BASS DIFFUSION MODEL FOR NEW PRODUCT FORECASTING

<u>Ref</u>: "A new product growth model for consumer durables" by Frank Bass, 1969, Management Science Journal.

- <u>Product forecasting</u> is the science of predicting the degree of success a new product will enjoy in the marketplace. To achieve this, the forecasting model needs to consider product awareness, distribution of the product, price fulfilling market needs that other products unmet, and competitive advantages.

- Bass model is one way to forecast <u>new product sales and technology</u>. The Bass product diffusion model is a classic tool in <u>marketing science literature</u>. It has been used to predict the market shares of newly introduced products (also mature ones).

☒ <u>Key Features</u>:

- The Bass model is a useful tool for forecasting the adoption (first purchase) of an innovation (more generally, a new product) for which no closely competing alternatives exist in the marketplace. <u>Eg.</u> iPhone sales forecast.
- It embeds a "contagion process" to characterize the spread of word-of-mouth between those who have adopted the innovation and those who have not yet adopted the innovation.

☒ <u>Assumptions</u>:

- The model can forecast the long-term sales pattern of new technologies and new durable products under two types of conditions:

(a) The firm has recently introduced the product or technology and has observed its sales for a few time periods; or

(b) The firm has not yet introduced the product or technology, but its market behavior is likely to be similar to some existing products or technologies whose adoption pattern is known.

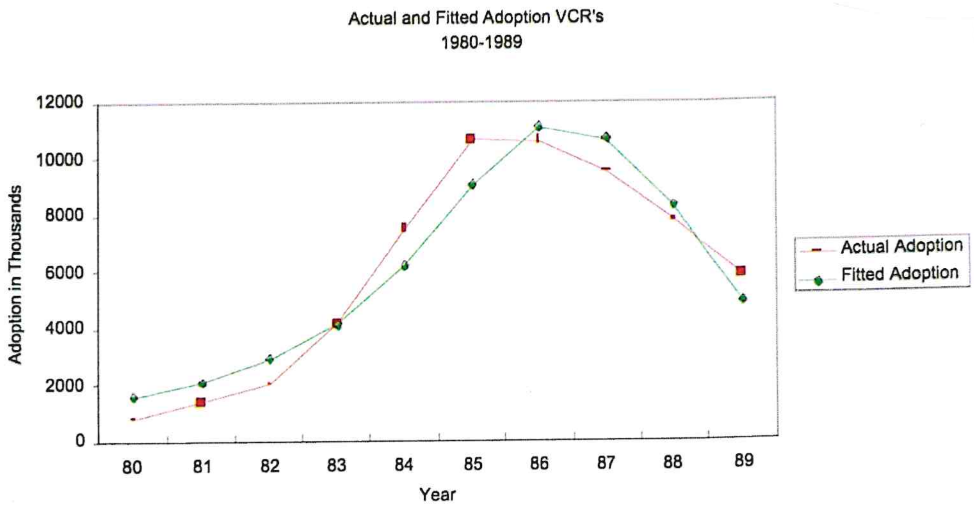
☒ R packages for easy implementation: "<u>diffusion</u>", "<u>DIMORA</u>".

The main idea of the model is that the adoption rate of a product comes from two sources:

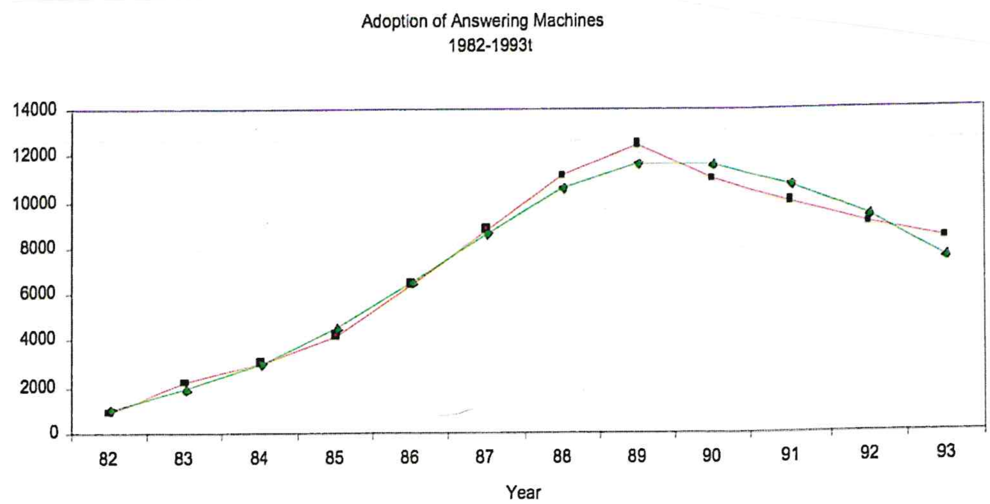
1. Propensity of consumers to adopt the product independent of social influences to do so. (INNOVATION)
2. The additional propensity to adopt the product because others have adopted it (influence of the early adopters becomes a driving force to adopt the product). This is also called network effect. (IMITATION)

- The Bass model shows how the information of the first few periods of sales data may be used to develop a fairly good forecast of future sales. The model can also be used to forecast the cashflows of a start-up (to determine its valuation) as well.

- Historical Examples:



— These are some classic example from the literature where the Bass model provided very good forecasts of the ramp up in product adoption as a function of the two resources described above.



— In practical session, we will work with iPhone sales data to make sales forecasts and finding sales peak.

Bass Model: Formulation

Define the cumulative probability of purchase of a product from time zero to time t by a single individual as $F(t)$. Then, the probability of purchase at time t is the density function denoted as $f(t) = F'(t)$.

The rate of purchase at time t, given no purchase so far, is given by

$$H(t) = \frac{f(t)}{1-F(t)}$$ (matches with the concept of hazard rate in survival analysis)

Modeling this is just like modeling the adoption rate of the product at a given time t.

Bass (1969) suggested that the adaption rate can be defined as

$$\frac{f(t)}{1-F(t)} = p + qF(t), \quad \text{---(1)}$$ (p: coefficient of innovation, q: coefficient of imitation)

where p is the independent rate of customer adopting the product and q as the imitation rate because it modulates the impact from the cumulative intensity of adoption, $F(t)$.

Hence, if we find p and q for a product, we can forecast its adoption over time, and thereby generate a time path of sales.

Solving the model for $F(t)$ and $f(t)$:

We write (1) as:

$$\frac{dF/dt}{1-F} = p + qF$$

and note that $F(0) = 0$.

$$\Rightarrow \frac{dF}{dt} = (p+qF)(1-F);$$

Integrating: $$\int \frac{1}{(p+qF)(1-F)}\, dF = \int dt = t + c_1 \quad \text{———— (2)}$$

We write
$$\frac{1}{(p+qF)(1-F)} = \frac{A}{p+qF} + \frac{B}{1-F}$$
$$= \frac{A+pB+F(qB-A)}{(p+qF)(1-F)}$$

This implies that $A+pB=1$ and $qB-A=0$.

Solving we get $A = \frac{q}{(p+q)}$ and $B = \frac{1}{(p+q)}$.

From (2), $$\int\left(\frac{q/(p+q)}{p+qF} + \frac{1/(p+q)}{1-F}\right)dF = t + c_1$$

$$\Rightarrow \frac{\ln(p+qF)}{p+q} - \frac{1}{p+q}\ln(1-F) = t + c_1$$

$$\Rightarrow \frac{\ln(p+qF)-\ln(1-F)}{p+q} = t + c_1 \quad \text{——— (3)}$$

At $t=0 \Rightarrow F(0)=0$ and $c_1 = \frac{\ln p}{p+q}$.

Substituting c_1 in (3), we solve for F as
$$F(t) = \frac{p\left(e^{(p+q)t}-1\right)}{pe^{(p+q)t}+q} = \frac{e^{(p+q)t}-1}{e^{(p+q)t}+\frac{q}{p}} \quad \text{——— (4)}$$

After differencing, we get
$$= \frac{1-e^{-(p+q)t}}{1+\frac{q}{p}e^{-(p+q)t}}.$$

$$f(t) = \frac{dF}{dt} = \frac{e^{(p+q)t}\, p(p+q)^2}{[pe^{(p+q)t}+q]^2} \quad \text{——— (5)}$$

Therefore, if the target market is of size m, then at each t, the adoptions are simply given by $m \times f(t) = s(t)$

NOTE: From (1), we can also write
$$\frac{dF}{dt} = p + (q-p)F - qF^2 \quad \text{——— (6)}$$

This is very similar to a Riccati Equation with constant coefficients:

ODE (Riccati): $y'(x) = q_0 x + q_1 x\, y(x) + q_2(x) y^2(x)$.

(1724)

Check: From (6): $f(t) = p + (q-p)F(t) - q(F(t))^2$.

Calibration: How do we get coefficients p and q?

Given we have the current sales history of the product, we can use it to fit the adoption curve.

- Sales in any period are: $s(t) = m f(t)$.
- Cumulative sales up to time t are: $S(t) = mF(t)$.

Substituting for $f(t)$ and $F(t)$ in the Bass equation (1) gives:

$$\frac{s(t)/m}{1 - S(t)/m} = p + q\,S(t)/m$$

$$\Rightarrow s(t) = \left[p + q\,S(t)/m\right]\left[m - S(t)\right].$$

Therefore, $s(t) = \beta_1 + \beta_2\, S(t) + \beta_3\, S(t)^2$ ——— (7) (Bass model)

with $\beta_1 = pm$, $\beta_2 = q - p$, and $\beta_3 = -\frac{q}{m}$.

Eqn. (7) is a regression equation with coefficients $\beta_0, \beta_1, \beta_2$ to be used to determine the values of m, p, q!

$$\beta_2 = q - p = -m\beta_3 - \frac{\beta_1}{m}$$

$$\Rightarrow \beta_3 m^2 + \beta_2 m + \beta_1 = 0$$

$$\Rightarrow m = \frac{-\beta_2 \pm \sqrt{\beta_2^2 - 4\beta_1\beta_3}}{2\beta_3} \quad \text{——— (8)}$$

and we can use m in (8) to solve for

$$p = \frac{\beta_1}{m} \quad \text{and} \quad q = -m\beta_3.$$

Sales Peak: It is easy to calculate the time at which adoptions will peak out. Differentiating $f(t)$ w.r.t. t and set to 0, i.e.,

$$t^* = \arg\max_t f(t)$$

which is equivalent to the solution to $f'(t) = 0$.

Differentiating (5) w.r.t. t, and solving for t, we get

$$t^* = \frac{-1}{p+q} \ln(p/q) = \frac{\ln q - \ln p}{p+q}.$$

[If negative, then sales has no peak]

Trading off p and q:

The peak formula is: $t^* = \dfrac{\ln q - \ln p}{p+q}$

$$= \frac{1}{p} \cdot \frac{\ln(q/p)}{(1+q/p)}$$

$$= \frac{1}{p} \cdot \frac{\ln x}{1+x}, \text{ taking } \frac{q}{p} = x.$$

Differentiating $\dfrac{\partial t^*}{\partial x} = \dfrac{1}{p}\left[\dfrac{1}{x(1+x)} - \dfrac{\ln x}{(1+x)^2}\right]$

$$= \frac{1+x-x\ln x}{px(1+x)^2}.$$

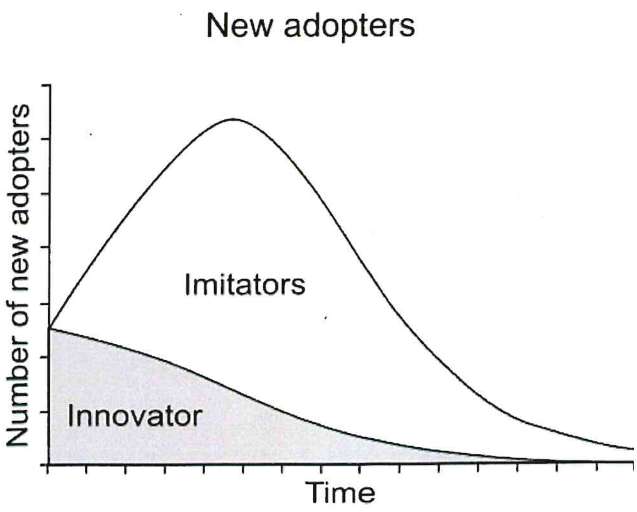
$q > p > 0$, otherwise t^* will be negative (periodic sales decline from launch with no peak). Therefore, the sign of $\dfrac{\partial t^*}{\partial x}$ is same as:

$$\text{sign}\left(\frac{\partial t^*}{\partial x}\right) = \text{sign}(1+x-x\ln x), \quad x > 1.$$

The above nonlinear equation has a root when $1+x-x\ln x = 0$
i.e., $x \simeq 3.59$.

Therefore, $\dfrac{dt^*}{dx} < 0$ when $x > 3.59$
> 0 when $x < 3.59$

Generalized Bass model paper suggests, if $p \in (0.01, 0.03)$ and $q \in (0.2, 0.4)$ then the discrete-time and continuous-time forecasts are very close.



Time to work on iPhone sales data.

Relationship with probability distributions:

Bass diffusion model in (5) is:

- When $q = 0$, reduces to exponential distribution.
- When $p = 0$, reduces to logistic distribution.

Generalized Bass Model (with pricing):-

Ref: Bass, F., Trichy, K. and Jain, D. (1994). Why the Bass model without decision variables, Marketing Science.

Bass found that his model fits the data without considering pricing and advertising variables (critical management decision variables). Generalized Bass model extend the model to the following equation:

$$\frac{f(t)}{1 - F(t)} = \left[p + qF(t) \right] x(t),$$

where $x(t)$ stands for current marketing effort (allowing the consideration of effort in the model).

However, both Bass model and Generalized Bass model come from a deterministic differential equation, extensions to stochastic case is done in literature. Bayesian inference in Bass model is done in "Boatwright and Kamakura (2003). Bayesian model for prelaunch sales forecasting of recorded music, Management Science.

- Limitations: The Bass model has been extensively applied for understanding how successful innovation have been diffused through the population. However, in forecasting contexts, most past data (from analogs) describe how successful innovations have diffused through the population, therefore, would produce favourable forecasts for any new product (success bias in the forecasts). Another limitation is that we can estimate model parameters well from data only after making several observations of the actual sales. However, by this time, the firm has already made critical investment decisions.