1) Given: $y = A + Bx + Cx^2$ — (1)

$\Rightarrow$ for point $(1,1)$

eq (1) becomes.

$$1 = A + B + C \quad \text{—(1)}$$

Ily for $(2,-1)$

$$-1 = A + 2B + 4C \quad (2)$$

Ily for $(3,1)$

$$1 = A + 3B + 9C \quad \text{—(3)}$$

$\Rightarrow$ We have

$$A + B + C = 1$$
$$A + 2B + 4C = -1$$
$$A + 3B + 9C = 1$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Consider: $[A \; B]$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & -1 \\ 1 & 3 & 9 & 1 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_2$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 2 & 8 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

$\Rightarrow \quad 2C = 4$

$$\boxed{C = 2}$$

$B + 3C = -2$

$$\boxed{B = -8}$$

$$\boxed{A = 7}$$

$$\boxed{y = 7 - 8x + 2x^2}$$

2) $\begin{bmatrix} 2 & 5 & 2 & -5 \\ 4 & 12 & 3 & -14 \\ -10 & -29 & -5 & 38 \\ 10 & 21 & 21 & -6 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 + 5R_1$
$R_4 \to R_4 - 5R_1$ $= \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & -4 & 5 & 13 \\ 0 & -4 & 11 & 19 \end{bmatrix}$

$R_3 \to R_3 + 2R_2$
$R_4 \to R_4 + 2R_2$ $\begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 9 & 11 \end{bmatrix}$

$R_4 \to R_4 - 3R_3$ $\begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix}$

$\Rightarrow U = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix}$

$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ 5 & -2 & 3 & 1 \end{bmatrix}$

3) $T(x,y,z) = (x+2y-z,\ y+z,\ x+y-2z)$

$\Rightarrow T\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}1\\0\\1\end{bmatrix} = 1\begin{bmatrix}1\\0\\0\end{bmatrix} + 0\begin{bmatrix}0\\1\\0\end{bmatrix} + 1\begin{bmatrix}0\\0\\1\end{bmatrix}$

$T\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}2\\1\\1\end{bmatrix} = 2\begin{bmatrix}1\\0\\0\end{bmatrix} + 1\begin{bmatrix}0\\1\\0\end{bmatrix} + 1\begin{bmatrix}0\\0\\1\end{bmatrix}$

$T\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}-1\\1\\-2\end{bmatrix} = -\begin{bmatrix}1\\0\\0\end{bmatrix} + \begin{bmatrix}0\\1\\0\end{bmatrix} - 2\begin{bmatrix}0\\0\\1\end{bmatrix}$

$\Rightarrow \boxed{T = \begin{bmatrix}1 & 2 & -1\\0 & 1 & 1\\1 & 1 & -2\end{bmatrix}}$

Now

$R_3 = R_3 - R_1$

$\begin{bmatrix}1 & 2 & -1 & b_1\\0 & 1 & 1 & b_2\\0 & -1 & -1 & b_3-b_1\end{bmatrix}$

$R_3 \to R_3 + R_2$

$\begin{bmatrix}1 & 2 & -1 & b_1\\0 & 1 & 1 & b_2\\0 & 0 & 0 & b_3-b_1+b_2\end{bmatrix}$

$\therefore$ basis of $C(A) = \{(1,0,1), (2,1,1)\}$

basis of $C(A^T) = \{(1,2,-1), (0,1,1)\}$

basis of $N(A^T) = \{(-1,1,1)\}$

Now

$U = \begin{bmatrix}1 & 2 & -1\\0 & 1 & 1\\0 & 0 & 0\end{bmatrix}$

$R_1 \to R_1 - 2R_2$

$\begin{array}{ccc}\begin{bmatrix}1 & 0 & -3\\0 & 1 & 1\\0 & 0 & 0\end{bmatrix}\\ x \quad y \quad z\end{array}$

$\Rightarrow$ $x = 3z$

$y = -z$

$\Rightarrow$ basis of $N(A) = \{(3,-1,1)\}$

(iii) $\begin{vmatrix}1-\lambda & 2 & -1\\0 & 1-\lambda & 1\\1 & 1 & -2-\lambda\end{vmatrix} = 0$ $\quad \cancel{4}+9-2\sqrt{3}$

$\lambda^3 - 3\lambda = 0$

$(\lambda^2-3)\lambda = 0$

$\boxed{\lambda = 0, \pm\sqrt{3}.}$

$\Rightarrow$ If $\lambda = 0$.

$[A-\lambda I][x] = 0$

$\Rightarrow \begin{bmatrix}1 & 2 & -1\\0 & 1 & 1\\1 & 1 & -2\end{bmatrix}$

$\frac{x}{\begin{vmatrix}2 & -1\\1 & 1\end{vmatrix}} = \frac{-y}{\begin{vmatrix}1 & -1\\0 & 1\end{vmatrix}} = \frac{z}{\begin{vmatrix}1 & 2\\0 & 1\end{vmatrix}}$ $\Rightarrow \frac{x}{3} = \frac{y}{1} = \frac{z}{1}$

$\boxed{e_1 = k_1\begin{bmatrix}3\\1\\1\end{bmatrix}}$

If $\lambda = \sqrt{3}$.

$\begin{bmatrix}1-\sqrt{3} & 2 & -1\\0 & 1-\sqrt{3} & 1\\1 & 1 & -2-\sqrt{3}\end{bmatrix}$

$\frac{x}{\begin{vmatrix}2 & -1\\1-\sqrt{3} & 1\end{vmatrix}} = \frac{-y}{\begin{vmatrix}1-\sqrt{3} & -1\\0 & 1\end{vmatrix}} = \frac{z}{\begin{vmatrix}1-\sqrt{3} & 2\\0 & 1-\sqrt{3}\end{vmatrix}}$ $\Rightarrow \frac{x}{1+\sqrt{3}} = \frac{y}{1-\sqrt{3}} = \frac{z}{10}$

$\boxed{e_2 = k_2\begin{bmatrix}1+\sqrt{3}\\1-\sqrt{3}\\10-2\sqrt{3}\end{bmatrix}}$

If $\lambda = -\sqrt{3}$

$\Rightarrow \begin{bmatrix}1+\sqrt{3} & 2 & -1\\0 & 1+\sqrt{3} & 1\\1 & 1 & -2+\sqrt{3}\end{bmatrix}$

$\frac{x}{\begin{vmatrix}2 & -1\\1+\sqrt{3} & 1\end{vmatrix}} = \frac{-y}{\begin{vmatrix}1+\sqrt{3} & -1\\1 & 0\end{vmatrix}} = \frac{z}{\begin{vmatrix}1+\sqrt{3} & 2\\1 & 1+\sqrt{3}\end{vmatrix}}$

$\Rightarrow \frac{x}{\cancel{1}-\sqrt{3}} = \frac{y}{1+\sqrt{3}} = \frac{z}{10+2\sqrt{3}}$

$\boxed{e_3 = k_3\begin{bmatrix}1-\sqrt{3}\\1+\sqrt{3}\\10+2\sqrt{3}\end{bmatrix}}$

$\frac{3}{2} - \frac{1}{2}$

0

$\frac{3}{2} + \frac{1}{2}$

5

iv) $T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$

$\Rightarrow q_1 = \frac{r_1}{\|r_1\|} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$q_2 = \frac{e_1}{\|e_1\|}$

$e_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} - \frac{\frac{1}{\sqrt{2}}[1, 0, 1]\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}}{} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 3/2 \\ 0 \\ 3/2 \end{bmatrix}$

$= \begin{bmatrix} 1/2 \\ 0 \\ -1/2 \end{bmatrix}$

$\Rightarrow$

$\Rightarrow q_2 = \sqrt{2} \begin{bmatrix} 1/2 \\ 0 \\ -1/2 \end{bmatrix}$

$\boxed{q_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}}$

$q_3 = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + \frac{3}{2} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$1 + \frac{3}{2}$ $= \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$

$\boxed{q_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$

$\cancel{Q = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{6} & \\ 0 & \sqrt{2/3} & \\ 1/\sqrt{2} & -1/\sqrt{6} & \end{bmatrix}}$

$\boxed{Q = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 0 \\ 0 & \sqrt{2/3} & 0 \\ 1/\sqrt{2} & -1/\sqrt{6} & 0 \end{bmatrix}}$

$R = Q^T T$

$\boxed{R = \begin{bmatrix} \sqrt{2} & 3/\sqrt{2} & -\frac{3}{\sqrt{2}} \\ 0 & \sqrt{3/2} & \sqrt{3/2} \\ 0 & 0 & 0 \end{bmatrix}}$

Scanned with CamScanner

5A):

$$x_1 + x_2 + 3x_3 + 4x_5 = 0$$

$$\Rightarrow x_1 = -x_2 - 3x_3 - 4x_5$$

$$= \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} x_2 + \begin{pmatrix} -3 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} x_3 + \begin{pmatrix} -4 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} x_5$$

$$\Rightarrow B = \begin{bmatrix} -1 & -3 & 0 & -4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{5\times 4}$$

$$\because \begin{bmatrix} 1 & 1 & 3 & 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = 0$$

$$\Rightarrow \boxed{B = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 0 \\ 4 \end{bmatrix}}$$

$$\Rightarrow Q = A(A^T A)^{-1} A^T$$

$$Q = \begin{bmatrix} 26/27 & -1/27 & -1/9 & -4/27 \\ -1/27 & 26/27 & -1/9 & -4/27 \\ -1/9 & -1/9 & 6/9 & -12/27 \\ -4/27 & -4/27 & 4/9 & 1/27 \end{bmatrix}$$

$$\boxed{\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}}$$

$$P_V + P_{V^\perp} = I$$

$$\Rightarrow P = \begin{bmatrix} +1/27 & 1/27 & 1/9 & 4/27 \\ +1/27 & 1/27 & 1/9 & 4/27 \\ 1/9 & 1/9 & 1/3 & 4/9 \\ 4/27 & 4/27 & 4/9 & 26/27 \end{bmatrix}$$

Scanned with CamScanner

6) $A = \begin{bmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{bmatrix}$

$\Rightarrow |a| > 0$

$\Rightarrow a_1 > 0 \quad \Rightarrow a^2 - 4 > 0$

$a < -2$ or $a > 2$

$a(a^2-4) - \cancel{4}(a-2) + \cancel{4}(2-a) \geq 0$

$(a-2) \quad a(a+2) - 8 = 0$

$a^2 + 2a - 8 = 0$

$(a+4)(a-2) > 0$

$a < -4 \quad a > 2$

$\Rightarrow a \in (2, \infty) \cup (-\infty, -4)$

$f = x^T A x$

$f = 2x^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3$

$$\boxed{\Rightarrow A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}}$$

7) $A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$

$\Rightarrow A^TA = \begin{bmatrix} -3 & 6 & 6 \\ 1 & -2 & -2 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$

$= \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix}$

$\Rightarrow \lambda^2 - 90\lambda + 0 = 0$

$\boxed{\lambda = 0, 90}$

$\lambda = 0 \quad e_1 = k_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$\lambda = 90. \quad e_2 = k_2 \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

$V = \frac{1}{\sqrt{10}} \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix}$

$\Rightarrow U_1 = \frac{AV_1}{\cancel{\lambda_1}} = 0$

$U_2 = \frac{AV_2}{\lambda_2} = \frac{1}{30} \begin{bmatrix} +10 \\ -20 \\ -20 \end{bmatrix}.$

$u_3 : 10x - 20y - 20z = 0$

$x = 2y + 2z.$

$x = 4, y = 1, z = 1$

$\Rightarrow U = \begin{bmatrix} 1/3 & 4/\sqrt{18} \\ -2/3 & 1/\sqrt{18} \\ -2/3 & 1/\sqrt{18} \end{bmatrix}$

$\cancel{\Rightarrow A = \frac{1}{\sqrt{10}} \begin{bmatrix} 1/3 & 4/\sqrt{18} \\ -2/3 & 1/\sqrt{18} \\ -2/3 & 1/\sqrt{18} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 3\sqrt{10} & 0 \\ & \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix}_{2\times 2}}$

$\boxed{A = \frac{1}{\sqrt{10}} \begin{bmatrix} 1/3 & 4/\sqrt{18} \\ -2/3 & 1/\sqrt{18} \\ -2/3 & 1/\sqrt{18} \end{bmatrix} \begin{bmatrix} \sqrt{90} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix}}$