

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

$$f(\hat{\beta}_0, \hat{\beta}_1) = \sum_{i=1}^n \varepsilon_i^2 : \text{mínima}$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \bar{X})(y_i - \bar{Y})}{\sum_{i=1}^n (x_i - \bar{X})^2} = \frac{\sum_{i=1}^n x_i y_i - n\bar{X}\bar{Y}}{\sum_{i=1}^n x_i^2 - n\bar{X}^2}$$

$$\hat{\beta}_0 \sim N\left(\beta_0; \sqrt{\sigma_\varepsilon^2 \left(\frac{1}{n} + \frac{\bar{X}^2}{\sum_{i=1}^n (x_i - \bar{X})^2}\right)}\right)$$

$$\hat{\beta}_1 \sim N\left(\beta_1; \sqrt{\frac{\sigma_\varepsilon^2}{\sum_{i=1}^n (x_i - \bar{X})^2}}\right)$$

$$Z = \frac{\hat{\beta}_0 - \beta_0}{\sqrt{\sigma_\varepsilon^2 \left(\frac{1}{n} + \frac{\bar{X}^2}{\sum_{i=1}^n (x_i - \bar{X})^2}\right)}} \sim N(0; 1)$$

$$Z = \frac{\hat{\beta}_1 - \beta_1}{\sqrt{\frac{\sigma_\varepsilon^2}{\sum_{i=1}^n (x_i - \bar{X})^2}}} \sim N(0; 1)$$

$$\underbrace{\sum_{i=1}^n (y_i - \bar{Y})^2}_{\text{Suma de Cuadrados Total (SCT)}} = \underbrace{\sum_{i=1}^n (\hat{y}_i - \bar{Y})^2}_{\text{Suma de Cuadrados Explicada (SCE)}} + \underbrace{\sum_{i=1}^n (y_i - \hat{y}_i)^2}_{\text{Suma de Cuadrados Residual (SCR)}}$$

$$SCE = \sum_{i=1}^n (\hat{y}_i - \bar{Y})^2 = \hat{\beta}_1^2 \sum_{i=1}^n (x_i - \bar{X})^2 = \hat{\beta}_1^2 \left(\sum_{i=1}^n x_i^2 - n\bar{X}^2 \right)$$

$$R^2 = \frac{SCE}{SCT} = \frac{\sum_{i=1}^n (\hat{y}_i - \bar{Y})^2}{\sum_{i=1}^n (y_i - \bar{Y})^2} = \frac{\hat{\beta}_1^2 \sum_{i=1}^n (x_i - \bar{X})^2}{\sum_{i=1}^n (y_i - \bar{Y})^2} = \hat{\beta}_1^2 \left(\frac{\sum_{i=1}^n x_i^2 - n\bar{X}^2}{\sum_{i=1}^n y_i^2 - n\bar{Y}^2} \right)$$

$$\hat{\sigma}_\varepsilon^2 = \frac{SCR}{n-2} = \frac{SCT - SCE}{n-2}$$

$$H_0: \beta_0 = 0$$

$$H_1: \beta_0 \neq 0$$

$$t^e = \frac{\hat{\beta}_0 - \beta_0}{\sqrt{\hat{\sigma}_\varepsilon^2 \left(\frac{1}{n} + \frac{\bar{X}^2}{\sum_{i=1}^n (x_i - \bar{X})^2} \right)}} \sim t_{n-2}$$

$$\hat{\beta}_0 \pm t_{1-\frac{\alpha}{2}; n-2} \sqrt{\hat{\sigma}_\varepsilon^2 \left(\frac{1}{n} + \frac{\bar{X}^2}{\sum_{i=1}^n (x_i - \bar{X})^2} \right)}$$

$$H_0: \beta_1 = 0$$

$$H_1: \beta_1 \neq 0$$

$$t^e = \frac{\hat{\beta}_1 - \beta_1}{\sqrt{\frac{\hat{\sigma}_\varepsilon^2}{\sum_{i=1}^n (x_i - \bar{X})^2}}} \sim t_{n-2}$$

$$\hat{\beta}_1 \pm t_{1-\frac{\alpha}{2}; n-2} \sqrt{\frac{\hat{\sigma}_\varepsilon^2}{\sum_{i=1}^n (x_i - \bar{X})^2}}$$

$$H_0: y_0 = 0$$

$$H_1: y_0 \neq 0$$

$$\hat{y}_0 = \hat{\beta}_0 + \hat{\beta}_1 x_0$$

$$\hat{y}_0 \sim N \left(y_0; \sqrt{\sigma_\varepsilon^2 \left(1 + \frac{1}{n} + \frac{(x_0 - \bar{X})^2}{\sum_{i=1}^n (x_i - \bar{X})^2} \right)} \right)$$

$$t^e = \frac{\hat{y}_0 - y_0}{\sqrt{\hat{\sigma}_\varepsilon^2 \left(1 + \frac{1}{n} + \frac{\bar{X}^2}{\sum_{i=1}^n (x_i - \bar{X})^2} \right)}} \sim t_{n-2}$$

$$\hat{y}_0 \pm t_{1-\frac{\alpha}{2}; n-2} \sqrt{\hat{\sigma}_\varepsilon^2 \left(1 + \frac{1}{n} + \frac{(x_0 - \bar{X})^2}{\sum_{i=1}^n (x_i - \bar{X})^2} \right)}$$