XX50215 Statistics for Data Science

Problems 3

1. Given X has the geometric pmf $f_x(x) = \frac{1}{2}\left(\frac{2}{3}\right)^x$, x = 0, 1, 2, ... Determine the probability distribution of Y = X/(X+1). Note that here both X and Y are discrete random variables. To specify the probability distribution of Y, specify its pmf.

2. Show that the following function is a cdf and find $F_x^{-1}(y)$.

$$F_x(x) = \begin{cases} 0 & if \ x < 0 \\ 1 - e^{-x} & if \ x > 0 \end{cases}$$

3. Consider a sequence of independent coin flips where the probability of each being heads is p.

   a. Define a random variable X as the length of the run of either heads or tails started by the first trial. (e.g. X = 4 if the sequence is HHHHT or TTTTH.)

   b. Find the distribution of X and EX.

4. If a couple decides to continue having children until a daughter is born. What is the expected number of children of the couple?

5. Find the moment generating function (mgf) corresponding to f(x) = 1/c, 0 < x < c.

6. Does a distribution exist for $M_x(t) = 1/(1 - t)$, |t| < 1?

   If yes, find it. If no, prove it.

7. A median of a distribution is a value m such that P(X <= m) >= 1/2 and P(X >= m) >= 1/2.

   If X is continuous, m satisfies

$$\int_{-\infty}^{m} f(x)dx = \int_{m}^{\infty} f(x)dx = 1/2.$$

   Find the median of the distribution f(x) = 3x², 0 <x<1.

8. Suppose the pdf $f_x(x)$ of a random variable X is an even function.

   Show that:

   a. X and –X are identically distributed.

   b. $M_x(t)$ is symmetric about 0.

   Note: fx(x) is an even function of $f_x(x) = f_x(-x)$ for every x.