XX50215 Statistics for Data Science

Problems 2

1. Give an example of a discrete random variable and a contiguous random variable.

2. If B \ A is the set of elements in B but not in A, known as the difference, verify the following identity:

$$A \setminus B = A \setminus (A \cap B) = A \cap B^c$$

3. The Smiths have two children. At least one of them is a boy. What is the probability that both children are boys?

4. In the game of dominoes, each piece is marked with two numbers. The pieces are symmetrical so that the number pair is not ordered (so, for example, (2, 6) = (6, 2)). How many different pieces can be formed using the numbers 1, 2, …, n?

5. Suppose that 5% of men and 25% of women are colour-blind. A person is chosen at random and that person is colour-blind. What is the probability that the person is male? You can assume there are equal number of males and females.

6. Prove the following functions are cdfs.

   a. $\frac{1}{2} + \frac{1}{\pi}\tan^{-1}(x)$, $x \in (-\infty,\infty)$

   b. $e^{-e^{-x}}$, $x \in (-\infty,\infty)$

7. A particular powerstations generating load peaks each day. Suppose that the low load is set at 1 and the peak load Y has distribution function

$$F_y(y) = P(Y <= y) = 1 - \frac{1}{y^2},\ 1 <= y <= \infty$$

   a. Verify that $F_y(y)$ is a cdf.
   b. Find $f_y(y)$, the pdf of Y.
   c. If the low load is reset to 0 and we use a unit of measurement that is $1/10^{th}$ of that given previously, the peak load becomes Z = 10(Y-1). Find $F_z(z)$.

8. The Monty Hall Problem

   "Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?"

   This is a rather famous problem that has generated much debate. If you've not see it before think about your own answer before you Google the solution.