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* Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

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Finding the core: Network structure in interbank markets

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Preliminary: please do not quote without prior consultation

Abstract

This paper investigates the network structure of interbank markets, which has proved to be important for financial stability during the crisis. First, we describe and map the interbank network in the Netherlands, an exception in the literature because of its small and open banking environment. Secondly, we follow recent analyses of interbank markets of Germany and Italy in estimating the Core Periphery model, using data for the Netherlands instead. We find a significant Core Periphery structure and discuss model selection. The overall analysis opens up new opportunities for systemic risk assessments of the interbank market, especially as more granular data is becoming available for the eurozone.

Keywords: Interbank networks, Core-periphery, loan intermediation

JEL classifications: G10; G21; L14.

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1. Introduction

Understanding complex interbank markets is crucial for managing financial stability, as became clear during the financial crisis. Whereas the relevance of the network structure prior to crisis was mentioned only infrequently, it has now caught the attention of both academics (Tirole [30]) and policy makers (Haldane [22]). Systemic risk and contagion have become keywords in finance, and debate on their precise content and implications is ongoing.

The network structure has two important implications for policy makers. First, the network structure is ultimately determined by deeper market forces that need to be unravelled in order to anticipate monetary policy responses. Traditionally, the interbank market is considered as a market that banks can use to co-insure against – idiosyncratic – liquidity shocks (cf. Bhattacharya and Gale [5]). Interbank exposures can also be seen as a peer-monitoring device (Flannery [16], Rochet and Tirole [27]) and may thus improve market discipline. In a broader context, some have argued that relationships matter in the interbank market. Cocco et al. [10] show that borrowers pay a lower interest rate on loans from banks with whom they have a stronger relationships. Such relationship will be reflected in the network structure.

Secondly, the actual distribution of links between banks affects the stability of the systems and the possible contagion after large shocks. In a seminal contribution, Allen and Gale [2] use stylised examples showing that the fragility of the system depends crucially on the structure of interbank linkages. If a network is ‘complete’, i.e. all nodes (banks) are connected to all other nodes, a shock to a single bank can easily be shared between the banks and thus the stability of the system is safeguarded. If instead the network becomes clustered, spillover of some of the nodes can become substantial. The examples in Allen and Gale [2] are clearly simplified and subsequent research has shown that many other aspects are relevant. For instance, Gai et al. [18] build a model around unsecured claims, repo activity and shocks to collateral haircuts. They show that systemic liquidity crises

as seen in the 2007-2008 crisis can arise with funding contagion spreading throughout the network.¹

Given limitations in data sources, the empirical analysis of the relevance of financial networks is still lagging theoretical work. In the literature so far, network structures have been mapped for several countries and these empirical networks have been used for (deterministic) stress test exercises.² As noted by Upper [31] in an overview of this literature, the estimated contagious effects are limited. This is not surprising as the data generally only covers a single market and because behaviour conditional on defaults does not change.³ The –limited– empirical work that makes use of the tools of network theory, and comes closest to our approach, is discussed in the next section.

The goal of this paper is to find empirical methods providing a stronger connection between observed interbank markets and theoretical models. To this end we will analyse a long running panel of bank links for the Netherlands. We will estimate measures of complex system network theory as well as a Core Periphery (CP) structure recently suggested by Craig and von Peter [13]. Moreover, we will test these attributed network properties formally and see whether the data contains enough information to select one model over the other.

Getting a better picture of the network structure will be a crucial step in developing systemic risk assessments of the interbank market. The idea of the Core Periphery model a small set of Core banks is highly connected, while Periphery banks are not connected with each other but only to the Core. Figure 1 gives a preview of the results. It shows that there are

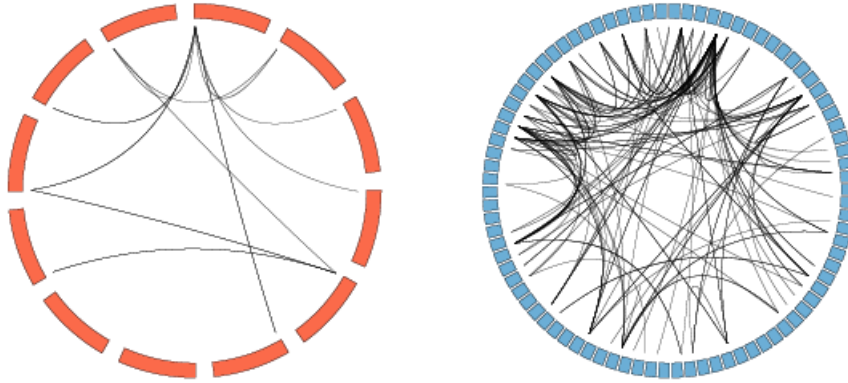
¹Other contributions have been made by, amongst others, Ahnert and Nelson [1] and Castiglionesi et al. [9], although these models do not explicitly model the network structure but rather the exposure to a common shock.

²See for instance Upper and Worms [32], van Lelyveld and Liedorp [33], and Degryse and Nguyen [14].

³See Co-Pierre Georg (Universidad Carlos III de Madrid, Multilayer Financial Networks) for efforts to model multi-layered markets.

few missing links in the Core, and also relatively few existing links in the Periphery.

Figure 1: Errors of the CP model (right) for the Dutch interbank market on 2005Q1 in the core (left) and in the periphery (right). For the core banks, the errors are the missing links, whereas for the periphery the links between any two periphery banks are drawn.



The remainder of this paper is structured as follows. First we place our contribution within the existing network literature in Section 2. Section 3 describes our data set and empirical approach. Section 4 contains our main estimation results including a discussion on testing and model selection. In Section 5, we briefly relate our results to financial stability implications. Section 6 concludes.

2. Related network literature

Mathematical network or graph theory has been applied to many different fields such as biology, technological networks, and information science.⁴ One

⁴Newman [26] provides a comprehensive review.

of the earliest theoretical models of a network was introduced by Erdős and Rényi [15]. In their random graphs, each possible link between any two nodes can occur with a certain independent and identical probability.

The benchmark Erdős-Rényi model has severe limitations in empirical applications: it doesn't allow for some nodes to have a very high degree of connections. This property of a long tail in the degree distribution has been widely observed and led to the development of scale-free models by Barabási and Albert [4], where the probability of forming another link increases proportionally with degree. Theoretically, a scale-free network displays a power law in the degree distribution.

Currently, the empirical work on the interbank exposures almost exclusively builds on the scale-free model. In one of the earliest descriptions of interbank network topology, Boss et al. [7] fit power laws on two different regions in the degree distribution for Austria. Using data from Brazil, Cont et al. [12] connect a systemic risk measure from the stress test exercises with local network characteristics, after calculating various properties of the network including the scale-free parameter. Finally, Martínez-Jaramillo et al. [25] also find large degree heterogeneity in the Mexican interbank market. Taking a broad perspective, it seems that still a lot of research is necessary to have an insightful model of the interbank network.

In reviewing the interbank network literature, it is useful to contrast interbank exposures treated in this paper with overnight interbank transactions. Whereas interbank exposures refer to stock variables on the balance sheet, interbank transactions are flow variables. Fricke and Lux [17] summarise the findings of the overnight interbank literature, and name explicitly the apparent scale-free distributions as the main finding.

However, the statistical support for many of these claims has recently been called into question (Stumpf and Porter [29]). The problem boils down to the fact that scale-freeness is an asymptotic property, and even in the largest datasets there are few observations of extreme degrees. In the present context of the Dutch interbank market, with around 100 banks, it is even

more difficult to test for scale-freeness.

The difficulty with the random graph and the scale-free network is that they are purely stochastic, as links are formed following some probability distribution. For many social and economic environments, it might be useful to also consider network formation models where nodes form links strategically. Goyal [21] shows that under a large set of assumptions, equilibrium networks may arise in which there is one single 'star'-node to which all other nodes are connected. Babus [3] applies a related model to financial markets and also finds a star network, which is claimed to reproduce a qualitative feature of subset of core banks that intermediate between other banks.

Whereas the stochastic network models can be said to be too random, the strategic models are too little random, because the models ultimately result in a star structure. In the following two subsections, we discuss two different mechanisms that generalise of the star structure: Preferential Attachment and Core Periphery. The aim is to maintain the asymmetry in degree but in a less extreme way than the star. Both mechanisms can be important for interbank markets. In Section 4 we will discuss which of the two mechanisms, if any, is found to be more important in the data.⁵

2.1. Preferential Attachment

Preferential Attachment refers to the mechanism that was introduced in Barabási and Albert [4] to generate scale-free networks. In a similar spirit, Jackson and Rogers [23] propose a network growth model where each period a new link is added with a fixed outdegree, connecting to existing nodes depending on their current indegree. For banks something similar may apply, in the sense that they want to interact with a reliable counterparty that is

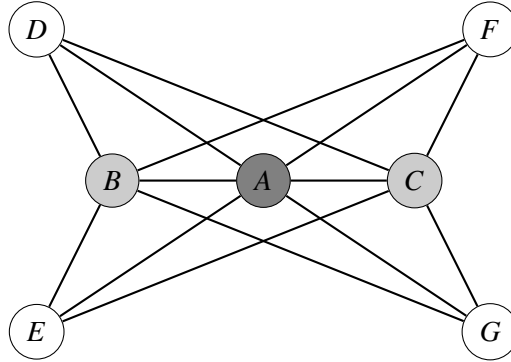
⁵Another related concept is k -cores: the sets of nodes with at least degree k . As will become clear soon, nodes in higher k -cores are more likely to belong to the Core (and not to the Periphery), but the interpretation of the Core is economically more interesting.

used by many other banks.⁶

As an extreme form of Preferential Attachment, Cohen-Cole et al. [11] use a network model with strategic complementarities. Banks are assumed to simultaneously engage in producing 'loan quantities', and every bank wants to be connected to the most active players. The resulting network outcome is a nested split graph. This model therefore predicts a single but strong condition that has to be satisfied:

Condition PA: Every bank is either unconnected or connected to all banks with a degree higher than his own.

Figure 2: Example of a nested split graph as perfect Preferential Attachment



In the example of Figure 2, bank A has 6 links, B and C 5 links and D, E, F and G 3 links. It can easily be checked that the condition is satisfied. The darker filled nodes corresponds with the more preferable counterparties. Note that there is some freedom in nested split graphs, as e.g. additional links from D to F and from E to G could be added that would not violate the condition.

⁶Note that Preferential Attachment is essentially a mechanism that does not depend on the direction on the link, whereas the direction (i.e. borrowing or lending) is crucial in understanding an interbank relationship.

Cohen-Cole et al. [11] are the first to estimate a microfounded network model on interbank data. The main problem with this representation is the interpretation of the link in the interbank market. Instead of a strategic complementarity, a loan between banks can perhaps better be interpreted as fulfilment of liquidity needs of borrowers, who are happy to be served by a single lender if possible. It is therefore the question to what extent the mechanism of Preferential Attachment will be a useful property for describing the data.

2.2. Core Periphery structure

Another possible generalisation of the star network is a Core Periphery network, where there are multiple Core banks that intermediate between the Periphery banks, which in turn do not interact with each other. In the social sciences, the formal notion of Core Periphery structures was introduced by Borgatti and Everett [6]. Craig and von Peter [13], followed by Fricke and Lux [17], have recently applied this concept to interbank markets.

The Core Periphery structure is grounded in two implicit assumptions. First, there are not supposed to be different communities of banks that cluster together for geographical and historical reasons.⁷ Community detection can be seen as complementary to Core Periphery structures. Within the small open banking sector of the Netherlands, community structure is not expected to play an important role. Second, there is an imposed distinction between systemically important and unimportant banks. Fricke and Lux [17] also consider and estimate a continuous version of the Core Periphery structure; however, the conclusion is that this adaptation of the model alone does not increase the fit of the model.

To test the concept of an interbank Core Periphery network in a quantitative way, Craig and von Peter [13] introduce a strict definition. In a perfect

⁷Boss et al. [7] show that different communities in the Austrian interbank market correspond to federal provinces. See also Girvan and Newman [19].

Core Periphery structure, the following three conditions are satisfied:

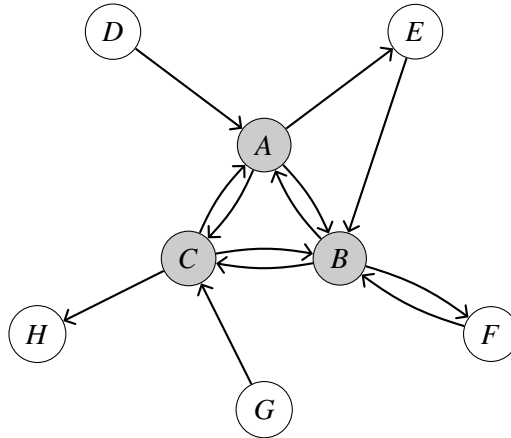
Condition CP1: Core banks are all bilaterally linked with each other.

Condition CP2: Periphery banks do not lend to each other.

Condition CP3: Core banks both lend to and borrow from at least one periphery bank.

These requirements are illustrated in Figure 3. Banks A, B and C are core banks and all lend to each other bilaterally. These banks also intermediate between the remaining periphery banks: for example, bank A intermediates between D and E. Note that Condition CP3 still leaves a lot of freedom in how the links between Core and Periphery are arranged. Some periphery banks, like bank E and F, may lend and borrow at the same time, as long as they are not connected to other periphery banks.

Figure 3: Example of a perfect Core Periphery structure



Of course, in practice there is generally not a perfect division between the core and the periphery in the data. Any chosen set of core banks will generally have errors within each of the two layers, for example the absence

of links between core banks A and B (violating Condition CP1) or the presence of links between periphery banks D and E (violating Condition CP2). Furthermore, a core bank that does not borrow from the periphery, for example bank C after its link from G is removed (violating Condition CP3), produces errors for every periphery bank to which it could have lent.⁸ For any classification of core banks and periphery banks, the number of errors with respect of the (perfect) CP model can be counted.

The optimal core is defined by the core producing the smallest number of errors, and finding the optimal core is similar to running a regression. The domain contains all possible cores, so that the number of core banks does not have to be specified. The Hamming distance, as the chosen fitness measure is sometimes called, is known to produce for some pathological networks cores that are not connected. This danger could be circumvented by using a Pearson correlation measure of fit, as done by Boyd et al. [8]. However, in our context we will be quite sure that the networks intuitively correspond to a core periphery structure, as the core will be very dense.

For small networks, the solution can easily be found by an exhaustive search. However, because the size of this domain rises exponentially with the number of banks, Craig and von Peter [13] propose a sequential optimisation algorithm with a reduced, linear running time. Starting with an initial random partition, the algorithm iteratively improves the outcome by moving banks generating most errors to the other group. Craig and von Peter [13] devote considerable attention to check the robustness of this ‘greedy’ algorithm, in order to assure that the core found does not depend on the initial partition.

Craig and von Peter [13] estimate the Core Periphery model in the German interbank market. With a data set of roughly 1800 banks over 68

⁸This way of counting errors is convenient, as it imposes a restriction on the core-periphery links. Alternatively, each violation of Condition 3 could count for 1 error. This would yield the more conventional Rand distance and would put less emphasis on the relation between the core and periphery, leading to bigger optimal cores.

quarters (1991Q1-2007Q4), they show that the model gives a good fit (producing few errors) and is stable over time. We will apply the same method to the Dutch data, placing more emphasis on the significance of the model and comparing it with the Preferential Attachment mechanism described previously.

3. Data description

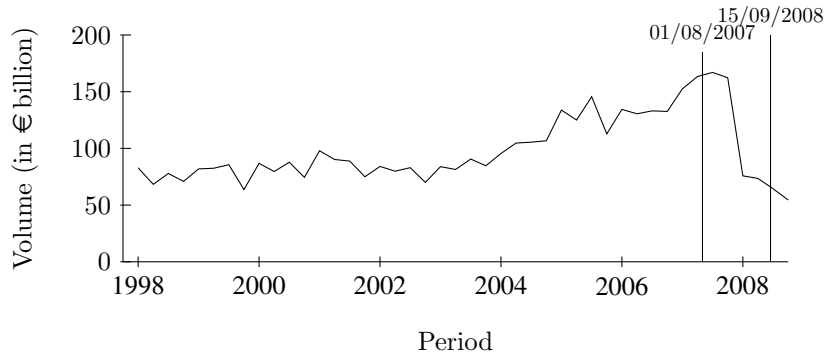
Financial institutions interact on many levels. Sometimes these interactions are very short-lived with contractual obligations expiring before the end of the trading day. Other contracts, such as for instance swaps, can be long-running and can last up to 30 years. Our data combines all of these interactions on a quarterly reporting frequency. We use prudential reporting of balance sheet positions of Dutch banks. Each quarter banks have to report the interbank assets and liabilities to the market as a whole. We limit the sample to exposures up to one year. In addition, banks have to report exposures to their largest counter parties. Assuming that the distribution of interbank exposures is equal to the distribution of claims in general, given by the large exposure reporting, we construct a matrix of interbank exposures for the Dutch banking market.

Data are available from 1998Q1 until 2008Q4 with the number of reporting banks varying between 91 and 102. About 50 banks report every quarter during the sample period. To get a feel for the data we show the overall domestic market volume in Figure 4.⁹ In the long run the volume has been growing, dropping markedly in the financial crisis. The two vertical lines indicate 1) the beginning of the crisis in August 2007 once the subprime mortgages made the headlines, and 2) the bankruptcy of Lehman Brothers

⁹Unfortunately, we have to restrict our attention to only the Dutch banks as information about exposures between foreign counterparties is not available to us. This limitation leads to retain about 15% of the total interbank exposures in the Netherlands. The growth in total volume is relatively high and, interestingly, the drop after the bankruptcy of Lehman Brothers is more pronounced. Possibly this can be explained by a 'return to home market'-argument.

on 15 September 2008. Computed network measures such as path length tell a similar story: the network became more connected over time with a marked reversal due to the crisis.

Figure 4: Domestic market volume over time, 1998Q1-2008Q4



In order to apply unweighted or binary network models to our dataset, we reduce our weighted links to binary links. We use the same €1.5 million threshold as Craig and von Peter [13] to determine the most important links. For every period a sparse network remains of about 8% of the possible number of links. This makes the Dutch interbank network much denser than the German network (where less than 1% of possible links materialise). It seems that the main difference of the German market compared is the large number of small banks, with relatively less links above the threshold.

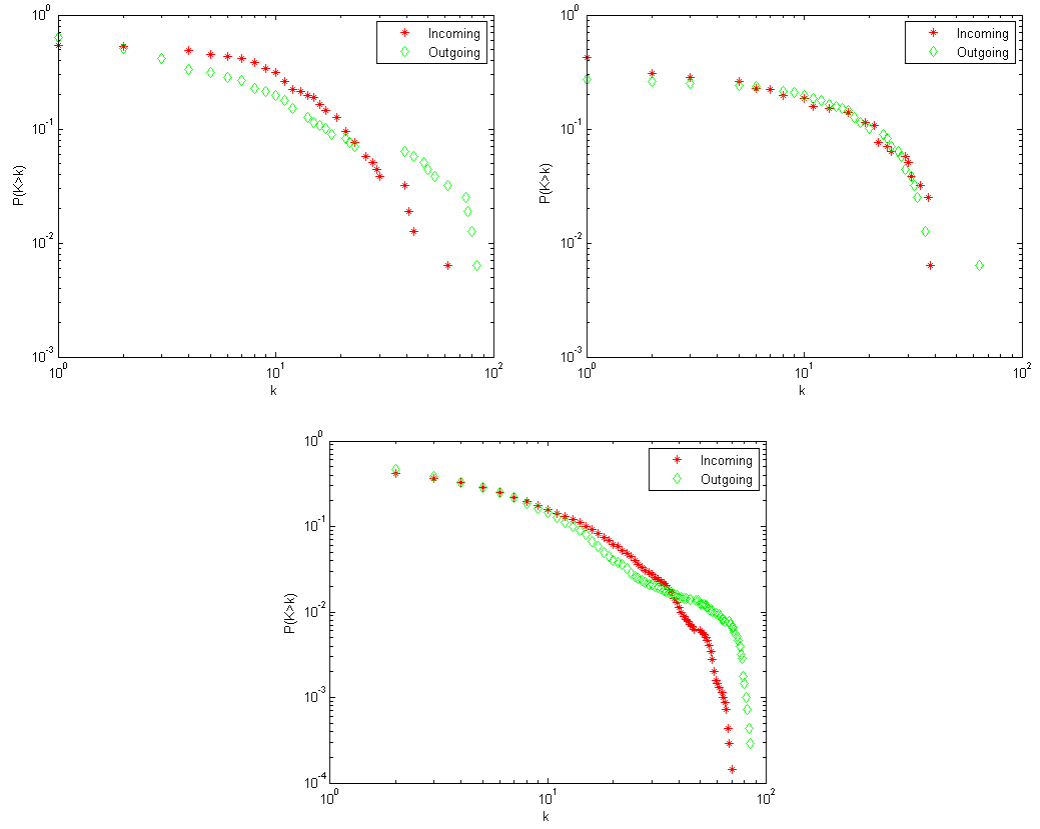
Imposing any threshold necessarily involves some arbitrariness. In the present context, a higher threshold will result in fewer qualifying links and consequently a smaller Core. However, this dependency does not change the qualitative feature of the interbank market consisting of two groups. A more interesting question is how the threshold value affects the explanatory power of the Core Periphery model. We found that this effect is moderate, although the fit of the model could be improved by choosing a higher threshold. More characteristics of the data can be found in Appendix A.

The first property of a binary network to check is the degree distribution. Indeed, the distribution of the number of links per node is the distinguishing feature of scale-free networks compared to random graphs and was also analysed by Boss et al. [7] and later studies. We show selected distributions of in- and outdegree in Figure 5 (i.e. the first period, the last period and the overall aggregate over all periods).¹⁰ By convention these distributions are plotted as the inverse cumulative distribution (showing the proportion of nodes with degree higher than k) on a log-log scale. Scale-free networks have in the limit infinitely many nodes displaying a power law in the degree distribution, corresponding to a straight line in such plots.

The plots do not suggest a scale-free network and we therefore did not try to fit a power law on these degree distributions. Rather, the distribution of indegree looks like a concave decreasing function, similar to the binomial distribution of the Erdős-Rényi random graph or the exponential distribution of random growth models. However, for almost all periods there is a jump in the outdegree distribution indicating more heterogeneity in outdegree than in indegree. This discontinuity in degree is a first indication to a clear distinction between Core and Periphery.

¹⁰ Boss et al. [7] only consider the overall degree distribution. However, its interpretation is not clear because each data point refers to multiple banks over all periods.

Figure 5: In- and outdegree distributions for 1998Q1 (left), 2008Q4 (right) and over all periods (down)



4. Estimation results

Given our empirical network, we now want to find which of the two mechanisms best describes the data: Preferential Attachment or Core Periphery. For our procedure we first estimate the Core Periphery model, finding the number of core members and errors for every period. Secondly, we simulate networks with Preferential Attachment, using a data generating process that takes into account the size and density of the actual network. For each of these simulated networks, we calculate the error score of the Core Periphery structure, and then construct an empirical distribution function. Finally, we use the error score as a test statistic, to test whether the actual number of errors is likely to be drawn from the empirical distribution function.¹¹

4.1. Estimating the core

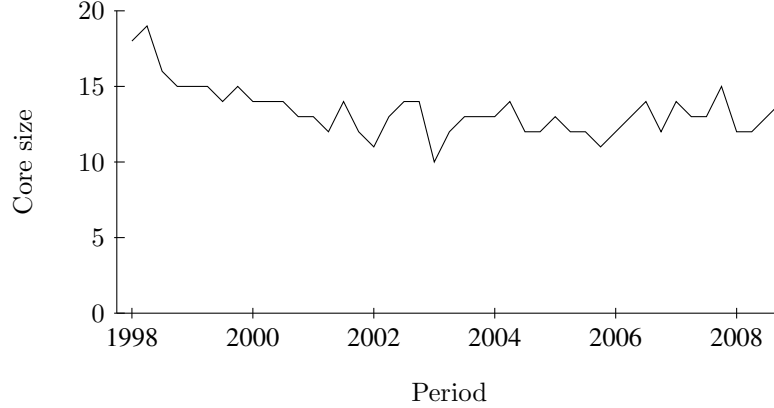
As our first step, we use the sequential optimisation algorithm developed by Craig and von Peter [13].¹² In our dataset with around 100 banks, the optimal core varies between 10 and 19 banks. Figure 6 plots the core size per period. Although in the first three periods a relatively large core is found of respectively 18, 19 and 16 banks, the core size thereafter stays relatively stable between 10 and 15 banks.

There are two points to be noted with respect to the core size. First, the relative core size is closely related with the density of the network. As the density of the network clearly depends heavily on the data construction, one should therefore be careful not to take the core size too literally. Second, there is no correlation with the total volume in the interbank mar-

¹¹This procedure is in the same spirit as a PE-test for non-nested regression models. Having fit one of the two models, it tests whether the other model still has additional power. To fully replicate this idea we would also have to generate a empirical distribution function from CP-networks on a test-statistic for Preferential Attachment. However, both methods for generating randomised CP-networks and for testing Preferential Attachment are not yet developed.

¹²The original code for identifying the core was graciously provided by Ben Craig and Goetz von Peter.

Figure 6: Number of Core banks over time, 1998Q1-2008Q4



ket: although volumes almost doubled, the higher number of links above the threshold stayed relatively stable which leads to a stable core size.

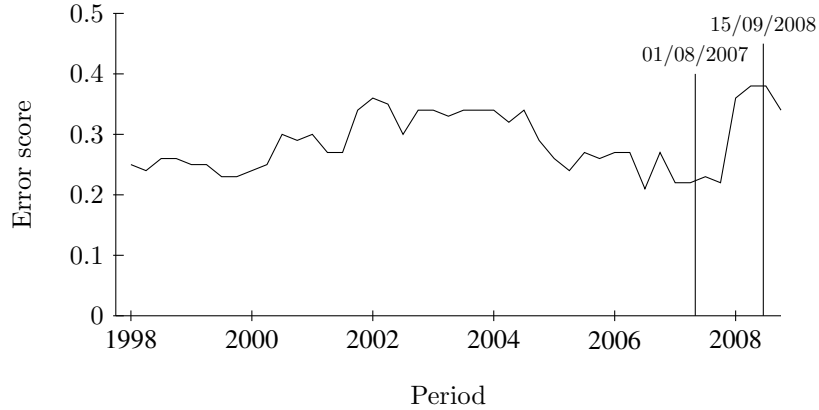
To compare across periods, the *error score* of the optimal core expresses the number of errors compared to the CP model as a proportion of the number of actual links. This expression should not be considered as a percentage, because errors occur due to both the absence and presence of links. Rather, the point is that error scores above 1 indicate that the CP model gives more errors than an empty network without links. Having an error score below 1 is therefore a first requirement in evaluating the fit of the CP model.

In Figure 7, the error score is shown to be between 0.21 and 0.38, with an average of 0.29. The fit is worse than found in the German market (around 0.12), but better than in the Italian equivalent (0.42).¹³ In Appendix B we visualise the fit of the CP model in the first period. We also plot all errors in both the Core and the Periphery for all 44 periods to show that most of

¹³We also tried a higher threshold of €1 billion, in which case the density of the Dutch interbank market is 0.5%, very close to the density in the German market. The average error score decreases to 0.15.

the errors occur in the periphery, where the perfect CP structure dictates a very large empty subgraph.

Figure 7: Error score over time, 1998Q1-2008Q4



Interestingly, the fit is clearly worse in the last four quarters, after the subprime crisis had started to spread. This might be an indication that the CP structure, as far as it is a good description of the Dutch market, dissolved during the crisis. This is further supported by the more regular degree distribution in the last period (Figure 5) and the large number of errors in the core (Figure B.16), particularly in period 44. Using a longer dataset up to 2010 in the Italian market, Fricke and Lux [17] established a significant structural break in 2008Q4, and a similarly deteriorated fit of the model.

We also calculate the transition matrix between the states of being in the core and in the periphery, and being inactive in the interbank market ('Exit'). Most importantly, the transition from core to core indicates that on average 83% of the core banks stay in the core the next period. As we found that the number of core banks is quite stable, the flow from and to the core is in absolute terms almost equal. The higher persistence in the

periphery merely reflects that it consists of much more banks.

$$\begin{pmatrix} & \text{Core} & \text{Periphery} & \text{Exit} \\ \text{Core} & 83\% & 16\% & 1\% \\ \text{Periphery} & 2\% & 96\% & 2\% \\ \text{Exit} & 0\% & 2\% & 98\% \end{pmatrix}$$

4.2. Significance

In Table 1, the empirical results of the CP model for the Netherlands are summarised and compared to Germany and Italy. So far, we haven't said anything about the significance of the Core Periphery structure, or whether it has more explanatory power than Preferential Attachment. These questions will be answered by both simulated distribution functions and a recently proposed analytical method.

Table 1: Comparing CP model for the Dutch interbank market to Germany (Craig and von Peter [13]) and Italy (Fricke and Lux [17]).

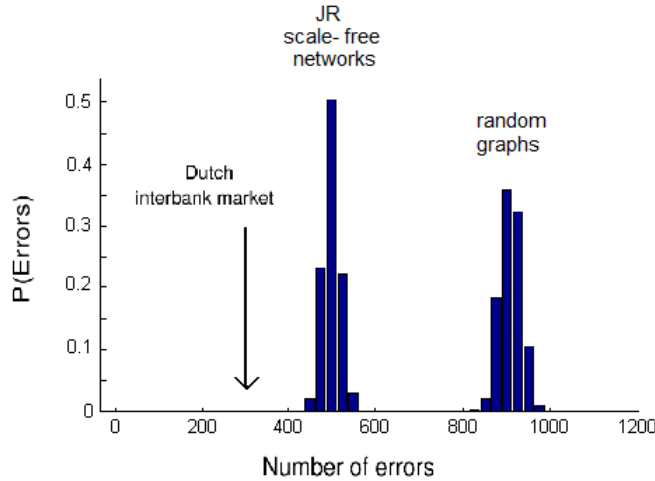
		Netherlands	Germany	Italy
Description	Total number of banks	100	1800	± 120
	Network density	8%	0.4%	$\pm 15\%$
	Average number of core banks	± 15	± 45	± 30
	Average core size	$\pm 15\%$	$\pm 2.5\%$	$\pm 25\%$
Fit	Error frequency, as % of links	29%	12%	42%
	Transition prob. core \rightarrow core	83%	94%	83%

Figure 8 plots the empirical distribution function for a 1000 generated Erdős-Rényi random graphs and a 1000 scale-free networks. Regarding the Erdős-Rényi random graphs, the Dutch interbank data fits much better to the CP model and this is not surprising. If the probability of forming a link is completely random, i.e. not dependent on the number of already existing links, the generated networks have a barely skewed degree distribution. In fitting these random networks to a CP model, an optimal core is found with a core size that is too small. Note that the probability of randomly ending

up in a perfect CP structure (with zero errors) is theoretically positive, but practically negligible for finite numbers of replications.

For the scale-free networks, we have used the growth model of Jackson and Rogers [23]. Although growth models are generally used to find the limiting distribution, we let the model grow until we have the required number of nodes (e.g. 103 for 1998Q1). In each step a node is added with a fixed outdegree, such that the ultimate density will be identical to the actual network. The actual error score is much lower than the entire distribution of the generated scale-free networks.¹⁴ This means that the hypothesis that the actual network is drawn from the distribution of scale free networks is rejected in favour of the CP mechanism.

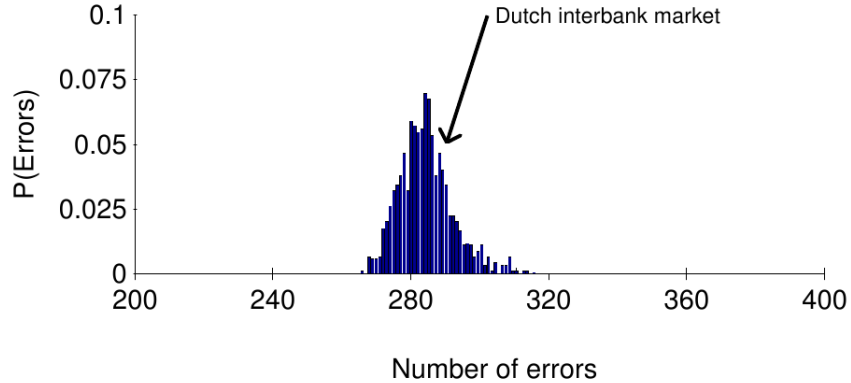
Figure 8: Empirical distribution function of number of errors for 1998Q1, 1000 Jackson Rogers scale free networks and 1000 random graphs.



¹⁴A drawback of the chosen data generating process is that it requires a significant number of initial nodes that are fully connected to each other, to make the procedure operational. This leads to higher core sizes, and less randomness in the simulation because of the small sample. Fricke and Lux [17] chose a procedure to generate scale-free networks based on Goh et al. [20]

As typical in the complex network literature, it is also possible to test whether the network outcomes are significant given the same degree distribution, i.e. under the *configuration model*. Typically this is done by the so-called local rewiring algorithm, where two separated links in the actual network are broken and the nodes are reconnected reversely. The distribution of the error score under the configuration model indicates whether or not, given the degree distribution, there is additional evidence for the CP structure caused by the 'ordering' of the links. We do not find this evidence, because the larger part of the empirical distribution is left of the error score of the actual network (Figure 9). However, this test cannot give any indication of whether the CP structure is plausible in itself.¹⁵

Figure 9: Empirical distribution function of number of errors for 1998Q1, 1000 permutations without destroying the degree distribution.

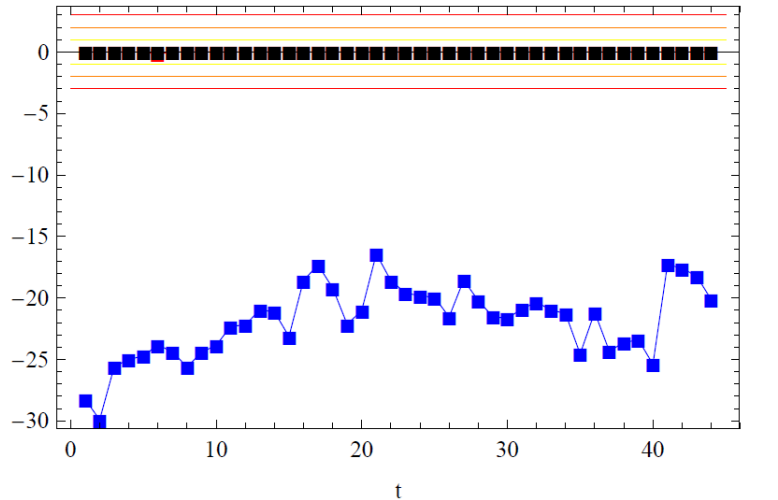


So far we restricted ourselves to the significance of the model in the first period only. The reason is that the simulation methods is computationally very demanding, because (1) many iterations of the local rewiring algorithm are required for every permutation of the original network and (2) for

¹⁵In fact, in a perfect CP structure, there are no possible permutation possible that would generate errors from this structure. The empirical distribution function would therefore consist of a single point, not providing any information.

every simulated network the optimal core has to be found. Squartini and Garlaschelli [28] have recently developed an analytical method that can randomize the network more efficiently, on every period given the density (the random graph model) or the degree distribution (the configuration model).¹⁶ Figure 10 shows the z-score over all periods: the observed number of errors expressed in standard deviations from the expected value under both null models.¹⁷

Figure 10: Z-scores for the number of errors under the random graph model (blue, down) and the configuration model (black, up).



The analysis with the more advanced method confirms the earlier conclusions. We favour the Core Periphery structure over Random graphs or Preferential Attachment models, because the error score in the data is much higher than is expected by networks generated by these models. The good fit seems to be fully attributed to the degree distribution, with a small subset of banks having many links, removed sharply from the majority of banks

¹⁶Unfortunately, this method cannot be used to generate scale-free networks.

¹⁷We are much indebted to Tiziano Squartini and Diego Garlaschelli for constructing this plot.

having only a few. In fact the z-score under the configuration model is in most periods exactly 0, indicating that the actual number of errors matches its expectation.

5. Stability implications

In this final section, we investigate how belonging to the core is related to bank characteristics. As Craig and von Peter [13] already showed, Core banks tend to be the larger banks. However, as Figure 11 shows, the Core also tends to have a much lower buffer, defined as the Tier1 capital over the total assets.¹⁸

Figure 11: Average buffer, 1998Q1-2008Q4

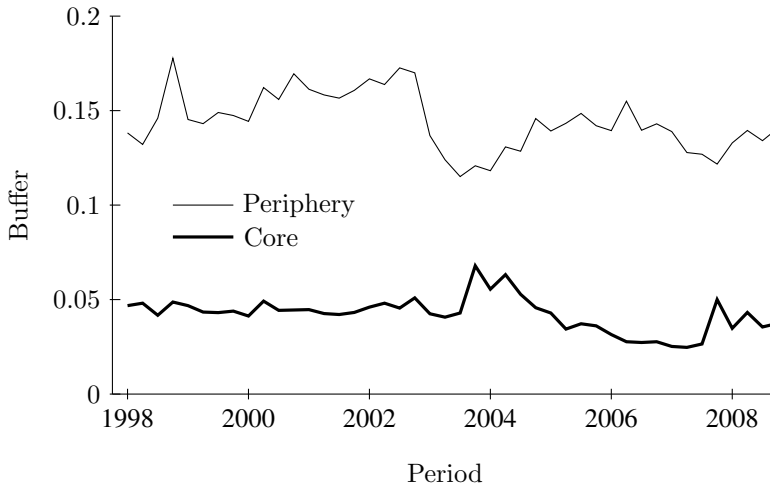


Figure 11 plots the average buffer over the respective Core and Periphery members every period. To test for the difference more formally, we stack the panel data to cross section¹⁹ and regress the buffer on a time-varying constant and the Core dummy. The coefficient for Core membership is estimated

¹⁸Data is the same as in Liedorp et al. [24].

¹⁹This simplification is justified because we are only interested in the intrinsic difference

at -0.102, indicating a 10.2 percentage point decrease in the conditional mean of Core bank buffers relative to Periphery bank buffers. The coefficient is significant at the 1% level, the R^2 equals 0.035 and the F-statistic equals 2.77, which is significant as well on every reasonable level.

6. Conclusions

The Core Periphery model is an useful tool to represent interbank data, and recently researchers have started to apply the model to available datasets. Using a Dutch panel of bank connections from 1998Q1 until 2008Q4, we find an optimal core of around 15 out of the 100 active banks. While the point estimates of the Core Periphery model for our dataset (error score, core persistence) are less convincing than for the German interbank market, they are significant. The estimated representation is found to be better than random graphs or scale-free networks: neither complete randomness nor Preferential Attachment seems to fit the Core Periphery structure sufficiently.

We also showed that the Core Periphery model affects financial stability. Not only is the Core important for providing interbank lending throughout the Periphery, it also tends to have lower relative buffers. These findings suggest two complementary reasons to impose higher regulatory requirements on the systemically important Core banks. The Core Periphery structure deteriorated considerably during the financial crisis. The general analysis so far opens up new opportunities for systemic risk assessments of the interbank market, especially as more granular data is becoming available for the eurozone.

between Core and Periphery banks, and not in dynamic effects of banks moving into or out of the Core.

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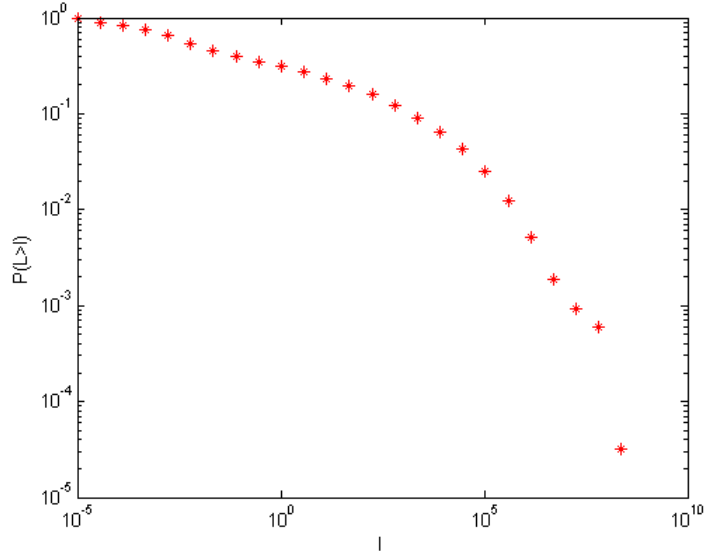
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Appendix A. Other characteristics of the data

In this appendix we present additional characteristics of the data that are not fundamental to our discussion, but provide a better overall understanding of the Dutch interbank market. First of all, we consider the loan size distribution over all periods. Figure A.12 plots this inverse cumulative distribution (of loans in €1000) in log-log scale. The plot shows that around 92% of the links is estimated below $10^0 = 1 * \text{€}1000$. As these small loans are due to the data construction (see below) rather than actual banking agreements, it is important to use some sort of threshold value as we do in our analysis.

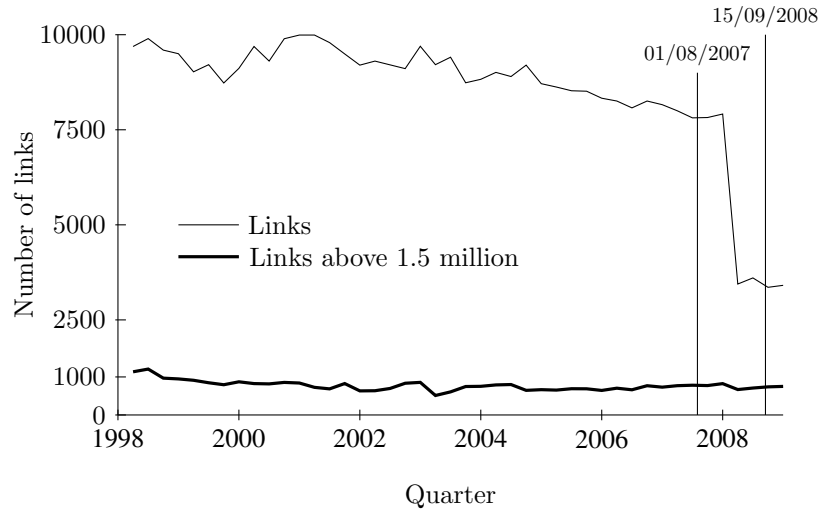
Moreover, the figure shows a wide dispersion of loans within the sample. In fact, we could estimate a power law distribution on the right tail to see whether loan sizes in the Netherlands display scale free behaviour. In an early analysis of interbank network structure, Boss et al. [7] fit power laws to the Austrian loans.

Figure A.12: Loan size distribution over all periods



Now we turn to the effect of imposing the threshold. In Figure A.13, both the total number of links and the 'significant' links above the threshold of €1.5 million are presented. Interestingly, the total number of links shows a sharp drop after the beginning of the financial crisis in August 2007 where almost half of the links disappear permanently. This might also be connected with the way the data is constructed. For the smaller loans, it is assumed that the totals of the interbank assets and liabilities for a given bank (observable from the balance sheet) follow the same distribution over other banks as claims in general. Before the bankruptcy of Lehman Brothers in September 2008, the payment system between banks was already under stress, and this might have caused a direct effect on the estimation of the small loans.²⁰

Figure A.13: Number of links over time, 1998Q1-2008Q4



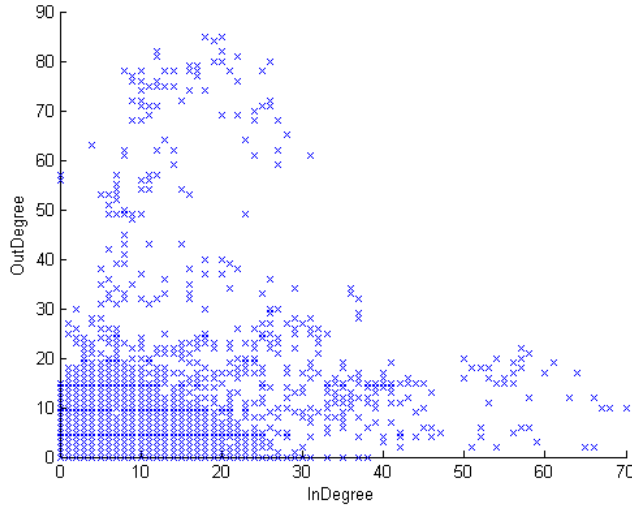
Importantly, the number of links above the threshold shows a much milder drop because of the crisis. The steady number of 'significant' links is therefore a much more reliable source of information. Also note that

²⁰See Liedorp et al. [24] for a full description of the construction of the data set.

while only around 1000 links remain, the remaining links cover practically all market volume (more than 99.5%). This observation is in accordance with the wide dispersion of link size as noted above. Thus in terms of volume only considering big loans is much less restrictive than our ignorance of the identities of foreign counterparties.

Given the threshold of €1.5 million, we can also find the correlation between in- and outdegree of the nodes. It would seem that the correlation should be quite high, as banks that are exposed to many banks should also rely on many other banks. However, we find a relatively low correlation coefficient of 0.40.²¹ Figure A.14 reveals that periods exist in which banks have either an extreme in- or outdegree, most probably because of other balance sheet considerations in that particular period.

Figure A.14: In- and outdegree per bank over all periods

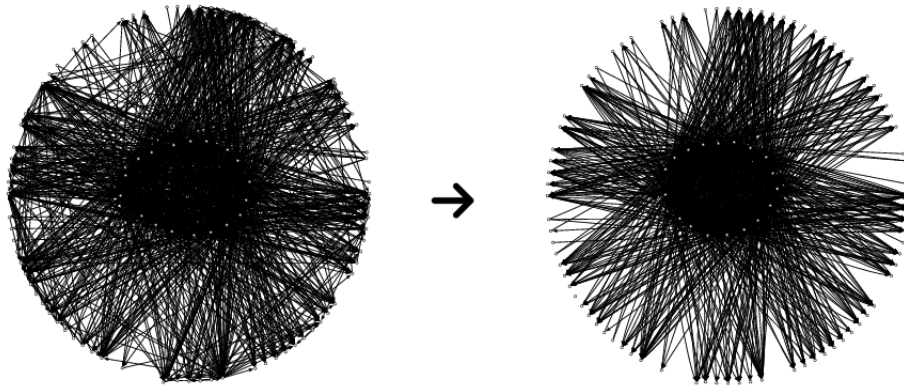


²¹Fricke and Lux [17] estimate an asymmetric version of their continuous CP model, allowing for difference in 'in-coreness' and 'out-coreness'. This interesting extension in fact leads to improved fit of their data.

Appendix B. Visualising errors in the CP model

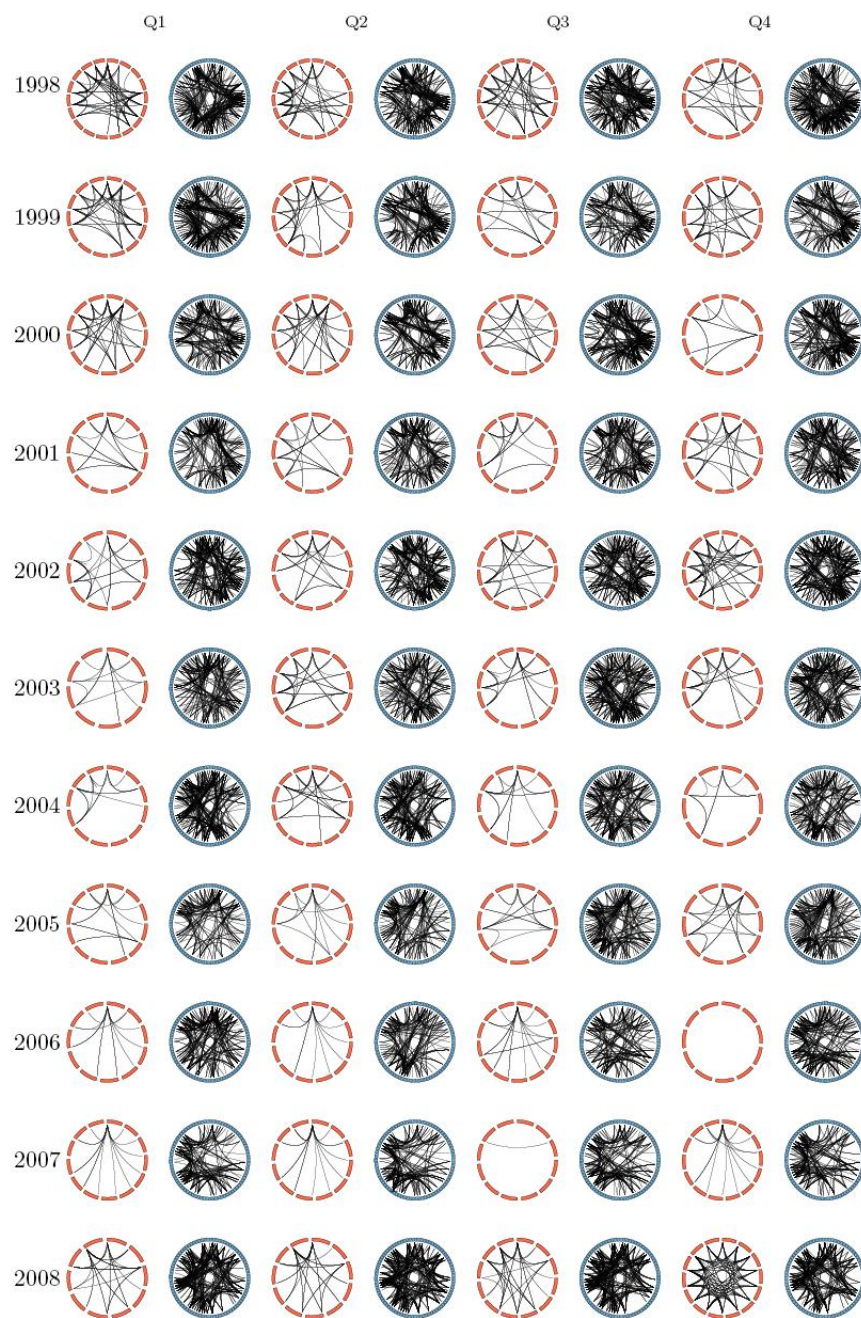
This appendix shows two visualisations of the Core Periphery model. Note that even for a relative small number of 100 banks, network plots very quickly become inscrutable. Figure B.15 shows the interbank linkages of Dutch banks in 1998Q1, where the estimated Core banks are placed in an inner circle within the circle of Periphery banks.²² It is visible that the fitted network incorporates a clear structure, while retaining the general nature of the data. Figure B.16 focuses on the errors in all periods. For the core banks, the errors are the missing links, whereas for the periphery the links between any two periphery banks are drawn.

Figure B.15: Plot of the Dutch interbank market on 1998Q1 (left) and its fit on the CP structure (right). The outer circle represents Periphery banks; the inner circle of small white dots Core banks.



²²These plots were made with the Pajek program.

Figure B.16: Errors in the CP structure: Core (red, left) and Periphery (grey, right)



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