OFR WORKING PAPER

OFFICE OF FINANCIAL RESEARCH

16-14 | December 20, 2016

Interbank Contagion: An Agent-based Model Approach to Endogenously Formed Networks

Anqi Liu
Stevens Institute of Technology
aliu@stevens.edu

Mark Paddrik
Office of Financial Research
mark.paddrik@ofr.treasury.gov

Steve Yang
Stevens Institute of Technology
steve.yang@stevens.edu

Xingjia Zhang
Stevens Institute of Technology
xzhang21@stevens.edu

Interbank Contagion: An Agent-based Model Approach to Endogenously Formed Networks*

Anqi Liu†
Mark Paddrik‡
Steve Yang§
Xingjia Zhang¶

December 20, 2016

Abstract

The potential impact of interconnected financial institutions on interbank financial systems is a financial stability concern for central banks and regulators. A number of algorithms/methods have been developed to extrapolate latent interbank risk exposures. However, most use highly stylized network models and reconstruction methods with global optimality lending allocation approaches such as maximizing entropy or minimizing costs. This paper argues that U.S. bank lending and borrowing decisions are largely suboptimal and performance-driven. We present an agent-based model to endogenously reconstruct interbank networks based on 6,600 banks' decision rules and behaviors reflected in quarterly balance sheets. The model formulation reproduces dynamics similar to those of the 2007-09 financial crisis and shows how bank losses and failures arise from network contagion and lending market illiquidity. When calibrated to post-crisis data from 2011-14, the model shows the banking system has reduced its likelihood of bank failures through network contagion and illiquidity, given a similar stress scenario.

Keywords: Interbank lending market, agent-based simulation, contagion risk, network topology, financial crisis

JEL Classification Numbers: D85, G17, G21, L14

*The authors would like to thank Randall Dodd, Gregory Feldberg, Katherine Gleason, Benjamin Kay, Blake LeBaron, Matthew McCormick, Sofia Priazhkina, Stacey Schreft, Kevin Sheppard, Stathis Tompaidis, and Julie Vorman. We also thank participants at the 2016 International Finance and Banking Society conference in Barcelona, the XI Annual Seminar on Risk, Financial Stability and Banking held by Banco Central do Brasil, and Bank of Canada and Payments Canada Workshop on the Modelling and Simulation of Payments and Other Financial System Infrastructures for their valuable comments. Additionally we would like to thank the OFR's ETL, Applications Development, Data Services, Legal and Systems Engineering teams. **Note:** Views and opinions expressed are those of the authors and do not necessarily represent official positions or policy of the OFR or the U.S. Department of the Treasury.

†Stevens Institute of Technology, Financial Engineering Division, email: aliu@stevens.edu.

‡Office of Financial Research, U.S. Department of Treasury, email: Mark.Paddrik@ofr.treasury.gov.

§Stevens Institute of Technology, Financial Engineering Division, email: steve.yang@stevens.edu.

¶Stevens Institute of Technology, Financial Engineering Division, email: xzhang21@stevens.edu.

1 Introduction

Recent experiences since the financial crisis suggest that the existing understanding of systemic risk may not adequately capture latent fragility and shock propagation. The impact of shocks or any disturbances in the financial sector cannot be assessed without relying on a systemwide perspective on how different institutions interact, how system complexities evolve, and how the endogenous behaviors of different agents converge.

Prior to the 2007-09 crisis, financial regulators placed less emphasis on assessment of systemwide characteristics of networks and risks within them (Hałaj and Kok (2013)), and instead focused on microprudential risk of individual institutions. Of particular interest since then has been the topic of interconnectedness of financial institutions because of the unknown probability of contagion between institutions. Additionally, though less discussed in the network literature, is endogenous network formation, where institutions have to decide whether creating a financial relationships is attractive.

Both these concerns were seen in the U.S. interbank lending market, one of the most immediate sources of liquidity for banks, during the crisis. Afonso et al. (2011) show that the interbank market behaved with a heightened concern for counterparty risk that reduced liquidity and increased the cost of financing for weaker banks. Banks overall were less likely to lend liquid assets to each other. Large banks, which play a central role in this market, increased their liquidity buffers (Berrospide (2012)), forcing medium and small banks to look for new sources of liquidity.

As a result of these events, network-based representations of the interbank market have begun to be studied. Some central banks have even started using network models related to contagion (Bank of Korea (2012), European Central Bank (2013), Anand et al. (2014), Martinez-Jaramillo et al. (2014)). Due to the fact that in practice the interbank networks often remain unobserved, several algorithms/methods have been developed to extrapolate interbank risk exposures and network structures to consider contagion risks.

Existing methods for interbank network reconstruction and consequential contagion modeling have two major shortcomings: (1) most models tackle the problem with highly stylized structures including some agent-based approaches; and (2) the assumption of some type of optimal decisions in bank lending and borrowing is broadly applied. The reality is that banks are more performance-driven, and their performances are individually optimal but are collectively suboptimal (Acharya (2009)). Banks are autonomous decision makers with various constraints. A performance-driven agent would respond or adapt to market changes to achieve its performance objectives, which are dynamic in nature. This characteristic is largely not present in a pure network optimization setting, and when these networks are used for stress testing, results would likely exhibit significant deviation from reality.

For example, the two leading methods in reconstructing interbank networks using assets and liabilities reported in financial balance sheets assume a globally optimal allocation of lending. The first one, known as the *Maximum Entropy* (ME) method, effectively assumes that banks diversify their exposures by spreading their lending and borrowing across all other banks (Upper and Worms (2004), Upper (2011), Elsinger et al. (2013)). The key concept here is that maximum entropy is optimal from an information-theoretic perspective, but empirical studies show interbank networks are typically sparse (Cocco et al. (2009)). The second method is aptly named *Minimum Density* (MD), which minimizes the number of links necessary for distributing a given volume of loans on the interbank market (Anand et al. (2015)). In contrast to the Maximum Entropy method, it is based on the economic rationale that interbank linkages are costly to maintain. Both Maximum Entropy and Minimum Density present two extreme cases in backtracking interbank networks. When used to stress test the system, the Minimum Density method provides lower bound, while the Maximum Entropy method offers upper bound to the number of linkages (Anand et al. (2015)). The real network structure lies in between the two.

This paper uses historical financial data from the U.S. Federal Financial Institutions Examination Council (FFIEC) to build a large scale agent-based model (ABM) to represent all the banks at a 1:1 scale of the U.S. banking system. Bilateral exposures are represented by different asset maturities, such as overnight debts (federal funds), short-term and long-term debts. Bank lending and borrowing behaviors are based on statistics of individual banks and general behavior patterns from the empirical findings. This framework reconstructs an interbank exposure network using agent-driven decisions that are then compared with and validated against the existing empirical findings, as well as other existing interbank network construction algorithms.

The model is additionally validated by calibrating it to the pre-crisis FFIEC data and running Monte Carlo simulations. The simulations demonstrate that modeled bank failures follow similar dynamics and outcomes as those seen in 2007-09. The analysis introduces systemic shocks that cause a correlated collapse of asset holdings across the system in the ABM to induce system contagions. Finally, the model is recalibrated with post-crisis banking data, and the simulated results of running a similar shock are compared to those of the pre-crisis results.

The first contribution this paper makes to the current literature includes introducing a mechanism that endogenously forms a financial architecture using individual bank performance objectives derived from balance sheets. This framework examines the network resilience of the financial architecture to bank defaults and contagion while dynamically allowing the banks to form new relationships and reform the network.

This paper makes a second contribution by presenting a model for stresstesting the banking system that incorporates indirect losses from contagion-driven insolvency and illiquidity. The power of this methodology is demonstrated by examining how the banking system performs before and after the 2007-09 financial crisis under a shock similar to that of the crisis, and how different aspects of the shock propagate defaults. It could additionally give regulators a platform to test new regulations and policies that either target or impose network structure in a dynamic environment, such as the Federal Reserve's recent proposal for single-counterparty credit limits (Federal Reserve System (2016)). As this rule is meant to constrain large counterparty relationships, determining what the new network equilibrium would look like is important in identifying how the rule would improve financial stability.

The paper is structured as follows. Section 2 reviews current literature related to systemic risk and interconnectedness, interbank networks topology, and extrapolation techniques. Section 3 discusses U.S. banking financial data used in this study. Section 4 summarizes the methodology used to construct the agent model and to incorporate autonomous behaviors of the agents. Section 5 explains the validation of the model. Section 6 presents model experiments and results. Finally the paper concludes in Section 7 by assessing the results and the methodology's contributions.

2 Background

This section delves into four key aspects of modeling interconnectedness in the U.S. banking system: (1) modeling interconnectedness as it relates to systemic risk, (2) the topology of the interbank networks for short and longer term lending characterized by previous studies, (3) current methods and practices for recovering network structure, and (4) ABM as a method for endogenously determining how networks form under stress.

2.1 Systemic Risk and Endogenous Networks

Among the many factors contributing to the financial crisis of 2007-09, the role of the growing interconnectedness of the global financial system is perhaps the least well understood (Glasserman and Young (2015)). Pioneering works by Allen and Gale (2000) and Eisenberg and Noe (2001) highlighted the importance of financial interconnectedness and systemic risk and the crisis exposed the fact that regulators and market participants had limited information to examine financial networks and identify risk channels.

Many models have highlighted how interbank network data could be used to examine the spread of contagion (Wells (2004); Iori et al. (2006); Elliott et al. (2014); Acemoglu et al. (2015a)). However, little work has considered how financial networks endogenously form and change as new market participants enter, defaults occur, or new policies are enacted. The answer to the question of how to use strategic network formation can be traced to seminal works of Jackson and Wolinsky (1996) and Bala and Goyal (2000). This literature focuses on how agents trade off the costs and benefits of creating links with one another and characterizes the set of networks that are formed in equilibrium. More recent works by Acemoglu et al. (2015b) have looked at how endogenous network formation can impact systemic risk, and Gofman (2016) has developed these themes by calibrating network formation based on network features seen in agent trading decisions.

2.2 Interbank Network Topology

The interbank network's structure is of interest to central banks and regulators concerned with bank bilateral exposures and the implications they pose in periods of stress. Research so far has focused on the overnight funding market because of data accessibility. Boss et al. (2004), Iori and Gabbi (2008), and Roukny et al. (2014) investigated the interbank market in Austria, Italy, and Germany, respectively, and discovered similar network features of the banking system in those countries. These features include: (1) sparsity and short average distance among nodes, (2) heterogeneous degree count among nodes that follows a power law distribution, (3) small clustering, and (4) small world properties. Fewer studies have looked at the total network including overnight transactions, short-term loans, and long-term loans in the aggregate due to the lack of data.

Cont et al. (2013) investigated the Brazilian banking system based on balance sheets with complete interbank exposures. Their findings suggest that connectivity properties of the total network are consistent with those of overnight transaction networks. This similarity is due in part to the preference seen in lending practices between large and small banks. Cocco et al. (2009) documents that smaller banks, which normally have higher default risk, tend to rely on large banks when borrowing funds. Large banks prefer to borrow funds with familiar counterparties to reduce interest payments. Though this may create similar network features between the loan maturity networks, the combination of both loan types is an important determinant of interbank lending liquidity (Bargigli et al. (2015)).

2.3 Interbank Network Extrapolation

Interbank networks are seen as fundamental channels for contagion and systemic risk today. But in practice, most interbank networks remain unobserved because interbank loans are generally arranged over the counter and data are not centrally collected in most countries. As a result, several methods have been developed to approximate the network with available data. These methods do so by estimating networks from balance sheet lending and borrowing. The predominate approach is the Maximum Entropy method that has a simple risk-sharing mechanism that implicitly assumes perfect competition, i.e. all banks are equally willing to accept an equal share of risk (Upper and Worms (2004)). However, interbank networks have been sparse, because interbank activity is based on relationship banking (Cocco et al. (2009)). Smaller banks are limited by the number of linkages they can maintain (Craig and Von Peter (2014)), as it is costly to manage a large and diversified set of lending and borrowing relationships.

As a result, many different algorithms have been used to manage linkage formation by including optimizing features for different network measures.[1] The Basel Committee on Banking Supervision (2015) compared many of these algorithms and found the Minimum Density (Anand et al. (2015)) to be one of the most accurate estimators for interbank networks, although it has a bias toward

[1] Alternative methods suggested in the literature include Anand et al. (2015), Baral and Fique (2012), Battiston et al. (2012), Tarashev et al. (2011), Hałaj and Kok (2013), and Mastrandrea et al. (2014)

underestimating the total number of linkages.

2.4 Agent-Based Modeling in Interbank Networks

As an alternative to the network theoretic-based approach, ABM offers flexibility and enhances fidelity to the real data observed. Many attempts have been made to apply this approach to interbank contagion problems. By its definition, ABM is a simulation framework comprised of autonomous agents with interacting behaviors, connections between agents, and an exogenous environment (Macal and North (2010)). In contrast to statistical and mathematical models, ABMs have advantages in replicating real social phenomena, adaptive agent behaviors, and information diffusion among agents (Macy and Willer (2002), Gilbert and Terna (2000)). These features provide an ideal platform for modeling endogenous network formation through behavior-based rules.

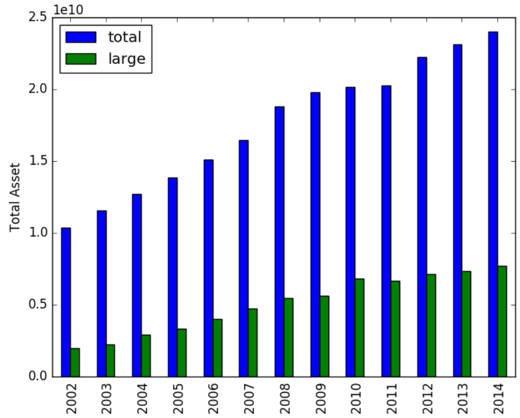
ABMs have been used for systemic risk evaluation in the past (Streit and Borenstein (2009), Bookstaber et al. (2014)). Within the banking system more specifically, ABMs have been applied on top of network topologies to explore contagion risk among banks (Georg (2013), Ladley (2013)). In addition, further extension has replicated multi-layered network structures hinging on multiple types of interbank loans. Kok and Montagna (2013) investigated contagion risk among large EU banks and discovered nonlinearities in the shock propagation.

3 Data

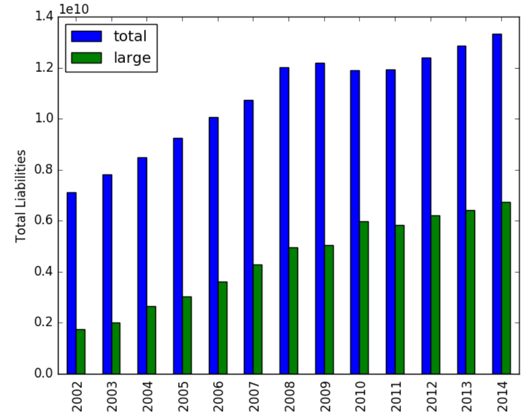
U.S. national banks, state member banks, insured state nonmember banks, and savings associations are required to submit quarterly financial reports to the FFIEC known as the *Federal Financial Institutions Examination Council Reports of Condition and Income*.[2] The balance sheet and income statements disclosed on the form show each bank's business model and lending-borrowing practices. These are used to derive the interbank market structure. The data sample used in this paper covers 14 years, from March 2001 to December 2014, and includes reports from just over $10,000$ active and failed banks.

Figure 1 has bank balance sheet statistics for the sample period. Figure 1a and 1b show assets and liabilities held by all U.S. banks and by the 10 largest U.S. banks. There is steady growth

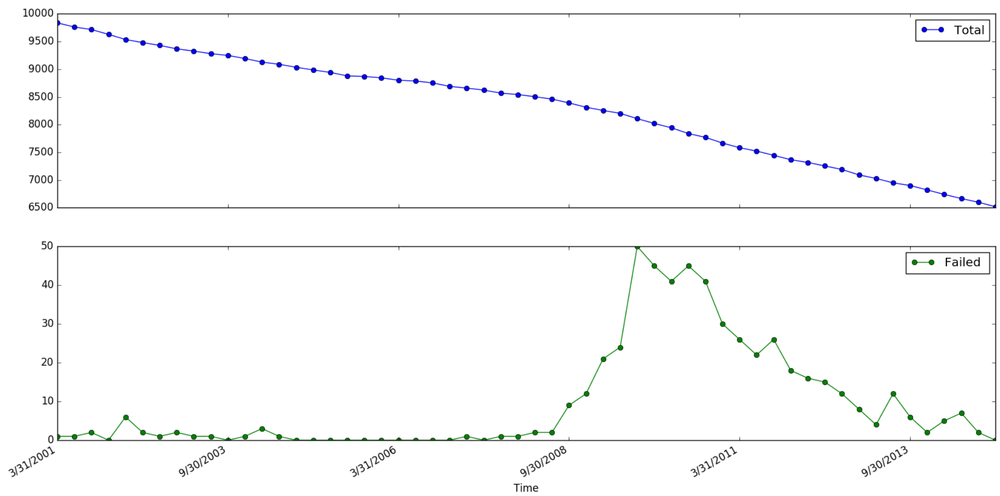
[2]In the case of bank holding companies, the data represents only balance sheet information associated with the commercial bank part of the company.



(a) Total Assets of Banks



(b) Total Liabilities of Banks



(c) Number of Banks and Bank Failures

Figure 1: Bank Sample Plots: Assets, Liabilities, and Number.

Notes: The top two bar charts show the total assets and liabilities of all banks in the sample and the largest 10 banks for the first quarter of each year. The bottom two line charts plot the numbers of banks and the number of failed banks in each quarter. *Source*: Federal Financial Institutions Examination Council Reports of Condition and Income.

on both sides of the balance sheet with exception of the period after the 2007-09 crisis. During the crisis, there was an initial increase in total bank balance sheet values, followed by a flattening during 2010 and 2011, before the positive linear growth trend pre-crisis reemerged.

Figure 1c shows the total number of banks, and the number of banks that failed. In contrast to an increase in the aggregate balance sheet of the banks, there is a steady decrease in the number of banks due to consolidation. By the end of the analysis period in 2014, the total number of banks had decreased by nearly a third. There is little impact on the trend of this decrease throughout the crisis. However, the number of bank failures, which occur when a bank is unable to meet its obligations to depositors and lenders, substantially increased beginning in the third quarter of 2008, peaked in mid-2010, and slowly decreased through 2014.

3.1 Interbank Lending Markets

Because interbank lending markets fund the most immediate sources of bank liquidity, a source of concern during the financial crisis, bank regulators are interested in monitoring these markets. When stress rises in these markets, it can lead to insufficient bank liquidity and inadequate allocation of capital and risk sharing between banks (Afonso et al. (2014)). The FFIEC data show interbank lending on an overnight, short-term, and long-term basis by the amount of federal funds, federal securities, and interbank loans each institution has on its balance sheet.

How banks use interbank markets depends on their liquidity needs. For example, Afonso et al. (2014) show that large banks have a lower liquidity and higher leverage than small banks. Table 1 shows the average percentage of a bank's balance sheet that each lending and borrowing activity represents during different three-year periods. Banks on average use the overnight market to lend and use the short-term market to borrow.

Considering how these markets have changed in terms of bank balance sheets pre-, during- and post- crisis, there is a noticeable decrease in how important these markets are on both sides of the balance sheet. Both short-term and long-term lending and borrowing in the post-crisis period are half of what they were prior to the crisis. Overnight borrowing is one-fourth of its pre-crisis size and overnight lending has declined marginally.

3.2 Large and Small Banks

The interbank lending market is a mix of two types of banks: small end-user banks that need to borrow or lend, and large banks that act as intermediaries to the flow of lending and borrowing needs. Previous research has distinguished these two groups using bank asset sizes (Afonso et al. (2014)).

Banks are separated into a large bank group or a small bank group, based on total assets on

Table 1: Interbank Lending and Borrowing as a Percentage of the Balance Sheet

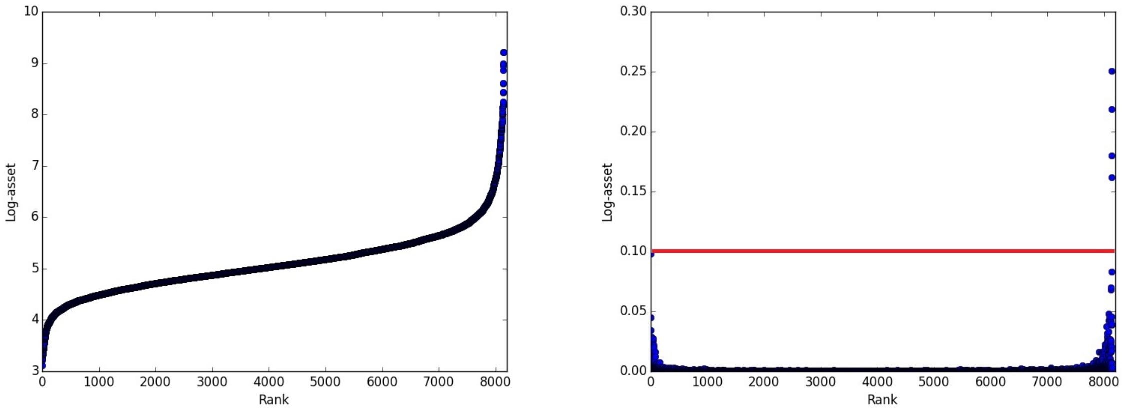
Year	Overnight		Short-term		Long-term	
	Asset	Liability	Asset	Liability	Asset	Liability
Pre-Crisis						
2002	5.13	0.53	0.15	1.06	0.11	0.32
	(7.19)	(3.37)	(1.93)	(3.19)	(1.73)	(3.64)
2004	4.62	0.60	0.17	1.07	0.11	0.28
	(7.30)	(3.60)	(2.45)	(3.28)	(1.97)	(3.50)
2006	4.44	0.79	0.14	1.11	0.09	0.21
	(8.16)	(4.56)	(2.19)	(3.36)	(1.70)	(2.74)
Crisis						
2007	5.51	0.61	0.18	1.14	0.08	0.13
	(8.96)	(4.39)	(2.58)	(3.12)	(1.62)	(1.97)
2008	5.33	0.69	0.16	1.16	0.09	0.15
	(8.42)	(4.44)	(2.50)	(3.12)	(1.67)	(2.28)
2009	3.23	0.45	0.12	1.05	0.10	0.25
	(5.63)	(3.64)	(1.63)	(2.74)	(1.86)	(2.69)
Post-Crisis						
2010	2.57	0.28	0.09	0.99	0.09	0.17
	(2.13)	(3.00)	(1.51)	(2.64)	(1.73)	(2.20)
2012	2.13	0.14	0.10	0.87	0.09	0.10
	(4.75)	(2.04)	(1.56)	(2.45)	(1.78)	(1.92)
2014	1.56	0.16	0.06	0.76	0.09	0.05
	(4.09)	(1.94)	(1.11)	(2.17)	(1.86)	(0.55)

Notes: This table shows the mean and standard deviation (in parentheses) of the percentage that balance sheet interbank lending and borrowing contribute to assets and liabilities.
Source: Federal Financial Institutions Examination Council Reports of Condition and Income.

their balance sheets over time. First, banks are ranked by assets. The next step calculates the differences of logarithmic total assets between two adjacent banks in the ranking, as shown in Figure 2. All banks above a threshold of 0.10, depicted by the red line in Figure 2b, are considered large. In Figure 2, four banks are above the threshold line.

We use quarterly financial reports from 2001 to 2014 to separate banks into large and small types. Some banks switch between the two groups in different time periods, but four banks consistently appear in the large bank group: Bank of America, Citibank, J.P. Morgan Chase Banks, and Wells Fargo Bank.

There is a distinct difference in how the two groups behave in the interbank market. Large banks borrow and lend more than small banks. In terms of overnight lending, large banks lend



(a) Log(Asset) of Banks **(b)** Difference of Log(Asset)

Figure 2: Bank Categorization By Asset Value.

Notes: This figure shows the total U.S. bank asset distributions from March 2001 to December 2014.
Source: Federal Financial Institutions Examination Council Reports of Condition and Income.

1.4 times more than small banks. Large banks also borrow over five times more than small banks, meaning that large banks prefer to borrow from small banks, which is consistent with the empirical findings of Cocco et al. (2009). In the short-term market, large banks lend just over six times more than small banks but do similar amounts of borrowing. In the long-term market, large banks do eight times more lending and borrowing than their small bank counterparts.

Table 2: Interbank Lending and Borrowing: Large and Small Banks

Type	Overnight		Short-term		Long-term	
	Asset	Liability	Asset	Liability	Asset	Liability
Large	7.2	2.52	0.92	2.09	1.25	1.99
	(13.47)	(3.29)	(1.05)	(2.33)	(1.93)	(1.68)
Small	5.33	0.49	0.15	2.09	0.14	0.28
	(7.25)	(3.37)	(1.98)	(3.09)	(2.08)	(3.61)

Notes: This table shows the mean and standard deviation of the percentage of the balance sheet interbank lending and borrowing.
Source: Federal Financial Institutions Examination Council Reports of Condition and Income.

4 Model

This section presents an ABM approach to simulate the U.S. interbank lending system. The model is constructed by incorporating bank-level decisions on lending and borrowing based on individual bank preferences, allowing the inference of interbank networks based on observed bank lending and borrowing behaviors. The remainder of this section covers how the banks' balance sheets and objectives are modeled and is followed by how each bank's lending and borrowing needs are matched to endogenously form lending networks.

4.1 Banks

The model considers a single economy, populated by risk-neutral banks that can only lend to each other. Each bank has a balance sheet made up of assets, A, and liabilities, L, represented in Table 3. On the assets side, banks make interbank loans that include overnight market, ON^l, short-term, ST^l, and long-term, LT^l maturities, as well as cash and cash equivalents, C, and other assets, OA.[3] On the liabilities side, banks have interbank loans borrowed in the overnight market, ON^b, short-term, ST^b, and long-term, LT^b market, as well as equity, E, and other assets, OA.

Table 3: Description of the Bank's Balance Sheet

Assets, A		**Liabilities,** L	
Overnight lending: federal funds, ON^l	Interbank lending	Overnight borrowing: federal funds, ON^b	Interbank borrowing
Short-term lending: federal securities, ST^l		Short-term borrowing: federal securities, ST^b	
Long-term lending: loans due from banks, LT^l		Long-term borrowing: loans due to banks, LT^b	
Cash and balance due, C		Other liabilities, OL	
Other assets, OA		**Equity,** E	

Notes: This description of a bank's balance sheet focuses on major bank lending and borrowing channels, i.e. overnight, short-term, and long-term markets. The rest of the balance sheet is condensed into cash or other assets and liabilities. The notations introduced here of the balance sheet will be used throughout this paper.
Source: Authors' model.

[3] Cash equivalents include Federal Reserve bank deposits and deposits held at other banks.

Bank lend and borrow decisions are based on individual preferences, which are a function of financial ratios derived from their balance sheets. Because a bank has several different lending and borrowing channels to select from, it uses a combination of ratios that, when maintained in unison, keep constant its interbank lending and borrowing preferences for overnight, short-term, and long-term debts (see Table 4). Additionally a bank uses the equity multiplier, the ratio of its total assets to its equity, to control its balance sheet for the degree of leverage desired.

Table 4: Bank Balance Sheet Ratio

Equity Multiplier	$\frac{E_i}{A_i}$
Overnight Lending, Borrowing	$\frac{ON_i^l}{A_i}$, $\frac{ON_i^b}{L_i}$
Short-term Lending, Borrowing	$\frac{ST_i^l}{A_i}$, $\frac{ST_i^b}{L_i}$
Long-term Lending, Borrowing	$\frac{LT_i^l}{A_i}$, $\frac{LT_i^b}{L_i}$

Notes: This table lists all the features of the balance sheet that bank i targets in determining how to allocate its lending and borrowing demand from period to period.
Source: Authors' model.

In each period, a bank evaluates its current ratio against its target ratios to determine how much it needs to lend and/or borrow. For example, if bank i's current overnight lending-to-asset ratio is lower than its target, it will want to find a borrower to lend to in the overnight market. Likewise, if bank i's current overnight borrowing-to-liability ratio is lower than its target, it will want to find a lender to borrow from in the overnight market. Once the bank reaches all its targets, it will no longer want to lend or borrow in any of the three markets.

There are two types of banks in the model, large and small, and they are differentiated in two ways. First, large and small banks have different balance-sheet characteristics and interbank lending practices that are important to capture in constructing their balance sheets, as discussed in Section 3.2.[4] Second, large banks are intermediaries for lending and borrowing, which makes them

[4]We split the data sample across large and small banks to ensure that when we parametrize the models through sampling the data is drawn from similar bank distributions.

attractive to banks looking for a correspondent. This will be discussed in the following section.

4.2 Interbank Market Activities

In each period, banks repay debts so they can make new lending and borrowing decisions that will evolve the interbank system over time. This procedure has four behaviors: 1) interbank lending-borrowing, 2) debt payments, 3) failed bank defaults, and 4) balance sheet updates.

4.2.1 Bank Lending-Borrowing

可参考用于交易主体行为分析的参数

If a bank needs to lend or borrow in ON^b, LT^b, or ST^b during a period, it goes through a scoring system to determine with whom to do this new activity. This procedure is done by assigning two scores to each bank: a size score, S^{size}, and a relationship score, S^{relation}. The size score is meant to capture the preference of banks to do business with larger banks with more assets. This is calculated here as a bank's assets less existing counterparties' average assets:

$$\begin{aligned} S^{\text{size}}_{i,j}(t) &= \log A_j(t) - \frac{\sum_{k,k\neq i} \log A_k(t-1)\mathbb{I}_{i,k}(t-1)}{\sum_{k,k\neq i} \mathbb{I}_{i,k}(t-1)}, \\ \mathbb{I}_{i,k}(t) &= \begin{cases} 1, \text{ If } i \text{ and } k \text{ are have a relationship at period } t \\ 0, \text{ Otherwise,} \end{cases} \end{aligned} \tag{1}$$

where $S^{\text{size}}_{i,j}(t)$ is the size score of bank j evaluated by bank i in period t, A_j is the total assets of bank j, and $\mathbb{I}_{i,k}(t)$ is a binary variable for keeping track of previous debt obligations. The relationship score captures a bank's tendency to keep existing relationships. In each model period, this score decreases according to a decaying function and increases if new debt is settled:

$$\begin{aligned} S^{\text{relation}}_{i,j}(t=0) &= \begin{cases} \log D_{i,j}(0), \text{ If } i \text{ and } j \text{ have initialized debts} \\ 0, \text{ Otherwise,} \end{cases} \\ S^{\text{relation}}_{i,j}(t>0) &= \begin{cases} S^{\text{relation}}_{i,j}(t-1) + \log D_{i,j}(t), \text{ If } i \text{ and } j \text{ set new debts} \\ (1-\eta) S^{\text{relation}}_{i,j}(t-1), \text{ Otherwise,} \end{cases} \end{aligned} \tag{2}$$

where $S^{\text{relation}}_{i,j}(t)$ is the relationship score of bank j evaluated by bank i, in period t, $D_{i,j}(t)$ is the new debt between bank i and bank j in period t, and η is the memory decaying parameter, which we set to a default value of 0.1. Finally, a bank uses the two scores in combination, $S^{\text{total}}(t)$, to rank from whom it wants to borrow.

Each bank, knowing its borrowing target, first sends one borrowing request at a time to each large bank in order to obtain its desired funding. If the bank's borrowing target is not fulfilled by the large banks, the bank will then send one request at a time to each small bank it has had a previous lending relationship with, in order of largest to smallest $S^{\text{relation}}_{i,j}(t)$. Finally if the targeted amount is still not fulfilled, the borrowing bank then contacts each small bank it has not contacted (i.e. not had a previous lending relationship with), in order of largest to smallest $S^{\text{size}}_{i,j}(t)$.[5] Once a bank has contacted all potential borrowers, and has not been able to fulfill its target, the bank may suffer a liquidity default on its balance sheet if it does not have enough equity.

When a bank receives a borrowing request, it must decide two things: 1) whether to provide new loans to requesting borrowers and 2) how much to lend. Two primary factors affect bank lending preferences. Each bank with space in its ON^l, LT^l, or ST^l follows a similar scoring system described in Equation 3 with respect to its potential borrowers. Accordingly, a bank chooses to lend by going through each request until its lending target is satisfied or there are no more requests to fill.[6]

$$S^{\text{total}}_{j,i}(t) = \omega S^{\text{relation}}_{j,i}(t) + (1-\omega) S^{\text{size}}_{j,i}(t), \tag{3}$$

and $S^{\text{total}}_{j,i}$ is the score that lender j assigns to borrower i. $S^{\text{total}}_{j,i}$ is the weighted average of the relationship score and size score of bank i. Equal weights are set to these scores ($\omega = 0.5$). However, a lending bank does not agree to every borrowing request, even if it has the capacity, and uses an S-shaped function, $p(S^{\text{total}}_{j,i})$, to assess the chance that lending bank j settles new debts to borrowing bank i, where

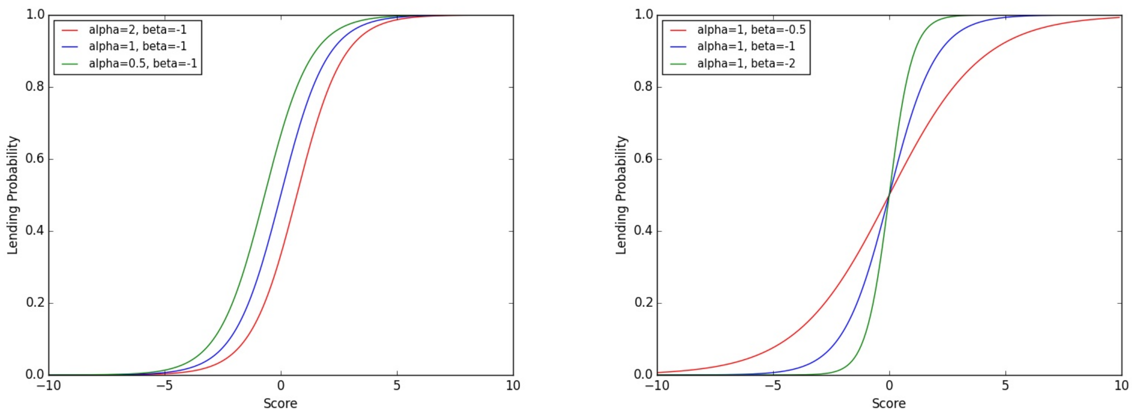
$$p(S^{\text{total}}_{j,i}(t)) = \frac{1}{1 + \alpha \times \exp\left(\beta \times S^{\text{total}}_{j,i}(t)\right)}; \tag{4}$$

where $p(S^{\text{total}}_{j,i})$ is the probability that i lends to j, and α and β are two parameters that control the intercept and slope, respectively. In this function, α is a positive real number. The larger the number, the lower probability of lending to a bank scoring 0 (see Figure 3a). To present

[5] During this process banks are selected (at random) to send a request to borrow from next available borrower, according to this preference algorithm.

[6] These scores evolve with time as the balance sheets of banks do.

different preferences of large banks and small banks, values are chosen from the uniform distribution $U(0.3, 0.5)$ for large banks and from the uniform distribution $U(0.9, 1.1)$ for small banks. This approach allows more lending from large banks to small banks. β is a negative real number, and the larger it is the slower the probability moves from 0 to 1 (see Figure 3b). In other words, a larger β means a tighter lending policy such that fewer borrowers get loans. Default values are chosen for banks from the uniform distribution $U(-1.1, -0.9)$.



(a) Sigmoid function sensitivity to α **(b)** Sigmoid function sensitivity to β

Figure 3: Lending Probability Determined by Sigmoid Function.
Source: Authors' analysis.

A lending bank follows a uniform distribution to determine the fraction it wants to lend from its available lending limit. The lower value between the one determined by the lending bank and requested by the borrowing bank is set as the new debt size.

4.2.2 Lending Repayment Rate

Banks make payments on their debts, P_{ON}, P_{ST}, and P_{LT}, at the beginning of each period and receive debt payments, R_{ON}, R_{ST}, and R_{LT}. Given that this paper's modeling period is based on quarterly data, the following assumption is made about repayment frequency. As overnight debts are paid daily, all lending agreements are reset each period. As most short-term loans are made for less than three months (Sheldon et al. (1998)), the majority of short-term lending is repaid within one period. A uniform distribution of $U(99\%, 100\%)$ is used to represent the percentage of payments to short-term debts. Long-term loans usually are repaid in less than one year, or four periods, so 75 percent of outstanding loans continue to exist. This paper uses $U(25\%, 100\%)$ to represent the percentage of payments to long-term debts.

4.2.3 Bank Failures and Defaults

A bank may run into a critical condition when it is not able to fulfill borrowing requests for liquidity needs or there is an exogenous impact on the banking system. The model sets bank failure conditions for defaults due to insolvency or liquidity. When the equity of bank i is negative, it faces solvency default (Equation 5). If the bank does not have enough assets to pay its debts at maturity, it will have a liquidity-driven default (Equation 6).

$$E_i(t) < 0, \tag{5}$$

$$C_i(t) < ON_i^p(t) + ST_i^p(t) + LT_i^p(t), \tag{6}$$

where $ON_i^p(t)$, $ST_i^p(t)$, and $LT_i^p(t)$ are bank i' payments of overnight borrowing, short-term borrowing, and long-term borrowing on period t.

When bank k fails at time t, it will default on its interbank borrowing. Bank k's lender i, will write down the assets and realize loss WD. The write-down percentage is associated with the type of lending. Overnight lending is not covered, so the full amount of the loan is counted as a loss. Short-term and long-term loans are collateralized so the lender writes down a percentage from a uniform distribution and realizes a loss in equity.

$$\begin{aligned} WD_{ON}(i,t) &= ON_{ik}^l(t-1), \\ WD_{ST}(i,t) &= U(0.0, 0.2) ST_{ik}^l(t-1), \\ WD_{LT}(i,t) &= U(0.0, 0.2) LT_{ik}^l(t-1), \end{aligned} \tag{7}$$

where $WD_{ON}(i,t)$, $WD_{ST}(i,t)$, $WD_{LT}(i,t)$ are bank i's total write-downs on period t, and $ON_{ik}^l(t)$, $ST_{ik}^l(t)$, and $LT_{ik}^l(t)$ are outstanding debts from bank k to bank i on period t.

4.2.4 Balance Sheet Updates

A bank evaluates current interbank lending and borrowing positions, calculates its net income, and updates its balance sheet at the end of each quarter or period. Activities in the overnight, short-term, and long-term markets are recorded according to the rules in defined in Table 5, and new values are updated in each period.

The income recognition process is simplified by estimating net income based on bank equity in each period. To calculate net income, an empirical distribution $Beta(17, 36, -0.1, 0.3)$ is used for the return-on-equity ratio (ROE) based on bank-reported data. In each simulation cycle, net income is derived from returns calculated by multiplying ROE (drawn from the empirical distribution) by the current equity. Net income will then be recognized as the most liquid assets – cash and balance due – on the balance sheet. A bank allocates the income according to its target ratios in the next period.[7]

With current balance sheet values from the previous period, the new balance sheet entries are updated for the next period by adding all the interbank lending and borrowing values and recognizing the net income in the current period.

[7] As part of the simplification, we skip the direct calculation of the interest earned from interbank lending for two reasons: a) for each period, income from interbank debts is a very small portion of bank profits, and b) a majority of the interest earned is captured in the periodic profits and losses through ROE.

Table 5: Balance Sheet Updates on Interbank Debts

	Lending	Borrowing	Paying	Receiving	Writing-down
Overnight	$ON^l + D^l_{ON}$ $C - D^l_{ON}$	$ON^b + D^b_{ON}$ $C + D^b_{ON}$	$ON^b - P_{ON}$ $C - P_{ON}$	$ON^l - R_{ON}$ $C + R_{ON}$	$ON^l - WD_{ON}$ $E - WD_{ON}$
Short-term	$ST^l + D^l_{ST}$ $C - D^l_{ST}$	$ST^b + D^b_{ST}$ $C + D^b_{ST}$	$ST^b - P_{ST}$ $C - P_{ST}$	$ST^l - R_{ST}$ $C + R_{ST}$	$ST^l - WD_{ST}$ $E - WD_{ST}$
Long-term	$LT^l + D^l_{LT}$ $C - D^l_{LT}$	$LT^b + D^b_{LT}$ $C + D^b_{LT}$	$LT^b - P_{LT}$ $C - P_{LT}$	$LT^l - R_{LT}$ $C + R_{LT}$	$LT^l - WD_{LT}$ $E - WD_{LT}$

Notes: This table shows balance sheet value changes from different market activities. D^l_{ON}, D^l_{ST}, and D^l_{LT} are new overnight, short-term, and long-term lending. D^b_{ON}, D^b_{ST}, and D^b_{LT} are new overnight, short-term, and long-term borrowing. P_{ON}, P_{ST}, P_{LT} are overnight, short-term, and long-term debt payments. R_{ON}, R_{ST}, R_{LT} are payments collected from overnight, short-term, and long-term lending. WD_{ON}, WD_{ST}, and WD_{LT} are write-downs of overnight, short-term, and long-term lending.
Source: Authors' model.

5 Model Validation

Validation exercises confirm that the model produces an interbank market resembling the real market based on individual bank decisions on lending and borrowing. The model is first validated based on bank balance sheet ratios and interbank lending network properties by comparing its results to those empirically observed using data from 2001 to 2006. Second the ABM methodology's performance is compared to other algorithmic methods in selecting network linkages and creating stylized facts. Lastly, the model's network topology features are compared to those observed in other papers.

5.1 Bank Balance Sheets Validation

Banks make lending and borrowing decisions based on many different factors, but this study focuses on two aspects: risk and behavior. Balance sheet information is used to measure bank decisions. Two ratios are used to indicate risk: the liquidity ratio and the leverage ratio. Another two ratios are defined to measure the interbank lending and borrowing behaviors. All four ratios are defined in equations (8, 9, 10, and 11)

$$\text{Leverage Ratio} = \frac{A_i}{E_i} \tag{8}$$

$$\text{Liquidity Ratio} = \frac{C_i}{A_i} \tag{9}$$

$$\text{Interbank Lending Ratio} = \frac{ON_i^L + LT_i^L + ST_i^L}{A_i} \tag{10}$$

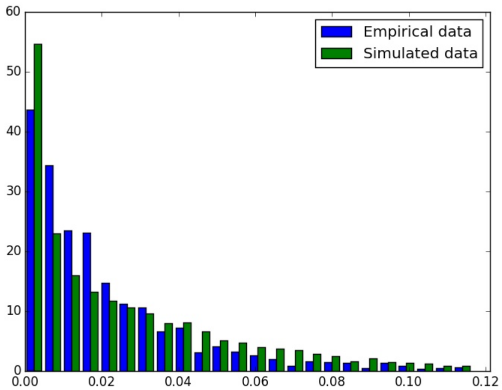
$$\text{Interbank Borrowing Ratio} = \frac{ON_i^B + LT_i^B + ST_i^B}{L_i} \tag{11}$$

Given these measures, the model is first initialized based on 2001 financial data. The distribution of the four selected ratios is validated according to simulation data of 20 quarters and empirical data from 2001 to 2006 (see Figure 4). A comparison of the distributions of the four observed versus simulated ratios shows that from a balance sheet perspective, the simulation closely resembles real bank lending and borrowing behaviors.

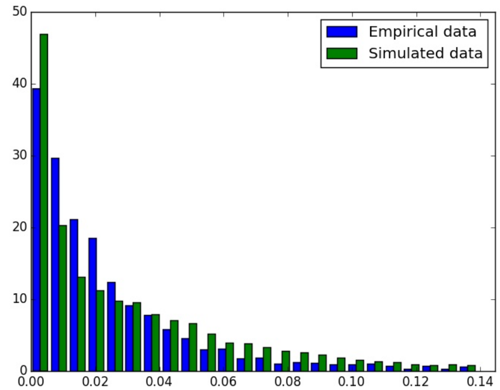
5.2 Comparison to Maximum Entropy and Minimum Density Algorithms

Interbank lending networks are generated based on an ABM that reflects lending and borrowing behaviors using FFIEC balance sheet data. The results are compared with two established interbank network reconstruction approaches from previous studies: Maximum Entropy (ME) and Minimum Density (MD) methods. Both set certain optimization rules inferring interbank exposures from observable marginals. However, as discussed earlier, the results generated from these two methods do not resemble the real market's network properties, and can only serve as lower and upper bounds.

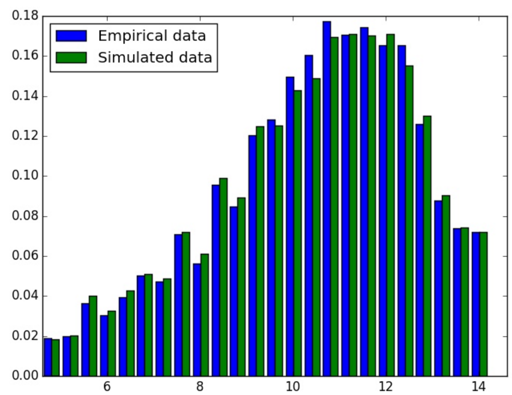
Following this paper's earlier methodology, repeated simulations are run with parameters based on 6,600 U.S. banks' financial data from 2001 to 2006. In each of the 30 simulations, the interbank network topology is initialized using the ME algorithm and the simulation model until the network properties stabilize at a steady state, allowing the calculation of the interbank network properties. The initial bank networks are also constructed using both ME and MD methods, and interbank properties computed. The three most robust measures of network topology are evaluated, i.e. degree distribution, clustering, and average path of the networks, generated by all three approaches.



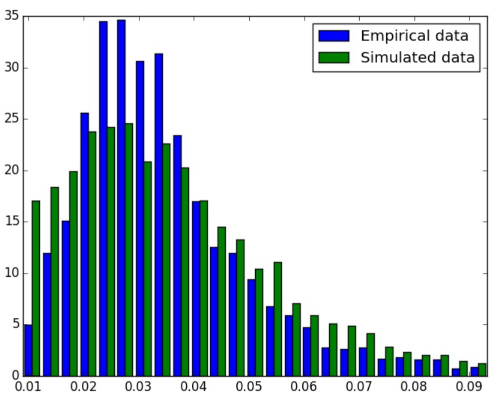
(a) Interbank Lending Ratio



(b) Interbank Borrowing Ratio



(c) Leverage Ratio



(d) Liquidity Ratio

Figure 4: Bank Categorization Through Asset Value.

Notes: The figure shows the comparison of ratio distribution (histogram) between the real bank financial data and the simulated results. The leverage ratio and the liquidity ratio are used as measures of bank risk. Interbank lending and borrowing ratios are used to quantify interbank lending practices.
Source: Federal Financial Institutions Examination Council Reports of Condition and Income; Authors' model.

Additionally, the power law exponent of the degree distribution is assessed to examine the characteristics of the reconstructed networks. Results show the model sits between the ME and MD methods (see Table 6).

The degree distribution presents differences of network connections more clearly. In networks generated by the model, the majority of agents create less than 10 links, and very few agents create as few as 1 or 2 links. This can be observed from the probability density function (PDF) in Figure 5. On the other hand, the ME method distributes interbank exposures so widely that the degree

Table 6: Comparison of Network Properties (average of 30 simulations)

	Average degree	Clustering coefficient	Power law	Average path
Maximum Entropy	476.73	0.80	2.31	1.93
Model	14.78	0.36	2.39	2.11
Minimum Density	2.71	0.02	3.14	4.89

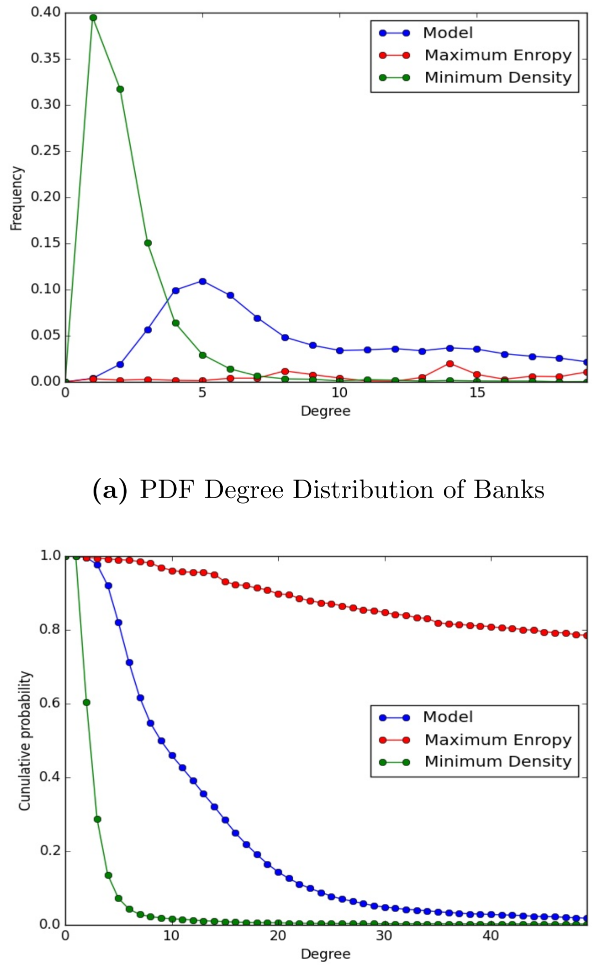
Notes: This table shows the network properties generated by the three methods for 6,600 U.S. banks. For the Maximum Entropy and Minimum Density methods, these properties are generated offline using bank balance sheets, and the average numbers are presented accordingly. For the ABM, these properties are based on the average of 30 simulations.
Source: Authors' calculations.

measure typically is meaningless. The MD method generates comparatively fewer links, which is evident from Figure 5. However, the average degree generated by MD is at the lower end with a value of 2.71, while the average degree generated by ME is at the higher end with a value of 476.73.

The clustering coefficient, the propensity of nodes to form cliques, is informative. The local clustering coefficient averages the probabilities that two neighboring nodes are connected (Jackson (2008)). The MD method produces a value of 0.02 and gives the appearance that local clustering cannot be found, meaning the MD method tends to generate star-like networks, while the ME method seems to be at the other extreme with a high number of links creating a nearly complete network. That suggests that both ME and MD methods fail to preserve local clustering. Our ABM produces a reasonable middle ground that is also close to results obtained in a study of the German interbank network (Anand et al. (2015)).

The average path – the average number of steps along the shortest paths for all possible pairs of network nodes – measures how efficient borrowers are at finding lenders through the network. Empirical studies find that the average path in interbank networks is between 2 and 3 in length (Boss et al. (2004), Bargigli et al. (2015)). The MD method generates a relatively large number (4.89), while the ME method is lower than the observed empirical range. The ABM here produces an average of 2.31, which is within the range documented for Austrian and German interbank networks.

Lastly the power law degree distribution exponent of the networks are generated by the three methods and a linear regression in a log-log plot of the cumulative distribution is used to obtain the



(a) PDF Degree Distribution of Banks

(b) CDF Degree Distribution of Banks

Figure 5: Comparison of Degree Distribution.

Notes: This figure displays the degree distribution in both probability density function (PDF) and cumulative density function (CDF). The degree distribution from the ABM is based on 30 simulations and is represented by the blue line. In the CDF plot, it sits between the Maximum Entropy and Minimum Density methods. *Source*: Authors' calculations.

power-law exponent (like the ones reported for the degree distribution) for the networks generated by the three methods. The ME method has a power law exponent of 2.31; the MD method, 3.14; the agent-based method, 2.19. The ME and agent-based methods produce values consistent with a scale-free network structure which range between 2 and 3 (Choromański et al. (2013)), while the MD appears on the high end. However, when additionally examining the logarithm degree distributions, in Figure 6, we find the ME method produces an interbank network closer to a complete network, and does not fit well to a constant power exponent.

Overall, as Anand et al. (2015) pointed out, the true network structure should lie between the results from the ME and MD methods. This experiment demonstrates the model produces a reasonable network structure that is well-bounded by the established ME and MD methods.

5.3 Network Properties Validation

A network properties comparison is performed between the simulated overnight lending market and the U.S. federal funds market.[8] Soramäki et al. (2007) evaluated 6,600 banks' transactions using 2006 Fedwire data and documented the empirical network structures. Here, 100 simulations are conducted with the same number of agents (see Table 7) and compared to the Soramäki et al. (2007) findings.

A comparison of two networks, based on the same set of statistics as in Section 5.2, shows a good overall match in the three average aggregate statistics. The clustering coefficient shows the weakest match, suggesting that the model may have a stronger propensity to form lending relationships between large and small banks than the real market does.

Table 7: U.S. Federal Funds Market Interbank Network Property Comparison

	Number of Nodes	Average Degree	Clustering Coefficient	Power Law
U.S. Federal Funds Market	6,600	15	0.53	2.15
Model (100 simulations)	6,600	14.78	0.36	2.39

Notes: This table lists the key network measures between the real U.S. federal funds market and the simulation results.
Source: Soramäki et al. (2007); Authors' calculations.

[8] This is the only U.S. lending market category that has had an empirical network analysis which the authors are aware.

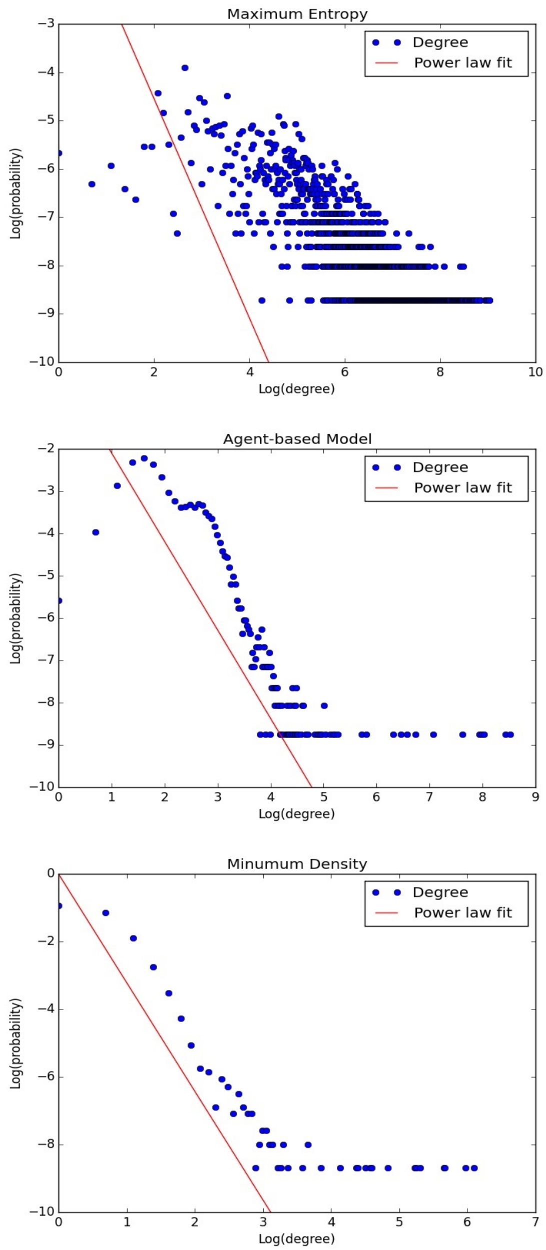


Figure 6: Comparison of Power Law Fit.

Notes: These figures show log-log power law fitting of the degree distribution of the interbank networks generated by the Maximum Entropy, Agent-based Model, and the Minimum Density methods.
Source: Authors' calculations.

6 Model Experiments

This section examines the informativeness of the ABM in replicating the impact of stress. A stress similar to the 2007-09 financial crisis is applied to bank balance sheets to see how effectively the model can match the number of the actual number of banks that failed. Post-crisis balance sheet data are then applied to compare how banks would fair. Lastly, this section examines the impact of contagion in bank failures brought on through either illiquidity in the interbank lending markets or insolvency due to write-downs of failed interbank loans.

6.1 The 2007-09 Financial Crisis

The ABM simulates the banking system dynamics and allows for the discovery of potential contagion of bank failures due to exogenous shocks. We run an experiment to replicate the 2007-09 financial crisis and show a simulated market response. From 2003 to mid-2007, banks increased their debt burden from rising home prices. When the housing bubble burst, it triggered a domino effect of bank defaults leading into the 2007-09 financial crisis.

Many stylized shocks have been developed to replicate the financial crisis. This simulation shocks the real estate lending part of the balance sheet, part of OA to trigger the model's systemic shock. To capture the impacts of falling home prices on bank loans, the 2006 ratio of real estate loans to other assets used. The simulation assigns real estate loans to each bank based on the distribution of empirical real estate loans. Exogenous shocks are triggered for 29 quarters, corresponding to the U.S. housing price drop from 2007 Q1 through 2014 Q1. The value of shocks is defined as the House Price Index (HPI) return.[9] At each time period, banks write down their real estate loans according to housing prices. For example, a 2 percent loss on the HPI triggers a shock of 2 percent in the model, with every bank recording a 2 percent loss on the housing loans on its balance sheet.[10]

The housing price shock experiment is applied over 30 model quarters, and each quarter the number of bank failures is recorded. The experiment is run 20 times and then plotted against real bank failures from 2007 Q2 to 2014 Q4 reported by the FFIEC (see Figure 7). The results show a sudden increase in bank failures during the housing price crash and a recovery period after 2011

[9] Source: Federal Housing Finance Agency

[10] We assume for simplicity of the model that banks are forced to reevaluate their real estate loans on a quarterly basis.

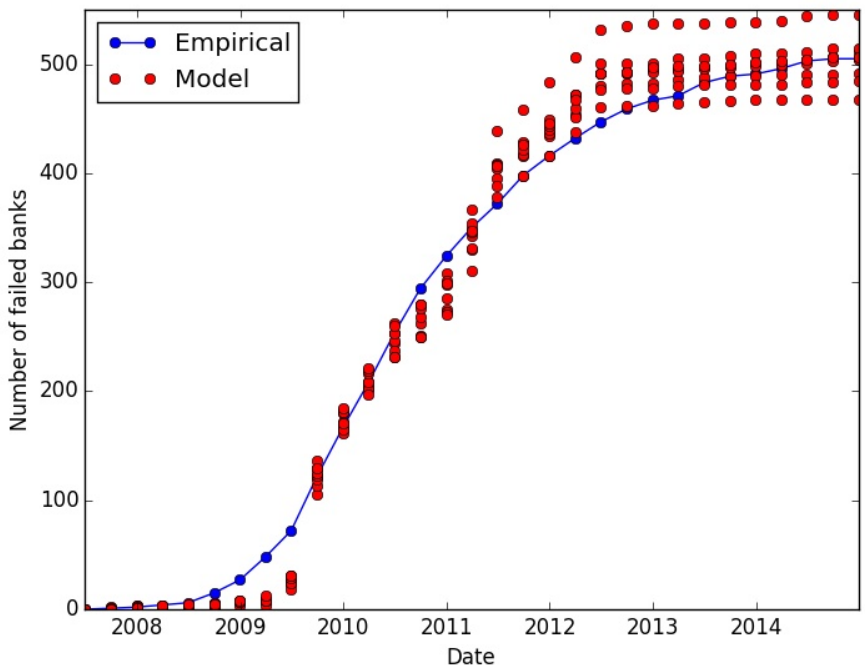


Figure 7: Validation of Failed Banks in the 2007-09 Financial Crisis.

Notes: This figure shows the simulation of bank contagion during the 2007-09 financial crisis. The blue line represents the cumulative number of bank failures from 2007 Q2 to 2014 Q4. The red dots represent the number of failed banks from 30 simulations.
Source: Federal Financial Institutions Examination Council Reports of Condition and Income; Authors' model.

that closely resembles the actual bank failures.

6.2 Pre-Crisis versus Post-Crisis Interbank Network Comparison

Bank balance sheets have changed since the financial crisis due to new policies and regulations, but it is unclear how these changes have impacted the robustness and resilience of the interbank system. Is the post-crisis banking system in better condition than the pre-crisis system? To answer that question, an experiment examines the impact on the interbank exposures and financial contagion in the post-crisis era.

The model is calibrated with bank financial data from March 2011 to December 2014. In addition to changes in interbank lending and borrowing ratios presented in Table 1, other ratios also shifted after the financial crisis (see Figure 8). In particular, the balance due from the Federal

Reserve Banks (FRB) is 10 times bigger than in 2001, and the overnight lending ratio has dropped 50 percent since the crisis. Overall, the FRB injected more liquidity into the system, and banks reduced their balance sheet ratios after the crisis. The latter should lead to a more robust and resilient interbank network structure. The recalibrated model follows the same bank decision rules and activity procedures as in the previous experiment, but with post-crisis data.

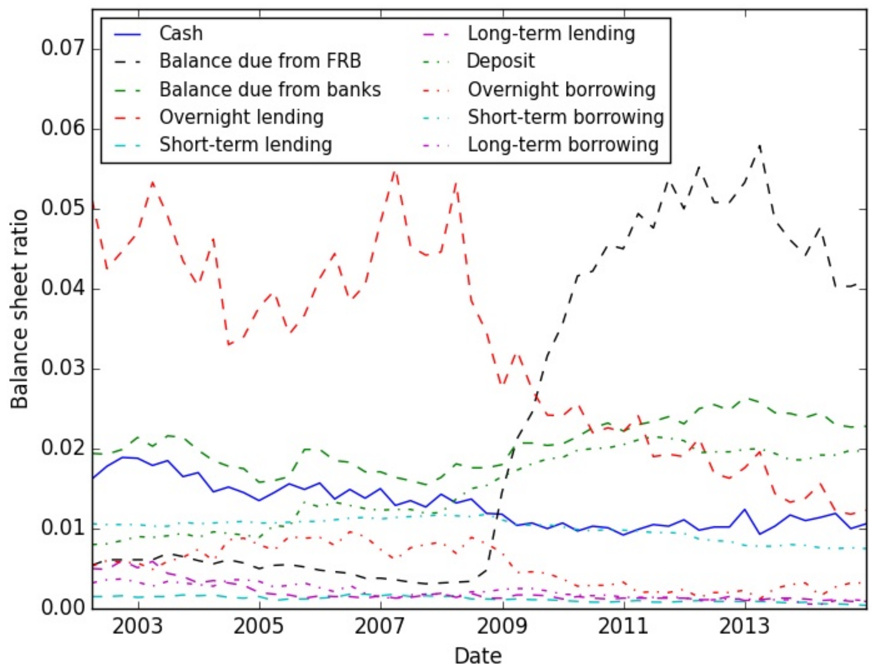


Figure 8: Average Balance Sheet Ratios (2002-14).

Notes: This figure shows average balance sheet ratios each quarter from 2002 to 2014.
Source: Federal Financial Institutions Examination Council Reports of Condition and Income.

In comparing the network properties between pre-crisis and post-crisis models, Table 8 shows a number of network topology changes in the overnight market but little in the short-term and long-term markets. The overnight network remains a scale-free network with similar power law exponents and the average path length. Both the average degree and clustering coefficient are reduced to one-third of the pre-crisis level, indicating a much lower number of interbank connections. Overall, the post-crisis overnight interbank network appears sparse compared to pre-crisis. For the short-term and long-term debt markets, the networks remain relatively sparse, but clustering coefficients increase a little, which indicates that although banks tend to have fewer connections, the tendency to form tightly knit groups has increased in the post-crisis era. Therefore, the post-crisis network structure reduces the chance of transmitting shocks to the rest of the system when one bank fails. However, the contagion may become more concentrated as the result of the sparse connections.

Table 8: Interbank Network Topology

	Average degree	Clustering coefficient	Power law	Average path
Overnight				
Pre-Crisis	14.78	0.36	2.39	2.11
Post-Crisis	5.33	0.13	2.45	3.09
Short-term				
Pre-Crisis	1.04	0.43	2.44	2.30
Post-Crisis	1.04	0.53	2.29	2.21
Long-term				
Pre-Crisis	2.42	0.40	2.14	2.44
Post-Crisis	2.42	0.57	2.15	2.28

Notes: The table presents the two balance sheet driven models of pre- and post-crisis banks using 6,600 representative U.S. banks. The Overnight market is the only one where a substantive difference in the network structure can be seen by looking at the four network statistics.
Source: Authors' model.

The shocks' impact in the post-crisis network showed the number of failed banks dropping from 500 to 370, a 25 percent decrease at a steady state (see Figure 9). This result is consistent with the network topology analysis that suggests higher stability in a post-crisis network. Both the pre-crisis and post-crisis bank failure curve share the same inflection point, yet the post-crisis failure slope is much smaller than the pre-crisis one. It shows that at the beginning of the contagion, the post-crisis shock transmission rate is higher than the pre-crisis scenario. Toward the end of the contagion, the post-crisis shock transmission rate is smaller than the pre-crisis scenario.

This shift in the bank failure pattern can be explained by the network topological changes in the post-crisis era. The post-crisis network has a concentration of exposures onto fewer links. This means that the greater loss transmitted by a given link is more likely to exceed the capital of the lending bank and cause its default. At the same time, the concentration effect is balanced by the fact that the scope of contagion is somewhat limited by the sparsity of the network, A lower number of linkages also reduces the channels allowing the propagation of losses. At the beginning of the contagion, the post-crisis network has more bank failures than the pre-crisis network. Toward the end of the contagion, the post-crisis network has fewer bank failures than the pre-crisis network. This is consistent with the observation of Allen and Gale (2000) that complete networks tend to have less contagion effects early, while incomplete networks generated higher contagion effects quickly. Overall, the post-crisis network is more resilient to the same types of shocks, the contagion rate is relatively mild and slower, and bank failures are reduced by 25 percent compared with the pre-crisis network.

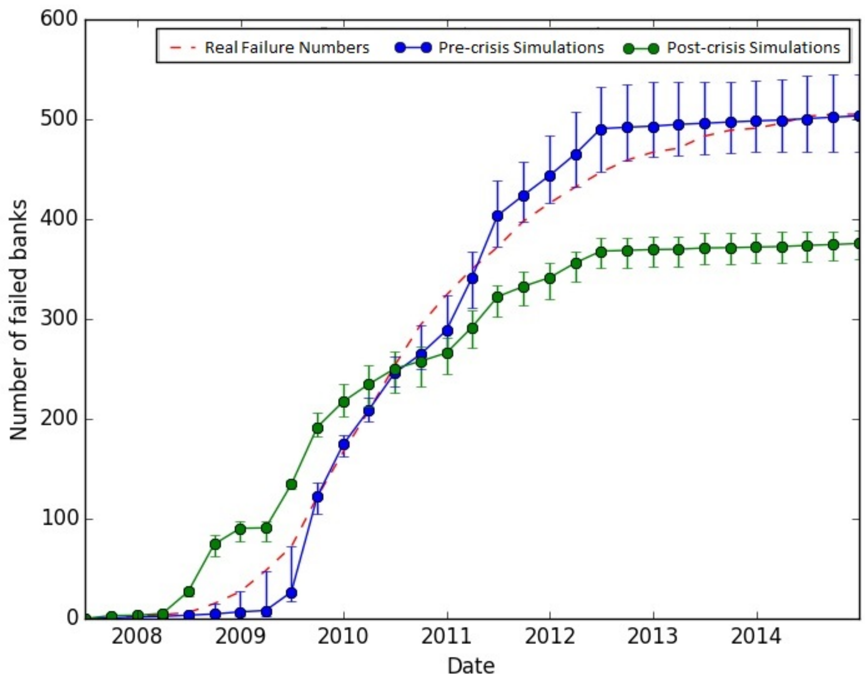


Figure 9: Number of Failed Banks.

Notes: The number of empirically observed failed U.S. banks during the 2007-09 financial crisis period vs. the average number bank failures obtained from model simulations using pre-crisis and post-crisis balance sheet data.
Source: Federal Financial Institutions Examination Council Reports of Condition and Income; Authors' model.

6.3 Interbank Network Contagion

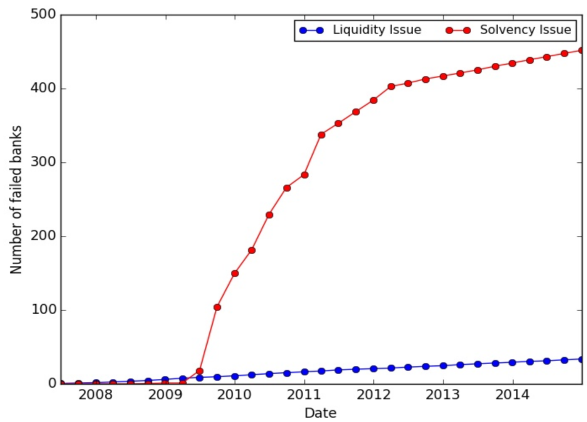
As mentioned earlier, bank failures can be caused by two types of risks in our model: insolvency risk and illiquidity risk. Real estate values, during 2007-09 are an example of the direct effects of the shock creating insolvency failures, due to the negative impact real estate had on many bank balance sheets. These failures could also trigger contagion-driven failures through the interbank lending network either through loan write-downs, which could cause further insolvency failures, or through the loss of funding availability causing illiquidity-driven failures.

The model produces an average number of bank failures of 505, with 465 due to insolvency and 40 due to illiquidity. These findings are in line with what is seen as the major driver of bank failures in 2007-09, real estate loans (Lee and Yom (2016)). They suggest that the direct effects of asset losses were a substantial part of the bank failures, and that there was substantive increase in demand for interbank borrowing. The imbalance of liquidity supply and demand created by the crisis causes further losses and creates sizable liquidity risk to the system (see Figure 10).

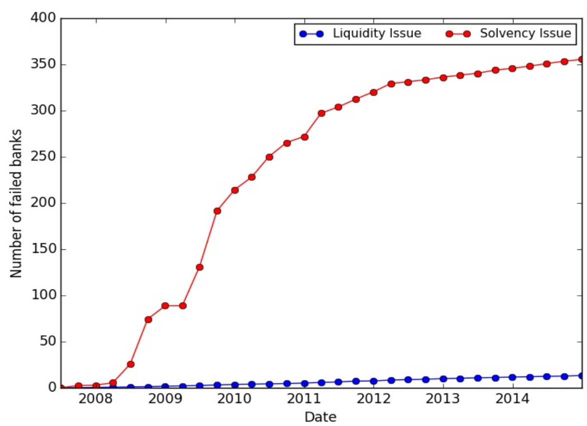
In comparing pre-crisis and post-crisis scenarios, the latter is found to have a smaller percentage of losses from illiquidity. This is in part due to the overall decrease in bank failures, seen in the post-crisis scenario, leaving the interbank lending market in a better condition to continue funding.

Lastly, the model examines the percentage of loss due to insolvency attributable to the direct shock of real estate losses versus indirect losses from interbank loan write-downs from bank failures. Figure 11 shows the portion of losses of failed banks in the before-crisis model run. The majority of failed bank losses are attributable to direct real estate losses, through the HPI shock. However, in the early quarters of the model runs, the portion of failed bank losses from write-downs is quite high (10-20 percent) as a share of early failed bank losses. This subsides as banks make new lending decisions and endogenously rearrange the network, reducing further contagion. Contagion is a significant cause of bank defaults only in the short term because banks will naturally reassess lending decisions and endogenously move away from poor performing loans, if given the opportunity.

The post-crisis scenario applied to the model shows near-zero losses from write-downs. That suggests limited contagion effects would result from the interbank lending market based on the decrease in overall interbank lending that has happened post-crisis.



(a) Pre-crisis



(b) Post-crisis

Figure 10: Number of Failed Banks by Liquidity Risk and Solvency Risk

Notes: The figure presents the cumulative number of bank failures we observe in the model under the pre-crisis and post-crisis scenarios, broken down by liquidity versus solvency risks.
Source: Authors' model.

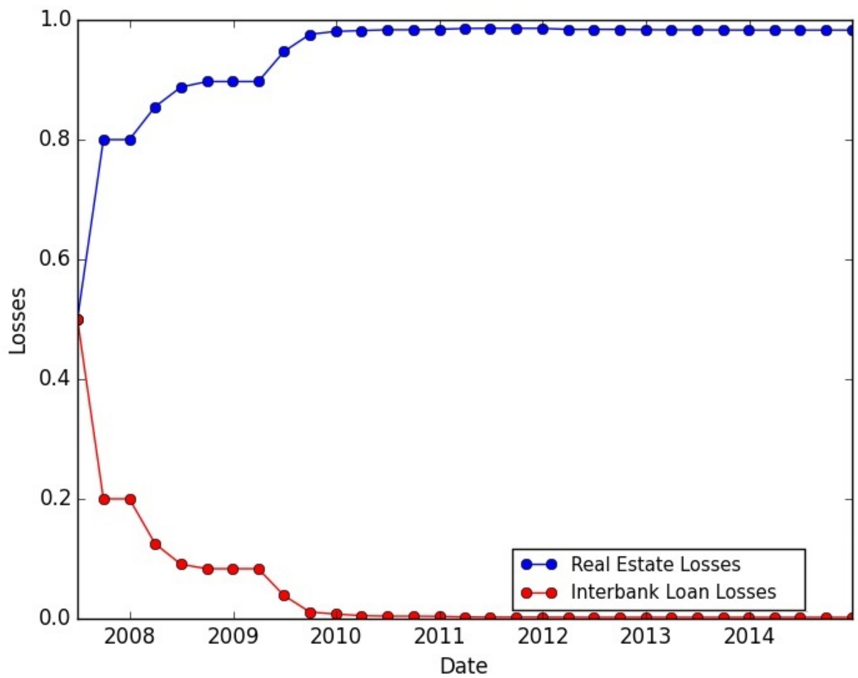


Figure 11: Pre-Crisis Failed Bank Losses From Real Estate, Interbank Loans

Notes: The figure presents the portion of average failed bank losses across time attributable to real estate losses and interbank loan losses.
Source: Authors' model.

7 Conclusion

This paper presents an agent-based approach, using bank balance sheet data, to model the U.S. interbank lending network. This dynamic model has embedded quarterly financial data reported to the FFIEC that reflects actual bank behaviors and performance-based decisions to endogenously reconstruct interbank networks.

We evaluated the model against two well-established interbank network reconstruction methods, the Maximum Entropy and Minimum Density algorithm approaches, and showed our agent-based method gives results closer to actual financial network structures. The resemblance to the real interbank networks is demonstrated through several network measures such as average degree, clustering coefficient, average path, and power distribution.

The model can show contagion risk while conditionally reformulating the network as bank failures occur. In one exercise, the model was calibrated to banks' balance sheets in the pre-crisis period of 2001-06, then correlated real estate loan loss shocks were added to the system. The model successfully replicated the market contagion and confirmed bank failure patterns during and after the 2007-09 crisis. A second exercise calibrated the model with post-crisis data from 2011-14, examined the network property differences, and compared the contagion effect with pre-crisis results. We find that in the post-crisis era, banks have less counterparty exposures as shown by a sparser network interbank structure than before the crisis. Furthermore, the post-crisis era network is more resilient to correlated asset write-down shocks and has fewer bank failures.

Overall, the methodology presented here is an alternative tool to better understand the contagion impact and network transitions in a bank network. The model provides a vehicle for bank regulators to stress test the interbank system by examining the severity of outcomes. It also could allow regulators to test new regulations and policies that either target or impose network structures, such as the Federal Reserve's recent proposal for single-counterparty credit limits (Federal Reserve System (2016)).

References

Acemoglu, D., Ozdaglar, A., and Tahbaz-Salehi, A. (2015a). Systemic risk and stability in financial networks. *American Economic Review*, 105(2):564–608.

Acemoglu, D., Ozdaglar, A. E., and Tahbaz-Salehi, A. (2015b). Systemic risk in endogenous financial networks. Working paper.

Acharya, V. V. (2009). A theory of systemic risk and design of prudential bank regulation. *Journal of Financial Stability*, 5(3):224–255.

Afonso, G., Kovner, A., and Schoar, A. (2011). Stressed, not frozen: The federal funds market in the financial crisis. *The Journal of Finance*, 66(4):1109–1139.

Afonso, G., Kovner, A., and Schoar, A. (2014). Trading partners in the interbank lending market. Federal Reserve Board of New York Staff Report 620.

Allen, F. and Gale, D. (2000). Financial contagion. *The Journal of Political Economy*, 108(1):1–33.

Anand, K., Bédard-Pagé, G., and Traclet, V. (2014). Stress testing the Canadian banking system: a system-wide approach. Working paper.

Anand, K., Craig, B., and Von Peter, G. (2015). Filling in the blanks: Network structure and interbank contagion. *Quantitative Finance*, 15(4):625–636.

Bala, V. and Goyal, S. (2000). A noncooperative model of network formation. *Econometrica*, 68(5):1181–1229.

Bank of Korea (2012). Financial stability report. Technical report.

Baral, P. and Fique, J. P. (2012). Estimation of bilateral exposures–a copula approach. Technical report, mimeo.

Bargigli, L., Di Iasio, G., Infante, L., Lillo, F., and Pierobon, F. (2015). The multiplex structure of interbank networks. *Quantitative Finance*, 15(4):673–691.

Battiston, S., Puliga, M., Kaushik, R., Tasca, P., and Caldarelli, G. (2012). Debtrank: Too central to fail? Financial networks, the Fed and systemic risk. *Scientific Reports*, 2.

Berrospide, J. M. (2012). Bank liquidity hoarding and the financial crisis: an empirical evaluation. Finance and Economics Discussion Series, Federal Reserve Board.

Bookstaber, R., Paddrik, M. E., and Tivnan, B. (2014). An agent-based model for financial vulnerability. OFR Working Paper 14-05, Office of Financial Research.

Boss, M., Elsinger, H., Summer, M., and Thurner 4, S. (2004). Network topology of the interbank market. *Quantitative Finance*, 4(6):677–684.

Choromański, K., Matuszak, M., and Mikisz, J. (2013). Scale-free graph with preferential attachment and evolving internal vertex structure. *Journal of Statistical Physics*, 151(6):1175–1183.

Cocco, J. F., Gomes, F. J., and Martins, N. C. (2009). Lending relationships in the interbank market. *Journal of Financial Intermediation*, 18(1):24–48.

Cont, R., Moussa, A., and Santos, E. B. (2013). Network Structure and Systemic Risk in Banking Systems. In *Handbook on Systemic Risk*, pages 327–368.

Craig, B. and Von Peter, G. (2014). Interbank tiering and money center banks. *Journal of Financial Intermediation*, 23(3):322–347.

Eisenberg, L. and Noe, T. (2001). Systemic risk in financial systems. *Management Science*, 47(2):236–249.

Elliott, M., Golub, B., and Jackson, M. O. (2014). Financial networks and contagion. *American Economic Review*, 104(10):3115–3153.

Elsinger, H., Lehar, A., and Summer, M. (2013). Network models and systemic risk assessment. *Handbook on Systemic Risk*, 1:287–305.

European Central Bank (2013). A macro stress-testing framework for bank solvency analysis. *Monthly Bulletin*, August.

Federal Reserve System (2016). Single-counterparty credit limits for large banking organizations. Technical Report 2016-05386.

Georg, C. P. (2013). The effect of the interbank network structure on contagion and common shocks. *Journal of Banking and Finance*, 37(7):2216–2228.

Gilbert, N. and Terna, P. (2000). How to build and use agent-based models in social science. *Mind & Society*, 1(1):57–72.

Glasserman, P. and Young, H. P. (2015). How likely is contagion in financial networks? *Journal of Banking & Finance*, 50:383–399.

Gofman, M. (2016). Efficiency and stability of a financial architecture with too-interconnected-to-fail institutions. *Journal of Financial Economics*. forthcoming.

Hałaj, G. and Kok, C. (2013). Assessing interbank contagion using simulated networks. *Computational Management Science*, 10(2-3):157–186.

Iori, G. and Gabbi, G. (2008). A network analysis of the Italian overnight money market. *Journal of Economic Dynamics and Control*, 32(1):259–278.

Iori, G., Jafarey, S., and Padilla, F. G. (2006). Systemic risk on the interbank market. *Journal of Economic Behavior & Organization*, 61(4):525–542.

Jackson, M. O. (2008). *Social and Economic Networks*. Princeton University Press.

Jackson, M. O. and Wolinsky, A. (1996). A strategic model of social and economic networks. *Journal of Economic Theory*, 71(1):44–74.

Kok, C. and Montagna, M. (2013). Multi-layered interbank model for assessing systemic risk.

Ladley, D. (2013). Contagion and risk-sharing on the inter-bank market. *Journal of Economic Dynamics and Control*, 37(7):1384–1400.

Lee, Y. Y. and Yom, C. (2016). The entry, performance, and risk profile of de novo banks. Federal Deposit Insurance Corporation Center for Financial Research Paper 2016-03.

Macal, C. M. and North, M. J. (2010). Tutorial on agent-based modelling and simulation. *Journal of Simulation*, 4(3):151–162.

Macy, M. W. and Willer, R. (2002). From factors to factors: Computational sociology and agent-based modeling. *Annual Review of Sociology*, 28(1):143–166.

Martinez-Jaramillo, S., Alexandrova-Kabadjova, B., Bravo-Benitez, B., and Solórzano-Margain, J. P. (2014). An empirical study of the Mexican banking systems network and its implications for systemic risk. *Journal of Economic Dynamics and Control*, 40:242–265.

Mastrandrea, R., Squartini, T., Fagiolo, G., and Garlaschelli, D. (2014). Enhanced reconstruction of weighted networks from strengths and degrees. *New Journal of Physics*, 16(4):043022.

Roukny, T., Georg, C.-P., and Battiston, S. (2014). A network analysis of the evolution of the German interbank market. Deutsche Bundesbank Discussion Paper 22.

Sheldon, G., Maurer, M., et al. (1998). Interbank lending and systemic risk: An empirical analysis for Switzerland. *Swiss Journal of Economics and Statistics (SJES)*, 134(IV):685–704.

Soramäki, K., Bech, M. L., Arnold, J., Glass, R. J., and Beyeler, W. E. (2007). The topology of interbank payment flows. *Physica A: Statistical Mechanics and its Applications*, 379(1):317–333.

Streit, R. E. and Borenstein, D. (2009). An agent-based simulation model for analyzing the governance of the Brazilian financial system. *Expert Systems with Applications*, 36(9):11489–11501.

Tarashev, N., Drehmann, M., et al. (2011). Measuring the systemic importance of interconnected banks. Working paper, Bank for International Settlements.

The Basel Committee on Banking Supervision (2015). Making supervisory stress tests more macroprudential: Considering liquidity and solvency interactions and systemic risk. Working Paper 29.

Upper, C. (2011). Simulation methods to assess the danger of contagion in interbank markets. *Journal of Financial Stability*, 7(3):111–125.

Upper, C. and Worms, A. (2004). Estimating bilateral exposures in the German interbank market: Is there a danger of contagion? *European Economic Review*, 48(4):827–849.

Wells, S. (2004). Financial interlinkages in the United Kingdom's interbank market and the risk of contagion. *Bank of England Quarterly Bulletin*, 44(3):331.