

# End-to-End Learned Random Walker for Seeded Image Segmentation



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## Overview

In this work we:

- Learn edge diffusivities for the Random Walker Algorithm [2].
- Compute the derivative for backpropagation analytically.
- Speed up the training by using a sparse sampling strategy.
- Show results in different microscopy domains.
- Obtain state-of-the-art seeded segmentation on CREMI [1] dataset.

## Mathematical Background

In the Random Walker Algorithm pixel assignments  $Z_U$  are computed by solving a discrete Dirichlet problem, whose boundary conditions are defined by seeds.

- Inference can be performed by solving

$$L_U Z_U = -B^T Z_M.$$

- Backpropagation can be expressed analytically as

$$L_U \frac{\partial Z_U}{\partial w} = -\frac{\partial L_U}{\partial w} Z_U - \frac{\partial B^T}{\partial w} Z_M.$$

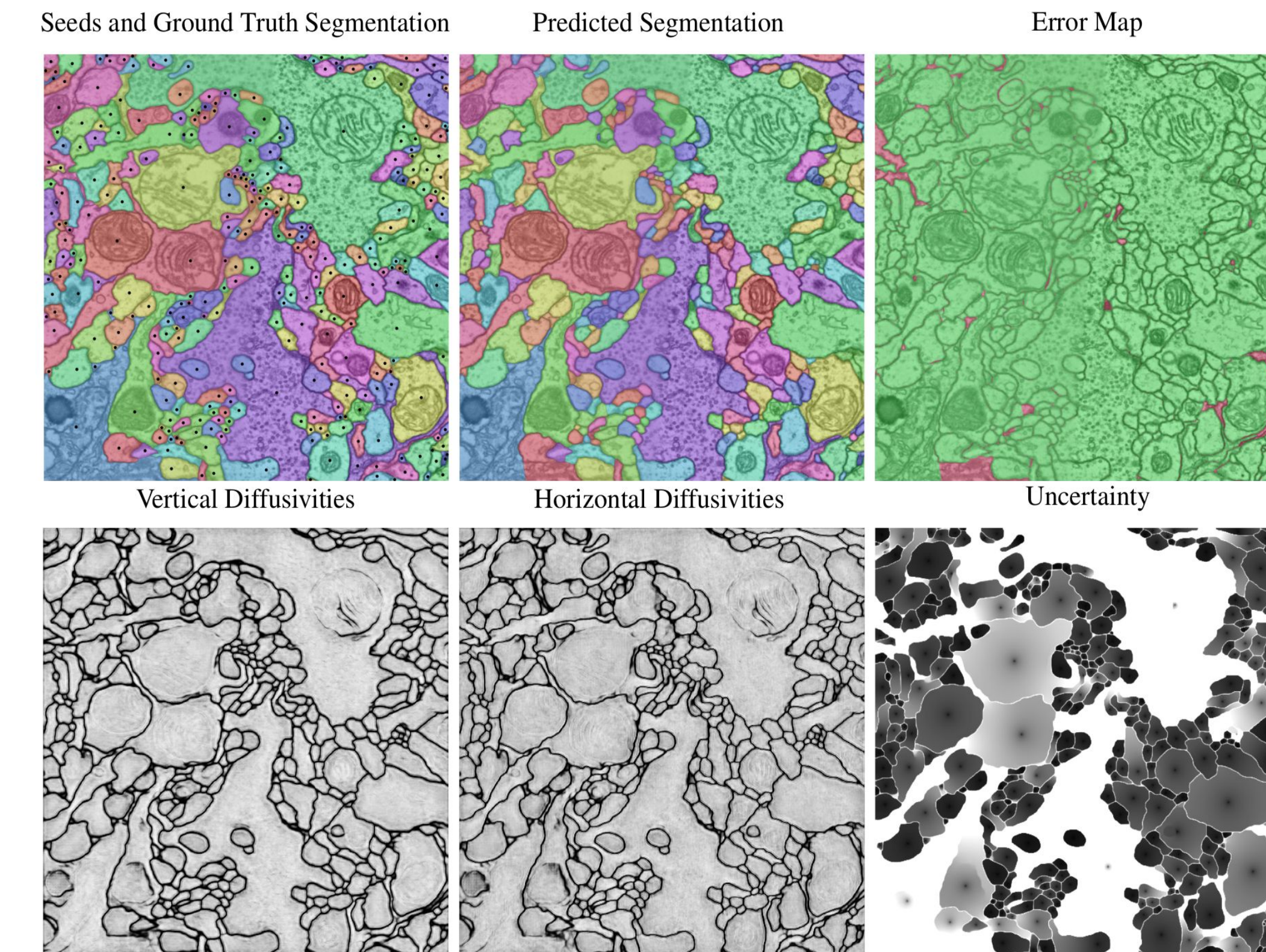
## Gradient Sampling & Pruning

The exact gradient requires the solution of a prohibitively large number of sparse linear systems of equations.

We simplify the computation by:

- Randomly select  $n$  edges for which we solve the corresponding linear systems.
- Compute the gradient for a single representative label per pixel,

$$\arg \max_a \left| \left( \frac{\partial l(Z_U^*, Z_U)}{\partial Z_U} \right)_{i,a} \right|.$$



## Results

We tested our approach on the CREMI [1] dataset, a segmentation challenge on Electron microscopy data. With oracle seeds, we obtained state of the art results.

The pipeline proved to be also competitive on Confocal Microscopy data.

Mathematical Notation:

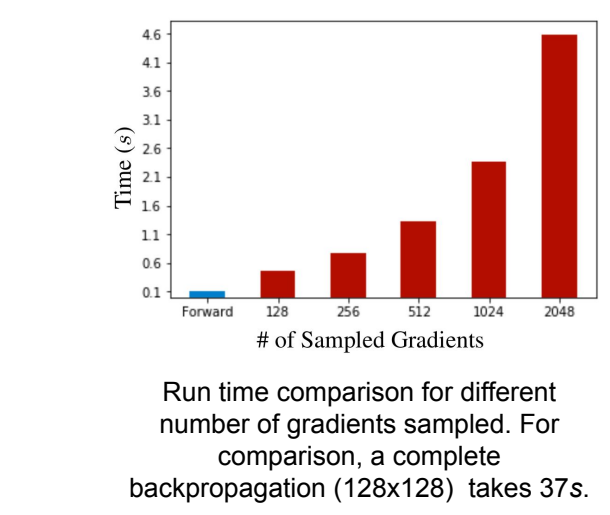
$Z_{i,a}$  = probability of vertex  $i$  having label  $a \in \mathcal{L}$

Laplacian:

$$L_{i,j} = \begin{cases} -w_{i,j} & \text{if } i \sim j \\ \sum_{k \sim i} w_{i,k} & \text{if } i = j \\ 0 & \text{else,} \end{cases} \quad L = \begin{pmatrix} L_M & B \\ B^T & L_U \end{pmatrix}$$

Complete Backpropagation:

$$\frac{\partial l(Z_U^*, Z_U)}{\partial \Theta} = \frac{\partial l(Z_U^*, Z_U)}{\partial Z_U} \frac{\partial Z_U}{\partial w} \frac{\partial w}{\partial \Theta}$$

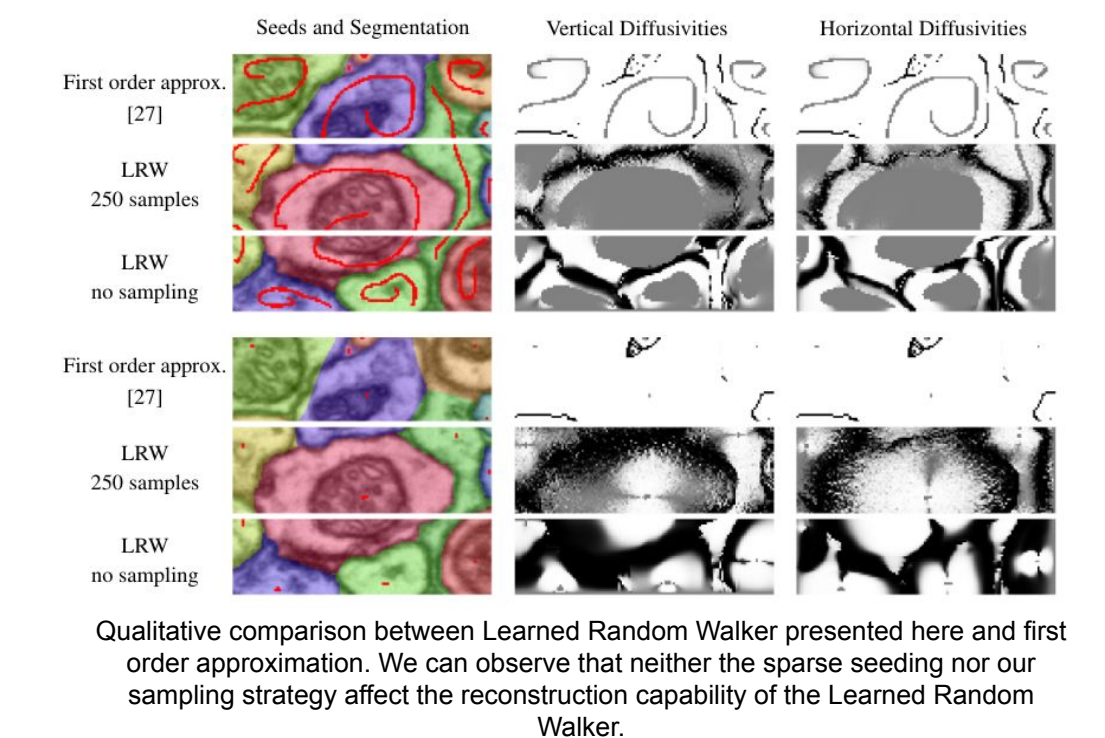


VOI	LRW with log barrier	LRW with side loss
CREMI A	0.076 ± 0.023	0.062 ± 0.021
CREMI B	0.220 ± 0.094	0.193 ± 0.089
CREMI C	0.272 ± 0.077	0.232 ± 0.081
Total	0.189 ± 0.109	0.162 ± 0.102

Quantitative comparison of the Learned Random Walker with log barrier and with side loss by looking at the means and standard deviations over the test set. Lower is better.

VOI	WS	LWS	RW	LRW
Cremi A	0.075 ± 0.024	—	0.177 ± 0.015	<b>0.062 ± 0.021</b>
Cremi B	0.211 ± 0.080	—	0.362 ± 0.086	<b>0.193 ± 0.089</b>
Cremi C	<b>0.209 ± 0.074</b>	—	0.421 ± 0.091	0.232 ± 0.081
Total	0.165 ± 0.091	0.376 ± 0.034	0.320 ± 0.127	<b>0.162 ± 0.102</b>

Quantitative comparison of Seeded Watershed on a good boundary probability map, Learned Watershed [3], Random Walker on a good boundary probability map and Learned Random Walker on the seeded CREMI [1] challenge. Lower is better.



Qualitative comparison between Learned Random Walker presented here and first order approximation. We can observe that neither the sparse seeding nor our sampling strategy affect the reconstruction capability of the Learned Random Walker.

## References

[1] CREMI. Miccai challenge on circuit reconstruction from electron microscopy images, 2017. <https://cremi.org>

## Acknowledgements

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[2] Leo Grady. Random walks for image segmentation. IEEE Trans. Pattern Anal. Mach. Intell. 2006.

[3] Steffen Wolf, Lukas Schott, Ulrich Kothe, and Fred A. Hamprecht. Learned watershed: End-to-end learning of seeded segmentation. ICCV 2017.

