

Quality assured full-waveform inversion: Ensuring starting model adequacy

Nikhil Shah*, Mike Warner, Tenice Nangoo, Adrian Umpleby, Ivan Stekl, Jo Morgan, Lluís Guasch
(Imperial College London)

Summary

Successful full-waveform inversion (FWI) of 3D seismic data typically requires low-frequency content in the field data coupled with an accurate starting velocity model. In these circumstances, two fundamental questions always arise: (1) is the starting model sufficiently accurate given the data that are available, and (2) will the inversion iterate towards the global minimum, or will it instead become trapped locally leading to an erroneous final model?

We present a robust and objective means to answer both these questions. The diagnostic feature that we use to achieve this is the spatial continuity of the phase difference between the predicted and observed field data, extracted at a single low frequency, after windowing in time around early arrivals. We show proof of principle on a simple 2D synthetic example, and demonstrate the application of quality assured full-waveform inversion (QA-FWI) to a full 3D field dataset that shows significant velocity anisotropy.

Introduction

Full-waveform seismic inversion is increasingly being used as a method to build high-resolution 3D velocity models. Although the method has produced spectacular results on field datasets (Sirgue *et al*, 2009; Ratcliffe *et al*, 2011), FWI is not yet a particularly robust or stable technology. The method is difficult to quality control and quality assure; it is especially compromised by problems of cycle skipping that can lead the inversion towards an incorrect model that represents a local rather than global minimum in the data misfit function. The existence of these local cycle-skipped minima, and the difficulty of determining in practice whether a particular inversion is working as desired, is probably the principal limitation on the wider uptake of FWI as a routine production technology.

Cycle skipping is problematic during FWI because seismic data are oscillatory, and FWI is a local inversion scheme. FWI perturbs a starting model such that it immediately reduces the mismatch between observed and predicted seismic data. If the start model is good enough, and if the start frequencies are low enough, then these model perturbations will move the model in the right direction. However if the initially predicted data are more than 180° out of phase with the field data, then a shift in the arrival time of the model data, with the wrong sign, can nonetheless reduce the data misfit (Beydoun & Tarantola, 1988).

Methodology

The most commonly used measure of data mismatch is the L_2 norm $\|u(t) - d(t)\|^2$ where $u(t)$ are the modeled, and $d(t)$ are the observed data, both of which are functions of time, and the norm involves a sum over all times and over all live source and receiver positions. This norm uses information from all arrivals, at all frequencies, and it incorporates both phase and amplitude information. However for successful global convergence in FWI, the lowest wavenumbers in the model must be inverted correctly during the earliest iterations, and $u(t)$ and $d(t)$ must always remain within less than half a cycle of each other. Consequently, practical FWI schemes invert early arrivals before late, low frequencies before high, and phases before amplitudes.

To assess the quality of the inversion, we therefore calculate the phase difference between the observed and predicted data, at the lowest useable frequency present in the field data, after windowing the data in time using a Gaussian window centered on the early arrivals. In practice, we do not pick first-break times from the field data; rather we use arrival times calculated from the start model to control the position of the Gaussian window.

The phase difference is calculated, after windowing, by Fourier transforming the data at a single low frequency to generate the complex-valued wavefields U and D . The phase residual ϕ is then defined as $\phi = \text{phase}(UD^*)$ where the asterisk implies complex conjugation, and phase indicates that we take the principal value of the phase of a complex number; the extracted phase difference ϕ will necessarily lie between $\pm 180^\circ$.

To assess both the adequacy of the start model, and to assess performance during each full-waveform iteration, we plot the phase difference ϕ as a function of source and receiver position. In 2D datasets, this plot is also two-dimensional. In 3D datasets, the plot is four dimensional, but it can often be conveniently viewed as a 3D volume or as a sequence of 2D images. If the combination of start model and field data is not cycle skipped, then ϕ will vary smoothly and consistently in space. However, if the model is cycle skipped, then there will be sudden, spatially consistent, 360° jumps in phase. These phase jumps are easy to observe in low-frequency windowed field data, and they are immediately diagnostic of a potential problem. The QA-FWI plots are unambiguous to interpret, and can be used to assure the quality of both the start model, and subsequent iterations.

Quality assured FWI

Simple 2D synthetic example

We illustrate the method using band-limited synthetic seismic data generated from the 2D Marmousi model; we then try to invert these data using two starting models, one of which is cycle skipped. Figure 1(a) shows the two start models. Figure 1(b) shows the windowed phase difference ϕ generated by each of these models at 3 Hz. The horizontal axis shows source position, the vertical axis shows receiver position, the leading diagonal represents zero offset, and source-receiver reciprocity ensures that the plots are symmetric about the diagonal.

Since the data have been windowed about the early arrivals, then provided that the source wavelet is a reasonable estimate at low frequency, we can be sure that the phase difference close to zero offset will not be cycle skipped. On both plots in Figure 1(b), the value ϕ of along the leading diagonal is close to zero.

In the left-hand plot in Figure 1(b), which represents an adequate starting model, ϕ varies smoothly and continuously away from zero offset over the entire dataset. In contrast, in the right-hand plot, the phase difference varies smoothly until it reaches values close to -180° at which point it jumps discontinuously to $+180^\circ$. This jump corresponds to the sharp boundary between blue and red, marked by the dotted line. Seismic data lying beyond this boundary are cycle skipped, and if they are included in the inversion, they will tend to cause the model to move in the wrong direction, potentially with catastrophic consequences for the veracity of the model.

Figure 1(c) shows final FWI results for the two models, and Figure 1(d) shows the true model. For the left-hand model, FWI has been successful. There are of course limitations in the resolution and accuracy of the results, but these are related directly to limitations in the bandwidth and aperture of the original data and do not result from cycle skipping.

In contrast, for the right-hand model, FWI has gone badly wrong. Both the shallow and deeper parts of the model show a mixture of true structure and cycle-skipped artifacts. However, if we did not know the true model, then it would be difficult to determine this from either the model or from the evolution of the L_2 -norm misfit function. The plot of ϕ in Figure 1(b) however provides an instant visual indicator of the existence of a potential problem for FWI.

Full 3D field data example

We have applied this method to a shallow-water 3D OBC field dataset from the North Sea. Unlike the simple synthetic example above, this field dataset is affected by all the features that characterize real data: noise, missing traces, unknown wavelet, attenuation, elastic effects, and

realistic bandwidth. The data also show evidence for significant anisotropy with epsilon values of up to 20%.

The field data were prepared for FWI as described in Nangoo *et al* (2012), and a high-quality anisotropic velocity model was used to begin the inversion (left side of Figure 2). Figure 2a shows a horizontal slice through this starting model at a depth of 1200 m. Figure 2(b) shows the corresponding time-windowed phase difference ϕ , for one 3D receiver gather, plotted at 3.6 Hz. The receiver is located at the center of the circular structure; close to this, at short offset, the phase difference is close to zero.

Moving away from the receiver in Figure 2(b), the phase difference ϕ initially remains close to zero, then decreases, but it does not reach -180° . Moving to longer offsets, the phase difference increases again smoothly through 0° , becoming increasingly positive. At the longest offsets of greater than 8 km, in the extreme bottom left of Figure 2(b), the data become cycle skipped as evidenced by the sudden jump from red to blue, that is from $+180^\circ$ to -180° .

To invert these data, we might try to improve the velocity model, or we might choose to exclude the cycle-skipped arrivals. In fact in this case, we chose to invert the data including the cycle skipped arrivals, in the hope and expectation that the large majority of the data would be capable of countering the deleterious effects of the small volume of cycle skipped data. This is of course a dangerous assumption, and one that we must validate.

Figure 2(c) shows the phase difference obtained after the first iteration. If the inversion is successful, then it should show, as it does, that the phase difference shrinks towards zero over most of the domain. Significantly, we see that the cycle-skipped region, at bottom left, is now no longer cycle skipped. Evidently, the cumulative changes that FWI has made to the model have correctly reduced the phase differences even where the data were cycle-skipped.

Continued iteration continues to improve the model, and the phase difference continues to drop, Figure 2(d). The final model is reached in Figure 2(e) after about fifty iterations. The shallow gas cloud in these data, shown by the central low-velocity region, is now imaged sharply. This FWI velocity model has been shown to improve PSDM of these data significantly (Ratcliffe *et al*, 2011) and to match sonic logs (Nangoo *et al*, 2012).

The right side of Figure 2 shows the same sequence of plots, but for the start model in Figure 2(f); here the low-velocity gas cloud was not well resolved, and the anisotropy was not correct. Figure 2(g) shows the corresponding phase differences. Away from the receiver, the phase difference becomes increasingly negative, and as the phase jumps from blue to red, the data are cycle skipped.

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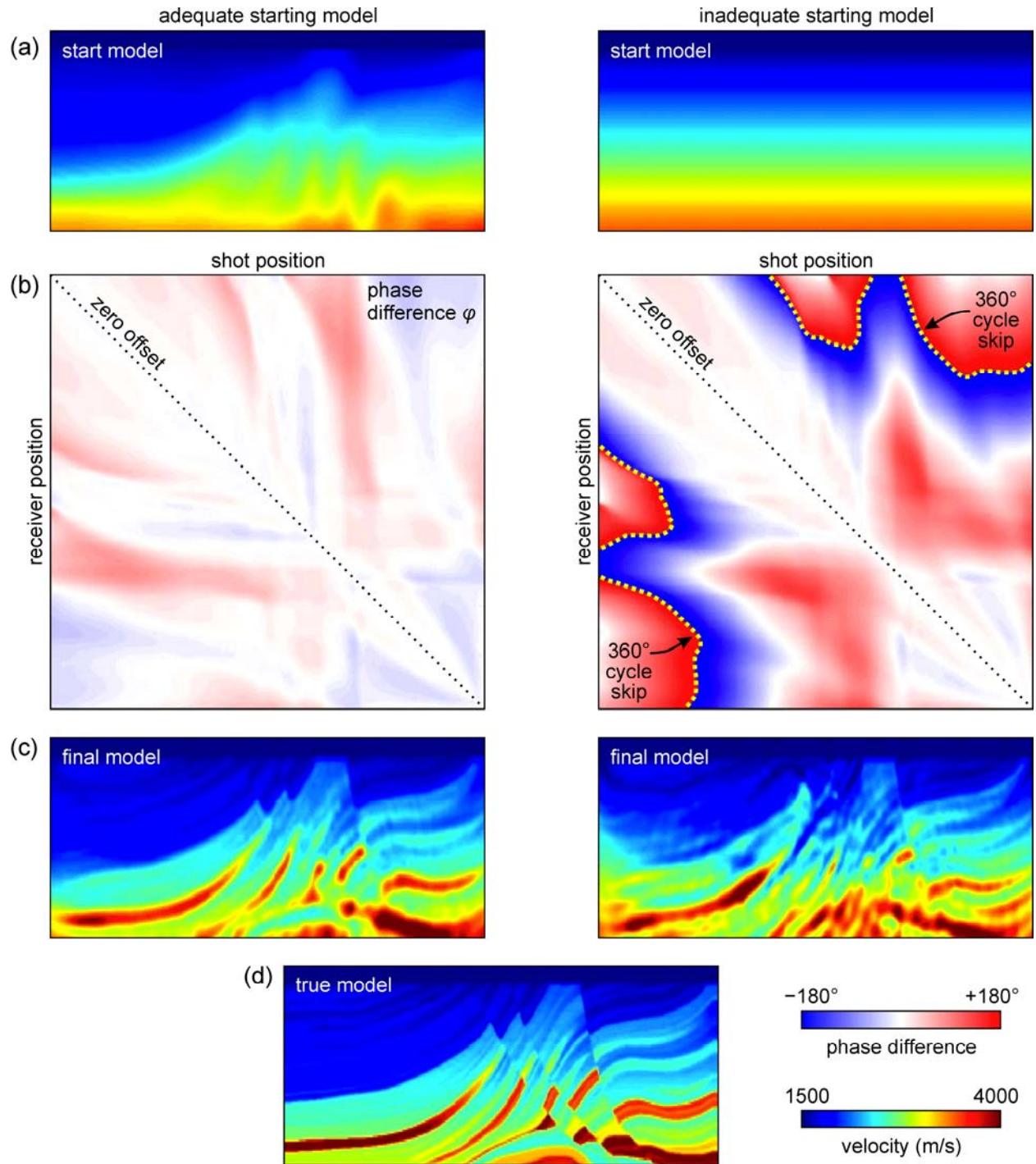


Figure 1 Proof of principle on a simple 2D synthetic dataset. Left panels show FWI beginning from an adequate starting velocity model; right panels show the same plots when beginning from an insufficiently accurate starting model. (a) Starting velocity models. (b) Phase difference ϕ between true data and predicted data as a function of shot and receiver position. (c) Final models produced by FWI. (d) True velocity model.

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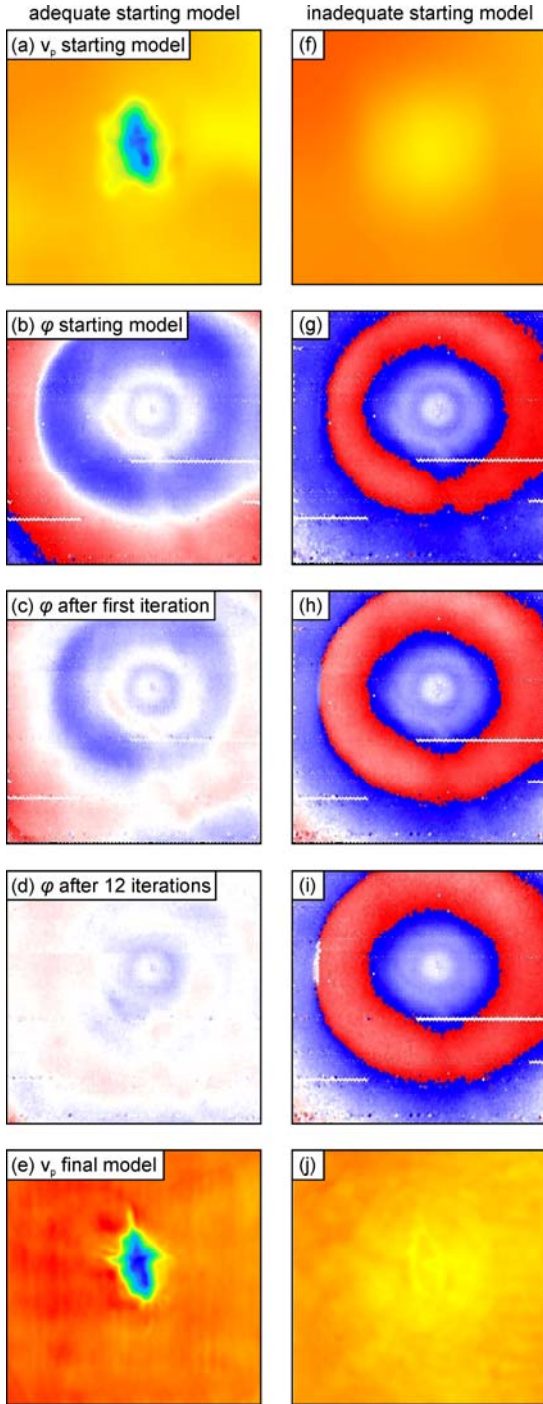


Figure 2 Inverting 3D field data using adequate and inadequate v_p starting models. Phase difference plots of ϕ are for one receiver. Left: Data are just cycle skipped, but recover after one iteration. Right: Data remain cycle skipped throughout the inversion.

Moving towards even longer offsets, the cycle skipping is reversed, and the longest-offset data are not cycle skipped. Given the large region of cycle-skipping, inverting these data is unlikely to be successful. Figure 2(h) demonstrates this. In this figure, although the L_2 norm has dropped, and the predicted time-domain data will now more closely match the field observations, the data are evidently still cycle skipped. Indeed, although the average phase difference has reduce somewhat, the region of cycle skipping has actually marginally increased.

Figure 2(g) provides an immediate diagnostic suggesting that FWI is unlikely to be successful using these data and this model. Figure 2(h) confirms this, providing an equivalent diagnostic for the failure of FWI at the first iteration. The process does not recover, Figure 2(i). The final model in Figure 2(j) is erroneous even though it does reduce data mismatch and begins to image the gas cloud.

We have shown the principle in Figure 2 using a single receiver gather. In practice we use many gathers together, and these are straightforward to analyze as a 3D volume.

Conclusion

Full-waveform inversion is not yet a robust and routine production method. A significant impediment to its wider adoption is the difficulty of avoiding cycle-skipped local minima and of detecting when the results have been compromised by these effects. In principle, these effects can be detected by manually comparing predicted and field data in the time domain, but this is usually error-prone and ambiguous.

In contrast, here we have demonstrated an unambiguous and robust procedure, using the spatial continuity of windowed, low-frequency, phase differences, that provides effective quality assurance for full-waveform inversion (QA-FWI). The method can diagnose both the starting condition, and the performance of FWI at each iteration.

Our experience of inverting 3D field datasets, leads us to suspect that many FWI results, both published and proprietary, do contain the unsuspected and detrimental effects of cycle skipping. The cost of QA-FWI is minimal, and its application is automated. We therefore recommend its use as a standard part of any FWI methodology, and we look forward to its routine adoption as a diagnostic tool that helps to move FWI from the arcane to the mundane.

Acknowledgements

We thank the PL044 partnership: ConocoPhillips Skandinavia AS, Total E&P Norge AS, ENI Norge AS, and Statoil Petroleum AS for use of their field data. The study was sponsored by the Fullwave consortium.