Statistics for Data Science – Differentiation Tutorial

Notes on differentiation

The rate of change of *f(x)* with respect to *x* is measured by the derived function, denoted *f'(x)*.

Where y = f(x), notations for the derived function include *f'(x), f', y'(x), dx/dy, df/dx*

This maps to the gradient of the function f(x) at x.

Rules

There are a number of rules that you can apply to a function to calculate its derivative.

Constant Rule

$$\frac{d}{dx}(a) = 0$$

$$\frac{d}{dx}(ax) = a$$

Constant Multiple Rule

$$\frac{d}{dx}(ax^n) = nax^{n-1}$$

Sum/difference Rule

$$\frac{d}{dx}\big(f(x) + g(x)\big) = \frac{d}{dx}(f(x)) + \frac{d}{dx}(g(x))$$

Exponential/Logarithmic

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}b^x = b^x \ln(b)$$

$$\frac{d}{dx}\ln(x) = 1/x$$

Sine,cosine,arctan

$$\frac{d}{dx}(\sin(x)) = \cos(x)$$

$$\frac{d}{dx}(\cos(x)) = -\sin(x)$$

$$\frac{d}{dx}\big(tan^{-1}(x)\big) = \frac{1}{1+x^2}$$

Chain Rule

$$\frac{d}{dx}(f\big(g(x)\big) = \frac{d}{dg}f(g) \times \frac{d}{dx}g(x)$$

Product Rule

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$

Multiple Rule

$$\frac{d}{dx}\big(f(ax)\big) = a.f'(ax)$$

This is a special case of the chain rule.

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{\frac{d}{dx}(f(x))\cdot g(x) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$$

Law of indices

$$\frac{d}{dx}\left(\frac{a}{\sqrt{x}}\right) = \frac{d}{dx}\left(\frac{a}{x^{1/2}}\right) = \frac{d}{dx}(ax^{-1/2})$$

Examples

$$\frac{d}{dx}(5x^4) = 4 \times 5 \times x^{4-1} = 20x^3$$

$$\frac{d}{dx}\left(5x^{\frac{3}{7}}\right) = \frac{3}{7} \times 5 \times x^{\frac{3}{7}-1} = -\frac{105}{49}x^{-\frac{4}{7}}$$

$$\frac{d}{dx}(2x^4 + 5x^2 + 3x + 4) = 8x^3 + 10x^1 + 3 + 0$$

$$\frac{d}{dx}\left(\sqrt{2x^4 + 5x^2 + 3x + 4}\right) = \frac{8x^3+10x+3}{2\sqrt{2x^4+5x^2+3x+4}}$$ (Using chain rule)

Try These

$$\frac{d}{dx}(5x^{-1})$$

$$\frac{d}{dx}(x^{\frac{2}{3}})$$

$$\frac{d}{dx}(5x^{-\frac{2}{3}})$$

$$\frac{d}{dx}\left(2x^{2/3} - 3x^{-2} + 7x - 2\right)$$

$$\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)$$

$$\frac{d}{dx}\left(\frac{1}{2} + \frac{1}{\pi}tan^{-1}(x)\right)$$

$$\frac{d}{dx}(1 - e^{-x})$$

$$\frac{d}{dx}(e^{-e^{-x}})$$

Check out www.derivative-calculator.net