\lambda}\frac{d \sin\theta_k}{c}} && \omega = \frac{2\pi c}{\lambda} \\
&= s_k(t) \underline{e^{-j(m-1)\frac{2\pi d \sin\theta_k}{\lambda}}} \\
&= s_k(t) \underline{a_m(\theta_k)} && m = 1, 2, \ldots, M
\end{aligned}
$$

Signal Model

Received signal at *m-th* antenna:

$$y_m(t) = \sum_{k=1}^{\#of\ sources\ K} a_m(\theta_k) s_k(t) + \underbrace{n_m(t)}_{\text{measurement noise}}, \quad m = 1, 2, \ldots, M$$

Received signal in vector form:

$$\underbrace{\begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_M(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{-j\varphi_1} & e^{-j\varphi_2} & \cdots & e^{-j\varphi_K} \\ \vdots & \vdots & \vdots & \vdots \\ e^{-j(M-1)\varphi_1} & e^{-j(M-1)\varphi_2} & \cdots & e^{-j(M-1)\varphi_K} \end{bmatrix}}_{\varphi_k = \frac{2\pi d}{\lambda} \sin\theta_k} \begin{bmatrix} S_1(t) \\ S_2(t) \\ \vdots \\ S_K(t) \end{bmatrix} + \begin{bmatrix} n_1(t) \\ n_2(t) \\ \vdots \\ n_M(t) \end{bmatrix}}_{\mathbf{y}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t)}$$

Steering matrix: $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_K)]$

Sparse Signal Reconstruction Based DoA

Sparse representation of the signal vector $\mathbf{s}(t)$:

$$\mathbf{A}^{sparse} = [\mathbf{a}(\hat{\theta}_1), \mathbf{a}(\hat{\theta}_2), \ldots, \mathbf{a}(\hat{\theta}_{N_\theta})]$$

$\hat{\boldsymbol{\theta}} = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_{N_\theta}] = [1^\circ, 2^\circ, 3^\circ, \ldots, 180^\circ]$ *All possible directions*

$$\boxed{\mathbf{y} = \mathbf{A}^{sparse}\mathbf{s}^{sparse}(t) + \mathbf{n}(t)}$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_M(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{-j\hat{\varphi}_1} & e^{-j\hat{\varphi}_2} & \cdots & e^{-j\hat{\varphi}_{N_\theta}} \\ \vdots & \vdots & \vdots & \vdots \\ e^{-j(M-1)\hat{\varphi}_1} & e^{-j(M-1)\hat{\varphi}_2} & \cdots & e^{-j(M-1)\hat{\varphi}_{N_\theta}} \end{bmatrix}}_{\hat{\varphi}_k = \frac{2\pi d}{\lambda}\sin\hat{\theta}_k} \begin{bmatrix} s_1^{saprse}(t) \\ s_2^{saprse}(t) \\ \vdots \\ s_{N_\theta}^{saprse}(t) \end{bmatrix} + \begin{bmatrix} n_1(t) \\ n_2(t) \\ \vdots \\ n_M(t) \end{bmatrix}$$

$\mathbf{s}^{sparse}(t)$ is ***sparse***(only a few # of non-zero entries)!!

Sparse Signal Reconstruction Based DoA

$$\mathbf{y} = \mathbf{A}_{M\times N_\theta}^{sparse}\mathbf{s}^{sparse}(t) + \mathbf{n}(t)$$

1) $M < N_\theta \Rightarrow$ infinitely many solutions for $\mathbf{s}^{sparse}(t)$

2) Choose the sparse solutions:

$$\boxed{\min_{\mathbf{s}^{sparse},\lambda} \left\|\mathbf{y}(t) - \mathbf{A}^{sparse}\mathbf{s}^{sparse}\right\|_2^2 + \lambda\left\|\mathbf{s}^{sparse}\right\|_0 \qquad (OPT\ 1)}$$

L-1 norm relaxation

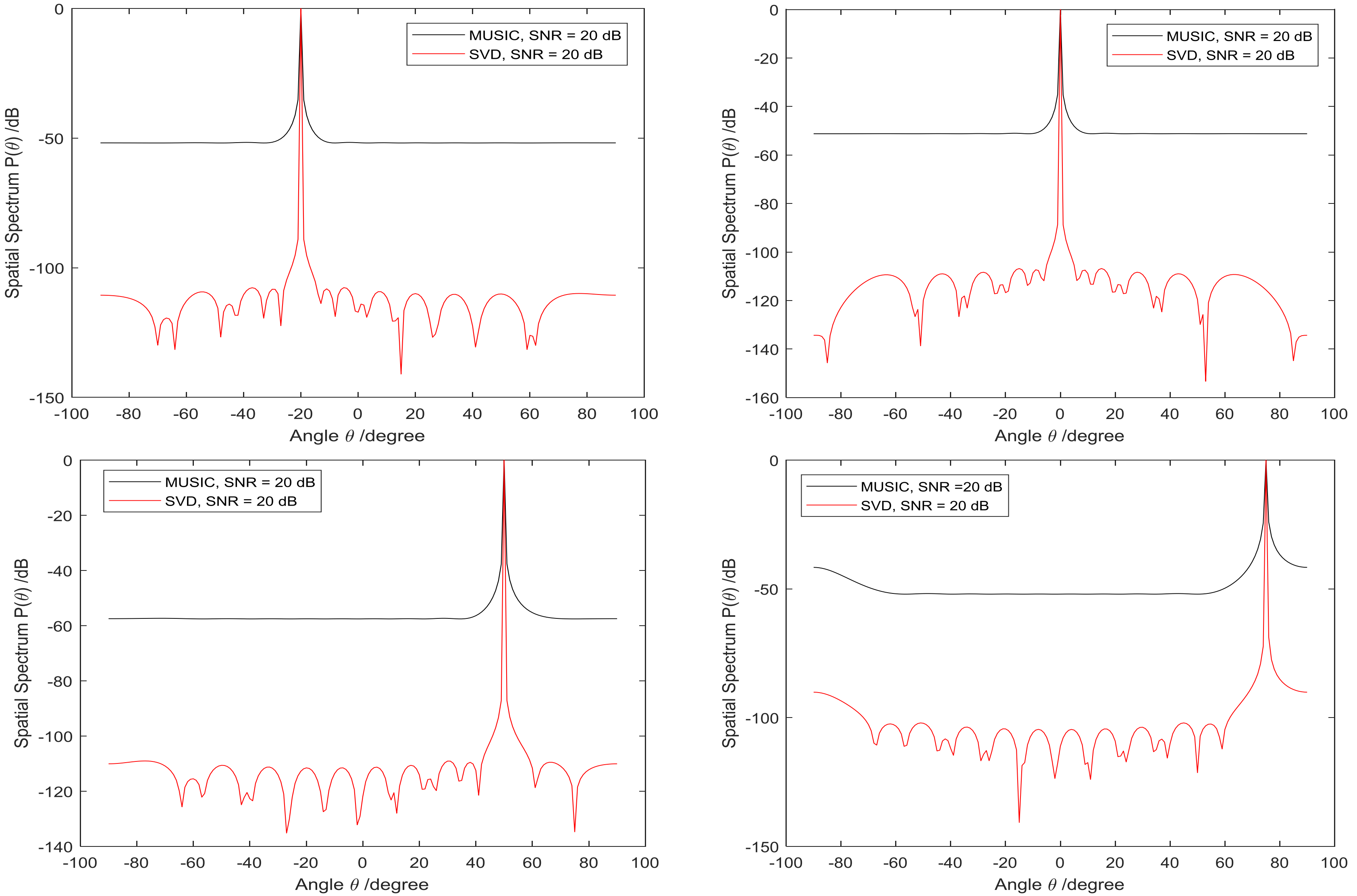
$$\boxed{\min_{\mathbf{s}^{sparse},\lambda} \left\|\mathbf{y}(t) - \mathbf{A}^{sparse}\mathbf{s}^{sparse}\right\|_2^2 + \lambda\left\|\mathbf{s}^{sparse}\right\|_1 \qquad (OPT\ 2)}$$

Second Order Cone (SOCP) opt. problem

Solve by MATLAB CVX tool

Simulation Result

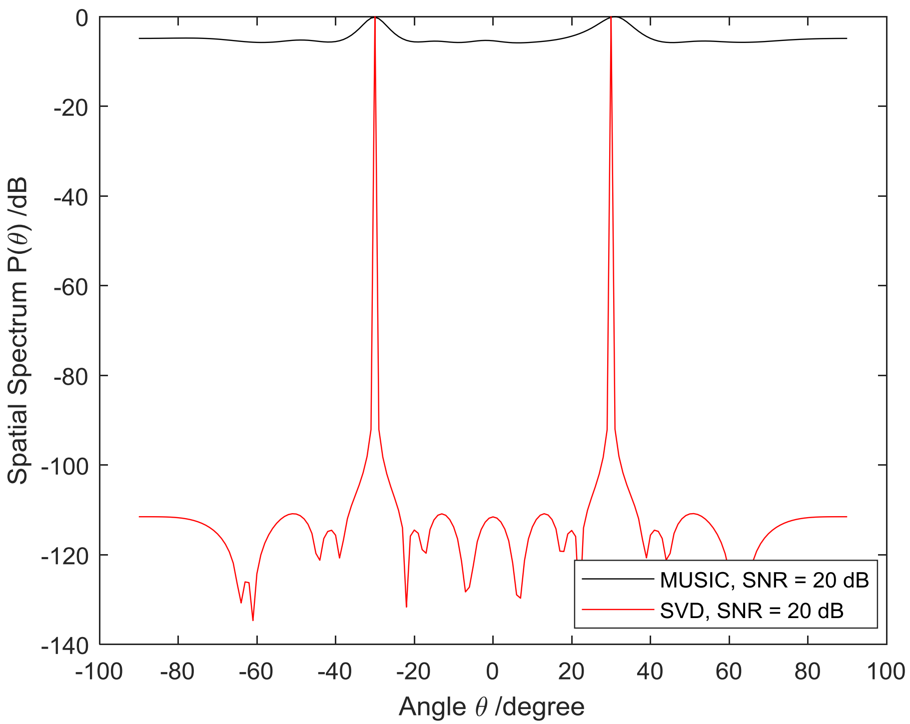
DOA= -20, 0, 50, 75 respectively

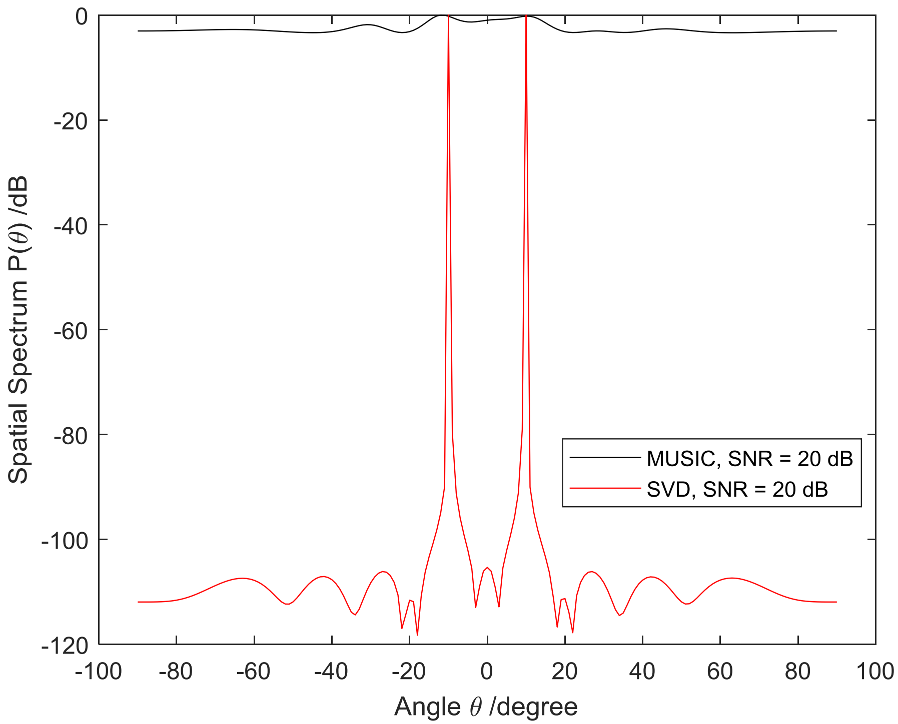


Simulation Result

w=[pi/3, pi/3],SNR= 20 dB, DOA =[-30 30],[-10 10] respectively

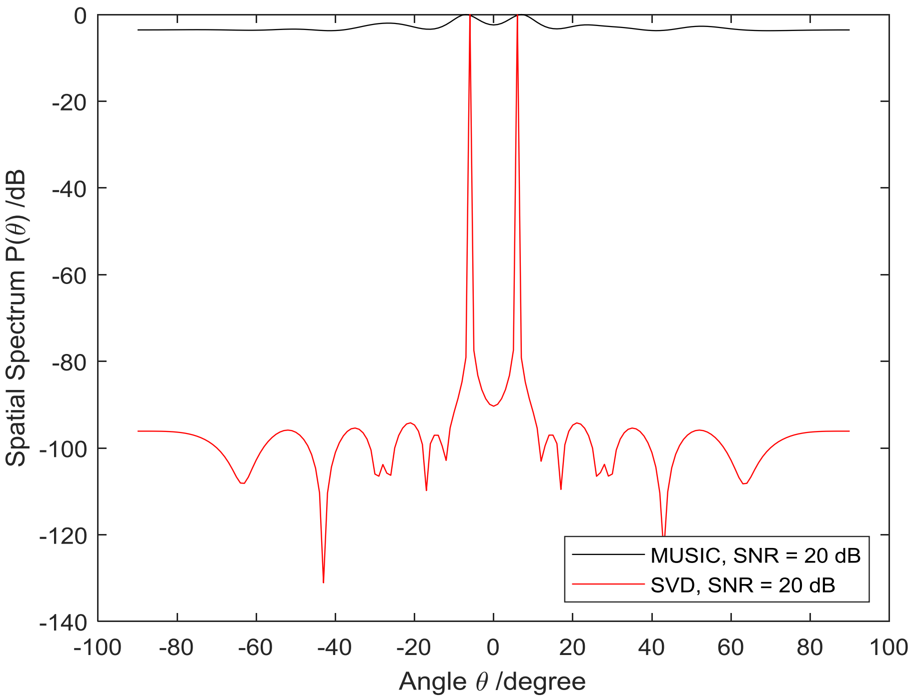
Frequency of the envelope signal $f_k(t)$

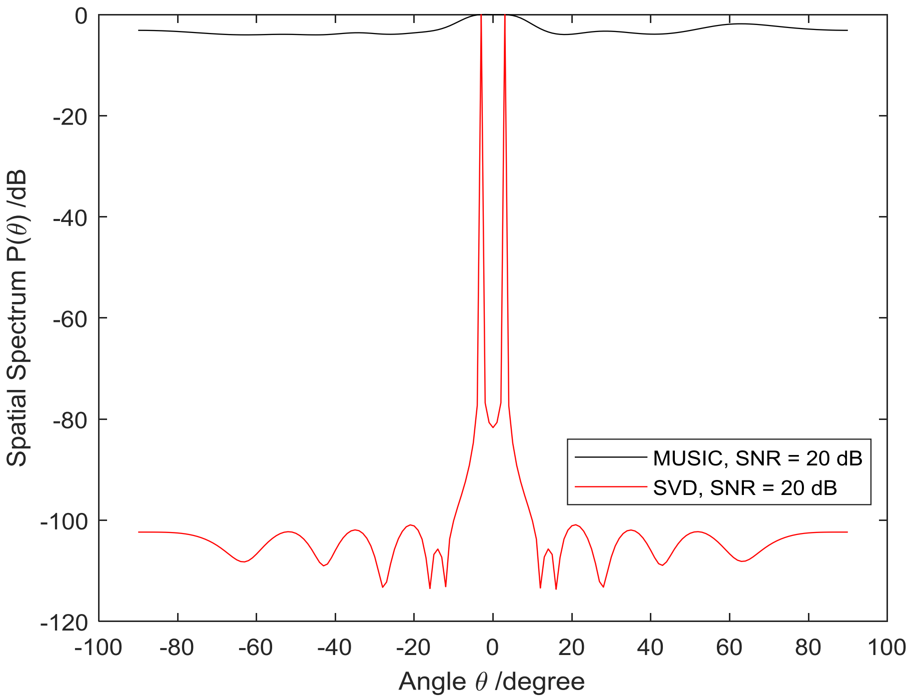




Simulation Result

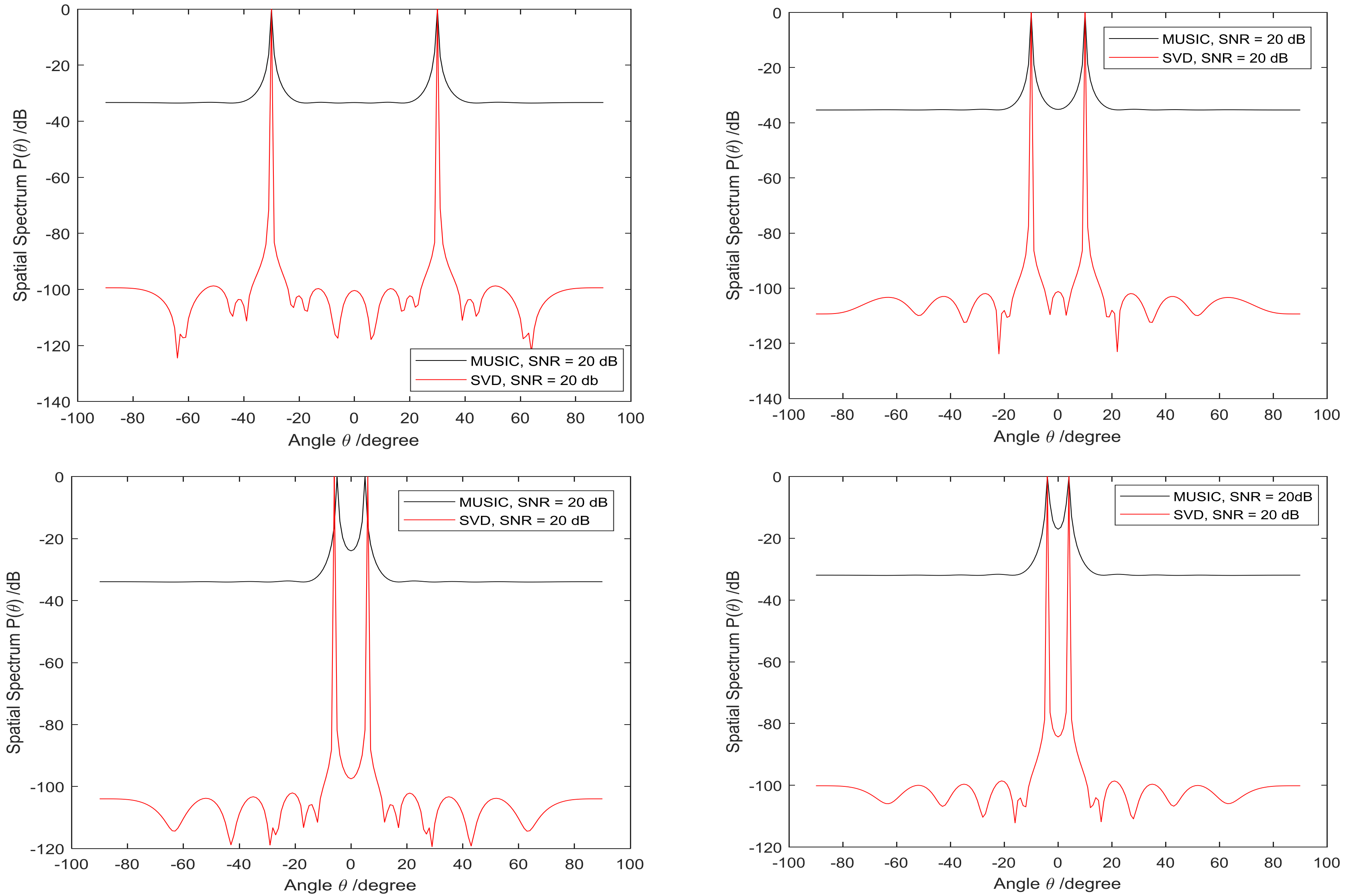
w=[pi/3, pi/3],SNR= 20 dB ,DOA =[-5 5],[-4 4] respectively





Simulation Result

w=[pi/3, pi/3.01],SNR= 20 dB ,DOA =[-30 30],[-10 10],[-5 5],[-4 4]



Conclusion

- The sparse signal reconstruction based DoA estimation approach has higher resolution (shaper peaks) than MUSIC algorithm.
- Sparse approach is more effective in dealing with correlated sources .
- Another advantage is that MUSIC requires the knowledge of the # of sources, but sparse approach doesn’t.

Thanks!!