Optimal Number of Startups in a Venture Capital Portfolio

Gonzalo Plaza Molina

4 April 2024

Mechanical Engineering MEng Individual Project 5

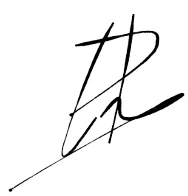
Personal Statement

This project is original, self-proposed and agreed with my supervisor, Prof. Filippo Menolascina. I maintained regular meetings with Filippo, where we discussed the progression of the study and where he provided occasional and useful guidance. Upon commitment, I was given access to relevant projects with analogies to mine, which served as a source of inspiration at the beginning of the journey.

Initially, I focused on acquiring the necessary background knowledge for the development of this project. This mainly involved learning fundamental finance concepts and the investigation of machine learning algorithms that could be utilised. The latter was especially challenging, considering my previous lack of knowledge for such field, so it required studying basic concepts and practices to be able to make decisions with sufficient judgement. Following this crucial step, I was able to select a suitable, state-of-the-art machine learning algorithm and write the backbone structure of the code. Following this, I was able to plan a work breakdown schedule for the remaining, and focus on research and the development of the model. The project progressed satisfactorily, with unpredictable delays common to any project, that did not have major consequences.

The main challenge in this project was the requirement to make a large number of important assumptions to simplify the existing problem. This necessitated faith in that the decisions made during the project would deliver meaningful results, given the computational demand of the SSTD3 algorithm, making it unsuitable for an iterative process between assumptions and results. Furthermore, the little access to private market information and the consequent dependence on a relatively small number of sources complicated the process of finding useful and exhaustive data to incorporate into the model. These challenges caused the planned work schedule to be tightened, thus leaving little room for manoeuvre.

Signed on the 4th of April 2024:
Gonzalo Plaza Molina



Executive Summary

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Gonzalo Plaza Molina
4th April 2024

Venture Capital (VC) funding is crucial for the success of many startups around the world, and thus contributes to technological and business innovation. Venture capital firms (VC firms) invest in young companies and hope for multiplying the value of a small proportion, given the high failure rates inherent in such nascent projects. The success of a VC firm's investment strategy largely depends on the number of startups within its investment portfolio, which determines the extent of risk diversification and resource dispersion.

The aim of this study is to conclude on an optimal number of startups for a VC portfolio, that VC firms can use as a benchmark for their investment strategies. For this, an agent-based model (ABM) is designed to simulate the real-world interaction dynamics between the agents, VC firms and startups, allowing the former to make investment decisions based on a number of parameters. VC agents are trained using a state-of-the-art machine learning algorithm: the Softmax Twin Delayed Deep Deterministic Policy Gradient (SSTD3). By encouraging VC firms in the model to optimise their investment strategies, it is expected that the VC portfolios resultant from such refined strategies consist of the ideal number of startups.

The results suggest that an optimal VC portfolio should be comprised of 23 startups. However, this number proved to be sensitive to a range of factors, most importantly the trade-off between diversification and diminishing returns to advice. The quantity of interest showed no correlation with the previous success and reputation of a VC firm, given the assumptions in the model. The results set the ground for a useful generic benchmark for the VC ecosystem, and further studies that could address some of the limitaitons of this model. These include the exclusion of essential factors that can influence the optimal number of startups in a VC portfolio, such as VC firm type or the startup maturity a VC firm targets; and simulated behaviours that could be refined, such as the management of human capital resources for startup advisory and due diligence.

Acknowledgements

I am indebted to my project supervisor, Prof. Filippo Menolascina, for his guidance and motivation during this journey. With his passion for the Venture Capital field, Filippo did not only contribute to my learning, but also to my enthusiasm for Venture Capital and Private Equity. Through him, I found the motivation for committing to this project and starting a professional career within the world of Finance.

List of Abbreviations

Abbreviation	Definition
ABM	Agent-Based Model
ANN	Artificial Neural Network
CDF	Cumulative Density Function
DRL	Deep Reinforcement Learning
FOIA	US Freedom of Information Act
IRR	Internal Rate of Return
MPT	Modern Portfolio Theory
PDF	Probability Density Function
RL	Reinforcement Learning
SOR	Sortino ratio
SSTD3	Swap Softmax Twin Delayed Deep Deterministic Policy Gradient
TDD	Target downside deviation
TVPI	Total Value to Paid-In Capital
VC	Venture Capital

List of Symbols

Symbol	Definition
Q	ratio of the market value of a firm to its book value
SR_p	Sharpe ratio of a portfolio
R_p	Return of a portfolio
R_i	Return of a startup
R_f	Risk-free rate
R_T	Target return for Sortino ratio
σ	Standard deviation of VC quality distribution, or OU noise parameter
σ_p	Standard deviation of returns of a portfolio
σ_i	Standard deviation of returns of a startup
$\sigma_initial$	Initial standard deviation of startup potential
σ_final	Final standard deviation of startup potential
σ_id	Standard deviation of idiosyncratic risk
σ_noise	Standard deviation of noise for due diligence
x_t	OU noise
ζ	OU noise parameter
d_t	OU noise parameter
B_t	Brownian motion
$\rho_{i,j}$	Correlation of returns between two startups
w_i	Weight of a startup in a portfolio, or weight of a value function
$Q^\pi(a, s)$	Value function for action *a*, state *s* and policy π
y_i	Target for training
γ	Discount factor for target estimation
$J(\phi)$	Expected long-term rewards
ϕ	parameters for actor network
θ	parameters for critic network
ϕ_T	parameters for target actor network
θ_T	parameters for target critic network
$[a, s, r, s']$	Transition containing an action, a state, a reward, and a new state
π	Policy followed by actor network
π_T	Policy followed by target actor network
τ	Target network update rate
q	VC quality
z	startup potential
μ	Mean of VC quality distribution, or OU noise parameter
ϵ	Shape parameter for skew normal distribution
ξ	Location parameter for skew normal distribution
ω	Sscale parameter for skew normal distribution
α	Parameter for power law distribution, or weight for time progression
β	Weight for time progression

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Word Count

Section	Number of Words
Introduction	257
Literature Survey	1956
Methodology	5240
Results and Discussion	2190
Conclusions	303
Total	**9946**

1 Introduction

Venture Capital (VC) firms play a pivotal role in the growth of startups by providing essential funding from raised funds, enabling these emerging companies to accelerate market entry, innovate, and create employment [1]. A VC firm's investment success depends on the risk inherent in its investment strategy, which in turn depends on the number of startups in its investment portfolio, or its portfolio size. Identifying the optimal number of startups for a VC portfolio is thus critical, influenced by variables such as available human capital for startup guidance, diversification approaches, the nature of the VC firm, investment types, and prevailing market conditions [1–4].

This study aims to derive the ideal portfolio size for VC firm , offering a benchmark to the VC industry to enhance investment strategies and thus, the success rate of VC-backed startups. Utilising an agent-based model (ABM), the study simulates real-world interactions between VC firms and startups. VC agents are trained through a state-of-the-art machine learning algorithm, specifically the Swap Softmax Twin Delayed Deep Deterministic Policy Gradient (SSTD3), which has been proven to outperform similar, preceding algorithms. They learn to refine their investment strategies by making investment decisions based on a set of observed parameters, and evaluating the outcomes. Fully trained VC firms should be able to follow optimal investment strategies based on the specific conditions and constraints within the developed environment. The resulting portfolios are expected to reveal the optimal number of startups for a VC portfolio. Python was used as the programming language for both the ABM and the SSTD3 algorithm.

2 Literature Survey

This review explores the underlying principles behind determining the optimal number of startups in a VC portfolio, evaluating VC portfolio performance, and understanding the SSTD3 algorithm. This sets the stage for the detailed exploration of the ABM in subsequent sections.

2.1 Optimal Venture Capital Performance

2.1.1 The Optimal Portfolio Size

A startup's success probability augments with an increased quality of advice and managerial effort from its VC investor. However, these diminish as the number of startups in a VC's portfolio increases, as the VC firm must distribute its limited resources [1, 3, 4]. The most successful VC firms achieve a higher startup success rate with a lesser effort in advisory, allowing a larger portfolio size while delivering effective guidance [5]. The optimal size of a VC portfolio balances a trade-off between the dilution of value-added advice and diversifying risks [3]. Portfolio diversification involves spreading risk by investing in low-correlated assets, and thus requires a minimum number of investments to be effective [4, 6].

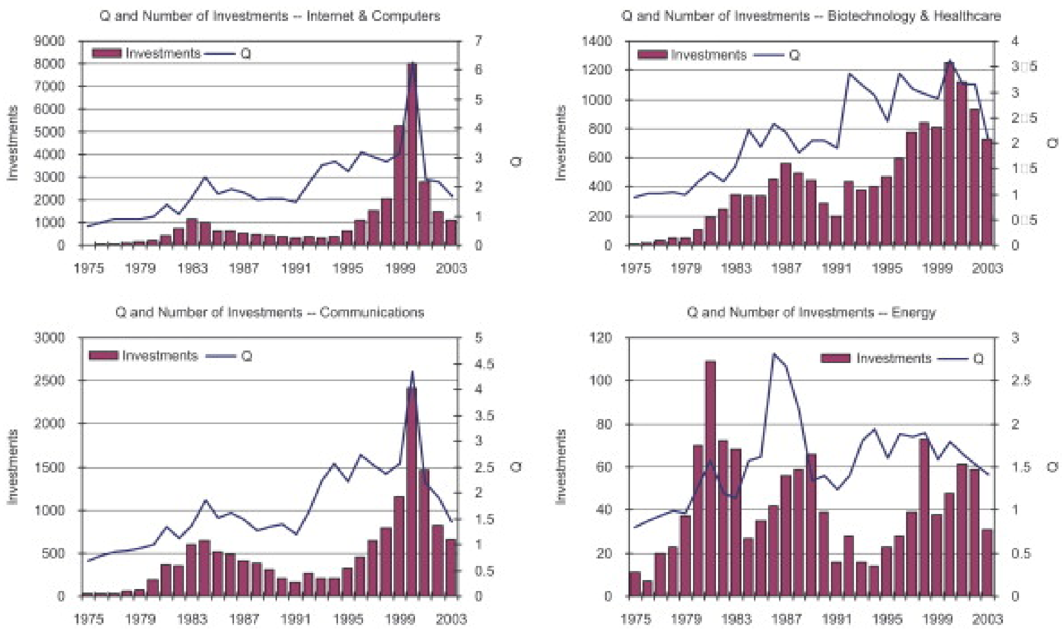

Figure 1: Weighted average Q and number of VC investments for selected industries. Q is the ratio of the market value of a firm to its book value. A larger Q signals a better market performance [7].

As Fig. 1 illustrates, VC investment activity is influenced by public market performance. Therefore, with the existence of local public markets and policies, such as grant schemes or financial regulations, VC investment activity, and thus startup success rates [5], vary across regions [8, 9]. This suggests that investing in different locations provides opportunities for diversification [10], and explains why VC portfolios tend to be larger in periods of greater stock market returns [3].

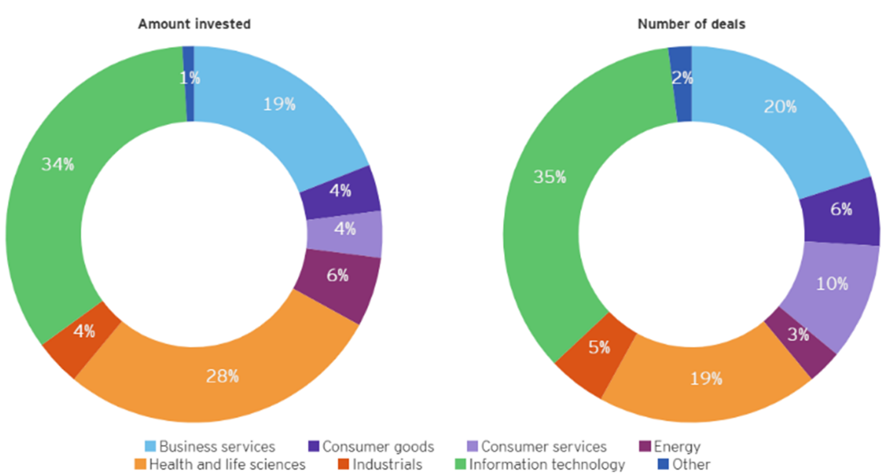

Figure 2: US VC investments across industries, Q2 2023. The differences between both charts conveys that the average size of investments varies across industries [11].

Fig. 1 also shows industry-specific VC trends. While diversification across industries can reduce risk, VC firms that focus and specialise on a narrow range of industries tend to outperform, due to their deep expertise and ability to identify promising opportunities [7, 12]. Nevertheless, the benefits of diversification suggest the need for an optimal balance between diversification and specialisation [6, 13].

In addition, portfolio size is influenced by VC investment characteristics, such as size or financing stage [3]. For instance, the magnitude of VC investments differs across industries, as Fig. 2 demonstrates. Furthermore, VC firms focusing on early-stage investments (younger startups) tend to manage larger portfolios than those focusing on later-stage investments [3, 4].

2.1.2 Startup Returns and Portfolio Performance

VC investments carry inherent risks due to the high failure rates of startups [14], with specific challenges such as conflicts among founders and principal-agent problems between VC firms and startups. These idiosyncratic risks are significant throughout a startup's development [15], and cannot be completely mitigated through diversification [6, 16].

VC firms also confront with informational uncertainties before and after investing, because of internal, company-specific knowledge within startups. This complicates effective advisory and return predictions [8, 17]. While startup screening reduces unknowns [8], it comes at the expense of time and resources that could be allocated to other potential investment opportunities, and is mainly limited by cost and time [17].

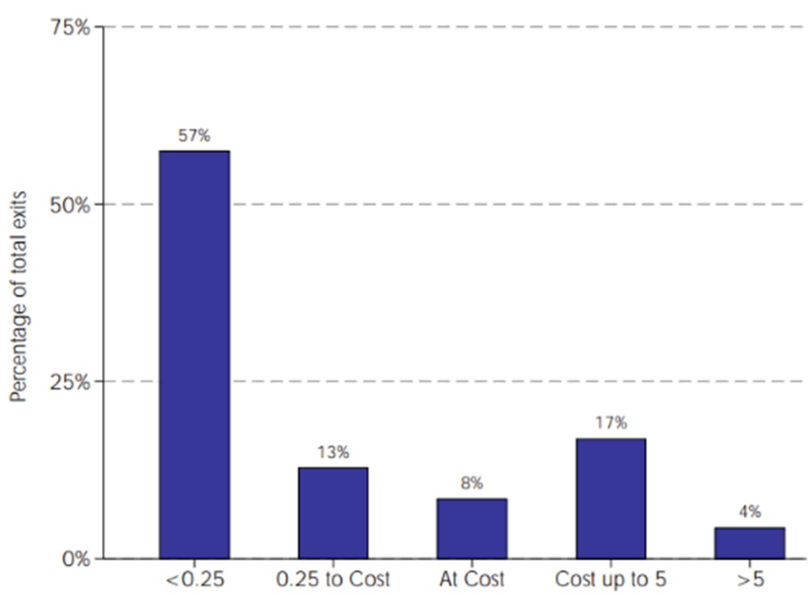

Figure 3: Distribution of value multiples of 2,065 early-stage VC investments between 1996 and 2015 by European-Investment-Fund-backed VC funds. A value multiples is the ratio of investment exit amount to initial investment, and "At Cost" includes all values such that $0.8 \leq value\,multiple < 1.2$ [18].

VC firms' investment success relies on a small proportion of startups that manage to proliferate and multiply their value over time. Entrepreneurship is thus governed by power law distributions where a few outliers represent a disproportionate amount of the returns on investment [14], as depicted below.

Regarding portfolio performance, the predominant metrics to measure performance are the Internal Rate of Return (IRR) and Total Value to Paid-In Capital (TVPI) [19]. IRR is the discount rate that sets the net present value of all cash flows to zero [20, 21], and TVPI is the ratio of the total value returned from investments to the total capital paid into them [19].

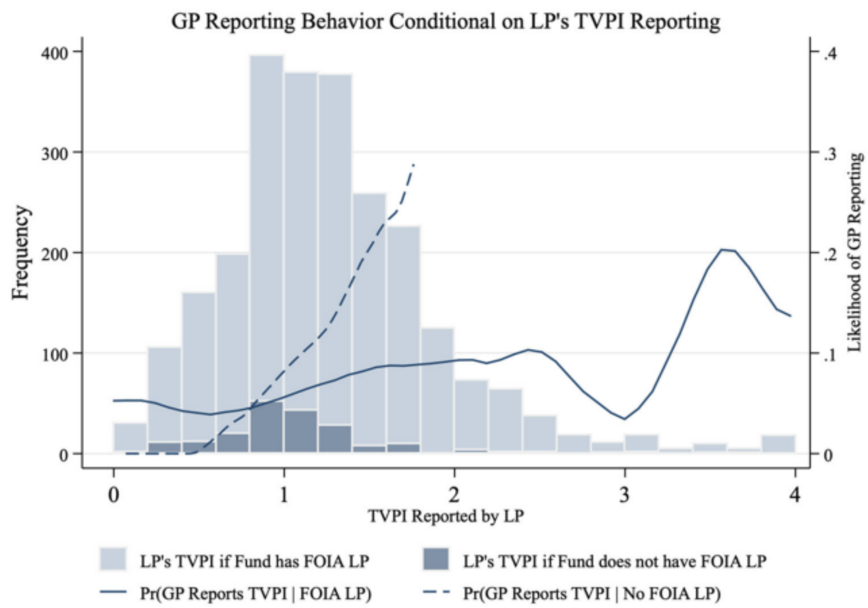

Figure 4: Reported TVPI frequency data from 2,841 VC funds between 2013 and 2017. LP is limited partner, GP is general partner, and FOIA stands for US Freedom of Information Act [19].

Measuring VC portfolio performance using IRR encounters important challenges. Firstly, it assumes that any cash inflows during the investment holding period are reinvested at the IRR rate, a presumption that is particularly problematic for startups, as they exhibit irregular cash flows. Secondly, IRR's ambiguousness and reliance on the timing of cash inflows may incentivise managers to manipulate this to artificially enhance performance [21]. Conversely, TVPI is not influenced by the time value of money and is not prone to these issues [19]. Fig. 4 displays a TVPI frequency distribution, indicating a distribution akin to a lognormal shape for this performance metric.

2.1.3 Diversification and Risk-Adjusted Performance

In portfolio construction, investors need to consider not just the expected returns of individual investments, but also their interrelationships. Markowitz's Modern Portfolio Theory (MPT) provides a framework for portfolio optimisation based on an investor's risk appetite and required rate of return, showcasing the importance of diversification as a hedge against the risks of highly correlated returns [6].

The Sharpe ratio (SR_p) is a widely used measure of risk-adjusted performance, quantifying how much excess return a portfolio provides per unit of risk [22, 23]:

$$SR_p = \frac{R_p - R_f}{\sigma_p} \tag{1}$$

Here, R_p is the portfolio return, σ_p is the standard deviation of returns (representing risk), and R_f is the risk-free rate, which is the return on a hypothetical risk-free asset [24]. By minimising the correlation between investment returns, diversification can mitigate risk [6], as reflected by the formula for expected standard deviation $E(\sigma_p)$ [25]:

$$E(\sigma_p) = \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1, i \neq j}^{n} w_i w_j Cov(R_i, R_j)}, \quad Cov(R_i, R_j) = \rho_{i,j} \sigma_i \sigma_j \tag{2}$$

Here, w_i is the weight of investment *i*, σ_i is the standard deviation of the returns for investment *i*, $Cov(R_i, R_j)$ is the covariance between the returns of investments *i* and *j*, and $\rho_{i,j}$ is their correlation coefficient. Nonetheless, the Sharpe ratio considers all deviations from the mean equally. This may not be suitable for VC investors, which are typically more concerned with downside risk [22, 23]. With VC investments, the Sortino ratio (SOR) empirically outperforms the Sharpe ratio as it specifically addresses downside volatility [22, 23]:

$$SOR = \frac{R_p - R_T}{TDD} \tag{3}$$

Here, R_T is the target return, often equated to the risk-free rate, and TDD is the target downside deviation, calculated as the root mean-square of the downside deviations from the target return.

2.2 Agent-Based Modelling with Machine Learning

2.2.1 Agent-based Modelling

Agent-based models (ABMs) are computational models where independent agents operate within a defined space and time, interacting with both their environment and each other [26, 27]. These models incorporate behavioural rules and spatial dynamics that may be derived from empirical data, balancing model complexity against the need for simplicity by sometimes excluding certain real-world behaviours or dependencies [26]. ABMs are particularly effective for complex problem-solving and understanding agent behaviours [26, 27], having proven their utility in entrepreneurship studies [28]. The integration of reinforcement learning (RL) with ABM represents a growing trend, enabling the creation of more sophisticated and adaptive agents [29].

2.2.2 Deep Reinforcement Learning and Actor-Critic Algorithms

In RL, agents interact with an environment in its current state and select actions that lead to new states, receiving rewards based on those actions and outcomes. The agents follow a policy in selecting actions, which is a mapping between states and probability distributions over actions. The agents learn by pursuing the optimal policy that maximises the expected rewards across all states [30–32].

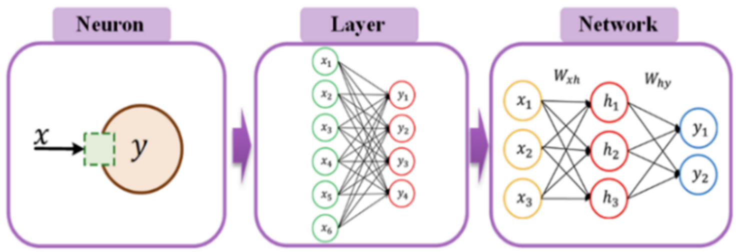

Figure 5: The hierarchical structure of an ANN. Output accuracy is enhanced with an increased number of layers. In the case of DRL, ANNs contain more than three hidden layers (layers between the input and output) [33].

In deep RL (DRL), Artificial Neural Networks (ANNs) determine the actions agents take and evaluate their outcomes for learning. These networks, illustrated in Fig 5, are characterised by parameters, namely weights and biases, that determine their output given a set of inputs [30, 33].

To maximise rewards, agents estimate the expected reward *r* for a given state *s*, policy π, and an action a through the value function $Q^{\pi}(s,a) = \mathrm{E}[r \mid s,a,\pi]$. The learning process aims to find an optimal policy that maximises $Q^{\pi}(s,a)$ for any set of actions and states. When combined with an ABM, agents can be trained for optimal behaviour [29, 30].

DRL often employs an actor-critic architecture with two ANNs: the actor network, with parameters ϕ, decides actions based on policy π_{ϕ}; and the critic network, with parameters θ, evaluates these decisions by estimating the value function [30, 32]. Training updates the networks' parameters to refine the policy selection process. At each timestep during the learning process, when an action *a* is taken from state *s*, receiving a reward *r* and leading to the next state *s'*, the transition is stored as a tuple [*s,a,r,s'*] in an experience replay buffer. A random sample of transitions drawn from the buffer is then used for training. This method reduces the variance of the learning updates, hence improving learning stability [30].

To optimise the critic network parameters, a separate set of networks is used to expedite convergence: the target actor π_{ϕ_T} and critic networks with parameters ϕ_T and θ_T, respectively. These are used to minimise the Bellman loss during training [32, 34]:

$$Bellman\ Loss = \frac{1}{M}\sum_{i=1}^{M}(y_i - Q^{\pi_\phi}(s_i,a_i))^2 \quad (4)$$

Here, M is the number of experiences $[s_i, a_i, r_i, s_i']$ sampled from the replay buffer, and y_i is the target, computed by assuming an immediate reward r_i and estimating the value function from the next state s_i':

$$y_i = r_i + \gamma Q^{\pi_{\phi_T}}(s_i', a_i') \tag{5}$$

Here, γ is a discount factor ($0 < \gamma < 1$) that allocates more importance to immediate rewards, and $Q^{\pi_{\phi_T}}(s_i', a_i')$ is estimated using the target critic network. Action a_i' is the action that would be taken following the target actor network's policy π_T from state s_i'. The Bellman loss is minimised using gradient descent and the critic network's parameters θ are updated accordingly [30, 32, 34].

Given a current state s_0, current action a_0 and actor with parameters ϕ, and expression for the expected long-term rewards can be derived [34]:

$$J(\phi) = \mathrm{E}\left[\sum_{k=0}^{\infty} \gamma^k r_k \mid s_0, a_o, \pi_\phi\right] \tag{6}$$

Here, *k* is the transition number in chronological order. In training, the gradient $\nabla_\phi J(\phi)$ is utilised to find the actor network's parameters ϕ that maximise long-term rewards, and update them accordingly. Subsequently, the target networks' parameters are refined through momentum updates [32, 34]:

$$\theta \leftarrow \tau\theta + (1-\tau)\theta_T \tag{7}$$

$$\phi \leftarrow \tau\phi + (1-\tau)\phi_T \tag{8}$$

Here, τ is the target network update rate ($0 < \tau < 1$), which smoothens and stabilises the learning process [31].

Some algorithms are prone to overestimation bias when computing the target y_i. Recent algorithms solve this problem by employing two target actor-critic network pairs and taking the minimum from both estimates [33]:

$$y_i = r_i + \gamma \min_{i=1,2}[Q^{\pi_{\phi_{Ti}}}(s_i', a_i')] \tag{9}$$

However, this approach can lead to a large underestimation bias [34].

2.2.3 SSTD3

The SSTD3 is a state-of-the-art actor-critic DRL algorithm, well-suited for continuous action spaces where actions span a continuous range. It addresses target over- and underestimation bias by utilising the Swap Softmax function [32]:

$$y_i = r_i + \gamma \cdot Swap\,Softmax[Q^{\pi_{\phi_{T1}}}(s_i', a_i'), Q^{\pi_{\phi_{T2}}}(s_i', a_i')] \tag{10}$$

The Softmax function computes a weighted average, allocating more weight (w_i) to larger values:

$$Softmax[z_1, z_2, ..., z_N] = \sum_{i=1}^{N} w_i z_i, \quad w_i = \frac{e^{z_i}}{\sum_{k=1}^{N} e^{z_K}} \tag{11}$$

Here, $z_1, z_2, \ldots, z_N$ represent general inputs. The Swap Softmax, in the case of the SSTD3 with two inputs, reverses weights to favour the smaller value:

$$Swap\,Softmax[Q_1, Q_2] = w_1 Q_2 + w_2 Q_1, \quad w_i = \frac{e^{Q_i}}{e^{Q_1} + e^{Q_2}}\,for\,i = 1, 2 \tag{12}$$

This swap is applied as the underestimation of the target is preferable to its overestimation, and it has been proven to avoid over- and underestimation bias [32].

The SSTD3 also employs asynchronous updates during training: the critic network's parameters are updated at each timestep, and the actor and target networks' parameters are updated at predefined intervals, effectively reducing errors and volatility during training [32, 33].

3 Methodology

To determine the ideal number of startups for a VC portfolio, an ABM was developed to simulate the VC ecosystem, incorporating realistic assumptions and interactions between VC firms and startups based on real-world data. By employing a SSTD3 algorithm, the model aims to refine VC investment strategies within the simulations, expecting to reveal the optimal VC portfolio size once the algorithm is fully trained. This section explains every decision made regarding the ABM and the SSTD3 algorithm.The general structure of the code is outlined in Appendix A.

3.1 Source of Data

Private entities like VC firms and startups have fewer public financial reporting obligations compared to public companies [35]. Therefore, specialised databases are required for accessing private market data. PitchBook [20], chosen for its extensive database and detailed records on VC firms and investments, serves as the primary data source for the model. Its selection was also influenced by a user-friendly interface, a vast database of over 39,000 VC funds, a comprehensive coverage of the VC market [36], and the availability of a university license for access.

The methodology for data extraction from PitchBook was constrained by the university license limitations and the variability in data disclosure among private entities. This variability in data availability can be noticed throughout the report, where different samples were necessarily used for different purposes, although the largest possible sample was selected in each case.

3.2 Outline of the Model

The developed ABM includes two agent types, VC firms and startups, set in a timeline where 100 VC firms start with fully raised funds and begin investing in startups, all simultaneously for simplicity. This timeline spans a ten-year period, mirroring the typical lifecycle of a VC fund [35]. It ends with VC firms realising their returns after divesting all of their ventures. Data from PitchBook on 1,030 VC investors [20] reveals that 89.5% manage a single fund and only 2.4% operate more than two. Hence, the model assumes each VC operates a single fund.

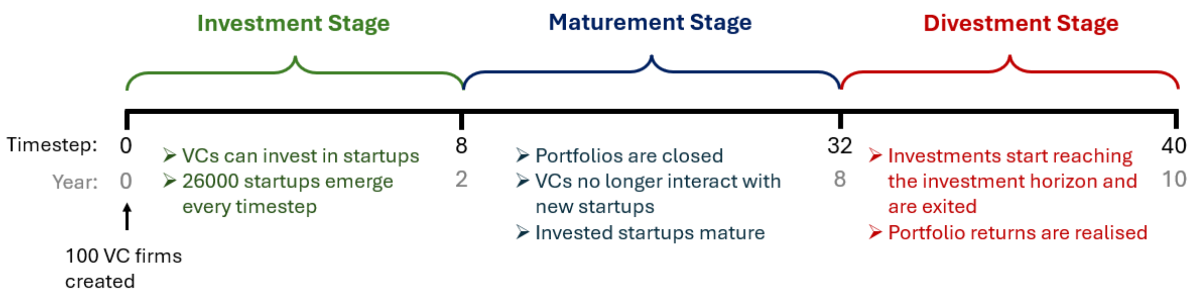

Figure 6: Timeline of the ABM outlining the three main stages.

The timeline is segmented into 40 intervals, each interval a quarter of a year. This simplifies any actions taken within a three-month period into a single step. New startups appear at each quarter, and VC firms decide on investments using the SSTD3 algorithm, serving as the decision maker. Investments are exited during the divestment stage, and the simulation terminates when all funds mature after ten years, allowing for the evaluation of each VC firm's portfolio performance. This simulation was repeated 50 times in a loop, with the algorithm starting with the parameters trained from the last simulation each time. This proved enough to fully train the SSTD3 algorithm, which converged to an optimal VC portfolio size.

3.3 Number of Agents and Investment Horizon

The ABM simulates the VC ecosystem with 100 VC firms, and 26,000 new startups launched each quarter. This mirrors the fact that, today, there are approximately 48,000 VC firms operating globally [37], and 12.5 million startups emerging quarterly worldwide [38]. Although the number of VC firms was scaled down to reduce computational demand, the ratio of operative VC firms to startups launched every quarter was maintained. Startups not funded during their first quarter were deemed as failures and removed from the model.

The duration a VC firm retains an investment for before exit, known as the investment horizon or holding period, differs significantly across investment types [35]. Nevertheless, the model generalises this duration to eight years for all investments to maintain simplicity. This was deemed reasonable considering VC investment horizons typically span between five and ten years [39].

This approach results in the timeline depicted Fig. 6, designed to ensure that all investments exit by the tenth year, thus allowing a two-year investment period in which VC firms can engage in new investments. Following this phase, the portfolios are considered locked, and startups continue to develop until they are divested at the end of their eight-year holding period.

3.4 Venture Capital Firm Attributes

VC firms and startups are characterised by attributes that serve multiple functions, such as determining how they interact with startups, estimating their due diligence capabilities in the case of VC firms, and assessing their performance. These attributes are covered in this and the following section.

3.4.1 VC Quality

VC quality is an attribute designed to encapsulate a VC firm's ability to make sound investment choices and provide valuable managerial advice to the startups within its portfolio, directly linked to its reputation [35]. As high capacity in these areas typically correlates with better returns [5], VC performance data was analysed to model a distribution of *VC quality* across all VC firms in the simulation. Due to its advantages over IRR, TVPI was selected as the reference performance metric.

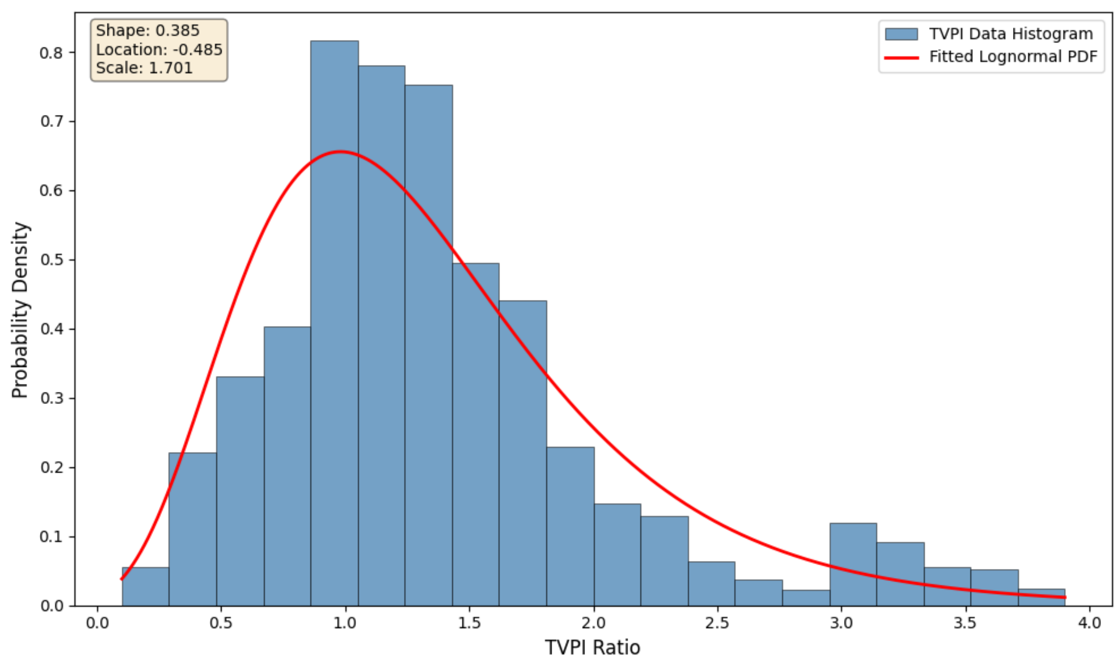

Figure 7: Reported TVPI frequency data from 2,841 VC funds between 2013 and 2017 [19] fitted into a PDF for *VC quality*. The 'shape', 'location', and 'scale' are the fitted parameters that are used as input to Scipy's *lognorm* function [40].

A probability distribution to *VC quality* was derived by fitting the TVPI distribution in Fig. 4 to a log-normal distribution, using the LP-reported data for funds with and without FOIA. The derived probability density function (PDF) is expressed as follows:

$$P(Q = q; \mu, \sigma) = \frac{1}{q\sigma\sqrt{2\pi}} exp\left[-\frac{(ln(q) - \mu)^2}{2\sigma^2}\right] \tag{13}$$

Here, $\mu = 1.347$, $\sigma = 0.733$ (mean and standard deviation, respectively), and *q* is the *VC quality*. This fitted distribution, shown in Fig. 7, was employed to randomly assign a TVPI value to each VC firm in the simulation. These values are then normalised between 0 and 1 to give *VC quality*. This proves convenient when modelling due diligence in a later section, which involves intense startup screening for informed investment decision making [35].

Normalisation is based on the upper boundary of the TVPI distribution's 99.8% confidence interval, calculated to be 5.11. This indicates that nearly all (99.9%) randomly generated TVPI values fall at or below this boundary, so 5.11 was used as the normalising factor. In rare cases where the normalisation process results in *VC quality* values exceeding 1, these values are adjusted down to 1 to maintain the model's consistency.

3.4.2 Investment professionals and Effort Available

In the model, the number of investment professionals within each VC firm plays a crucial role, determining its capacity or effort available for conducting due diligence on potential investments and managing portfolio companies. A distribution of investment professionals was applied to assign a random number of investment professionals to each VC firm in the simulation.

Table 1: Distribution for investment professionals in a VC firm, based on data for 15,831 VC firms with 25 investment professionals or less, representing 98.9% of all VC firms with available data [20]. These were deemed representative of the VC industry as some of the remaining 1.1% employ over 200 investment professionals, hence considered outliers.

# of Professionals	Rel. Frequency (%)	# of Professionals	Rel. Frequency (%)
1	30.42	14	0.38
2	21.95	15	0.29
3	15.31	16	0.24
4	9.66	17	0.19
5	6.49	18	0.17
6	4.30	19	0.12
7	2.82	20	0.11
8	2.23	21	0.12
9	2.56	22	0.14
10	1.26	23	0.10
11	0.83	24	0.12
12	0.66	25	0.11
13	0.41		

On average, a VC investment professional works 55 hours per week [41], translating to 715 hours per quarter. Given other necessary tasks such as networking and fundraising [42], only about 70% of this time, or roughly 500.5 hours quarterly, is dedicated to due diligence and advisory [43]. With this, the effort a VC firm can allocate to advisory and due diligence can be estimated, based on its number of investment professionals (e.g., 3 investment professionals gives 1501.5 hours).

Considering a startup requires about 45 hours of advice quarterly on average [43], the effort available just for due diligence can be estimated at each timestep, based on the current number of startups in the VC firm's portfolio. Knowing that a VC professional dedicates about 60 hours to due diligence per startup [44, 45], this framework allows to calculate the number of startups a VC firm can evaluate with due diligence each quarter. This is the number of startups a VC firm interacts with at each timestep in the model, which diminishes as its portfolio size grows.

If a VC firm has no effort available to interact with startups, it will not be able to make more investments. This realistically addresses a VC firm's limited resources, and the fact that advisory is a factor that tends to reduce the optimal VC portfolio size [1, 3, 4]. However, while the model assumes equal efficiency across all investment professionals, efficiency varies widely due to differences in experience levels, from senior partners to novice analysts. This assumption must be acknowledged and represents a limitation to the model, as experience and efficiency can vary significantly among and within VC firms [46].

3.4.3 Endowment

VC fund sizes vary widely, ranging from less than $50,000 to over $5 billion [20]. In this simulation, each VC firm is initially allocated a normalised endowment of 1 and invests fractions of this endowment in startups. This approach not only simplifies the model but is also well-suited for its objective, which is to estimate the optimal number of startups for a VC portfolio. While a larger endowment enables a fund to invest in more startups, it is argued that the ideal portfolio achieves the best risk-adjusted performance [23], which in turn relies on strategic diversification rather than investment capacity [6]. Although this approach seems logical, it overlooks the fact that small funds may be obliged to make small investments to reach a theoretical optimal portfolio size, thus constraining their investment options.

3.5 Startup Attributes and Performance

3.5.1 Startup Potential

Startup potential represents a startup's ability to generate returns in the future, derived in this model using annual revenue growth rate data. Revenue growth, reflecting the growth of a business's operational income, can be used as an indicator of potential firm performance [14]. While quality advice from the VC investor can positively influence startups' revenue growth [5], this is not an intrinsic characteristic of a startup and its potential; the quality of the business idea and the founder team's entrepreneurial skills are the most critical determinants of startup growth [47]. Therefore, revenue growth rate serves as a proxy for a startup's inherent potential.

Annual revenue growth data [20] includes extreme values that are unlikely sustainable. To avoid these, annual revenue growth rates that yield more than a 100x growth over an eight-year investment horizon, through compounding, were filtered out: $100^{\frac{1}{8}} - 1 = 77.8\%$. This threshold was selected because, while attainable, returns exceeding a 100 value multiple are uncommon even amongst successful VC investments [48]. So, annual revenue growth rates above 77.8% and below -77.8% (for consistency) were excluded.

Data from PitchBook must be extracted manually [20], so the analysis was conducted on 410 selected data points out of the available 4103, ensuring the random sample retained the original population's proportion of startup locations and industries.

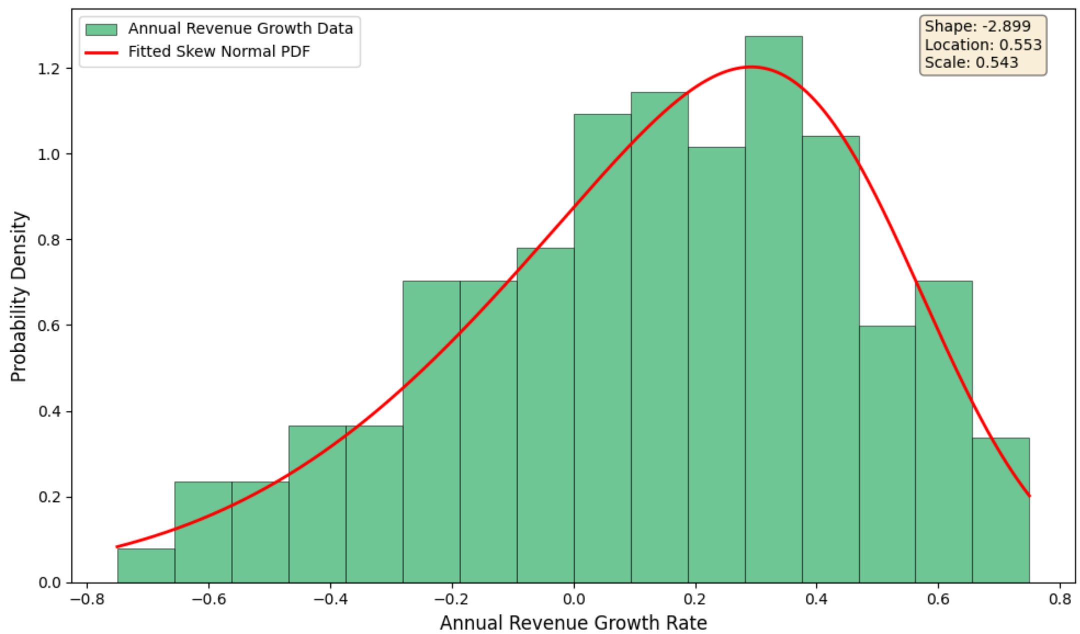

Figure 8: Annual revenue growth rate data on 410 startups [20] fitted into a PDF for *startup potential*. The 'shape', 'location', and 'scale' are the fitted parameters that are used as input to Scipy's *lognorm* function [49].

As illustrated in Fig. 8, the annual revenue growth rate distribution was fitted with a skew normal distribution, judged to best represent the population. The resulting PDF that was used to randomly allocate a potential to each startup [50]:

$$P(Z = z; \xi, \omega, \epsilon) = \frac{1}{\omega\sqrt{2\pi}} \left[-\frac{1}{2} \left(\frac{z-\xi}{\omega} \right)^2 \right] \left[1 + erf \left(\epsilon \left(\frac{z-\xi}{\omega\sqrt{2}} \right) \right) \right] \tag{14}$$

Here, $\epsilon = -2.899$, $\xi = 0.553$ and $\omega = 0.543$ are the shape, location and scale parameters, respectively, and *z* is startup potential. This PDF distribution for startup potential has a mean and standard deviation of 14.4% and 35.7%, respectively.

3.5.2 Subindustry

VC firms often specialise in specific industries to enhance their investment success [12], typically focusing on a limited range of sectors to strike a balance between specialization and diversification [13, 51]. This concurs with the data in Table 1, indicating that, probably, most VC firms cannot specialise in several industries with a low number of investment professionals. Hence, this model assumes VC firms invest in a single industry, so startups are not assigned an industry attribute.

Despite industry specialisation, diversification within industries can also enhance a portfolio's performance [52]. To allow for this possibility, the model randomly allocates each startup to one subindustry, any from *subindustry 1* to *subindustry 5*. A different startup potential distribution was derived for each subindustry, to allow VC firms to diversify by investing in startups with different risk-return profiles. Information Technology (IT), the most prevalent sector for VC investments (see Fig. 2), was selected for this. The IT industry includes five subindustries [20], and this is assumed to apply generally to all industries, for simplicity.

Table 2: Standard deviations of annual revenue growth rate data on 70 startups for each of the five subindustries within the IT industry [20], and their discrepancies form the mean. Five equally sized samples were randomly drawn for consistency, 70 being the largest possible size with the data available.

	Software	**Communications and Networking**	**Computer Hardware**	**IT Services**	**Semiconductors**	**Mean**
Standard deviation	29.5%	35.8%	32.6%	34.2%	39.6%	**34.3%**
Discrepancy from mean standard deviation	-4.9%	1.5%	-1.7%	-0.1%	5.3%	**0.0%**

Analysis was conducted on 70 startups' annual revenue growth rates from each IT subindustry (see Appendix B), using the same filtering criteria, and the standard deviation was calculated for each. Table 2 reveals that the IT services subindustry's standard deviation almost equals the average across all five subindustries, with the other four displaying approximately symmetrical discrepancies from this average. This structure was conveniently used in the model: subindustry 3 was assigned the global average startup potential distribution (derived in section 3.5.1), while all other subindustries were allocated similar distributions only with slightly different standard deviations and means. This approach introduces different risk-return profiles, while managing that all five distributions roughly average out to the global average startup potential distribution

Table 3: Means and standard deviations of each subindustry's italic(startup potential) distribution. The skewness remains intact across all distributions, so the shape parameter remains constant ($\epsilon = -2.899$), while the location and scale parameters vary [50].

Sub-industry	Equivalent IT subindustry	Discrepancy from mean standard deviation	Relative discrepancy	Standard Deviation	Mean	Location	Scale
1	Software	-5.0%	12.4%	30.7%	12.4%	0.475	0.466
2	Computer Hardware	-1.5%	13.8%	34.2%	13.8%	0.530	0.520
3	IT Services	0.0%	14.4%	35.7%	14.4%	0.553	0.543
4	Communications and Networking	1.5%	15.0%	37.2%	15.0%	0.576	0.565
5	Semiconductors	5.0%	16.4%	40.7%	16.4%	0.632	0.621

Table 3 illustrates how the discrepancies in Table 2 were conveniently rounded to ensure symmetry, and how adjustments in each subindustry's standard deviation were compensated by equivalent relative changes in the mean. This ensures the risk-return trade-off is accurately represented, as standard deviation and expected return should theoretically be roughly directly proportional [6].

Additionally, the model introduces diversification by exploiting the imperfect correlation of returns across subindustries [6], employing a correlation matrix to calculate portfolio variance and risk. Due to the absence of time-series startup return data [20], correlations were derived from US stock data, which are readily available. A limitation to this approach is that stock returns, in general, are only modestly correlated with VC returns [15]. Nevertheless, this relationship is more pronounced in the US [53], hence the choice of US stocks. .

Table 4: Correlation matrix for subindustries in the model. The US stock indices used are displayed with their equivalent subindustry. The quarterly values for each of these indices from 2014 to 2023 were used to derive the correlations (see Appendix C) [54, 55].

	S&P 500 Software Index	S&P 500 Technolgoy Hardware Select Index	S&P 500 IT Services Index	S&P 500 Communications Equipment Index	S&P 500 Semiconductor Materials & Equipment Index
	Subindustry 1	Subindustry 2	Subindustry 3	Subindustry 4	Subindustry 5
Subindustry 1	1.000	0.893	0.953	0.911	0.979
Subindustry 2	0.893	1.000	0.903	0.861	0.909
Subindustry 3	0.953	0.903	1.000	0.934	0.935
Subindustry 4	0.911	0.861	0.934	1.000	0.886
Subindustry 5	0.979	0.909	0.935	0.886	1.000

The stock indices in Table 4 were selected for their close alignment with the five IT subindustries previously analysed, ensuring consistency across the model.

3.5.3 Location

VC firms are not allocated to specific locations in this model for two key reasons. Firstly, VC firms invest predominantly in local ventures since these generally achieve greater success [10], suggesting that regional macroeconomic factors rarely contribute to diversification in most VC firms. Secondly, critical macroeconomic indicators, like real GDP growth, show a strong correlation across different areas. Consequently, in the case of international VC firms, macroeconomic factors are not expected to hugely impact diversification.

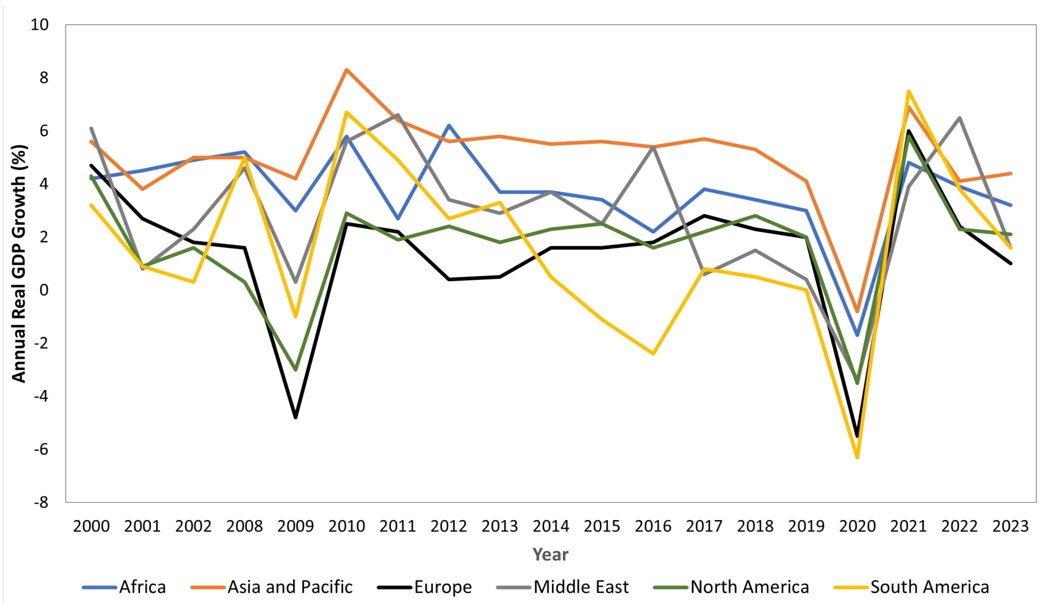

Figure 9: Annual real GDP growth for each world region between 2000 and 2023 [56]. The data depicts a strong, but not perfect, correlation between regions.

In practice, larger and more prestigious VC firms often engage in cross-border investments, and the diversification of their portfolio partially depends on macroeconomic considerations [10]. Furthermore, specific locations exist where startup success rates are higher than the average, such as San Francisco, US [57]. These factors are not captured in the model, representing a limitation. However, the decision to ignore these was made considering the complexity it would add against the marginal benefit it could provide, prioritising model simplicity.

3.5.4 Mapping Startup Potential into Returns

VC returns are distributed according to the following power law PDF, based on [58]:

$$P(R = r; \alpha) = (\alpha - 1)r^{-\alpha} \tag{15}$$

where $\alpha = 2.06$ is the power law parameter and r is the return. This is instrumental in mapping a startup's potential to its expected return in the model – a mapping between distributions. The cumulative density function (CDF) for the *startup potential* distribution is given by:

$$F_Z(z) = P_Z(Z \leq z) \tag{16}$$

Here, *z* is the startup potential. For mapping, the CDF for a specific startup potential, z_i, is input to the inverse of the returns PDF, yielding its corresponding expected return, $\mathrm{E}[R_i]$:

$$\mathrm{E}[R_i] = P_R^{-1}(P_Z(z_i)) \tag{17}$$

This method simplifies the calculation of expected returns for startups, relying on empirical data for return distributions. The SSTD3 algorithm utilises this process to guide investment decisions and evaluate portfolio performance. A main limitation of this approach is that revenues do not directly translate into returns. For instance, capital-intensive industries generally see lower returns on investment due to high operating costs [59]. Therefore, a greater revenue growth rate does not imply greater returns.

3.5.5 Progression in Time

Each enter the model with an initial potential based on the distributions developed, which evolves at each timestep until it is divested, modelled by fixing its potential. Their potential evolves at each timestep according to:

$$Potential_{t+1} = \alpha \times Potential_t + \beta \times Advisory + Idiosyncraticrisk \tag{18}$$

Here, α and β are weights. Eq. 18 models how a startup's growth is influenced by the advisory from its VC investors and idiosyncratic risks as the startup ages.

It was presumed that the quality of advice is directly linked to the VC firm's quality. Thus, the Advisory factor in Eq. 18 represents VC quality, adjusted so the mean VC quality aligns with the mean startup potential; the average VC quality should neither enhance nor reduce the average startup's potential. Given the mean normalized *VC quality* is 0.264, 12% higher than the mean startup potential of 14.4%, the following modification was employed:

$$Advisory = VC\,quality - 0.12 \tag{19}$$

The impact of advisory impact varies and it can be challenging to quantify, with studies indicating an increase in sales growth ranging from 6.2% [60] to 13.1% [61] annually due to quality advisory. Based on this range, it was assumed the optimal advisory boosts potential by 10% in 8 years (32 timesteps), as potential is mapped to returns once the investment horizon is reached. The maximum Advisory value is 0.88, as the maximum VC quality is 1. With this information and using the mean potential (from 14.4% to 24.4% in 32 steps with the best advisory), adequate values for α and β can be derived:

$$\mathrm{E}[Potential_{32}] = ((0.144\alpha + 0.88\beta) \times \alpha + 0.88\beta ... = \sum_{n=0}^{31} 0.88\beta\alpha^n = 0.244 \tag{20}$$

Using the constraints $\alpha + \beta = 1$ and $\alpha, \beta > 0$, Eq. 20 is solved using iteration to give $\alpha = 0.99545$ and $\beta = 0.00455$, to five decimal places. This formulation effectively simulates how advisory can impact a startup potential both positively and negatively [62]. The term for idiosyncratic risk is excluded from Eq. 20 because it follows a normal distribution with a mean of zero, presuming it could both positively and negatively impact investment returns, so it does not affect expected potential.

Arithmetic alpha is a metric that represents abnormal return above the expected [35]. Data spanning from 1987 to 2000 indicates an average annual VC return of 59% with an arithmetic alpha of 32% when correcting for selection bias, driven largely by idiosyncratic volatility [15]. Consequently, average returns should be 2.185 times greater than expected in the model, and conserving the near proportional risk-return relationship [6], the average volatility should be 2.185 times greater. Therefore, the average standard deviation for startup potential must be elevated from 35.7% to 78.4% before divestment. Assuming idiosyncratic risk is the sole responsible for the additional volatility:

$$\sigma_{id} = \sqrt{\sigma_{final}^2 + \sigma_{initial}^2} = \sqrt{0.784^2 + 0.357^2} = 0.698 \tag{21}$$

Here, σ_{id} is the standard deviation of idiosyncratic risk, $\sigma_{initial}$ is the standard deviation of a startup's initial potential for specific subindustry, and σ_{final} is the standard deviation of a startup's final potential (the average values for the two former are used here, as these vary with subindustry). However, as the *idiosyncratic risk* factor in Eq. 18 must be applied over eight years (32 timesteps), it is assigned a standard deviation of $\frac{0.698}{\sqrt{32}} = 0.123$.

3.6 Screening and Due Diligence

VC firms typically screen a large pool of startups, narrowing them down to a smaller group for subsequent due diligence [35]. The initial screening phase is simplified by randomly matching startups with VC firms. While this overlooks the possibility that an effective screening process may be part of the reason why top-performing VC firms tend to make better investments [62], the model focuses on the investment decision-making process, which most importantly involves due diligence [35].

VC firms are assigned startups until their due diligence capacity is fully utilised, forming a cohort of investment prospects at each timestep for each VC. Startups are limited to evaluations by a maximum of five VC firms – they can become part of a maximum of five cohorts – under the assumption that failing to secure investment after five interactions with VC firms leads to bankruptcy.

The model acknowledges that all VC firms, regardless of their success rate, encounter a certain level of uncertainty when assessing startup potential [63]. A noise with mean zero is applied to each startup's potential to output the startup potential that the investor perceives. This noise depends on the startup's inherent potential uncertainty ($\sigma_{initial}$), investor's *VC quality*, and the number of timesteps a startup has existed for (*startup age*):

$$\sigma_{noise} = \frac{2\sigma_{initial}(1.5 - VC\,quality}{\sqrt{startup\,age}} \quad (22)$$

Here, σ_{noise} is the standard deviation of the noise applied to startup potential, for a particular VC investor and startup. Doubling $\sigma_{initial}$ acknowledges the inherent risk in venture investments characterised by high failure rates [14], chosen arbitrarily, as well as the factor $(1.5 - VC\,quality)$. This factor models the influence of VC quality on the effectiveness of due diligence: best VC firms are able to make more accurate predictions [62], whereas the worst VC firms face increased uncertainty (by 50%). The discretionary adjustment for startup age accounts for the fact that VC firms become more accurate at forecasting the return of an investment as the startup matures [64].

3.7 Portfolio Performance

In the model, the SSTD3 algorithm continuously assesses the expected risk-adjusted performance of VC firms to optimise investment choices. Initially, the Sortino ratio was selected to measure risk-adjusted performance, considering its important advantage of addressing downside risk [22, 23]. Early in the simulation, however, VC firms are likely to have minimal investments with a significant chance that none underperform against the target (risk-free rate). This causes the Sortino ratio to malfunction by having denominator equal to zero, making it impractical for the model, so the Sharpe ratio was adopted instead.

For determining the risk-free rate, commonly represented by treasury yields with similar holding periods [[35], the average annual rate of the 10-year US Treasury Bill and the 10-year German bond from 2008 to 2024 was used. This resulted in an average annual rate of 1.98% (see Appendix D), which compounded over an 8-year investment holding period, yielded a risk-free rate of $17.0\% = (1 + 0.0198)^8 - 1$. The Sharpe ratio of any VC portfolio *p* in the model (SR_p):

$$SR_p = \frac{\mathrm{E}[R_p] - 0.17}{\mathrm{E}[\sigma_p]}, \quad where\,\mathrm{E}[R_p] = \sum_{i=1}^{n} w_i R_i \quad (23)$$

Here, $\mathrm{E}[R_p]$ and $\mathrm{E}[\sigma_p]$ are the expected return and standard deviation of portfolio *p*, respectively, w_i is the proportion of the total endowment invested in startup *i*, and R_i is the return of startup *i*. For the standard deviation of each startup's returns, necessary to calculate $\mathrm{E}[\sigma_p]$ as shown in Eq. 2, the standard deviation of each startup's final potential σ_{final} is taken, which depend on their subindustry:

$$\sigma_{final} = \sqrt{\sigma_{final}^2 + \sigma_{initial}^2} \quad (24)$$

The correlations between returns, displayed in Table 4, are also needed to compute $\mathrm{E}[\sigma_p]$.

3.8 Decision Making

The SSTD3 algorithm was selected to train the VC agents in the ABM due to its suitability for a continuous action space, and its ability to correct estimation biases inherent in other DRL algorithms, also outperforming them in effectiveness [32]. It assesses investment scenarios by analysing a set of parameters that define each investment prospect, mirroring the comprehensive evaluation a real-world VC would undertake for investment decision making. During training, VC agents learn to make better investment decisions based on these parameters.

Table 5: Parameters that define each investment prospect in the model. These can be divided into those specific to the prospect being assessed (1-2), those common amongst the investment prospect cohort (3-4), those portraying the portfolio of the VC agent making the investment decision (5-6), and those concerning the VC firm's characteristics and situation (7-10). Each startup potential is revealed to the algorithm after due diligence, and thus includes the corresponding noise that distorts the real value.

#	Parameter	Description
1	Prospect potential	Allows to forecast return on investment.
2	Mean prospect correlation with portfolio	Determines the potential diversification benefit.
3	Mean prospect potential in cohort	Allows a comparison of the prospect with its cohort, which may suggest whether there are other better opportunities.
4	Standard deviation of potential in cohort	
5	Mean portfolio potential	Allows a comparison of the prospect with the current VC firm's portfolio.
6	Standard deviation of potential in portfolio	
7	Percentage of due diligence capacity left	Supports the estimation of the number of investment opportunities the VC firm will be able to evaluate in the future.
8	Remaining of investment stage	
9	VC quality	Gives an indication of the accuracy of the *startup potential* observed.
10	Endowment left	Shows the endowment left to invest, between 0 and 1.

Table 6: Reward structure and investment decisions corresponding to each possible action by the SSTD3 algorithm.

Action	Investment Decision	Reward
$action < 0$	No investment	$-100 \times action$
$0 \leq action < 0.005$	No investment	0
$0.005 \leq action \leq 1$, $action \leq endowment\, left$	Investment in startup equal to the action selected	Sharpe ratio after investment – Sharpe ratio before investment
$0.005 \leq action \leq 1$, $action > endowment\, left$	No investment	$-100 \times (action - endowment)\, left$
$action > 1$	No investment	$-100 \times action$

The VC agents' actions are translated to investment decisions and rewards according to Table 6, which also. Actions represent the fraction of a VC's endowment invested in a startup, and the aim of the SSTD3 algorithm is to maximise the rewards VC agents receive.

Actions range from 0 to 1 to prevent impossible investment sizes, with penalties in the form of negative rewards for exceeding this range. Investments under 0.005, the minimum investment, yield no reward to prevent investments being unrealistically small. This was based on data from 13,267 VC firms [20], revealing that only 0.37% manage over 200 active investments, so this was chosen as the maximum number of startups a VC firm can invest in, in the ABM. No penalty is applied, however, as these actions as this allows the VC agents to not invest while avoiding a penalisation for it. Actions above the VC firm's remaining endowment incur a negative reward, proportional to the excess amount, to discourage over-investment.

If an investment is made, the reward is based on the improvement of the portfolio's risk adjusted performance, as Table 6 shows. This guides the VC agents towards optimising investment decisions to maximise risk-adjusted returns. Thus, once fully trained, this is expected to reveal the optimal number of startups in a VC portfolio. As rewards are calculated using startups' actual potential, but VC firms assess this potential disturbed by the noise of due diligence, the model introduces a realistic element of uncertainty in investment outcome predictions.

3.8.1 Algorithm and Hyperparameters

The pseudocode for the SSTD3 algorithm employed in this model is presented in the following.

Algorithm 1 SSTD3

for t=1 to *Number of actions* **do**
  Take an action *a* incorporating an exploration OU noise
  Obtain a reward *r* from the environment and transition to a new state *s'*
  Save transition [*s,a,r,s'*] in experience replay buffer
  Randomly draw a sample of transitions [*s,a,r,s'*] from the experience replay buffer
  $y_i = r_i + \gamma \cdot Swap\,Softmax[Q^{\pi_{\phi_{T1}}}(s'_i, a'_i), Q^{\pi_{\phi_{T2}}}(s'_i, a'_i)]$
  Update parameters θ_i of critic networks by using Bellman Loss
  if t % *policy delay frequency* == 0 **then**
    Update parameters ϕ_i of actor networks by using $\nabla_\phi J(\phi)$
    $\theta \leftarrow \tau\theta + (1-\tau)\theta_T$
    $\phi \leftarrow \tau\phi + (1-\tau)\phi_T$
  end if
end for

The model introduces noise to each action taken by the VC agents to encourage exploration and prevent early convergence on suboptimal solutions [65]. The Ornstein-Uhlenbeck (OU) process is used to generate correlated noises, which has been widely used in DRL algorithms and avoids cancelling out the overall learning dynamics [66]. The OU process is described by the following stochastic differential equation:

$$dx_t = \zeta(\mu - x_t)dt + \sigma dB_t \tag{25}$$

Here, μ, ζ and σ are constants, x_t is the generated noise, and B_t is the standard Brownian Motion [67], where $dB_t = \sqrt{dt}N(0, 1)$ [66]. The values $\mu = 0$, $\zeta = 0.15$, and $d_t = 0.002$ were chosen based on their effectiveness in financial portfolio optimization using DRL [65]. dB_t defines the magnitude of the noise generated as $x_0 = 0$ to prevent a bias towards a positive or negative noise. $\sigma = 0.028$ was selected to prevent the magnitude of most noise generated from exceeding 0.0025: considering that 95.4% of values sampled from a standard normal distribution have a magnitude lower than 2 [68], 95.4% of the generated noise magnitudes will remain below 0.0025 ($\sigma dB_t = 0.028\sqrt{0.0022 = 0.0025}$). The value 0.0025 was deemed a suitable limit given that the minimum investment amount in the model is 0.005, and a larger noise could force the algorithm to choose actions significantly greater than the minimum amount to avoid receiving negative rewards for going under zero. It also allows the agents to choose actions between 0 and 0.005, in order to reject an investment while avoiding being penalised with a negative reward.

Table 7: Reward structure and investment decisions corresponding to each possible action by the SSTD3 algorithm.

Hyperparameter	**Value**	**Description**
τ (Tau)	0.005	Target network update rate.
γ (Gamma)	0.99	Discount factor for training.
Number of hidden layers	4	Number of layers between the input and output.
Hidden layer size	256	Number of layers between the input and output.
Input dimensions	10	Number of parameters observed for each prospect.
Output dimensions	1	Number of actions taken for each prospect.
Buffer Size	100000	Capacity of experience replay buffer.
Sample size	64	Size of each sample drawn from the experience replay buffer.
Actor learning rate	0.0001	Rate at which the actor and critic parameters are updated [30, 33].
Critic learning rate	0.001	
Policy delay frequency	2	Interval for training the actor and target networks.
ζ (Zeta)	0.15	OU noise parameter.
μ (Mu)	0	OU noise parameter.
σ (Sigma)	0.028	OU noise parameter.
dt	0.002	Time differential.

Table 7 displays the values chosen for each hyperparameter for the SSTD3 algorithm. As the trained ABM converged to meaningful results, these hyperparameters remained unchanged.

4 Results and Discussion

This section outlines and discusses the results from the main simulation, sensitivity analyses for some main assumptions, the limitations of this study, and recommendations on how to address the latter in future studies.

4.1 Main Simulation

The SSTD3 algorithm's hyperparameters achieved convergence after just four simulations, making the initial plan of 50 reproductions more than sufficient.

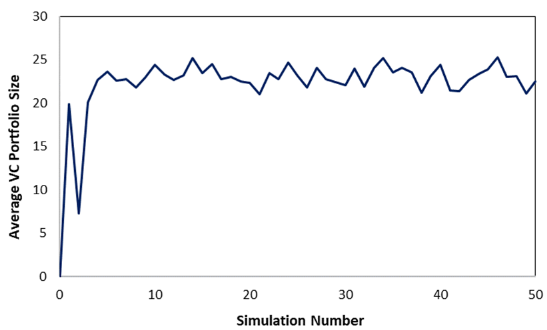

Figure 10: Convergence of the average VC portfolio size within the 50-loop main simulation. Average portfolio size remains within the same small range from simulation five onwards.

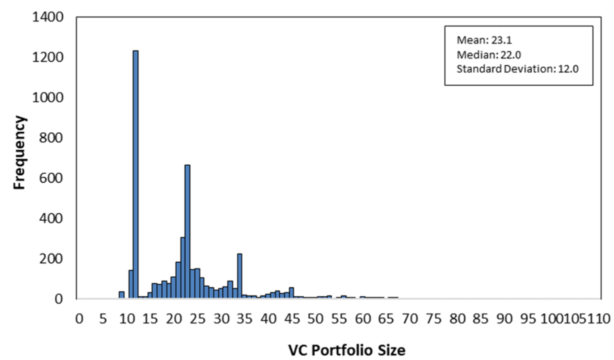

Figure 11: Histogram of VC portfolio size for all VC firms from main simulation number five onwards (4600 VC firms).

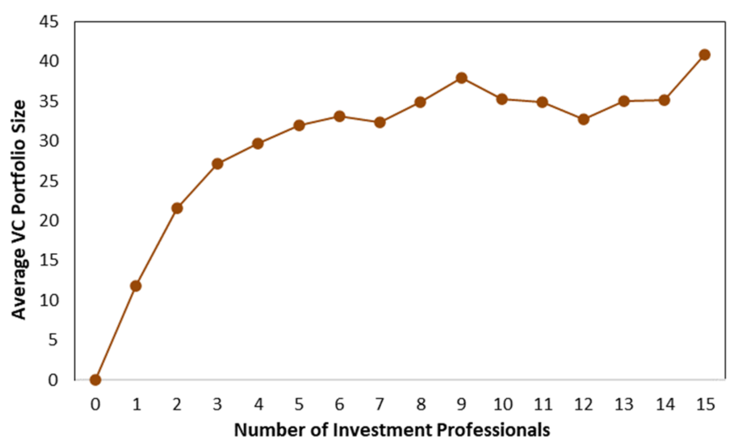

Figure 12: Correlation between the average VC portfolio size and the number of investment professionals, for VC firms from main simulation number five onwards, depicting a clear positive trend.

Analysis from the fifth simulation onwards, reflecting the converged solutions, reveals an optimal VC portfolio size of 23.1 startups on average, slightly above the real-world average of 15 to 20 [3, 69]. This discrepancy might arise from real VC firms' resource limitations, specifically the number of investment professionals, not allowing them to reach this optimal size not fully captured in the model. The results indicate a direct relationship between the number of investment professionals and the average VC portfolio size.

As it may be appreciated in Fig 12, 87% of all VC firms with one investment professional made exactly 12 investments, and 53.7% of all VC firms with two investment professionals made exactly 23 investments. This appears to be their respective maximum number of investments in the model, with no effort left to interact with more startups as it is all allocated to advising those already in the portfolio. It thus suggests that many agents would have sought to expand their portfolio if their human capital constraints did not exist. For further insight, the results were filtered to VC firms with five investment professionals or more.

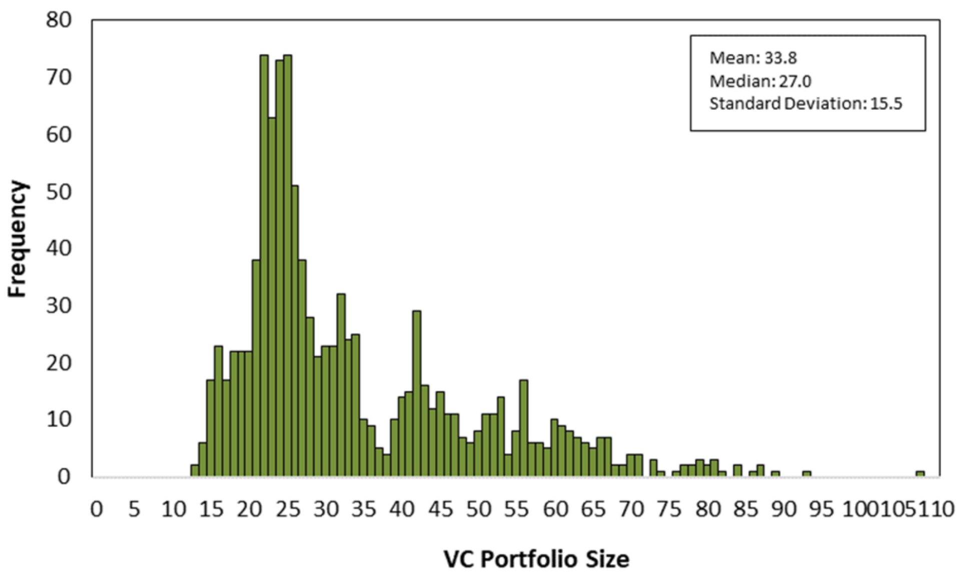

Figure 13: Histogram of VC portfolio size for all VC firms from main simulation number five onwards, including only VC firms with five investment professionals or more (1079 VC firms).

For VC firms with at least five investment professionals, the average portfolio size increases to 33.8, with a noticeable rise in the median from 22 to 27, and in the standard deviation from 12 to 15.5. While the rise in the mean and median concur with Table, a greater dispersion emphasises how VC firms with more investment professionals are not as constrained by their limited resources, and thus do not mostly make the maximum number of investments their resources allow them to.

The trend appreciable in Fig resembles the real VC ecosystem, where there is an evident positive correlation between the number of managers of a VC portfolio and portfolio size [3]. Clearly, the ABM has been able to model the need for a favourable balance between advisory effort and diversification, resembling how VC firms cease to make more investments to avoid diminishing benefits from quality advisory. In fact, in reality, while the average portfolio size for VC firms with less than seven managers around 12.3, this number ascends to 35.2 for VC firms with more than seven managers [3], almost coinciding with the corresponding portfolio size in Fig 13 (35.89).

In the model, once a VC firm uses up all of its effort in managing startups, there is none left to carry out due diligence, and thus they cannot interact with more startups. This is the reason behind 87% of all VC firms with one investment professional making exactly 12 investments. In reality, however, VC firms can always make more investments by paying with diminishing returns to advice, so this is not accurately modelled. Nevertheless, for addressing the advisory-diversification trade-off, which is one of the main drivers of the optimal VC portfolio size [1,3,4], the approach proved effective: once an additional startup incurs a significant reduction in the returns to advice per firm, the investment should not be made [1].

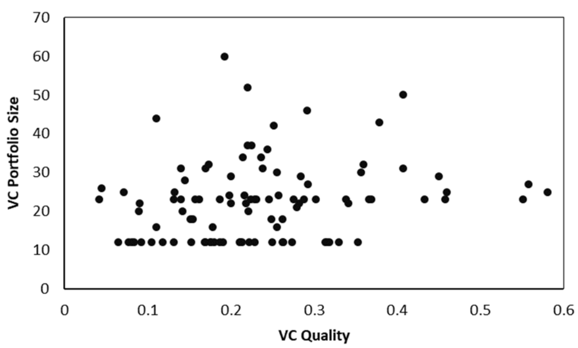

Figure 14: Relationship between *VC quality* and VC portfolio size for the last main simulation (100 VC firms). No clear correlation can be observed.

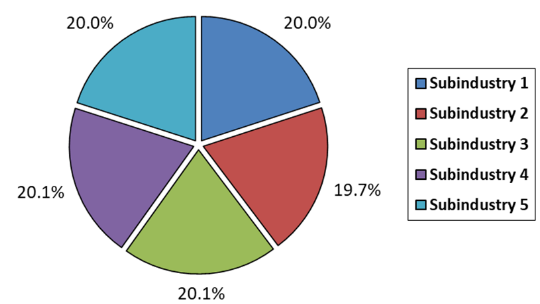

Figure 15: Proportion of investments in each subindustry, for all VC firms from the main simulation number five onwards (4600 VC firms). Almost identical percentages showcases the tendency of VC firms to diversify their investments to build an optimal portfolio.

Fig 15 illustrates VC firms' diversification across subindustries, even when some offer lower expected returns. This suggests agents have preferred the strategic choice by spreading risk across different risk-return profiles considering cross-correlations, rather than solely prioritising high expected returns. This behaviour supports the model's accuracy in representing VC firms' diversification strategies. In contracts to advisory capacity, advisory quality did not have an impact on the optimal VC portfolio size in the model, as Fig 14 depicts. A possible explanation is number of investment professionals being a limiting factor of huge importance, such that it eclipses any influence the quality of advisory might have on the statistic of interest. In reality, the best VC firms are able to deliver quality advisory with lesser efforts, allowing greater portfolio sizes with the same human capital resources [5]. Therefore, the lack of correlation could alternatively signify an oversimplification of the model in this aspect.

4.2 Removal of the Effort Attribute

Despite the success in addressing the advisory-diversification trade-off, VC agents' lack of freedom for investing in more startups once their effort capacity is full represents a limitation. Therefore, a second simulation was run by removing the effort attribute from VC firms, and thus the influence the number of investment professionals has on the ability of a VC firm to interact or make investments. This was done to deeper understand the influence of diversification, without considering advisory efforts and costs, on the optimal number of startups in a VC portfolio.

To manage computational demands, the simulation must limit interactions in some manner. VC-startup interactions were limited to 83 per quarter per fund, constant throughout the model (independent of the number of investments made), which is the due diligence capacity of 10 investment professionals in the model. This number was chosen as the maximum portfolio size in the results was 108, and 10 investment professionals are able to advise up to 111 startups based on the calculations in section, hence being an appropriate number that does not restrict VC agents whatsoever.

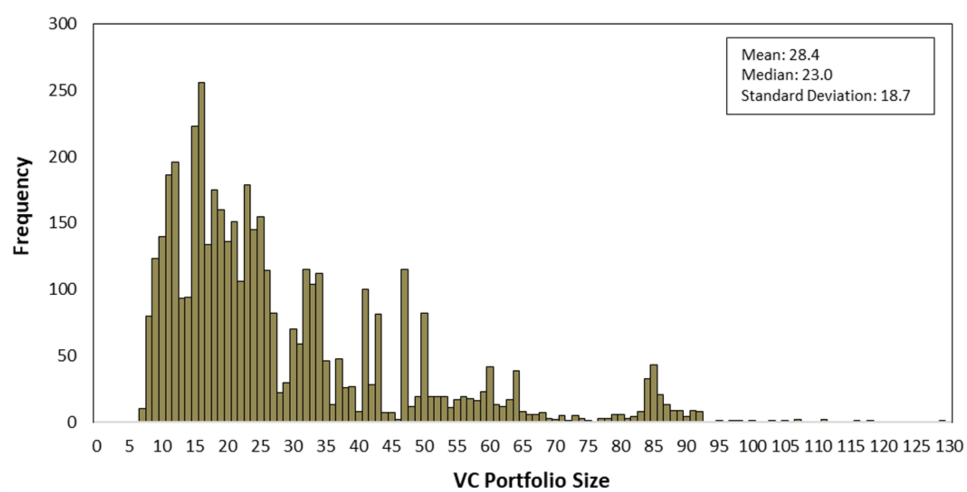

Figure 16: Histogram of VC portfolio size for the simulations with the effort attribute removed, and from simulation number five onwards (4600 VC firms).

The findings suggest an ideal VC portfolio comprises 28.4 startups when accounting for diversification but not the diminishing returns to advice, which is greater than the 23.1 from the main simulation. This agrees with the theory behind the conflict between diversification and advisory, as considering the resources available for advising portfolio firms reduces the optimal VC portfolio size [1, 3, 4]. However, Fig 16 suggests that if the advisory factor is removed, by focusing on VC firms with more employees, the average optimal VC portfolio size should be greater. This discrepancy is probably the result of the SSTD3 algorithm being biased in the main simulation. If it is strictly constrained by the number of investments it can make with the smaller VC firms, it may interpret that larger VC firms should leverage on their opportunity to make more investments, resulting in larger portfolios than the optimal.

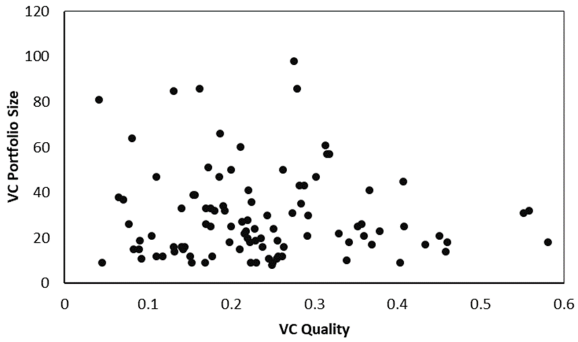

Figure 17: Relationship between *VC quality* and VC portfolio size with the effort attribute removed, only for the last simulation (100 VC firms). No clear correlation can be observed.

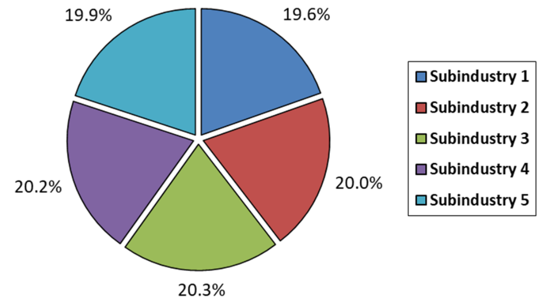

Figure 18: Proportion of investments in each subindustry, for the simulations with the effort attribute removed, and from simulation number five onwards (4600 VC firms). Almost identical percentages showcases the tendency of VC firms to diversify their investments to build an optimal portfolio.

When removing the effort attribute, the model has again been able to encourage diversification as Fig 18 evidences. The greater dispersion of these results when compared to the main simulation is probably the result of a greater freedom to invest in any number of startups. This is reflected by the greatest VC portfolio, 129, which is much greater than that in the main simulation, 108. In this case, again, no correlation between *VC quality* and optimal portfolio size can be observed. Having removed the influence of the number of investment professionals, this suggests that the model could indeed be oversimplified when handling the initial screening process, typical in the VC ecosystem [35].

A notable aspect of the simulation when removing the effort attribute is that 99.78% of VC firms invested 90% or more of their endowments, contrasting with only 53.26% doing so in the main simulation. This highlights a critical model issue: incorporating advisory efforts restricted many VC firms from fully utilising their endowments, diverging from the real-world scenario where VC firms often invest most or all their raised funds [35]. This is probably a result of the aforementioned strict "red line", in which VC firms cannot interact nor invest in more startups once their effort capacity is full. Thus, while the approach to balancing advisory efforts with diversification yielded meaningful and realistic outcomes, there is potential for improvement.

4.3 Lower Minimum Investment

A subsequent sensitivity analysis focused on adjusting the minimum investment for the rewards structure, this time setting it at 0.025. Following the same process, investments in this scenario fell below 0.005 but stayed above 0.025, unlike in the primary simulation where investments did not drop below 0.005.

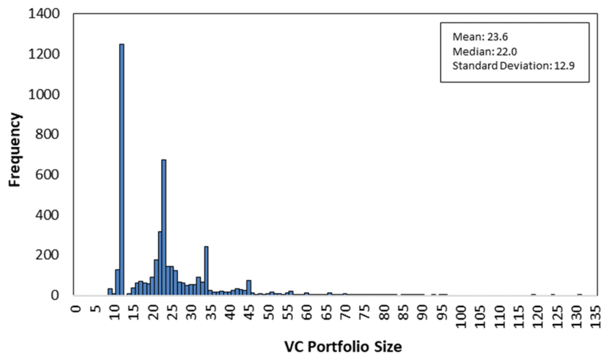

Figure 19: Histogram of VC portfolio size for the simulations with a minimum investment of 0.025, and from simulation number five onwards (4600 VC firms).

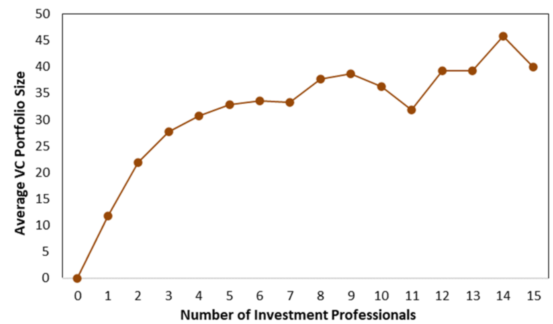

Figure 20: Correlation between the average VC portfolio size and the number of investment professionals, with a minimum investment of 0.025, and for VC firms from simulation number five onwards.

Although results were broadly consistent, a subtle yet critical variation emerged: the average optimal size of VC portfolios increased, primarily due to larger VC firms investing in a higher number of startups than observed in the main simulation, as illustrated in Fig. Removing VC firms with four or fewer investment professional from the analysis further accentuated the discrepancy between this simulation and the main.

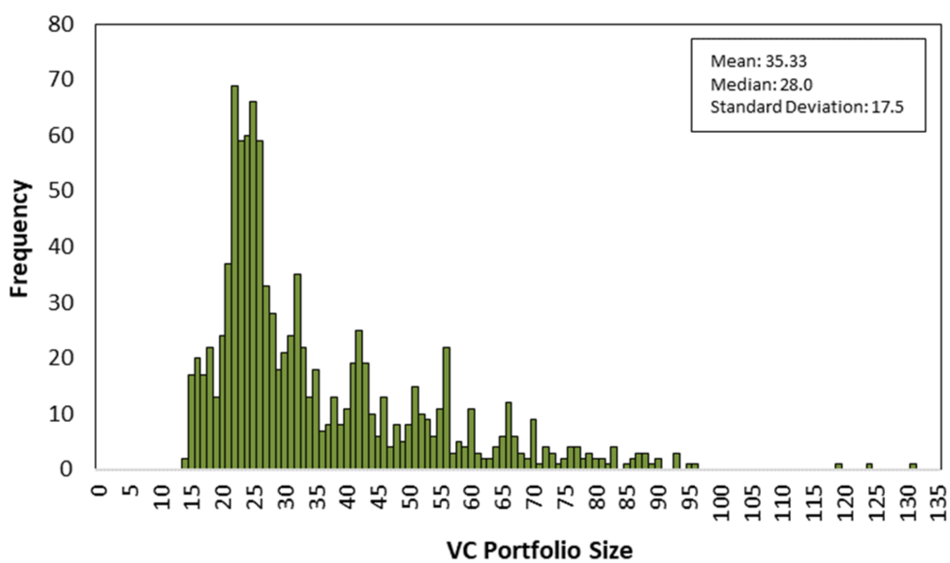

Figure 21: Histogram of VC portfolio size for all VC firms from simulation number five onwards, with a minimum investment of 0.025, and including only VC firms with five investment professionals or more (1039 VC firms).

These conclusions, however, do not necessarily reveal what the correct minimum investment for the model would be. The answer is likely dependent on the size of each VC fund and the funding needed by the startups they invest in, as these two factors are critical in estimating how much an endowment can be fractioned. This reveals a limitation in the ABM's design, which does not account for fund size, although it can have a subtle impact on the optimal number of startups in a VC portfolio

4.4 Limitations and Future Work

This study, aiming to model human investment behaviour, relies on a considerable amount of assumptions intended to simplify real-world scenarios which are otherwise very complex or almost impossible to replicate. For instance, the productivity of an VC investment professional can vary significantly [44–46], and this is very challenging to model without vast amounts of data from monitoring their behaviour. Hence, it was decided to simplify this aspect by assuming all employees are equally efficient and productive. These assumptions, mentioned throughout this report, introduce certain limitations in accurately simulating and optimising VC investment decisions.

Agency problems between VC firms and startups are factored into idiosyncratic risk, but their impact on the productivity or commitment levels of investment professionals is not taken into account. These factors may translate into a lower number of startups in a VC portfolio in real life [1, 3], which could partially explain why the optimal VC portfolio size, derived from the main simulation, is slightly greater than the real-world average.

The model omits several other factors that could influence the ideal portfolio size. It overlooks the empirical concave relationship between portfolio size and the share of profits retained by entrepreneurs [69], as well as the financial costs of managing firms, both of which negatively influence the investment capacity of a VC fund [1, 69]. Furthermore, empirical data shows that VC portfolio size varies with the type of investment (early versus late stage), type of VC, and market conditions [3]. While these exclusions allow for a manageable model, they suggest areas for future research. Future research could extend the scope of the model by, for example, varying the maturity of the startups when invested, thus modelling differences between earlier and later stage investments. On the other hand, future studies could also make the model more specific by focusing on a single type of VC, such as corporate VC firms, providing perhaps more applicable results to a specific VC sector.

The strict treatment of advisory efforts, as mentioned, have proven useful but susceptible of improvement. Modelling how VC firms manage their human capital, time, and financial constraints and how this affects the trade-off between diversification and managerial effort, without restraining VC agents' decision-making ability, could importantly contribute to the robustness of this study. Perhaps, a novel machine learning algorithm could be utilised to accurately mimic such a complex behaviour in a future study.

5 Conclusions

The ABM developed for this study reveals an optimal number of startups in a VC portfolio of 23, on average among the VC firms in the simulation, rounded to the nearest integer. The large dispersion in the results, specifically standard deviation of 12 for the optimal VC portfolio size, conveys that there is no exact answer for the question formulated in this study. This hypothesis is further supported by the reality that portfolio size is influenced by a wide range of factors, including VC fund type, investment strategies, market conditions, and location, amongst other [3, 4, 10]. Nevertheless, the model has concluded on an average optimal number, which is realistic and in accordance with VC portfolio sizes in reality, averaging between 15 and 20 startups [3, 69]. It thus sets a base and acts as a reference for further studies, as well as for VC firms in developing their investment strategies.

This model has shown how the optimal VC portfolio size is affected by many factors, with a strong focus on the trade-off between the diversification of investments and the diminishing returns when advising a larger number of startups. When the influence of the latter was ignored, the ABM output a greater optimal number of startups in a VC portfolio, specifically 28. This suggests that restrictions, caused by limited human capital efforts when delivering advisory services to startups, tend to encourage VC firms to reduce their number of investments, thereby sacrificing risk hedging ability for quality advisory. This tendency has been observed within the real VC arena [1, 3, 4], signifying the robustness of these conclusions. Further research that addresses the main limitations of the developed ABM, such as not differentiating between VC firm types or fund sizes, has the potential to deliver more concrete results that could be of further use to professionals in the VC ecosystem.

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Appendices

Appendix A

The main ABM is contained in the file ”ABM.py”. This file should be run while in the same folder as ”SSTD3.py” and ”Subindustry_correlation_matrix.xlsx”, as the code imports the contents of both. The libraries *numpy* and *torch* should be installed in the system that runs the model, and the results should all appear in the same folder the file is in. The results are in the form of three spreadsheets and four *.pth* files containing all trained network parameters.

The ”SSTD3.py” file contains several classes: The ”ReplayBuffer” class is used to characterise the replay buffer, the ”OUNoise” class is used to generate an Ornstein-Uhlenbeck nouse, the ”Softmax” executes the *Swap Softmax* function, the ”Actor” class contains all actor networks, the ”Critic” class contains all critic networks, and the ”Agent” class includes how the agent behaves, learns, and how the weights are stored to enable training.

The ”ABM.py” file contains all the code in the ABM, with a loop of 50 iterations. It first defines all parameters and assumptions. The ”VC” class defines VC agents and their attributes, the ”Startup” class defines startups agents and their attributes, the ”Activation_1” and ”Activation_2” classes create the startup and VC agents, respectively, to introduce them into the model, and the ”World” class executes each timestep for each simulation. The final lines collect all the data from each simulation and exports it to three separate spreadsheet files.

The ”Subindustry_correlation_matrix.xlsx” file contains the correlations between subindustries.

Appendix B

	Subindustry 1	Subindustry 4	Subindustry 2	Subindustry 3	Subindustry 5	
IT	Software	Communications and Networking	Computer Hardware	IT services	Semiconductors	
1	-12.39%	12.89%	-0.48%	-5.26%	-49.86%	
2	-25.00%	48.45%	7.35%	-0.06%	30.56%	
3	15.38%	9.98%	-25.45%	9.46%	41.93%	
4	4.56%	8.85%	57.19%	30.52%	39.57%	
5	66.67%	-12.64%	-28.83%	-10.13%	-5.35%	
6	31.82%	11.77%	-9.82%	25.64%	1.39%	
7	-25.00%	20.51%	27.65%	12.64%	34.18%	
8	38.44%	-31.86%	-7.90%	-1.24%	28.68%	
9	13.18%	-19.07%	-66.89%	-13.65%	-57.20%	
10	21.90%	-17.27%	-40.86%	-9.62%	-60.48%	
11	-3.60%	64.32%	-6.74%	15.93%	75.52%	
12	25.90%	-66.21%	73.49%	23.25%	-59.92%	
13	11.95%	67.09%	30.87%	-15.69%	8.52%	
14	-6.10%	-16.71%	5.60%	58.64%	68.59%	
15	0.00%	39.07%	21.97%	16.93%	-40.49%	
16	-4.61%	-1.57%	-55.60%	-13.23%	-64.48%	
17	18.11%	-3.76%	14.85%	32.73%	5.35%	
18	37.65%	13.41%	-9.11%	-12.65%	-0.37%	
19	21.03%	24.92%	25.44%	34.83%	20.35%	
20	7.07%	35.10%	42.92%	21.95%	47.35%	
21	37.24%	66.10%	70.26%	13.33%	7.79%	
22	2.15%	-35.48%	-10.01%	13.96%	23.67%	
23	9.95%	37.15%	11.64%	73.47%	7.08%	
24	17.87%	-41.15%	-22.54%	21.88%	41.96%	
25	66.67%	27.11%	-10.15%	-9.08%	-47.38%	
26	-34.79%	27.14%	-13.31%	46.71%	12.91%	
27	32.35%	30.51%	-8.19%	-7.57%	6.48%	
28	-13.59%	55.00%	19.15%	-9.75%	-35.54%	
29	17.65%	8.30%	7.17%	28.83%	-9.59%	
30	-65.38%	19.19%	61.07%	-42.40%	-55.65%	
31	-37.27%	10.50%	-8.08%	-44.49%	7.67%	
32	-43.85%	39.62%	46.54%	9.06%	10.66%	
33	28.57%	32.61%	77.19%	-74.67%	-6.54%	
34	2.90%	7.51%	-8.86%	-10.87%	22.81%	
35	29.69%	-11.58%	24.48%	40.68%	68.38%	
36	43.59%	55.81%	19.67%	64.50%	-64.56%	
37	37.47%	47.26%	38.37%	4.53%	-76.86%	
38	27.04%	21.32%	12.69%	12.25%	2.70%	
39	26.61%	-72.73%	43.32%	-3.31%	-33.48%	
40	-1.68%	69.70%	15.00%	30.28%	-0.72%	
41	31.26%	25.16%	12.94%	39.90%	75.00%	
42	0.93%	-53.34%	38.79%	-27.45%	38.67%	
43	18.26%	0.64%	1.81%	12.79%	-9.91%	
44	-6.46%	-0.94%	-22.00%	-70.13%	5.63%	
45	-10.01%	4.17%	-0.16%	-14.36%	-5.25%	
46	-12.56%	-38.64%	17.77%	-75.02%	-2.46%	
47	-2.16%	-20.49%	83.64%	52.02%	-1.57%	
48	18.57%	1.07%	-24.07%	-6.05%	41.70%	
49	62.79%	7.28%	13.85%	51.52%	-6.27%	
50	25.47%	50.68%	1.43%	10.07%	-49.71%	
51	0.00%	15.58%	20.42%	47.96%	35.11%	
52	29.04%	73.19%	-20.21%	-26.54%	-10.39%	
53	25.42%	26.63%	44.11%	1.21%	36.23%	
54	60.64%	21.64%	60.84%	9.57%	-34.29%	
55	52.83%	-4.69%	20.89%	25.03%	56.86%	
56	50.00%	8.64%	-8.95%	39.42%	50.00%	
57	25.95%	18.13%	21.37%	58.75%	5.56%	
58	4.00%	26.85%	41.04%	42.86%	44.60%	
59	70.32%	-12.08%	-6.31%	-47.63%	0.61%	
60	-13.04%	0.00%	75.00%	21.67%	59.13%	
61	-22.53%	-4.86%	23.72%	2.67%	51.07%	
62	-53.90%	-40.19%	38.54%	-42.77%	19.73%	
63	41.12%	-74.64%	-1.64%	-3.51%	70.05%	
64	-5.80%	-70.33%	-47.29%	-5.59%	24.97%	
65	-16.88%	-0.33%	53.24%	73.81%	48.15%	
66	4.04%	54.90%	-35.60%	-1.66%	43.28%	
67	-43.43%	14.37%	-5.63%	6.77%	-74.53%	
68	58.21%	-44.36%	47.66%	-44.56%	11.09%	
69	28.57%	-41.66%	25.49%	-73.64%	2.84%	
70	-22.84%	63.47%	-28.36%	-32.47%	67.82%	
	Subindustry 1	Subindustry 4	Subindustry 2	Subindustry 3	Subindustry 5	
	Software	Communications and Networking	Computer Hardware	IT services	Semiconductors	**Mean**
Standard deviation	29.5%	35.8%	32.6%	34.2%	39.6%	**34.3%**
Discrepancy from mean standard deviation	-4.9%	1.5%	-1.7%	-0.1%	5.3%	**0.0%**

Figure 22: Comparison between revenue growth rates of the five subindustries within the IT industry, used to calculate the standard deviations of each and their discrepancies.

Appendix C

	Subindustry 1	Subindustry 4	Subindustry 2	Subindustry 3	Subindustry 5
	Software	Communications and Networking	Computer Hardware	IT services	Semiconductors
Quarter	S&P 500 Software Index	S&P 500 Communications Equipment Index	S&P Technology Hardware Select Index	S&P 500 IT Services Index	S&P 500 Semiconductor Materials & Eq Index
Q1 2014	849.35	138.25	1225.59	230.65	386.37
Q2 2014	898.1	139.15	1292.61	232.88	436.13
Q3 2014	949.02	141.16	1193.81	237.26	471.22
Q4 2014	885.9	132.51	1302.06	238.25	474.51
Q1 2015	1018.68	144.45	1237.85	255.41	433.69
Q2 2015	989.85	141.55	1223.78	265.49	398.55
Q3 2015	1058.22	139.08	1023.88	262.46	412.63
Q4 2015	1054.22	113.56	1041.94	245.21	415.21
Q1 2016	1031.77	129.39	1089.88	263.41	458.85
Q2 2016	1141.53	141.35	1036.31	272.15	553.99
Q3 2016	1174.46	144.25	1182.13	275.12	595.37
Q4 2016	1250.09	146.55	1242.49	286.83	697.26
Q1 2017	1364.15	161.09	1378.86	303.42	836.73
Q2 2017	1474.82	151.22	1425.07	322.98	892.15
Q3 2017	1646.5	162.56	1539.85	354.76	1130.72
Q4 2017	1822.94	193.93	1546.18	393.98	1073.61
Q1 2018	1806.38	206.76	1529.3	382.84	1007.08
Q2 2018	1994.22	201.28	1585.63	408.92	1031.19
Q3 2018	2006.47	213.57	1658.14	397.44	738.91
Q4 2018	2017.03	216.21	1313.88	406.07	876.42
Q1 2019	2428.13	256.68	1564.06	476.37	1027.3
Q2 2019	2492.34	255.18	1625.98	502.59	1099.74
Q3 2019	2547.99	224.16	1624.12	491.29	1311.71
Q4 2019	2998.49	217.45	1820.72	541.17	1389.14
Q1 2020	3078.9	199.76	1306.23	487.18	1229.14
Q2 2020	3572.13	219.2	1526.72	551.89	1646.07
Q3 2020	3615.72	176.81	1579.39	523.23	1538.74
Q4 2020	4038.02	219.24	2156.75	579.29	2252.54
Q1 2021	4370.76	245.55	2533.21	680.02	2756.63
Q2 2021	4971.27	271.96	2606.45	703.92	2950.89
Q3 2021	5726.65	279.85	2428.41	638.54	2929.94
Q4 2021	5149.52	281.87	2526.12	636.31	2844.13
Q1 2022	4449.39	250.06	2215.92	561.47	2381.07
Q2 2022	4563.7	239.82	1739.3	561.11	2673.46
Q3 2022	3879.88	241.94	1614.32	537.26	2254.33
Q4 2022	4156.87	256.51	1697.61	559.62	2627.56
Q1 2023	4897.62	262.52	1980.45	551.06	2558.09
Q2 2023	5543.44	278.52	2296.11	613.43	3267.79
Q3 2023	5452.06	286.98	2154.8	590.55	2668.22
Q4 2023	6522.05	303.4	2357.1	730.5	3445.34

Correlation Matrix

	S&P 500 Software Index	S&P Technology Hardware Select Index	S&P 500 IT Services Index	S&P 500 Communications Equipment Index	S&P 500 Semiconductor Materials & Equipment Index
	Subindustry 1	Subindustry 2	Subindustry 3	Subindustry 4	Subindustry 5
Subindustry 1	1.000	0.893	0.953	0.911	0.979
Subindustry 2	0.893	1.000	0.903	0.861	0.909
Subindustry 3	0.953	0.903	1.000	0.934	0.935
Subindustry 4	0.911	0.861	0.934	1.000	0.886
Subindustry 5	0.979	0.909	0.935	0.886	1.000

Figure 23: Quarterly stock data used to calculate correlations between IT subindustries

Appendix D

US (10-year Treasury Bill)			Europe (10-Year German Bond)	
Year (1 Jan)	**Rate**		**Year (1 Jan)**	**Rate**
2008	3.67%		2008	4.11%
2009	2.87%		2009	2.93%
2010	3.63%		2010	3.38%
2011	3.42%		2011	2.91%
2012	1.83%		2012	1.89%
2013	2.02%		2013	1.53%
2014	2.67%		2014	1.80%
2015	1.68%		2015	0.49%
2016	1.94%		2016	0.53%
2017	2.45%		2017	0.31%
2018	2.72%		2018	0.44%
2019	2.63%		2019	0.20%
2020	1.51%		2020	-0.25%
2021	1.11%		2021	-0.56%
2022	1.79%		2022	-0.20%
2023	3.52%		2023	2.28%
2024	3.99%		2024	2.11%
Mean	2.56%		Mean	1.41%
Years	8		Years	8
8 years	1.224		8 years	1.118
	Overall Mean	1.98%		
	Years	8		
	8 years	1.170		

Figure 24: Risk free rates from 2008 to 2024 for US 10-year US Treasury Bill and 10-year German Bonds, used to estimate the risk-free rate.