STATISTICAL PROCESS CONTROL

Product :- An article or substance that is manufactured or refined for sale.

Examples :- Automobiles, Refrigerators, music systems, computer, etc.

Services :- A system supplying a public need such as transport, communications, or utilities such as electricity and water.

Examples :- Public Transport system, banking, railways, etc.

Quality :- Definitions :-

⟨1⟩ Fitness for use & Conformance to specifications/requirements.

⟨2⟩ (ISO 9000 Quality Management System)
The totality of features and characteristics of a product or service that bears its ability to satisfy stated or implied needs.

⟨3⟩ (Modern or Japanese approach)
Quality is inversely proportional to variability. The best quality product or service is the one with minimum variation in the performance or the one which gives uniform performance.

⟨4⟩ (Taguchi's Definition)
Quality is the loss to the society caused by a product after being shipped. According to Taguchi the best quality product is the one which causes minimum loss to the society at any time, everytime, till the end of time.

Quality Improvement means continuously reduce variation. Irrespective of carefully maintained or correctly designed every process have a certain amount of natural or inherent variability caused by combined effect of many small, essentially unavoidable causes.

Objective of SQC :- Quickly detect the occurrence of assignable causes so that Corrective actions may be undertaken before Unacceptable products are manufactured.

Control charts :- An on-line process monitoring technique used for statistical process control. Eventual goal is elimination of variability in process. May not be possible to completely eliminate variability but Control charts are very effective in reducing variability.

Stable or in control process :- A process operating with only chance cause of variation.

A process operating in the presence of assignable causes is out of control or unstable.

Control Chart :-
- A tool to ensure that process is stable or in control.
- A tool to detect the presence of assignable causes in the process
- Graphical display of a quality characteristic that has been measured or computed from sample versus the sample number or time.

A graphical tool with three horizontal lines
1. Lower Control Limit (LCL)
2. Center Line (CL)
3. Upper Control Limit (UCL)

In Control Chart :- (Walter A Shewhart)
- Central line represents the average value of the characteristic corresponding to in control state.
- Control limits are chosen such that if the process is in control nearly all the sample points will fall between them.
- As long as the points plot within the control limits, the process is assumed to be in control and no action is necessary.
- A point that plots outside of the control limits is interpreted as evidence that the process is out of control.
- Generally the plotted points in a control chart are joined with straight line segments to easily visualize how the process has evolved over time.
- Even if all points plot inside the control limits, if there is a systematic or non-random pattern, that could be an indication of out of control.
- If the process is in control, all the plotted points essentially have a random pattern.

Types of Control Charts :-

Variable Control Chart :- Used for monitoring variable quality characteristic. Variable characteristics can be conveniently describe using a measure of central tendency & variability. These are called Variable Control charts.

Attribute Control Chart :- Used for monitoring attribute quality characteristics. When the product is judged as conforming or non-confirming to requirements or when the count of non-conformities appearing in a product or unit is considered. Control charts for such characteristics are called attribute control charts.

Major reasons for the popularity of control charts:-

1. **Improves productivity**: Reduces scrap and rework so productivity increases, cost decreases and production capacity increases.
2. **Prevents Defects**: Helps to keep the process in control indicating do it right the first time. It is cheaper to build it right initially than sort out good units from bad later.
3. **Prevent unnecessary process adjustments**: Distinguishes between natural and abnormal variation. Unnecessary adjustments can deteriorate the process performance.
4. Control charts improves the process. Generally process don't operate in a state of statistical control. Use of control charts will identify assignable causes. Eliminating the causes will reduce variability & will improve process.

Out-of-control-Action-Plan (OCAP):— A flow chart or document describing the sequence of activities to be undertaken once assignable causes are detected.

Choice of Control charts:— (Use Normal Distribution)

For Normal Distn. between $\mu \pm 1\sigma$: 68.26% of values will lie

$\mu \pm 2\sigma$: 95.46% " " " "

$\mu \pm 3\sigma$: 99.73% " " " "

If characteristic x is normally distributed with mean μ & s.d. σ then $P(\mu - L\sigma \le X \le \mu + L\sigma) = \alpha$.

So, we choose $L = 3$.

$$\therefore P(\mu - 3\sigma \le X \le \mu + 3\sigma) = 0.9973.$$

$UCL = \mu + 3\sigma$; $CL = \mu$; $LCL = \mu - 3\sigma$.

Some Useful Definitions:-

Estimate: A numerical value of an estimator.

Estimator: A statistic corresponding to the parameters.

Point Estimator: A statistic that produces a single numerical value as estimate for the unknown population parameter.

- Point estimator should be unbiased (the expected value of the estimator should be same as the parameter value) & should have minimum variance.

NOTE:- Sample mean (μ) is the unbiased estimator of population mean & sample variance (s^2) is the unbiased estimator of population variance. But sample standard deviation is not an unbiased estimator of population standard deviation.

$$E(\bar{x}) = \mu \quad , \quad E(s^2) = \sigma^2.$$

Random Sampling :- (i) Variation within the items in a subgroup will be maximum.

(ii) Variation between items in different subgroups will be minimum

Rational Sampling :- (i) Variation within the items in a subgroup will be minimum.

(ii) Variation between items in different subgroups will be maximum.

Individual x & Moving Range Chart (x – MR Charts)

- Control chart with subgroup size 1 ($n=1$).
- Sample consists of an individual unit only.

Uses :- 1. When automated inspection & measurement is used. Every unit manufactured is measured so basis for rational subgrouping.

2. When the production rate is very slow. The long interval between observations will cause problems with rational subgrouping.

3. When the variation within the subgroup is almost negligible. The repeat measurements differ only because of laboratory or measurement error.

4. Multiple measurements are taken on the same unit.

Requirements :- The quality characteristic must be normally distributed. The process variability is estimated using MR.

Along with individual x chart, generally a control chart for moving range is also constructed.

Moving Range :- The range between two successive observations

$$MR_i = |x_i - x_{i-1}|$$

For x chart :-

$$UCL = \mu + 3\sigma$$
$$CL = \mu$$
$$LCL = \mu - 3\sigma$$

where, $\hat{\mu} = \bar{x} = \dfrac{x_1 + x_2 + \cdots + x_m}{m}$, $\hat{\sigma} = \dfrac{\overline{MR}}{d_2}$,

$$\overline{MR} = \frac{MR_1 + MR_2 + \cdots + MR_m}{m}.$$

$\therefore$ For individual x chart, the limits are :

$$UCL = \bar{x} + \frac{3}{d_2}\overline{MR}$$
$$CL = \bar{x}$$
$$LCL = \bar{x} - \frac{3}{d_2}\overline{MR}$$, for $n=2$, $d_2 = 1.128$.

For MR chart :-

$$UCL = \overline{MR} + 3d_3\frac{\overline{MR}}{d_2} = D_4\overline{MR}$$
$$CL = \overline{MR} = \overline{MR}$$
$$LCL = \overline{MR} - 3d_3\frac{\overline{MR}}{d_2} = D_3\overline{MR}$$

where, $D_3 = \left(1 - \frac{3d_3}{d_2}\right)$ and $D_4 = \left(1 + \frac{3d_3}{d_2}\right)$.

$\overline{X}$ & R Chart

Suppose a quality characteristic is normally distributed with mean μ and standard deviation σ. If $x_1, x_2, \dots, x_n$ is a sample of size n then the sample mean $\overline{x} = \frac{x_1 + \cdots + x_n}{n}$ is also normally distributed with mean μ & standard deviation $\sigma/\sqrt{n}$.

Methodology :- 1. Collect a sample of size m (m is at least 20 to 25).

2. Each sample contain n observations of quality characteristic (typically n is small 4, 5 or 6). n is called sub-group size.

3. Let $\overline{x}_1, \overline{x}_2, \dots, \overline{x}_m$ be the subgroup averages.

4. Let $R_1, R_2, \dots, R_m$ be the subgroup ranges.

5. The $\overline{X}$ chart is for subgroup averages.

6. The R chart is for subgroup ranges.

For $\overline{X}$ chart :-

$$UCL = \mu + \frac{3\sigma}{\sqrt{n}} = \overline{\overline{x}} + \frac{3}{d_2\sqrt{n}}\overline{R} = \overline{\overline{x}} + A_2\overline{R}$$
$$CL = \mu = \overline{\overline{x}} = \overline{\overline{x}}$$
$$LCL = \mu - \frac{3\sigma}{\sqrt{n}} = \overline{\overline{x}} - \frac{3}{d_2\sqrt{n}}\overline{R} = \overline{\overline{x}} - A_2\overline{R}$$

where, $\overline{\overline{x}}$ is the unbiased estimator of μ, given by

$$\overline{\overline{x}} = \frac{\overline{x}_1 + \overline{x}_2 + \cdots + \overline{x}_m}{m}.$$

Average Range is given by $\overline{R}$, $\overline{R} = \frac{R_1 + \cdots + R_m}{m}$.

Relative Range $W = \frac{R}{\sigma}$ and $E(W) = d_2$.

An unbiased estimator of σ is given by $\frac{\overline{R}}{d_2}$.

Also, $A_2 = \frac{3}{d_2\sqrt{n}}$ is available for different sample sizes in table of control chart constants.

For R chart :- Relative range $W = \frac{R}{\sigma}$ has mean $E(W) = d_2$, $Var(W) = d_3$.

Then $\hat{\mu}_R = \bar{R}$, $\sigma_R = d_3 \sigma$, $\hat{\sigma}_R = d_3 \cdot \frac{\bar{R}}{d_2}$ is an unbiased estimator of σ_R.

$$UCL = \hat{\mu}_R + 3\hat{\sigma}_R = \bar{R} + 3\, d_3 \cdot \frac{\bar{R}}{d_2} = D_4 \bar{R}$$

$$CL = \hat{\mu}_R = \bar{R} = \bar{R}$$

$$LCL = \hat{\mu}_R - 3\hat{\sigma}_R = \bar{R} - 3\, d_3 \cdot \frac{\bar{R}}{d_2} = D_3 \bar{R}$$

where, $D_3 = \left(1 - \frac{3d_3}{d_2}\right)$ and $D_4 = \left(1 + \frac{3d_3}{d_2}\right)$ are tabulated for different values of n.

$\bar{X}$ & s Chart

When subgroup size n is moderately large (say $n > 10$ or 12), Range may not be a good measure of variation. It is desirable to estimate variation using standard deviation.

Sample variance s^2 is an unbiased estimator of popl^n var. σ^2.

where, $s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$

We also have $E(s) = c_4 \sigma$, c_4 is a constant depends on 'n'.

$V(s) = \sigma^2(1 - c_4^2)$, $\sigma_s = \sigma\sqrt{1 - c_4^2}$.

For $\bar{X}$ chart:-

$$LCL = \hat{\mu} - \frac{3\hat{\sigma}}{\sqrt{n}} = \bar{\bar{x}} - \frac{3}{c_4\sqrt{n}}\bar{s} = \bar{\bar{x}} - A_3\bar{s}$$

$$CL = \hat{\mu} = \bar{\bar{x}} = \bar{\bar{x}}$$

$$UCL = \hat{\mu} + 3\frac{\hat{\sigma}}{\sqrt{n}} = \bar{\bar{x}} + \frac{3}{c_4\sqrt{n}}\bar{s} = \bar{\bar{x}} + A_3\bar{s}$$

where, $\hat{\mu} = \bar{\bar{x}} = \frac{\bar{x}_1 + \cdots + \bar{x}_m}{m}$ is an unbiased estimator of μ,

$\hat{\sigma} = \frac{\bar{s}}{c_4}$, where $\bar{s} = \frac{s_1 + s_2 + \cdots + s_m}{m}$, is an unbiased estimator of σ.

Also, $A_3 = \frac{3}{c_4\sqrt{n}}$ is available for different sample sizes in the table of control chart constants.

For s chart:- Estimate of mean, $\hat{\mu}_s = \bar{s}$

Standard deviation of s, $\sigma_s = \sigma\sqrt{1-c_4^2}$

$\therefore \hat{\sigma} = \dfrac{\bar{s}}{c_4}$ is an unbiased estimator of σ.

$$LCL = \hat{\mu}_s - 3\hat{\sigma}_s = \bar{s} - \frac{3\bar{s}}{c_4}\sqrt{1-c_4^2} = B_3\bar{s}$$

$$CL = \hat{\mu}_s = \bar{s} = \bar{s}$$

$$UCL = \hat{\mu}_s + 3\hat{\sigma}_s = \bar{s} + \frac{3\bar{s}}{c_4}\sqrt{1-c_4^2} = B_4\bar{s}$$

where, $B_3 = \left(1 - \frac{3}{c_4}\sqrt{1-c_4^2}\right)$ & $B_4 = \left(1 + \frac{3}{c_4}\sqrt{1-c_4^2}\right)$.

Note:- **For $\bar{X}$ & R chart:-** Process mean $= \bar{\bar{x}}$

Process s.d. $= \dfrac{\bar{R}}{d_2}$

For $\bar{X}$ & S chart:-

Process mean $= \bar{\bar{x}}$

Process SD $= \dfrac{\bar{s}}{c_4}$

For X & MR chart:- Process mean $= \bar{x}$

Process SD $= \dfrac{\overline{MR}}{d_2}$

Scrap $= P(X \leq LSL)$; Rework $= P(X \geq USL)$

Non-conforming = Scrap + Rework.

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Control charts for Attributes

- Many cases quality characteristic are not numeric.

Ex:- Classification of each item inspected as either confirming or non-conforming (defectives) to the specifications or requirements.

- **Types of Control charts for Attributes:-**

1. Control charts for nonconforming units (defectives)
2. Control charts for nonconformities (defects)

i.e. 1. Number of defectives chart (np chart)
Control charts for fraction non-conforming (p-chart)
: fraction defective charts.

2. Number of defects (c chart)
Defects per unit chart (u chart)

Usage of 1:- np chart is generally used when the subgroup size n is constant.
p chart is used when the subgroup size n is varying from sample to sample.

Usage of 2:- c chart is generally used when the subgroup size n (total inspected) is constant.
u chart is used when the subgroup size n is varying from sample to sample.

- **Control charts for Number of Defectives : np chart**

Used when subgroup size is constant.
Based on Binomial Distribution.
Number of Defectives are plotted on the chart.
If a random sample of n units of a product is selected and if D is the number of units of product that are non conforming, then D has a binomial distribution with parameters n and p, $\hat{p} = \frac{D}{n}$.

$$E(D) = np, \quad V(D) = np(1-p),$$

$$SD(D) = \sqrt{np(1-p)}.$$

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Control limits are:- $LCL = \mu - 3\sigma = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$

$$CL = \mu = n\bar{p}$$

$$UCL = \mu + 3\sigma = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

Estimate of $p = \hat{p} = \bar{p} = \sum_{i=1}^{m} D_i / mn$, where m is the number of samples.

- Control charts for Fraction Defectives : p chart

Used when subgroup size n is not constant.
Based on Binomial distribution.
Fraction of defectives are plotted on the chart.
If a random sample of n units of a product is selected and if D is the number of units of product that are non-conforming, then D has a binomial distribution with parameter n & p.

$$\hat{p} = \frac{D}{n}.$$

$$E(\hat{p}) = \frac{E(D)}{n} = \frac{np}{n} = p$$

$$V(\hat{p}) = V\left(\frac{D}{n}\right) = \frac{1}{n^2} \cdot np(1-p) = \frac{p(1-p)}{n}.$$

$$SD(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}$$

Control limits are:-

$$LCL = \mu - 3\sigma = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$CL = \mu = \bar{p}$$

$$UCL = \mu + 3\sigma = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}},$$

Estimate of $p = \hat{p} = \bar{p} = \sum_{i=1}^{m} D_i \Big/ \sum_{i=1}^{m} n_i$, where m is the number of samples.

- **Control charts for Number of Defects : c chart**

Used when subgroup size is constant.
Based on Poisson Distribution.
Number of defects are plotted on the chart.
If a random sample of size n units of a product is selected and if X is the number of non-conformities, then X has a poisson distribution with parameter c.

$$E(X) = c$$
$$V(X) = c$$
$$SD(X) = \sqrt{c}.$$

Control limits are:-
$$LCL = \mu - 3\sigma = \bar{c} - 3\sqrt{\bar{c}}$$
$$CL = \mu = \bar{c}$$
$$UCL = \mu + 3\sigma = \bar{c} + 3\sqrt{\bar{c}}$$

Estimate of $c = \hat{c} = \bar{c} = \sum_{i=1}^{m} x_i / m$, where m is the number of samples.

- **Control charts for Defects per unit : u chart**

Used when sample size is not constant.
Based on Poisson Distribution.
Defects per unit (X/n) are plotted on the chart.
If a random sample of n units of a product is selected and if X is the number of non-conformities, then X has a Poisson Distribution with parameter c.
Nonconformities per unit (X/n) is denoted by u.
Estimate of $u = \hat{u} = \bar{u} = \sum_{i=1}^{m} X_i \Big/ \sum_{i=1}^{m} n_i$, where m is the number of samples.

$$E\left(\frac{X}{n}\right) = \frac{c}{n} = u.$$
$$V\left(\frac{X}{n}\right) = \frac{1}{n^2}.c = \frac{u}{n}.$$
$$SD\left(\frac{X}{n}\right) = \sqrt{\frac{u}{n}}$$

Control limits are:-
$$UCL = \mu + 3\sigma = \bar{u} + 3\sqrt{\frac{\bar{u}}{n_i}}$$
$$CL = \mu = \bar{u}$$
$$LCL = \mu - 3\sigma = \bar{u} - 3\sqrt{\frac{\bar{u}}{n_i}}.$$

- Some more out of control cases:-

1. 9 consecutive values are in one side of center line.
2. 6 consecutive values are steadily increasing or decreasing.
3. 2 out of 3 values > 2SD from center line (same side)
4. 4 out of 5 values > 1SD from center line (same side).

Note:- The basic methods of SPC and capability analysis have been in use for over 50 years.
The basic methods of SPC are called Shewhart Control Charts.
Motivated by the success of basic techniques, increased emphasis on

— variability reduction,
— yield enhancement,
— process improvement,

lead to development of many new techniques for SPC.

- Disadvantages of Shewhart Control Charts:-

1. At any point of time, the decision is made only based on the last point plotted on the chart.
2. Generally ignores the information given by the entire sequence of plotted points.
3. This makes Shewhart control charts relatively insensitive to small shifts in the process — on order of 1.5σ or less.

Alternatives to Shewhart Control charts are:—

- Cumulative sum (CUSUM) control charts
- Exponentially weighted moving average (EWMA) control charts

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STATISTICAL PROCESS CONTROL

Definition of SPC :-

A powerful collection of problem solving tools useful for achieving process stability and reducing variability.

Two types of Variation:-

1. Chance cause of variation:-

- Variations of small magnitude
- Difficult to identify
- Difficult to eliminate
- Integral part of the process
- known as natural or allowable cause of variation.

2. Assignable cause of variation :-

- Variations of large magnitude
- Represents an unacceptable level of process performance
- Known as special cause of variation
- possible to identify -
- possible to eliminate.

[Att:- Prob. of occurance is very low but it appears.]

SPC

$\overline{X}$-Rchart Exercise

Q. Xbar - Rchart : Example : –

The table below presents 9 subgroups of four measurements on inside diameter (ID) of a part processed in a turning machine ? Set up Xbar and R charts on this process. Verify that the process is in statistical control ?

Sample No.	Hour	X_1	X_2	X_3	X_4	Mean $\overline{x}$	Range R
1	8.00	5.00	5.01	4.98	5.00	4.998	0.03
2	9.00	5.01	4.98	5.00	5.00	4.998	0.03
3	10.00	5.02	5.01	5.00	5.00	5.008	0.02
4	11.00	5.00	5.00	5.00	5.00	5.00	0.00
5	12.00	4.98	4.98	5.01	4.99	4.990	0.03
6	13.00	5.02	4.99	5.00	4.98	4.998	0.04
7	14.00	4.99	4.99	4.98	4.98	4.985	0.01
8	15.00	5.00	5.01	5.02	5.00	5.008	0.02
9	16.00	4.98	5.00	5.01	4.98	4.993	0.03

Process: Turning

Sample Size (N) : 9

Here $\overline{\overline{x}} = 4.997$

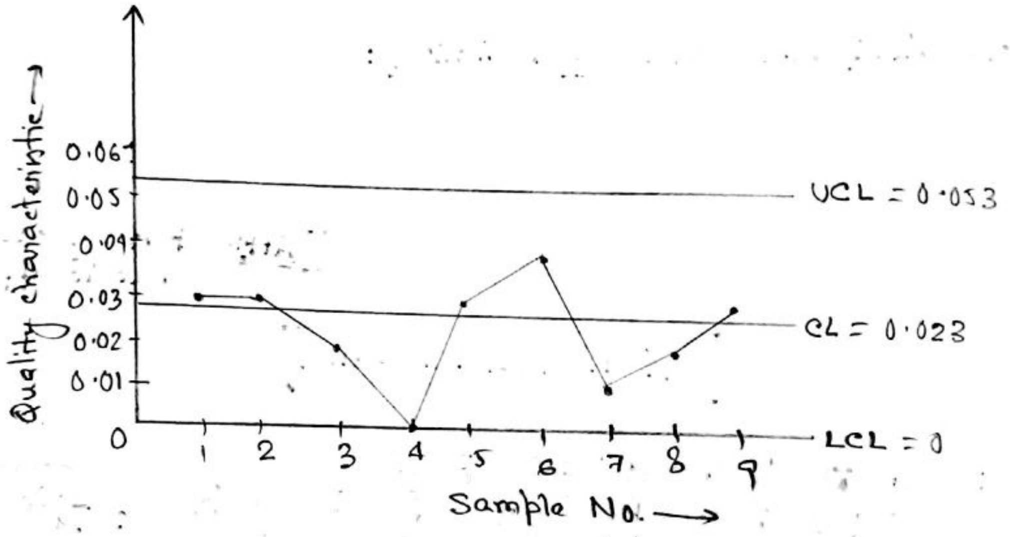
$\overline{R} = 0.023$

Characteristic: Diameter

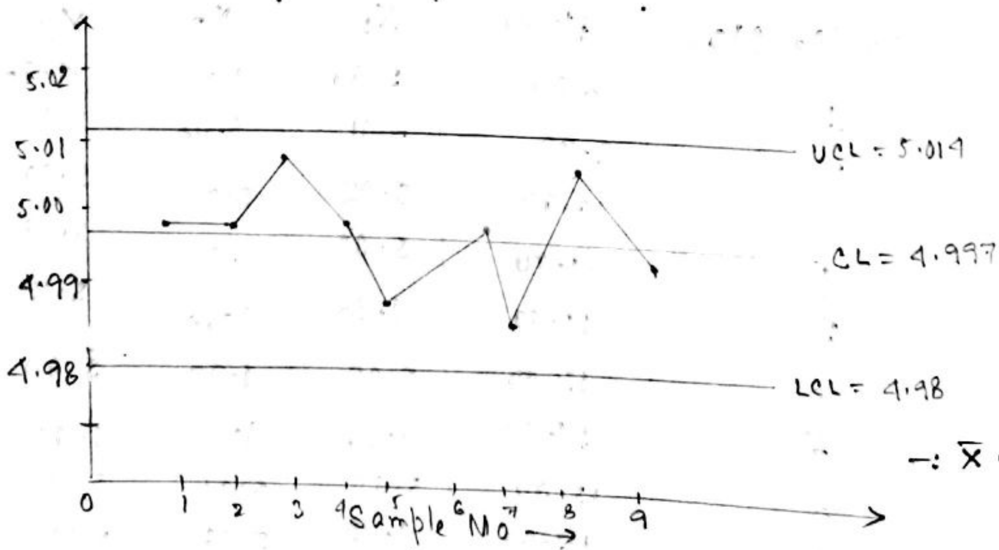
Subgroup Size (n) : 4

R chart:- $UCL = D_4\overline{R} = 0.023 \times 2.282 \quad, n=4.$
$= 0.053$
$CL = \overline{R} = 0.023$
$LCL = D_3\overline{R} = 0, \text{ since } D_3 = 0$

$\overline{X}$ Chart:-
$LCL = \overline{\overline{x}} - A_2\overline{R} = 4.98$
$CL = \overline{\overline{x}} = 4.997$
$UCL = \overline{\overline{x}} + A_2\overline{R}, \text{ since } A_2 = 0.729$
$= 5.014$



-: Graph of R- chart :-



-: $\overline{X}$ chart :-

Since, All the points in $\overline{X}$ & R chart lie within control limits, so the process is in control.

N.P.:- Control charts control stability but does not control characteristic.

$\bar{X}$-R Chart Exercise

Q. Sample of size $n=6$ items are taken from a manufacturing process at regular intervals. A quality characteristic is measured and $\bar{X}$ and R value are calculated for each sample. After 50 samples, we have

$$\sum_{i=1}^{50} \bar{x}_i = 2000 \quad \text{and} \quad \sum_{i=1}^{50} R_i = 200.$$

Assume that the quality characteristic is normally distributed.

(a) Compute control limits for the $\bar{X}$ & R control charts.

(b) Assume both charts exhibit control. Estimate the process mean & s.d.

(c) If the specification limits are 41 ± 5.0. What are your conclusions regarding the ability of the process to produce items within these specification.

(d) Assuming that if an item exceeds upper specification limit it can be reworked and if it is below lower specification limit it must be scraped. What is the % of scrap & rework?

<u>Solution</u>:- (a) <u>Control chart of R chart</u>:- $\bar{R} = \frac{200}{50} = 4$, $n=6$.

So, $D_3 = 0$, $D_4 = 2.004$.

So, $UCL = D_4\bar{R} = 4 \times 2.004 = 8.016$

$CL = 4$

$LCL = 0$

<u>Control chart for $\bar{X}$ chart</u>:- $\bar{\bar{x}} = \frac{2000}{50} = 40$, $\bar{R} = 4$.

$A_2 = 0.483$.

$UCL = \bar{\bar{x}} + \bar{R}A_2 = 40 + 4 \times 0.483 = 41.932$

$CL = 40$

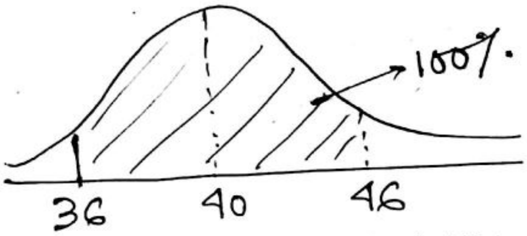
$LCL = 40 - 1.932 = 38.068$

(b) Process Mean $= \bar{\bar{x}} = 40$.

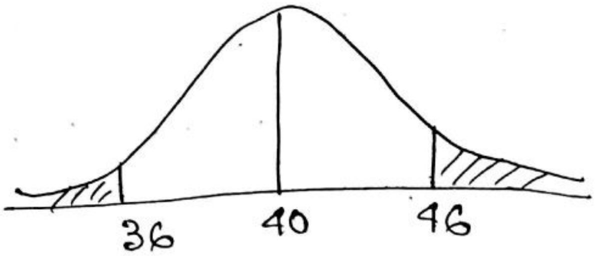
Process SD $= \frac{\bar{R}}{d_2} = 1.578$

(c) $USL = 46$

$LSL = 36$



If the area is 100% then we meet 100% customer satisfaction.



If there is some area gap, then it's not 100% satisfactory for customer.

P(the process produces items within the specifications)

$= P(36 \leq X \leq 46) = P(X \leq 46) - P(X \leq 36)$

$= P\left(\frac{X-\mu}{\sigma} \leq \frac{46-40}{1.578}\right) - P\left(\frac{X-\mu}{\sigma} \leq \frac{36-40}{1.578}\right)$

$= P(Z \leq 3.8023) - P(Z \leq -2.534)$

$= 0.9993 - 0.0057$

$= 0.99423$

i.e. 99.42% are under the limit of specifications.

(d) Scrap $= P(X \leq 36) = 0.57\%$

Rework $= 1 - P(X \leq 46) = 0.007\%$.

$\overline{X}$ - s chart Exercise

Q. Samples of $n=4$ items are taken from a manufacturing process of regular intervals. A normally distributed quality characteristic is measured and $\overline{X}$ & s values are calculated from each sample. After 50 subgroups have been analysed, we have

$$\sum_{i=1}^{50} \overline{x}_i = 1000, \quad \sum_{i=1}^{50} S_i = 72$$

(a) Compute the control limits for the $\overline{X}$ & S control charts.

(b) Assume that all points on both the control charts plot within the control limits, estimate the process mean & s.d..

(c) If the specification limits are 19 ± 4.0. Estimate the fraction non-conforming.

(d) Assume that if an item exceeds the USL it can be reworked & if it is below LSL it must be scrapped, then what's % of scrap & rework ?.

(e) If the process is centred at $\mu = 19$, what'd be the effect on % scrap & rework.

Solution:- (a) $\overline{\overline{x}} = \frac{1000}{50} = 20$, $\overline{S} = \frac{72}{50} = 1.44$

For S - chart:- $n=4$, $B_4 = 2.266$, $B_3 = 0$

$$UCL = B_4 \overline{S} = 3.2313$$
$$CL = 1.44$$
$$LCL = 0$$

For $\overline{X}$ - chart:- $n=4$, $A_3 = 1.628$

$$UCL = \overline{\overline{x}} + A_3 \overline{S} = 22.344$$
$$CL = \overline{\overline{x}} = 20$$
$$LCL = \overline{\overline{x}} - A_3 \overline{S} = 17.655$$

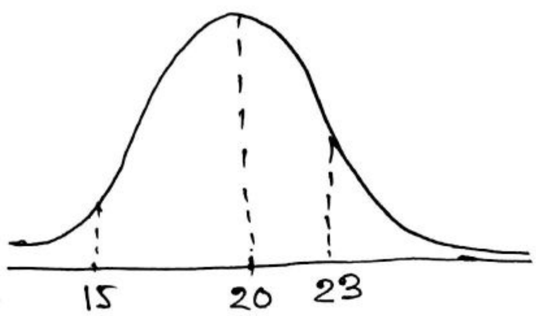
(b) Process mean, $\hat{\mu} = \overline{\overline{x}} = 20$

$$SD(\hat{\sigma}) = \frac{\overline{S}}{c_4} = 1.563$$

(c)

USL = 23

LSL = 15



$$P(15 \le x \le 23) = P(X \le 23) - P(X \le 15)$$
$$= P\left(\frac{x-\mu}{\sigma} \le \frac{23-\mu}{\sigma}\right) - P\left(\frac{x-\mu}{\sigma} \le \frac{15-\mu}{\sigma}\right)$$
$$= P\left(Z \le \frac{23-20}{1.563}\right) - P\left(Z \le \frac{15-20}{1.563}\right)$$
$$= P(Z \le 1.919) - P(Z \le -3.198)$$
$$= 0.9725 - .00071$$

(d) Scrap $= P(x \le LSL) = 0.00071$

Rework $= 1 - P(X \le 23) = 0.0275$

$\therefore$ Non-conforming = Scrap + Rework

$= 0.02828$

(e) If $\mu = 19$, the $P(X \le 23) = P\left(Z \le \frac{23-19}{1.563}\right)$

$= P(Z \le 2.55918)$

$= 0.99477$

$\therefore$ Rework $= 1 - 0.99477 = .00523$, i.e. $.523\%$

$$P(X \ge 15) = P\left(Z \ge \frac{15-19}{1.563}\right) = P(Z \ge -2.56)$$
$$= 0.00523$$

Scrap $= 0.00523$ i.e. 0.523%

$\therefore$ Non-conforming $= 1.04\%$

Individual X & MR chart

Q. The viscosity of a polymer is measured hourly. Measurements for the last 20 hours are shown as follows:

Test	Viscosity	MR	Test	Viscosity	MR
1	2838		11	3174	304
2	2785	53	12	3102	72
3	3058	273	13	2762	340
4	3064	6	14	2975	213
5	2996	68	15	2719	256
6	2882	114	16	2861	142
7	2878	4	17	2797	64
8	2920	42	18	3078	281
9	3050	130	19	2964	114
10	2870	180	20	2805	159

(a) Set up a control chart on viscosity and a moving range chart. Does the process exhibit statistical control.

(b) Estimate the process mean & standard deviation.

(c) The next five measurements on viscosity are: 3163, 3199, 3054, 3147 and 3156. Do these measurements indicate the process is in statistical control.

Solution:- (a) $\overline{MR} = \frac{2815}{19} = 148.157$

For $n=2$, $d_2 = 1.128$

For MR chart
$$\begin{cases} UCL = D_4 \overline{MR} = 148.16 \times 3.267 = 484.04 \\ LCL = D_3 \overline{MR} = 0 \end{cases}$$
$CL = 148.16$

[check chart behind] chart shows all the points are within the control limit.

Control limits for X chart:- $\bar{X} = 2928.9$

$$UCL = \bar{X} + \frac{3}{d_2}\overline{MR} = 3322.934$$
$$CL = 2928.9$$
$$LCL = \bar{X} - \frac{3}{d_2}\overline{MR} = 2535.86$$

Since all points are within the control limit. So the process is in control.

(b) Process mean is $\hat{\mu} = \bar{x} = 2928.9$, $\hat{\sigma} = \frac{\overline{MR}}{d_2} = \frac{148.157}{1.128} = 131.344$

(c) Yes, these 5 points indicate that the process is in statistical control.

$\overline{X}$ - s Chart Exercise

Q. The fill volume of soft-drink beverage bottles is an important quality characteristic. The volume is measured (approximately) by placing a gauge over the crown and comparing the height of the liquid in the neck of the bottle against a coded scale. On this scale, a reading of zero corresponds to the correct fill height. Fifteen samples of size $n=10$ have been analysed and given in the table. Set up $\overline{X}$ & s charts for this process.

Sample No.	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	$\overline{X}$	s
1	2.5	0.5	2.0	-1.0	1.0	-1.0	0.5	1.5	0.5	-1.5	0.5	1.3333
2	0	0	0.5	1	1.5	1	-1	1	1.5	-1	0.45	0.9265
3	1.5	1	1	-1	0	-1.5	-1	-1	1	-1	-0.1	1.1255
4	0	0.5	-2	0	-1	1.5	-1.5	0	-2	-1.5	-0.6	1.1738
5	0	0	0	-0.5	0.5	1	-0.5	-0.5	0	0	0	0.4714
6	1	-0.5	0	0	0	0.5	-1	1	-2	1	0	0.9718
7	1	-1	-1	-1	0	1.5	0	1	0	0	0.05	0.896
8	0	-1.5	-0.5	1.5	0	0	0	-1	0.5	-0.5	-0.15	0.8182
9	-2	-1.5	1.5	1.5	0	0	0.5	1	0	1	0.2	1.1832
10	-0.5	3.5	0	-1	-1.5	-1.5	-1	-1	1	0.5	-0.15	1.5284
11	0	1.5	0	0	2	-1.5	0.5	-0.5	2	-1	0.3	1.2065
12	0	2	-0.5	0	-0.5	2	1.5	0	0.5	-1	0.4	1.075
13	-1	-0.5	-0.5	-1	0	0.5	0.5	-1.5	-1	-1	-0.55	0.6852
14	0.5	1	-1	-0.5	-2	-1	-1.5	0	1.5	1.5	-0.15	1.2483
15	1	0	1.5	-1.5	1	-1	0	1	-2	-1.5	0.15	1.2704

Solution:- $\overline{\overline{X}} = 0.023$, $\overline{S} = 1.060$

For $\overline{X}$ chart:- $LCL = \overline{\overline{x}} - A_3\overline{S} = -1.0105$

$CL = \overline{\overline{x}} = 0.023$

$UCL = \overline{\overline{x}} + A_3\overline{S} = 1.0565$, $A_3 = 0.975$ for $n=10$.

For s chart:- $LCL = B_3\overline{S} = 0.301$; $B_3 = 0.284$, $B_4 = 1.716$

$CL = \overline{S} = 1.060$

$UCL = B_4\overline{S} = 1.819$

So, the process is in statistical control.

EXERCISE   np Chart

Q. 1. np Chart:- (used when subgroup size n is constant)

Inspection results of video of the month shipment to customers for 10 consecutive days are given in table. The number of inspection each day is constant and is equal to 1000. Construct np chart to control the defectives?

Sample Number	Number of defectives
1	47
2	42
3	48
4	58
5	32
6	38
7	53
8	68
9	45
10	37

Solution:- Subgroup size, $n = 1000$.

sample size, $m = 10$.

$$\bar{p} = \frac{\text{Sum of defectives}}{\text{Total checked}} = \frac{\Sigma D_i}{mn} = \frac{468}{10000} = 0.0468$$

np Control chart:-

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 66.84$$

$$CL = n\bar{p} = 1000 \times 0.0468 = 46.8$$

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 26.76.$$

Since one point is out of control limits, so the process is out of control.
Now remove that point.

Recalculate control limits for np chart:- $n = 1000$, $m = 9$.

$$\bar{p} = \frac{\Sigma D_i}{mn} = \frac{400}{9000} = 0.0444$$

$UCL = 63.995$
$CL = 44.444$
$LCL = 24.894$

p chart

Exercise:- The daily inspection results for electric carving knives are given below. Construct a control chart to monitor the process:

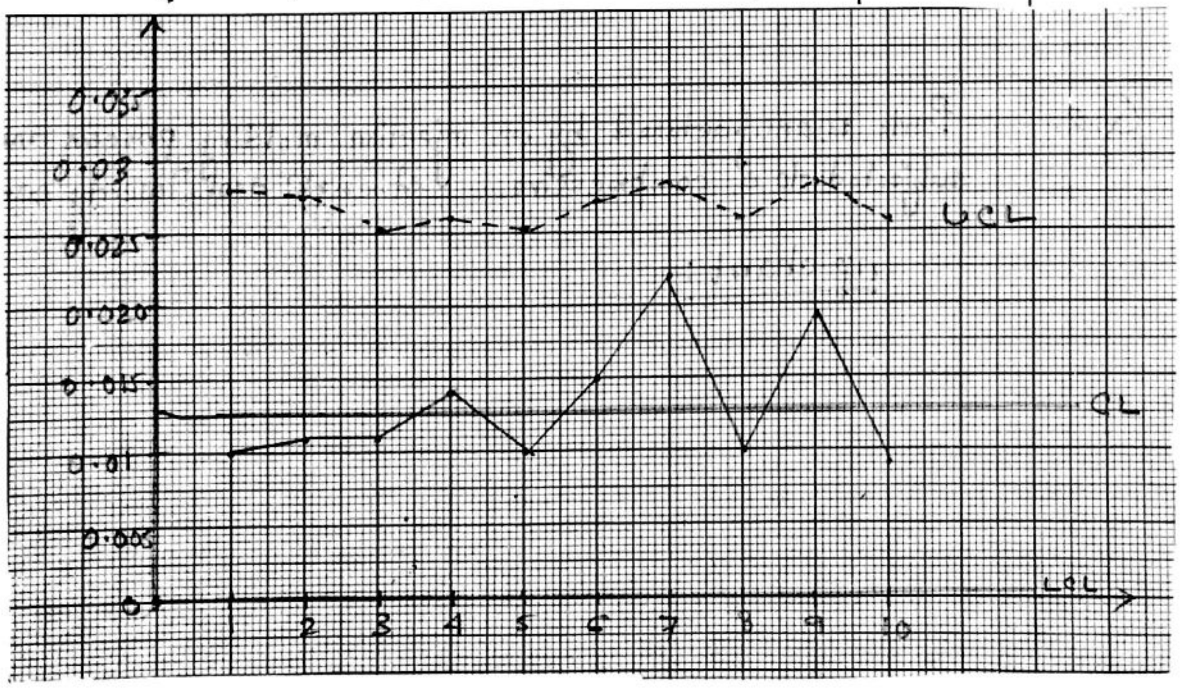
Sample No.	Number Inspected	Number of Defectives	$SD=\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}$	LCL	UCL
1	500	5	0.00502	0	0.0278
2	550	6	0.00479	0	0.0271
3	700	8	0.00424	0.00003	0.0255
4	625	9	0.00449	0	0.0262
5	700	7	0.00424	0.00003	0.0255
6	550	8	0.00479	0	0.0271
7	450	10	0.00529	0	0.0286
8	600	6	0.00458	0	0.0265
9	475	9	0.00515	0	0.0282
10	650	6	0.00440	0	0.0260

$$\bar{p} = \frac{\sum d_i}{\sum n_i} = \frac{74}{5800} = 0.0127586, = CL$$

Now, we have to calculate fraction defectives $= \frac{\text{\# of defectives}}{\text{\# of inspected}}$

Then plot the fraction defectives in their corresponding limits.

Sample No.	1	2	3	4	5	6	7	8	9	10
Fraction Defectives	0.010	0.011	0.011	0.014	0.010	0.015	0.022	0.010	0.019	0.009



C chart

Q. 100 product levels are inspected everyday for surface nonconformities. The data for the past 20 days is given below. Construct a suitable control chart to monitor the non-conformities :

Day	Number of Nonconformities	Day	Number of Nonconformities
1	22	11	15
2	29	12	10
3	25	13	33
4	17	14	23
5	20	15	27
6	16	16	15
7	34	17	17
8	11	18	17
9	31	19	19
10	29	20	22

Solution:- Sample size = 20, sub-group size = 100

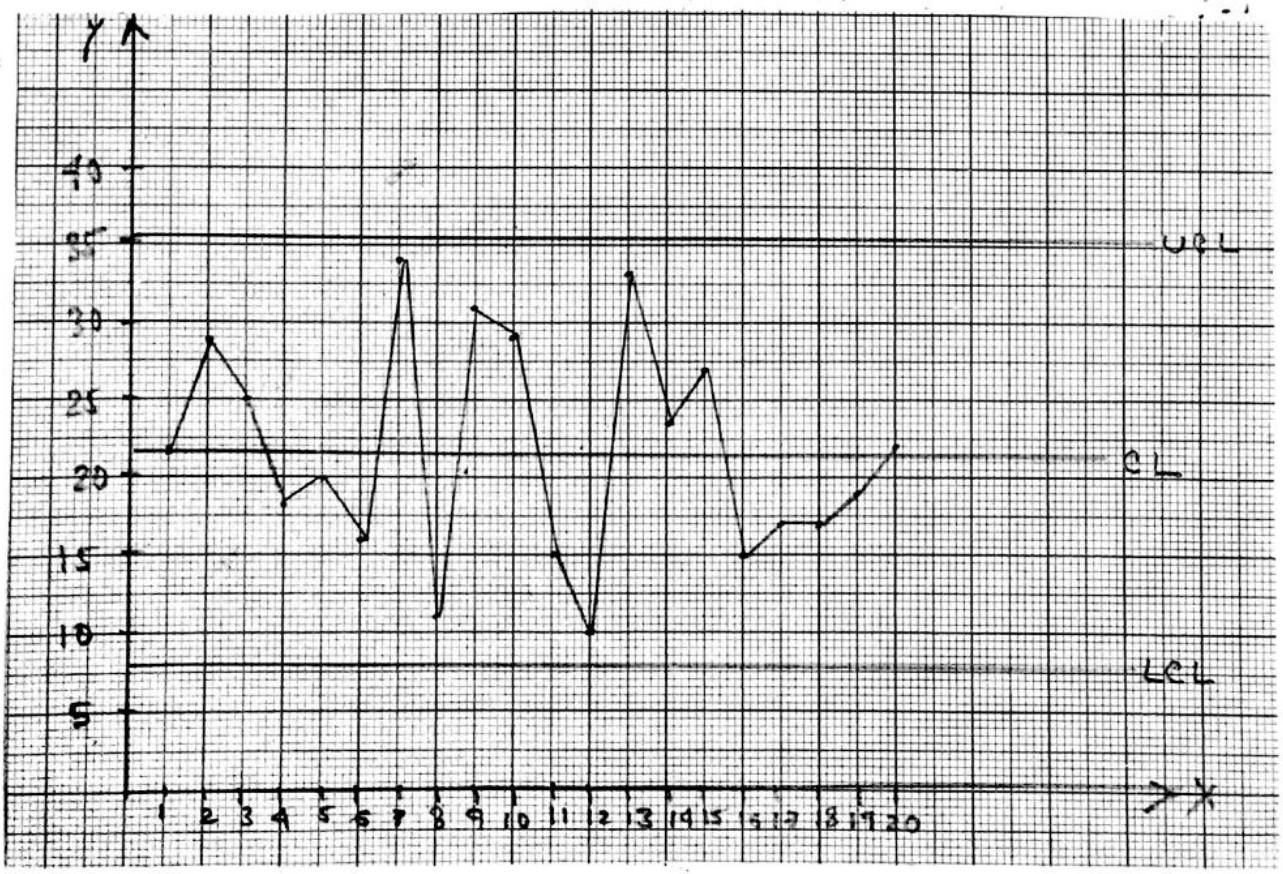
Mean = $\bar{c} = 21.6$

$SD = \sqrt{\bar{c}} = 4.647$

$\therefore$ $UCL = \bar{c} + 3\sqrt{\bar{c}} = 35.55$

$CL = 21.6$

$LCL = \bar{c} - 3\sqrt{\bar{c}} = 7.66$



1

u chart

The inspection results for the surface finish of rolls of white paper for 10 lots is given below. Construct a control chart to monitor the process.

Lot Number	Number Inspected (n_i)	Number of Defects (x_i)	$\sqrt{\frac{\bar{u}}{n_i}}$
1	10	45	0.605
2	10	51	0.605
3	10	36	0.605
4	9	48	0.637
5	10	42	0.605
6	10	5	0.605
7	10	33	0.605
8	8	27	0.676
9	8	31	0.676
10	8	22	0.676

Sol. $\bar{u} = \text{Mean} = \frac{\text{Sum of Defects}}{\text{Total inspected}} = \frac{340}{93} = 3.6559$

$SD = \sqrt{\frac{\bar{u}}{n}}$, $CL = 3.6559$; $UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$, $LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$

Sample Number	1	2	3	4	5	6	7	8	9	10
UCL	5.47	5.47	5.47	5.568	5.47	5.47	5.47	5.68	5.68	5.68
LCL	1.842	1.842	1.842	1.744	1.842	1.842	1.842	1.628	1.628	1.628

Plot Defects per unit $\left(\frac{X}{n}\right)$ in the control chart.

(u) Defects per unit	4.50	5.10	3.60	5.33	4.20	0.50	3.30	3.38	3.88	2.75

Assuming that the process has improved, so one point is below LCL. Same limits will be used for future control.

PROCESS CAPABILITY ANALYSIS

Ex.1. 20 data on acid content (mm) is given in the table below. If the specification on acid content is $0.70 + 0.2$ mm. Calculate Process capability?

0.85	0.75	0.80	0.65	0.75	0.6	0.8	0.7	0.75	0.6
0.8	0.75	0.7	0.7	0.75	0.75	0.85	0.6	0.5	0.65

Solution:- $USL = 0.9$, $LSL = 0.5$; (Units are in mm)

$Mean = 0.715$, $SD = 0.092$;

$$C_p = 0.725 = \frac{USL - LSL}{6\sigma} = \frac{0.90 - 0.50}{6 \times 0.092} = 0.72$$

$$C_{pl} = 0.78 = \frac{\mu - LSL}{3\sigma}$$

$$C_{pu} = 0.671 = \frac{USL - \mu}{3\sigma}$$

$$C_{pk} = 0.671 = \min(C_{pu}, C_{pl})$$

The process has not the potential and it is not capable.
So, We need to reduce the variation to make C_p & C_{pk} greater than 1.

Ex.2. The specification on coating thickness of powder coated panels is 80 microns ± 5 microns. A sample of 50 powder coated panels are randomly selected and thickness are measured. The data is given below.
Check whether the process is capable of meeting the specification?
If the coating thickness below the lower specification, then the panel can be reworked at a cost of \$5. Similarly if the coating thickness is more than upper specification, then also the panel can be reworked by removing the point & recoating it at a cost of \$20.
Suppose a batch of 120 panels are powder coated. Estimate the rework cost?

Thickness				
81.4	77.9	83.1	82.8	79.7
83.7	84.2	79	80.9	80.8
82.3	81.7	78.9	81.1	84.9
79.8	80.1	80	82.1	79.1
79.5	79	80.2	79.3	82.4
81.8	82.8	81.7	80.2	82.7
82.8	79.2	81.2	82.4	81.4
80.6	81.7	82.3	80.6	79.4
82.6	81.8	82	80.6	82.4
81.9	82.9	82.5	82.4	83.2

Solution:- USL = 85

LSL = 75

Mean = 81.34

SD = 1.551

$C_p = 1.074$, the process has the potential.

$C_{pl} = 1.362$

$C_{pu} = 0.786$

$C_{pK} = 0.786$, but it is not capable to meet specification.

$$\text{Below LSL} = P(x < LSL) = P\left(Z < \frac{LSL-\mu}{\sigma}\right) = P\left(Z < \frac{75-81.34}{1.551}\right)$$

$$= P(Z < -4.08)$$

$$= \Phi(-4.08)$$

$$= 0.00002 = 0.002\%$$

$$\text{Above USL} = 1 - P\left(Z > \frac{USL-\mu}{\sigma}\right)$$

$$= 1 - P(Z > 2.36)$$

$$= 1 - 0.99086 = 0.00914 = 0.914\%$$

Rework cost of 120 panels

	Number	Cost	Total cost
<LSL	0.0026	5	0.012941
>USL	1.0965	20	21.92992

Total Rework cost = 21.94.

Methods:-

1. Histogram Method:-

- Collect large sample of at least 100 observations on the quality characteristic under study.
- Draw Histogram.
- Judge based on Histogram whether the quality characteristic is normally distributed.
- If yes, Estimate process mean, $\hat{\mu} = \bar{x}$, $\hat{\sigma} = s$ is the estimated s.d.
- Estimate C_p and C_{pK}.

2. Control Chart Method:-

- Collect sample data in sub-groups.
- Construct $\bar{x}$ - R and $\bar{x}$ - s chart and check the stability of the process.
- Estimate process mean μ & s.d. σ from control charts

$$\hat{\mu} = \bar{\bar{x}} \quad , \quad \hat{\sigma} = \frac{\bar{R}}{d_2} \text{ or } \frac{\bar{s}}{c_4}.$$

- Estimate C_p & C_{pk}.

Example:- A high voltage power supply should have a nominal output voltage of 350V. A sub group of four units is selected each day and tested for process control purposes. The subgroup averages and ranges are computed and given in the next slide.

1. Set up xbar and R charts on this process. Is the process in statistical control?
2. Estimate the process mean and standard deviation?
3. If specifications are at 350V ± 2V, Estimate the process capability?
4. Assuming that if an item exceeds upper specification limit it can be reworked and if it is below lower specification limit it must be scraped, what is the percentage scrap and rework?

Sample	xbar	Range	Sample	xbar	Range
1	351.00	0.9	11	351.25	0.8
2	350.78	0.7	12	350.98	0.7
3	350.75	0.5	13	351.33	0.7
4	350.90	0.7	14	351.05	0.6
5	350.98	0.6	15	351.10	0.9
6	351.08	0.2	16	351.25	0.5
7	351.08	0.8	17	350.98	0.4
8	350.65	0.6	18	351.08	0.8
9	350.90	0.5	19	350.88	0.6
10	351.35	0.6	20	351.33	0.4

Solution:- 1. $\bar{R} = \frac{12.5}{20} = 0.625$

$D_3 = 0$, $D_4 = 2.282$

Rchart:-

$UCL = D_4 \bar{R} = 1.42625$

$LCL = D_3 \bar{R} = 0$

$CL = 0.625$

All points of R-chart are within control limits.

$\bar{X}$ chart:- $\quad \bar{\bar{X}} = 351.04$, $A_2 = 0.729$

$$UCL = \bar{\bar{X}} + A_2\bar{R} = 351.49$$

$$CL = \bar{\bar{X}} = 351.04$$

$$LCL = \bar{\bar{X}} - A_2\bar{R} = 350.58$$

2. The process is in control.

Process mean, $\hat{\mu} = \bar{\bar{x}} = 351.04$

Process SD, $\hat{\sigma} = \dfrac{\bar{R}}{d_2} = \dfrac{0.625}{2.059} = 0.3035$

3. Now, given specifications are:

$USL = 352$

$LSL = 348$

So, $C_p = \dfrac{USL - LSL}{6\sigma} = \dfrac{4}{6 \times 0.304} = 2.193$

$$C_{pu} = \frac{USL - \mu}{3\sigma} = \frac{352 - 351.04}{3 \times 0.304} = 1.053$$

$$C_{pL} = \frac{\mu - LSL}{3\sigma} = \frac{351.04 - 348}{3 \times 0.304} = 3.333$$

$$C_{pk} = \min\{1.053, 3.333\} = 1.05 > 1$$

So, it has potential to produce and is capability of doing it.

4. Scrap:- $P(x < LSL) = P\left(\dfrac{x-\mu}{\sigma} < \dfrac{LSL-\mu}{\sigma}\right)$

$$= P\left(Z < \frac{348 - 351.04}{0.304}\right)$$

$$= 0$$

Rework:- $P(x > USL) = P\left(\dfrac{x-\mu}{\sigma} > \dfrac{USL-\mu}{\sigma}\right)$

$$= P\left(z > \frac{352 - 351.04}{0.304}\right)$$

$$= 0.00079$$

$$= .079\%$$

$\therefore$ No scrap work, but 0.079% rework is there.

3. Probability Plot Method:— Used when the sample size is small or not sufficient to construct histogram.

- Collect sample data on the quality characteristic under study
- Construct the normal probability plot
- If the plotted points fall approximately on a straight line, then conclude that the quality characteristic follows normal distribution.
- Estimate process mean μ & s.d. σ from Normal Probability Plot as follows

$$\mu = 50^{th} \text{ percentile}$$
$$\sigma = 84^{th} \text{ percentile} - 50^{th} \text{ percentile}$$

- Compute C_p & C_{pk}.

Example:- The performance of the claims reimbursement process of finance department of a company is judged based on time (days) taken to reimburse employee expenses claims. The company wants to settle the claims within 25 days of submitting the documents. The data on cycle times (in days) of 30 randomly selected employee expense claims is given below. Check whether the process is capable of meeting the requirement?

5	5	16	17	14	12
8	13	6	12	11	10
18	18	13	12	19	14
17	16	11	22	13	16
10	18	12	12	12	14

Solution:-

Step-1:- Arrange the data in the ascending order.

Step-2:- Rank (i) the observations.

Step-3:- Compute the empirical cumulative d.f. $F(x) = \frac{i - 0.5}{n}$, where n is the total number of samples.

Step 4:- Plot X versus F(X) in a Normal Probability paper. If the plotted points fall approximately on a straight line, then the quality characteristic follows Normal Distn.

Step 5:- Compute the standard normal score z corresponding to F(x) using normal distn. tables as shown below:

Cycle Time(x)	i	F(x)	Z
5	1	0.017	−2.12
5	2	0.05	−1.65
6	3	0.0833	−1.38
8	4	0.1166	−1.20
10	5	0.15	−1.04
10	6	0.183	−0.90
11	7	0.217	−0.78
11	8	0.25	−0.67
12	9	0.283	−0.57
12	10	0.317	−0.47
12	11	0.35	−0.38
12	12	0.383	−0.30
12	13	0.417	−0.21
12	14	0.45	−0.12
13	15	0.483	−0.04
13	16	0.517	0.05
13	17	0.55	0.13
14	18	0.583	0.21
14	19	0.617	0.30
14	20	0.65	0.39
16	21	0.683	0.48
16	22	0.717	0.57
16	23	0.75	0.68
17	24	0.783	0.88
17	25	0.813	0.89
18	26	0.85	1.04
18	27	0.883	1.19
18	28	0.917	1.39
19	29	0.95	1.65
22	30	0.983	2.12

Step-6:- Plot X vs Z in an ordinary graph paper.
If the plotted points fall approximately on a straight line, then the quality characteristic follows normal distribution.

Note:- Try to draw the straight line connecting 25^{th} and 75^{th} percentiles.

Estimate process mean μ and s.d. σ from the normal plot as follows

$$\mu = 50^{th} \text{ Percentile}$$
$$\sigma = 84^{th} \text{ Percentile} - 50^{th} \text{ Percentile}$$

Percentile	F(x)	Z	Corresponding value (Y
50	0.5	0	13
84	0.84	0.99	17

$$\mu = 13$$
$$\sigma = 17 - 13 = 4.$$

USL = 25.
Since LSL is not defined. So we can't calculate C_p.

$$\text{For } C_{pK} = \min\{C_{pu}, C_{pL}\}$$

As C_p is not available, $C_{pK} = C_{pu} = 1$.

$$C_{pu} = \frac{USL - \mu}{3\sigma} = \frac{25 - 13}{3 \times 4} = 1.$$

So, the process is capable of meeting the requirements.

PROCESS CAPABILITY ANALYSIS

- An engineering study to estimate the capability of the process or to check whether a process is capable of meeting customer requirements.
- Expressed as Process Capability Indices or Ratios.

Common Process Capability Indices :-

1. Process Potential Index C_p (potential capability of the process).
2. Process Performance Index C_{pk} (actual capability of the process).

Process Potential Index C_p :- A methodology to check whether the process have the potential to meet the customer requirements.

Generally customer requirements are given as specification on product characteristics.

Example :- Specification on Heat treatment process :
Hardness should be within 55 ± 5 HRC

Customer requirements mean Variation allowed by the customer or Variation acceptable to customer.

The above example means that as long as Hardness of the heat treated jobs are between 50 HRC to 60 HRC, Customer is satisfied. So, Lower Specification Limit (LSL) = 50 HRC
Upper Specification Limit (USL) = 60 HRC

C_p : A process have the potential to meet customer requirement, if Total or natural variation in process < Allowed variation.

Process capability means Natural Variation in the process.

Definition of C_p :- If the quality characteristic is normally distributed with mean μ and standard deviation σ, then

Total variation: $\mu \pm 3\sigma$

Eg :- Suppose surface hardness achieved of induction hardened piston is normally distributed with mean 55HRC and SD 1HRC.

$$\mu = 55\text{HRC}, \quad \sigma = 1\text{HRC}$$

$$\therefore \text{Total Variation} = 55 - 3\times1 \text{ to } 55 + 3\times1$$
$$= 52\text{HRC to } 58\text{HRC}$$

Definition :- Ratio of allowed variation to Total variation,

$$C_p = \frac{\text{Allowed Variation}}{\text{Total variation}} = \frac{USL - LSL}{(\mu + 3\sigma) - (\mu - 3\sigma)}$$
$$= \frac{USL - LSL}{6\sigma}$$

A process has the potential to meet customer requirements if

total variation < allowed variation

$$6\sigma < (USL-LSL)$$

$$\therefore C_p > 1$$

Process Potential Index C_p : Issues

- C_p checks only whether the process has the potential to meet the requirements.
- C_p never checks whether the process is actually meeting requirements.

Example:- Process : Heat Treatment characteristic : Hardness

Specification: 55 ± 5 HRC

	Process 1	Process 2	Process 3
Mean (μ)	55	52	58
SD (σ)	1	1	1
USL−LSL	10	10	10
6σ	6	6	6
C_p	1.66	1.66	1.66

$\therefore$ $C_p = 1.66$ for all 3 processes.

So all 3 process have the potential to meet customer requirement but only Process 1 is meeting customer requirement. Hence process performance index is developed.

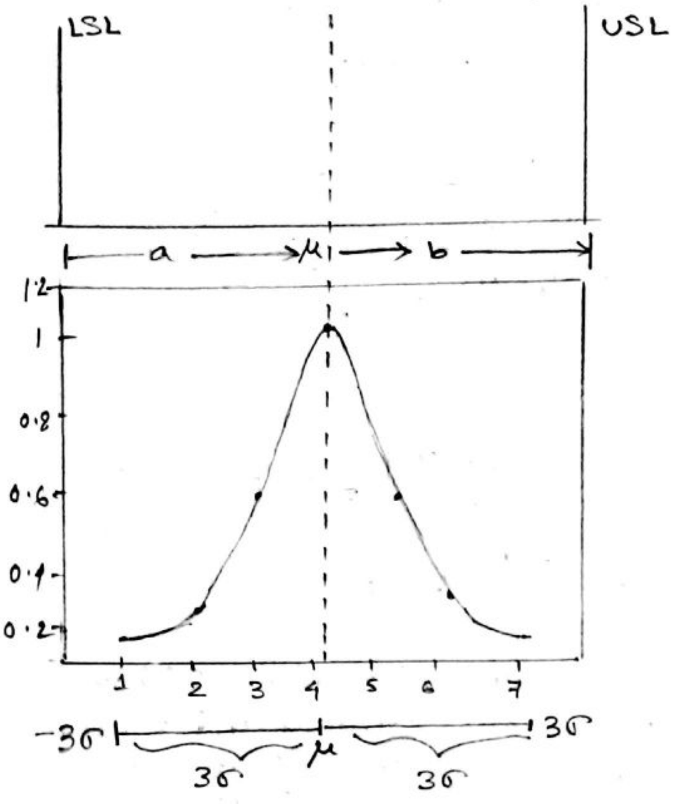
Process Performance Index, C_{pk} : Definition :-

$$C_{pk} = \text{Min}[C_{pl}, C_{pu}], \quad C_{pl} = \frac{\mu - LSL}{3\sigma}, \quad C_{pu} = \frac{USL-\mu}{3\sigma}.$$

C_{pk} checks whether the process is centered at the middle of specification.

$$C_{pk} < 1 \Rightarrow \text{Performance is not OK.}$$

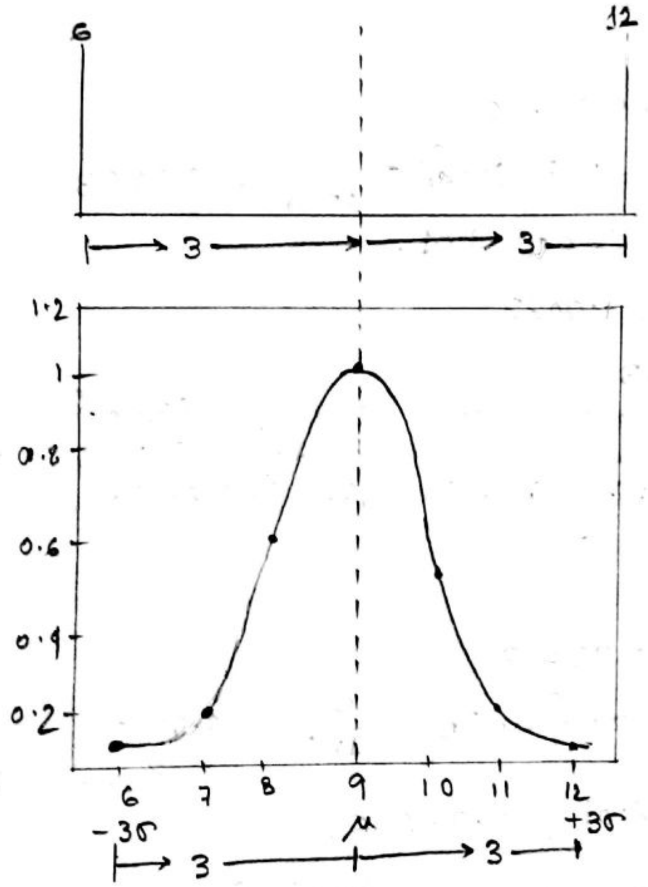
Graphical Representation:-



Example:-

$$C_{pl} = \frac{\mu - LSL}{3\sigma} = \frac{a}{3\sigma}$$

$$C_{pu} = \frac{USL - \mu}{3\sigma} = \frac{b}{3\sigma}.$$



Example:-

$USL = 12$, $LSL = 6$

$\mu = 9$ $\sigma = 1$

$C_{pu} = 3/3 = 1$

$C_{pl} = 3/3 = 1$

$C_{pk} = \text{Min}\{C_{pu}, C_{pl}\} = 1.$

$$C_p = \frac{(USL - LSL)}{6\sigma} = \frac{6}{6} = 1.$$

When Mean is at middle of specification $[(USL + LSL)/2]$ then $C_{pu} = C_{pl} = C_{pk} = C_p$

Otherwise, $C_{pk} < C_p$

When $C_{pk} < C_p$, performance is not optimum.

Process Capability of Non-Normal Characteristics- Approximate Method

Useful for large sample with size > 300

Uses the relationship between C_{pk} and fraction non-conforming

Fraction non-conforming above USL =

$$P(x > USL) = P\left[\frac{x-\mu}{\sigma} > \frac{USL-\mu}{\sigma}\right] = P[Z > Z_U]$$
$$= 1 - P[Z < Z_U]$$

where, $C_{pu} = \frac{USL-\mu}{3\sigma}$, $Z_U = \frac{USL-\mu}{\sigma}$; $C_{pu} = \frac{Z_U}{3}$.

Fraction non-conforming below LSL =

$$P(x < LSL) = P\left[\frac{x-\mu}{\sigma} < \frac{LSL-\mu}{\sigma}\right] = P[Z < Z_L]$$

where, $C_{pl} = \frac{\mu-LSL}{3\sigma}$, $Z_L = \frac{LSL-\mu}{\sigma}$; $C_{pl} = \frac{-Z_L}{3}$.

Ex.1. A company has to process every invoice within 24 hours. A random sample of 1200 invoices are selected and measured the time to process the invoice. The data shows that 2 out of 1500 invoices has taken more than 24 hours to process. Calculate the process capability?

Solution:- Sample size = 1500

No. of non-conforming > USL = 2

Fraction non-conforming $= P(Z > Z_u) = \frac{2}{1500} = 0.00133$

$P(Z < Z_u) = 0.9987$ $\therefore Z_u = 3.01$.

$\therefore C_{pu} = \frac{Z_u}{3} = \frac{3.01}{3} = 1.0033$

$\therefore C_{pk} = C_{pu} = 1.0033$

Ex.2. A back office wants to process at least 90 transactions hourly. The productivity for 1200 hours are measured and found that 3 out of 1200 cases, the productivity is below 90. Calculate process capability?

Solution:- Sample size = 1200

No. of non-conforming < LSL = 3

$P(Z < Z_L)$ = fraction non-conforming $= \frac{3}{1200} = 0.0025$

$\therefore Z_L = -2.81$

So, $C_{pL} = \frac{-Z_L}{3} = \frac{2.81}{3} = 0.93666$.

So, the office is not capable of producing.

MEASUREMENT SYSTEM ANALYSIS

Methodology to evaluate the capability of the measurement system.
Generally any activity involving measurements
— some variability will be inherent in the units or items measured.
— Remaining variability will result from the measurement system.

Example:- Human body temperature using a thermometer.

Day	Body Temperature	
	Self	Friend
1	98·6	97·5
2	97·6	97·9
3	102·1	101·8
4	102·2	102·1

Methodology to evaluate the capability of the measurement system:-

Major components are:-
1. Instrument or gauge used for measurement
2. Operators who use the instrument to measure the items

Objective:-
1. Determine how much of the total observed variability is due to the gauge or instrument.
2. Isolate the components of variability in the measurement system.
3. Access whether the instrument or gauge is capable.

- Variation in the data have two components:-
  - Variation in the process/ product
  - Variation in the measurement system

Total Variation = Product variation + Measurement System Variation

$$\sigma^2_{total} = \sigma^2_{product} + \sigma^2_{gauge}$$

Gauge Repeatability and Reproducibility (Gauge R & R):-

This is a methodology to estimate the measurement system variation.

Gauge R & R has two components:-
- Variation caused by operator (appraiser variation - AV)
- Variation caused by instrument (equipment variation - EV)

Repeatability (EV) :- • The variation due to the measuring instrument.

• The variation observed when the same operator measures the same sample repeatedly with the same instrument.

Reproducibility (AV) :- • The variation due to the measurement system.

• The variation observed when different operators measure the same sample using the same instrument.

So, $\sigma^2_{Gauge} = \sigma^2_{Reproducibility} + \sigma^2_{Repeatability}$

Gauge R & R : Data Collection :- (i) Collect at least 10 samples
(ii) Choose at least 2 operators for study
(iii) Allow each operator to measure each sample at least twice.

Methods :- • $\overline{X}$ - R chart method.
• ANOVA method.

$\overline{X}$ - R chart method :- Number of Operators : 2 = r
Number of Parts : 10 = n

Operator 1

Part	1	2	Mean	Range
1	21	20	20.5	1
2	24	23	23.5	1
3	20	21	20.5	1
4	27	27	27.0	0
5	19	18	18.5	1
6	23	21	22.0	2
7	22	21	21.5	1
8	19	17	18.0	2
9	24	23	23.5	1
10	25	23	24.0	2
			$21.9 = \overline{X}_1$	$1.2 = \overline{R}_1$

	Operator 2			
Part	1	2	Mean	Range
1	20	20	20	0
2	24	24	24	0
3	19	21	20	2
4	28	26	27	2
5	19	18	18.5	1
6	24	21	22.5	3
7	22	24	23	2
8	18	20	19	2
9	25	23	24	2
10	26	25	25.5	1
			$\overline{X_2} = 22.35$	$\overline{R_2} = 1.5$

Trials	K_1	K_2
2	.8862	.7071
3	.5908	.5231

$$\overline{R} = \frac{\overline{R_1} + \overline{R_2}}{2} = 1.35$$

$$\text{Repeatability (EV)} = K_1 \times \overline{R} = 0.8862 \times 1.35 = 1.19681$$

Overall variation between operators : $\left|\overline{X_2} - \overline{X_1}\right| = 0.45 = D$

$$\text{Reproducibility (AV)} = \sqrt{(D \times K_2)^2 - (EV^2/nr)}$$

$$= \sqrt{(0.45 \times 0.7071)^2 - (1.19681^2/10 \times 2)}$$

$$= 0.1739.$$

$$\text{Total Gauge R \& R} = \sqrt{\text{Repeatability}^2 + \text{Reproducibility}^2} = \sqrt{EV^2 + AV^2}$$

$$= \sqrt{1.19681^2 + 0.1739^2}$$

$$= 1.2094.$$

Part Variation:-

Part	Operator 1: 1	Operator 1: 2	Operator 2: 1	Operator 2: 2	Mean
1	21	20	20	20	20.25
2	24	23	24	24	23.75
3	20	21	19	21	20.25
4	27	27	28	26	27.00
5	19	18	19	18	16.00
6	23	21	24	21	17.25
7	22	21	22	24	17.25
8	19	17	18	20	18.50
9	24	23	25	23	23.75
10	25	23	26	25	24.75

Parts	K_3
2	0.7071
3	0.5231
4	0.4467
5	0.4030
6	0.3742
7	0.3534
8	0.3374
9	0.3249
10	0.3146

R_p : Mean max − Mean min $= 27 - 16 = 11$

Part Variation (PV) $= K_3 \times R_p = 0.3146 \times 11$
$= 3.4606$

Total variation $= \sqrt{(\text{Gage R \& R})^2 + (\text{Part Variation})^2}$

$= \sqrt{(1.2094)^2 + (3.4606)^2}$

$= 3.6658$

Source	SD	6XSD	% Study Var $= \frac{\text{GSD} \times 100}{\text{Total SD}}$
Repeatability	1.19681	7.18086	32.65
Reproducibility	0.1739	1.0434	4.74
Total Gauge R&R	1.2094	7.2564	32.99
Part Variation	3.4606	20.7636	94.40
Total Variation	3.6658	21.9948	100.00

Guidelines for accepting the Measurement System:—

Gauge R & R	Remark
Under 10%	Gauge system is satisfactory.
10% to 30%	May be acceptable based upon application, cost of gage, cost of repair etc.
Over 30%	Gauge system not satisfactory

ANOVA Method :-

Number of Auditors : 2
Number of calls : 5
Replication : 2

Appraisers

Part	A	B	Sum
1	50	55	215
1	54	56	
2	65	64	264
2	67	68	
3	75	79	308
3	76	78	
4	81	82	324
4	79	82	
5	95	96	381
5	94	96	
Sum	736	756	1492

(i) Interaction Sum table:-

Part	A	B
1	104	111
2	132	132
3	151	157
4	160	164
5	189	192

(ii) Correction factor:-

Grand total $T = 1492$

Total Count $N = 20$

Correction Factor $CF = \frac{1492^2}{20}$

$= 111303.2$

(iii) Sum of Squares:-

$\text{Total} = 50^2 + 56^2 + \cdots + 96^2 - CF = 3976.8$

$\text{Sample} = \frac{215^2 + 264^2 + \cdots + 381^2}{4} - CF = 3927.3$

$\text{Column} = \frac{736^2 + 756^2}{10} - CF = 20$

$\text{Interaction} = \frac{104^2 + 111^2 + \cdots + 192^2}{2} - SS_{sample} - SS_{column} - CF$

$= 7.5$

$\text{Within} = SS_{Total} - SS_{sample} - SS_{columns} - SS_{interaction} = 22.$

(iv) Degree of Freedom:-

Source	Formula	Degree of Freedom
Total	Total count − 1	19
Sample	No. of rows − 1	4
Columns	No. of columns − 1	1
Interaction	df of sample × df of Appraisers	4×1 = 4
Within	Total df − sample df − column df − Interaction df	10

(v) ANOVA Table construction:-

Source of Variation	SS	df	MS	F	Fcrit
sample	3927.3	4	981.825	446.2841	3.47805
Columns	20	1	20	9.0909	4.9646
Interaction	7.5	4	1.875	0.8523	3.478
Within	22	10	2.2		
Total	3976.8	19			

$MS = SS/df$

$$F(i) = \frac{MS_{(i)}}{MS_{within}}$$

Now, checking whether interaction F $\gtreqless$ Interaction Fcritical

Case I:- Since F = 0.85 < F crit = 3.478, Interaction is not significant. Modify ANOVA table:-

$SS_{within} = SS_{within} + SS_{interaction} = 22 + 7.5$
$= 29.5$

$DF_{within} = df_{within} + df_{interaction} = 10 + 4$
$= 14$

Modified table:-

$MS_{within} = SS_{within} / df_{within} = \frac{29.5}{14} = 2.107$

Source of Variation	SS	df	MS
Sample	3927.3	4	981.8
Columns	20	1	20
Within	29.5	14	2.107
Total	3976.8	19	

Variances:-

Equipment (EV) = MS_{within} = 2·107143

Appraiser (AV) = $(MS_{columns} - MS_{within}) / (\text{no. of parts} \times \text{no. of replications})$

$$= \frac{20 - 2{\cdot}107143}{5 \times 2} = 1{\cdot}7893$$

Part (PV) = $(MS_{simple} - MS_{within}) / (\text{no. of appraisers} \times \text{no. of replications})$

$$= \frac{981{\cdot}825 - 2{\cdot}107143}{2 \times 2} = 244{\cdot}9295$$

Gage R & R = EV + AV = 2·107143 + 1·7893 = 3·8964

Total = Gage R & R + PV = 3·8964 + 244·9295 = 248·8259.

	Variance	SD	6SD	% Study Var = $\frac{6SD \times 100}{6SD\ total}$
Gage R & R	3·8964	1·9739	11·8436	12·5137
EV	2·1071	1·4516	8·7096	9·2024
AV	1·7893	1·3376	8·0259	8·4799
PV	244·9295	15·6502	93·9013	99·2139
Total	248·8259	15·7742	94·6453	

Guidelines to accept gage:-

% Contribution of Gage R & R	Remark
Under 10%	Gage system is satisfactory
10% to 30%	May be acceptable based upon application, cost of gage, cost of repair, etc.
Over 30%	Gage system not satisfactory

Case II:- Assuming SS interaction = 70
i.e. When Interaction F > Interaction F critical

ANOVA Table:-

Source of Variation	SS	df	MS	F	Fcrit
Sample	3864.8	4	966.2	439.18	3.478
Columns	20	1	20	9.09	4.964
Interaction	70	4	17.5	7.95	3.478
Within	22	10	2.2		
Total	3976.8	19			

Variances:-

Equipment (EV) = MS_{within} = 2.2

Interaction (INT) = ($MS_{interaction}$ − MS_{within})/(no. of replications)

$= (17.5 - 2.2)/2 = 7.65$

Appraiser (AV) = ($MS_{columns}$ − $MS_{interaction}$)/(no. of parts × no. of replications)

$$= \frac{(20 - 17.5)}{5 \times 2} = 0.25$$

Part (PV) = (MS_{sample} − $MS_{interaction}$)/(no. of appraisers × no. of replications)

$$= \frac{981.825 - 17.5}{2 \times 2} = 237.175$$

Gage R & R = EV + AV + INT = 10.1

Total = Gage R & R + PV = 247.275

Complete Calculation:-

	Variance	SD	6SD	% contribution
Gage R & R	10.1000	3.178	19.0683	20.2102
EV	2.2	1.4832	8.8994	9.4324
INT	7.65	2.7659	16.5952	17.589
AV	0.25	0.5	3	3.1797
PV	237.175	15.4	92.4029	97.9364
Total	247.275	15.725	94.3499	

Conclusion:- Since % contribution of gage R & R = 20.21 > 10%, the measurement system may be acceptable.

Measurement System Analysis : Discrete

Example: The transaction monitoring process results for 2 auditors is given below. The results of expert (standard) is also given. Perform MSA and give your conclusions?

Transaction	Auditor 1	Auditor 2	Standard	Transaction	Auditor 1	Auditor 2	Standard
1	Pass	Pass	Pass	13	Pass	Pass	Pass
2	Pass	Pass	Pass	14	Fail	Pass	Pass
3	Fail	Pass	Pass	15	Fail	Fail	Fail
4	Pass	Fail	Pass	16	Fail	Fail	Fail
5	Fail	Fail	Fail	17	Fail	Fail	Fail
6	Fail	Pass	Fail	18	Pass	Pass	Pass
7	Pass	Pass	Pass	19	Pass	Pass	Pass
8	Pass	Pass	Pass	20	Pass	Pass	Pass
9	Pass	Pass	Pass	21	Fail	Pass	Pass
10	Pass	Pass	Pass	22	Pass	Pass	Pass
11	Fail	Fail	Pass	23	Pass	Pass	Pass
12	Pass	Pass	Pass	24	Pass	Pass	Fail
				25	Fail	Fail	Fail

Summarize the data as shown below:-

		Auditor 2		
		Pass	Fail	Total
Auditor 1	Pass	14	1	15
	Fail	4	6	20
	Total	18	7	25

∴ Observed agreement
= Sum of (Pass, Pass & Fail, Fail) cases
= 14 + 6 = 20

Calculation of Expected count :-

$$\text{Expected Count of cell } (1,1) = \frac{\text{Row 1 sum} \times \text{column 1 sum}}{\text{Total}}$$

$$\text{Expected Count (Pass, Pass)} = \frac{15 \times 18}{25} = 10.8$$

Between Auditor Analysis:-

Expected Count table:-

		Auditor 2	
		Pass	Fail
Auditor 1	Pass	10.8	4.2
	Fail	7.2	2.8

∴ Expected agreement
= Sum of (Pass, Pass & Fail, Fail) cases
= 10.8 + 2.8
= 13.6

Calculate kappa,

$$K = \frac{\text{No. Observed Agreement} - \text{No. Expected Agreement}}{\text{Total Observation} - \text{No. Expected Agreement}}$$

$$= \frac{20 - 13.6}{25 - 13.6}$$

$$= 0.5614$$

Kappa	Strength of Agreement
<0.00	Poor or None
0.00 – 0.20	Slight
0.21 – 0.40	Fail
0.41 – 0.60	Moderate
0.61 – 0.80	Substantial
0.81 – 1.00	Almost perfect

ACCEPTANCE SAMPLING

Inspection of raw material, semi finished products and finished products are part of quality assurance activity.

Acceptance sampling:- A sampling procedure to accept or reject products based on inspection.

Example:- 1. A company receives a shipment of product from a vendor.

2. A sample is taken from the lot or shipment and certain quality characteristic of the units in the sample is inspected.

3. Based on the results of inspection, a decision is made either to accept or reject the lot or shipment (lot sentencing)

4. Accepted lots are put into production & rejected lots may be returned to the vendor or subjected some other lot disposition action.

Acceptance Sampling:-

1. The purpose is to accept or reject the lot not to estimate quality.
2. Donot provide any direct quality control, Simple accepts or rejects lots.
3. Even if all lots are of same quality, sampling will accept some lots and reject others.

Approaches for lot acceptance or rejection —

1. Accept with no inspection
2. 100% inspection
3. Acceptance Sampling

Advantages:-

1. Less expensive because there is less inspection.
2. Less handling of products hence reduced damage.
3. Number of personnel required inspection is less.
4. Often reduces inspection error.
5. Return of entire lots not just the defectives in the sample often provides stronger motivation to the vendor to improve quality.

Disadvantages:-

1. Risk of accepting bad lots and rejection of good lots.
2. Requires documentation of planning and documentation of sampling procedure.
3. Not much information about the quality of the vendor's process is gained.

$\boxed{\text{Single Sampling Plan}}$:- Defined by sample size n and acceptance number c.

- Procedure :-

1. Select n items at random from the lot containing N items
2. Inspect the n items in the sample and count the defective items d.
3. If $d > c$, reject the lot. Otherwise accept the lot of N items.

- Example :- $N = 10000$, $n = 89$, $c = 2$.

1. Select 89 items randomly from the 10000 items in the lot.
2. Inspect and count the number of defectives d.
3. If $d > 2$, reject the entire lot of 10000. Otherwise accept the lot.

- Operating Characteristic Curve :- Measures the performance of a sampling plan.

— Plot the probability of accepting a lot P_a versus the lot fraction defective p (lot quality).

— Shows the discriminatory power of sampling plan.

— Shows the chance that a lot submitted with certain fraction defective will be accepted or not.

Construction of OC Curve :-

1. Vary the lot fraction defective (p) from 0 to 1.
2. Compute the probability of acceptance of the lot, P_a.
3. Plot p versus P_a in the graph.

Ex :- Let p be the fraction defective in a lot. Let a sample of size n is selected and inspected from the lot. The probability of getting d defectives is

$$P\{d \text{ defectives}\} = \frac{n!}{d!\,(n-d)!}\, p^{d} \cdot (1-p)^{n-d}$$

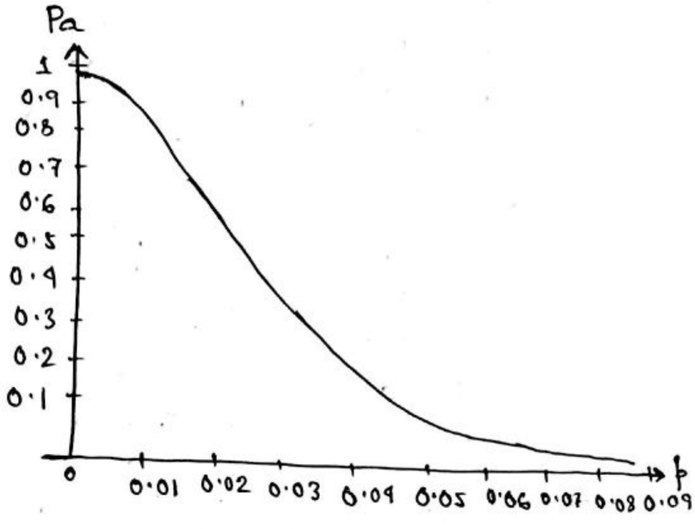
Lot is accepted if $d \le c$.

$$\text{Probability of acceptance} = P_a = P\{d \le c\} = \sum_{d=0}^{c} \frac{n!}{d!\,(n-d)!}\, p^{d} (1-p)^{n-d}$$

Example :- Suppose a product is shipped in lots of size $N = 5000$. The receiving inspection procedure used is single sampling plan with $n = 50$ and $c = 1$. Construct the OC curve of the plan? Compute AQL and LTPD for a producer's risk of $\alpha = 0.005$ and consumer's risk of $\beta = 0.1$?

<u>Solution:-</u> $N = 5000, \; n = 50, \; c = 1$

Fraction Defective (p)	Prob. of Acceptance (P_a)
0·005	0·9739
0·01	0·9106
0·02	0·7358
0·03	0·5553
0·04	0·4005
0·05	0·2794
0·06	0·1900
0·07	0·1265
0·08	0·0827
0·09	0·0532



→ OC curve :-

$\alpha = 0{\cdot}005$, $P_a = 0{\cdot}995$

$\beta = 0{\cdot}1$, $P_a = 0{\cdot}1$

Identify AQL (fraction defective p) corresponding to $P_a = 0{\cdot}995$ from graph.

Identify LTPD (fraction defective p) corresponding to $P_a = 0{\cdot}01$ from graph.

- <u>Average Quality Level (AQL):-</u>

A percent defective that is the base line requirement for the quality of the producer's product.

The producer could like to design a sampling plan such that there is a high prob. of accepting a lot that has a defect level less than or equal to the AQL.

<u>Producer's Risk:- (α)</u>

This is the prob. for a given (n, c) sampling plan, of rejecting a lot that has a defect level equal to the AQL.

The producer suffers when this occurs, because a lot with acceptance quality was rejected.

p	P_a
0	1
0·005	0·9739
0·0072	0·9494
0·01	0·9106
0·02	0·7358
0·03	0·5553
0·04	0·4005
0·05	0·2794
0·06	0·1900
0·07	0·1265
0·08	0·0827
0·09	0·0532
0·1	0·0338
0·11	0·0212
0·12	0·0131

AQL = 0·0072

LTPD = 0·013

- **Lot tolerance Percent Defective (LTPD):-** A designated high defect level that would be unacceptable to the customer. Consumer would like the sampling plan to have a low probability of accepting a lot with a defect level as high as the LTPD.

Consumer's Risk:- (β) This is the probability, for a given (n, c) sampling plan, of accepting a lot with a defect level equal to the LTPD.
The consumer suffers when this occurs, because a lot with unacceptable quality was accepted.

Rectified Inspection Program:-

The process of Screening (100%) inspection of rejected lot, reworking or replacing the defective items with good ones. This is usually done by the supplier or vendor.
The quality of outgoing lots with rectifying inspection will be better than that of incoming lot quality or the quality of the lot submitted for inspection.

- **Average Outgoing Quality (AOQ):-** Quality of the lot resulting from rectifying inspection.

Average value of lot quality that would be obtained over a long sequence of lots with rectifying inspection.
Suppose a lot of size N is subjected to acceptance sampling with a fraction defective p.
Let a sample of size n is randomly selected, inspected and counted the number of defectives d and all the defectives d will be replaced with good ones.
If $d > c$, then the lot is rejected.
If lot is rejected, then the remaining $N-n$ items are also inspected and all the defectives are replaced with good ones.
If lot is rejected, then all the N items in the lot will be good.
No defectives after the inspection.

Average Outgoing Quality (AOQ) Curve:- If the lot is accepted then $N-n$ items not inspected can contain defectives.
Since fraction defectives is p, the estimated number of defectives after inspection $= p(N-n)$.
If P_a is the chance of accepting the lot, then expected number of defectives after rectifying inspection $= P_a \cdot p(N-n)$

$$\text{Average Outgoing Quality, } AOQ = \frac{P_a p(N-n)}{N}$$

When N is large compared to n, then $AOQ \approx P_a p$.
Plot AOQ Vs. Incoming quality or fraction defective p.

Average Outgoing Quality Level (AOQL):- Highest outgoing fraction defective. Worst possible average quality result from rectifying inspection.

Average Total Inspection (ATI):- Let P_a be the probability of accepting a lot submitted for inspection. If the lot is accepted, then the number of items inspected is n.
If the lot is rejected, then the remaining $N-n$ items also inspected.
Then the chance of inspecting $N-n$ items (rejecting the lot) is $(1-P_a)$.

$$\text{Average Total Inspection, } ATI = n + (1-P_a)(N-n)$$

Average Total Inspection Curve:- Plot of ATI versum incoming quality or fraction defective of lot submitted for inspections p for specific N.

When incoming quality is very good then the lot will generally will be accepted and number of defectives is $N-n$ uninspected items also will be low.
Hence, outgoing quality also will be very good.

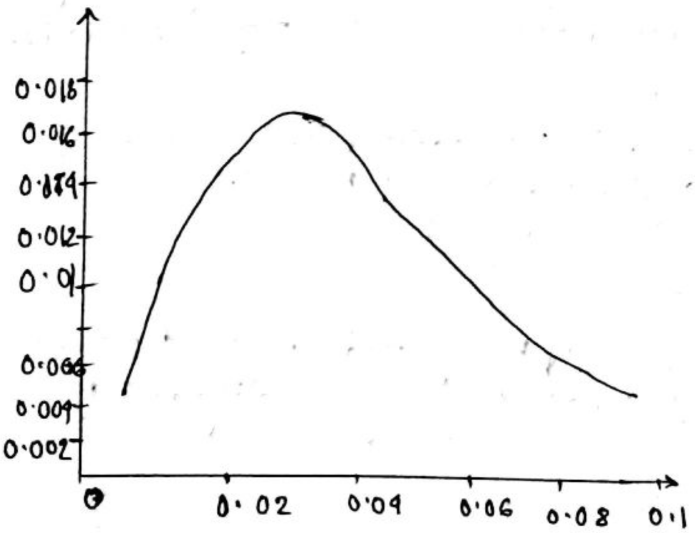
When incoming quality is very bad then the lots will generally be rejected and all defectives will be replaced with good ones. Hence outgoing quality will again be very good.

When incoming quality is average then the lot may or may not be accepted. Outgoing quality will be average and outgoing fraction defective will be reasonably high.

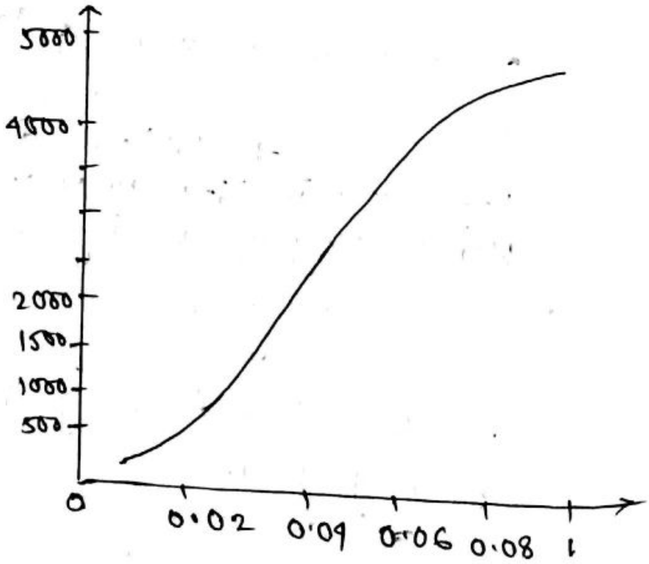
Ex. Suppose a product is shipped in lots of size N = 5000. The receiving inspection procedure used is single sampling plan with n = 50 and c = 1.
(i) Construct AOQ plot. (ii) Construct the ATI plot.

Solution:-

Fraction defective (p)	Prob. of Acceptance (Pa)	PaP	ATI
0.005	0.9739	0.0049	179
0.01	0.9106	0.0091	493
0.02	0.7358	0.00147	1358
0.03	0.5553	0.0167	2251
0.04	0.4005	0.016	3018
0.05	0.2794	0.014	3617
0.06	0.19	0.0114	4060
0.07	0.1265	0.0089	4374
0.08	0.0827	0.0066	4591
0.09	0.0532	0.0048	4737



-: AOQ curve:-



-: ATI Curve:-

Double Sampling Plan:- Decision to accept or reject the lot is taken based on two samples.

Procedure:- Let lot of size N is submitted for inspection.

1. Take a sample of size n_1 from the lot and count the defectives d_1 in the sample.
2. If $d_1 \le c_1$, the lot is accepted.
3. If $d_1 > c_2$, the lot is rejected.
4. If $c_1 < d_1 \le c_2$, then another sample of size n_2 is taken & count the defectives d_2 in sample 2.
5. If $d_1 + d_2 \le c_2$, then lot is accepted.
6. If $d_1 + d_2 > c_2$, then lot is rejected.

Notations:-
N: lot size
n_1: sample size on first sample
c_1: acceptance number of the first sample
n_2: sample size on second sample
c_2: acceptance number for both samples

Advantages:-

1. Reduces total amount of inspection, if decision is taken with first sample itself.
2. If lot is accepted or rejected based on first sample, the cost of inspection will be less.
3. Having a psychological advantage that a lot is given a second chance.

OC curve:- Measures the performance of a double sampling plan.
Plots the prob. of accepting a lot P_a versus the lot fraction defective p.
It shows that a lot submitted with certain fraction defective will be accepted or not.

$$P_a = P_a^{I} + P_a^{II}$$

P_a^{I} = Prob. of acceptance on the 1st sample = $P(d_1 \le c_1)$

$$= \sum_{d_1=0}^{c_1} \frac{n_1!}{d_1!\,(n_1-d_1)!} p^{d_1} (1-p)^{n_1-d_1}$$

P_a^{II} = Prob. of acceptance on the 2nd sample = $P[d_1 + d_2 \le c_2]$

where p is the fraction defective in the lot.

$$P_a^{I} = \sum_{d_1=0}^{c_1} \frac{\binom{Np}{d_1}\binom{N-Np}{n_1-d_1}}{\binom{N}{n_1}} \; ; \; P_a^{II} = \sum_{d_2=0}^{c_2-d_1} \sum_{d_1=c_1+1}^{c_2} \frac{\binom{Np}{d_1}\binom{N-Np}{n_1-d_1}\binom{Np-d_1}{d_2}\binom{N-n_1-Np+d_1}{n_2-d_2}}{\binom{N}{n_1}\binom{N-n_1}{n_2}}$$

Example:- Calculate the probability of acceptance for a double sampling plan for a lot of size $N = 5000$ with fraction defective $p = 0.05$. The parameters of the sampling plan are $n_1 = 50$, $c_1 = 1$, $n_2 = 100$, $c_2 = 4$?

Solution:- $P_a = P_a^{I} + P_a^{II}$

$$P_a^{I} = P(d_1 \le c_1) = 0.2794$$

$$P_a^{II} = P(c_1 < d_1 + d_2 \le c_2)$$

$$= P(d_1 = 2, d_2 \le 2) + P(d_1 = 3, d_2 \le 1) + P(d_1 = 4, d_2 = 0)$$

$$= P(d_1 = 2) \times P(d_2 \le 2) + P(d_1 = 3) P(d_2 \le 1) + P(d_1 = 4) P(d_2 = 0)$$

$$= (0.2611 \times 0.1183) + (0.2199 \times 0.0370) + (0.1360 \times 0.0059)$$

$$= 0.0308 + 0.0082 + 0.0008 = 0.0398$$

$$\therefore P_a = 0.3192$$

$$ASN = n_1 P_1 + (n_1 + n_2)(1 - P_1) \; ; \quad P_1 = P(d_1 \le c_1) + P(d_1 > c_2)$$

$$= P(d_1 \le 1) + P(d_1 > 4)$$

$$= 0.2794 + 0.1036$$

$$= 0.383$$

$$= 1 \times 0.383 + 5(1 - 0.383)$$

$$= 3.468$$

$$AOQ = \frac{[P_a^{I}(N - n_1) + P_a^{II}(N - n_1 - n_2)]\,p}{N} = 0.0157$$

$$ATI = n_1 P_a^{I} + (n_1 + n_2) P_a^{II} + N(1 - P_a)$$

$$= 3424$$

Average Sample Number (ASN):-

Number inspected = n_1, if lot is accepted or rejected in first sample

Number inspected = n_1+n_2, if a second sample is needed.

$$ASN = n_1 P_1 + (n_1+n_2)(1-P_1) \quad ; \text{ where,}$$

$$P_1 = P\{\text{lot is accepted in } 1^{st} \text{ sample}\} + P\{\text{lot is rejected in } 1^{st} \text{ sample}\}$$

$$= P(d_1 \le c_1) + P(d_1 > c_2)$$

ASN curve:- Compute ASN for various values of fraction defective p and plot ASN versus p.

Double Sampling plan – Rectifying Inspection :-

The rejected lots are screened 100%, all defectives are replaced with good ones and accept the lot.

Fraction defective p in the accepted lot after inspection.

Suppose a lot of size N with fraction defective p is submitted for inspection.

If the lot is rejected and subjected to rectifying inspection, then the fraction defective after inspection is 0.

If the lot is accepted in the first sample, then the uninspected $(N-n_1)$ may contain $(N-n_1)p$ defectives.

If the lot is accepted in the second sample, then the uninspected $(N-n_1-n_2)$ may contain $(N-n_1-n_2)p$ defectives.

The rejected lots are screened 100%, all defectives are replaced with good ones and accept the lot.

Average Outgoing Quality (AOQ):-

$$AOQ = \frac{\left[P_a^{I}(N-n_1) + P_a^{II}(N-n_1-n_2)\right]p}{N}$$

AOQ curve:- Plot of AOQ vs. various fraction defective p.

Average Total Inspection (ATI):-

1. If lot is rejected the entire lot N is inspected.
2. If lot is accepted in first sample, then n_1 items are inspected.
3. If lot is accepted in second sample, then n_1+n_2 items are inspected.

$$ATI = n_1 P_a^{I} + (n_1+n_2) P_a^{II} + N(1-P_a)$$

Multiple Sampling Plan:-

A multiple-sampling plan is an extension of double-sampling in that more than two samples can be required to sentence a lot. Example:-

Cumulative-Sampling Size	Acceptance No.	Rejection No.
20	0	3
40	1	4
60	3	5
80	5	7
100	8	9

Advantage:- The principal advantage of multiple-sampling plans is that the samples required at each stage are usually smaller than those in single or double sampling. So it is economical.

Procedure:- If, at the completion of any stage of sampling, the number of defective items is less than or equal to the acceptance number, the lot is accepted.

If, during any stage, the number of defectives equals or exceeds the rejection number, the lot is rejected; otherwise the next sample is taken.

The multiple-sampling procedure continues until the fifth sample is taken, at which time a lot disposition decision can be made.

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MILITARY STANDARD 105E Sampling Schemes

Developed during World War II
Same as ISO 2859 of International Organisation for Standardization (ISO).
Provides single, double and multiple sampling plans.

Type of Inspection:- 1. Normal Inspection
2. Tightened Inspection
3. Reduced Inspection

MIL STD 105E Sampling Schemes:-

Normal Inspection : Used at the start of the inspection activity
Tightened Inspection : When supplier's recent quality history has deteriorated.
Acceptance requirements under tightened inspection are more stringent.
Reduced Inspection : When supplier's recent quality history has been exceptionally good.
Sample size under reduced inspection is generally less than that under normal inspection.

Focal point is AQL.
Sample size is determined by lot size & inspection level.

Inspection Levels:- Level-II is designated as normal.
Level-I requires about one half the amount of inspection as that of level II and used when less discrimination is needed.
Level-III requires about twice as much inspection as that of level II and used when more discrimination is needed.

Special Inspection Levels:- S-1, S-2, S-3 and S-4
Used when small sample sizes are necessary and when large risks can be tolerated.

Switching Rules:-

Normal to tightened:- When two out of five lots have been rejected on original submission.

Tightened to Normal:- When five consecutive lots have been accepted on original submission.

Normal to reduced :- When all four of the following conditions are satisfied:
(a) Preceeding 10 lots have been in normal inspection & none of them has been rejected.
(b) Total number of defectives from the preceeding 10 lots is less than or equal to the applicable limit number specified in the standard.

(c) Production is at steady state, no issues like machine breakdown, material shortage, etc.

(d) Desirable by the authority.

Reduced to Normal:- When any of the following four conditions are satisfied:

(a) A lot is rejected

(b) When procedure terminated with neither acceptance or rejection criteria has been met.

(c) Production is irregular or delayed

(d) Other conditions warrant normal inspection.

Discontinue Inspection:- When 10 consecutive lots remain on tightened inspection. Action should be taken at supplier level to improve the quality.

MIL STD 105E – Procedure:-

1. Choose the AQL
2. Choose the inspection level (normally level II)
3. Determine lot size
4. Find appropriate sample code from sample size code letter table.
5. Determine appropriate type of sampling plan to use (single, double, multiple)
6. Enter the appropriate table to find the type of plan to be used.
7. Determine the corresponding tightened and reduced plans to be used when required.

Ex.1. A supplier ships a component in lots of size N = 3000. The AQL has been established for this product at 1%. Find the normal, tightened and reduced single sampling plans for this situation from MIL STD 105E, assuming that general inspection level II is appropriate?

Solution:- N = 3000, AQL = 1%, Level: II

Sample code level = K.

	c	n
Normal	3	125
Reduced	2	125
Tightened	1	50

Ex.2. A product is supplied in lots of size N = 10,000. The AQL has been specified at 0.10%. Find the normal, tightened and reduced single sampling plans from MIL STD 105E, assuming general inspection level II?

Solution:- N = 10,000

AQL = 0.1%

Sample code level = L

	n	c
Normal	200	0
Reduced	80	0
Tightened	200	0

DODGE-ROMIG Sampling Plans

Developed by H.F. Dodge and H.G. Romig.
Plans are available for single & double sampling.

Types of Sampling Plans:-

1. Plans for Lot Tolerance Percent Defective (LTPD) plans
2. Plans for Average Outgoing Quality Level (AOQL) plans

- LTPD Plans :- Plans are available for LTPD values of 0.5%, 1%, 2%, 3%, 4%, 5%, 7% and 10%.
  Knowledge of process average - average fraction defective (non-conforming) of the incoming product is necessary.

Procedure:-

1. Choose required LTPD
2. Determine lot size
3. Determine process average (fraction non-conforming)
4. Based on the lot size and process average, choose the sample size n and acceptance number c from the corresponding LTPD table.

Ex. A product is shipped in lots of size N = 2000. Find a Dodge-Romig single sampling plan for which the LTPD = 1%, assuming that the process average is 0.25% defective?

Solution:- N = 2000.
LTPD = 5%.
Process average = 0.25%
$n = 75$, $c = 1$, AOQL = 1.0

- AOQL Plans:- Plans are available for AOQL values of 0.1%, 0.25%, 0.5%, 0.75%, 1%, 1.5%, 2%, 2.5%, 3%, 4%, 5%, 7% and 10%.
Knowledge of process average - average fraction defective of the incoming product is necessary.
Only applicable when rectifying inspection is used.

Procedures:-

1. Choose required AOQL
2. Determine lot size
3. Determine process average (fraction non-conforming)
4. Based on the lot size and process average, choose the sample size n and acceptable number c from the corresponding AOQL table.

Ex. A company wish to find a single sampling plan for a situation where lots of size N = 8000 are shipped from a supplier. The supplier's process operates at a fall out level of 0.50% defective. The company want the AOQL from inspection activity to be 3%. Suggest the appropriate Dodge-Romig plan?

Solution:- N = 8000. P = process average = 0.50%.
AOQL = 2.0%

$n = 55$, $c = 2$, from Dodge-Romig plan.

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Pre-Control Chart :- (Setup Approval Chart)

- A technique used to detect shifts or upsets in the process which will result in producing non-conforming products or parts.
- Conventional control charts are used to detect shifts in process due to assignable causes or to ensure stability of the process.
- Based on Normal Distribution.
- Useful when $C_p \geq 1$, and $C_p = C_{pK}$.
- It is easy to reset the process or adjust the process mean.

The pre-control chart has two additional limits called Upper and Lower pre-control limits (UPCL and LPCL).

Let X be normally distributed quality characteristic with process mean μ and process standard deviation σ. Then

$$UCL = \mu + 3\sigma$$
$$UPCL = \mu + 1.5\sigma$$
$$CL = \mu$$
$$LPCL = \mu - 1.5\sigma$$
$$LCL = \mu - 3\sigma$$

Approximately, 86% of process output will lie inside $\mu \pm 1.5\sigma$ limits and 7% will lie in each region between PC and Control limits.

<u>Special Case</u>:- $C_p = C_{pK} = 1$.

$$UCL = USL$$
$$UPCL = \frac{USL + \mu}{2}$$
$$CL = \mu$$
$$LPCL = \frac{\mu + LSL}{2}$$
$$LCL = LSL$$

- <u>Working Rules</u>:-
  1. Start the process and check 1st product or item. If the 1st item is outside the control limits, reset the process.
  2. If an item is inside the control limits but outside the PC line, then check the next item.
  3. If the 2nd item is also outside the same PC line, reset the process.
  4. If 2nd item is inside the PC line then continue.
  5. If one item is outside a PC line & the next item is outside the other PC line, then the process variability is out of control. Investigate and take necessary corrective actions.

6. When five consecutive points are inside the PC line, shift to sampling.
7. During sampling donot adjust the process unless an item fall outside PC lines.
8. If the point is outside control charts, reset the process and proceed as in step 6.
9. If the point is within control limit but outside the PC line, then check the next item as in step 4.
10. When a process is reset, five consecutive items must fall within PC lines before changing to sampling.

Control Charts for multi stream process (MSP):—

Data at any point of time consists of measurements from several sources or streams, sources of streams are assumed to be identical.

It is possible to monitor and adjust each of the streams individeually or in small groups.

Use group control chart:– Plot only the largest and smallest $\overline{x}$ on $\overline{x}$ chart and only largest range is plotted on R chart.

Out of control cases:– 1. Output of one stream (or a few streams) has shifted off target.

2. Output of all streams has shifted off target.

Group Control Charts:- Suppose that the process has s streams and each stream has same target value and inherent variability. Distribution of measurement is well approximated by the normal. Sampling is prefered. Suppose sample size is n.

This process is repeated until m subgroups of samples have been taken.

So, $\overline{\overline{x}} = \dfrac{\Sigma\Sigma x_{ij}}{m \times s}$, $\overline{R} = \dfrac{\Sigma\Sigma R_{ij}}{m \times s}$.

$UCL = \overline{\overline{x}} + A_2\overline{R}$, $LCL = \overline{\overline{x}} - A_2\overline{R}$ for the $\overline{x}$ chart

$UCL = D_4\overline{R}$, $LCL = D_3\overline{R}$ for the R chart.

It is useful to examine the stream numbers on the chart.

ADVANCED CONTROL CHARTS

- **Cumulative Sum Control chart (CUSUM chart) :—**

  - Used when small shifts are important.
  - Uses all informations in the sequence of sample values
  - Highly effective for subgroup of size of $n = 1$.
  - Ensures the quality characteristics will be always on or close to the target.
  - Mostly used in chemical and process industries where subgroup size is often 1.
  - Highly suitable for modern industries with automated inspection or on line control.
  - Plots the cumulative sum of the deviation of sample values from the target value.

Working Rules:— 1. If the process shifts or drifts off the target value, then cusum will signal.

2. An adjustment is made to some control factors to bring the process back on target.

CUSUM Control Chart: Logic :—

1. Let the quality characteristic x has a normal distribution with mean μ and standard deviation σ.
2. Let μ_0 be the target value of x.
3. Accumulate the deviation from μ_0 that are above target with one statistic C^+.
4. Accumulate the deviation from μ_0 that are below target with one statistic C^-.

$$C_i^+ = \max\left[0,\ x_i - (\mu_0 + K) + C_{i-1}^+\right]$$

$$C_i^- = \max\left[0,\ (\mu_0 - K) - x_i + C_{i-1}^-\right]$$

where $C_0^+, C_0^- = 0$.

5. Reference value or allowance value K is chosen halfway between target value μ_0 and maximum allowed shift value μ_1, $K = \dfrac{|\mu_0 - \mu_1|}{2}$
6. Plot C^+ or C^- on the chart.
7. If either C^+ or C^- is beyond the decision interval H, reset the process $H = 5\sigma$.

Example:- The data on molecular weight taken hourly from a chemical process is given below:

Sample	x	sample	x
1	1045	11	1139
2	1055	12	1169
3	1037	13	1151
4	1064	14	1128
5	1095	15	1238
6	1008	16	1125
7	1050	17	1163
8	1087	18	1188
9	1125	19	1146
10	1146	20	1167

The target value of molecular weight is 1050. Design a cusum to quickly detect a shift of about 1.5σ.

Solution:-

Sample	x	Moving Range
1	1045	—
2	1055	10
3	1037	18
4	1064	27
5	1095	31
6	1008	87
7	1050	42
8	1087	37
9	1125	38
10	1146	21
11	1139	7
12	1169	30
13	1151	18
14	1128	23
15	1238	10
16	1125	13
17	1163	38
18	1188	25
19	1146	42
20	1167	21

MR Chart	
$\overline{MR}$	28.316
UCL	92.508
CL	28.316
LCL	0.000
σ	25.10

Compute μ_1, K & H :-

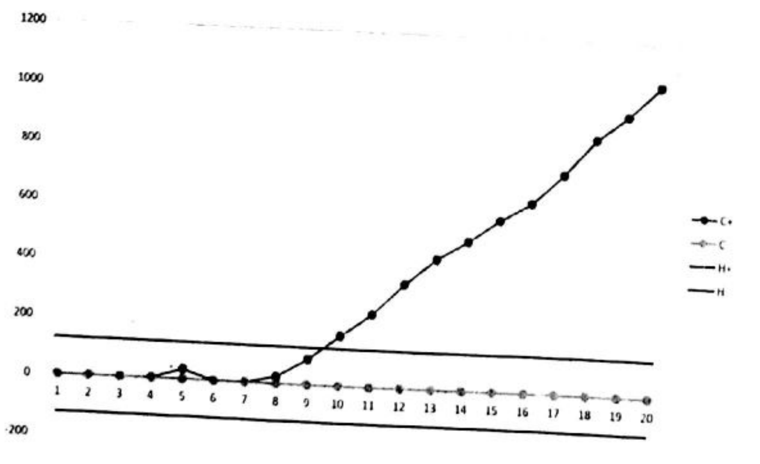
σ	25.10
μ_0	1050
μ_1	1075.10
K	12.55
H	125.5

Sample	x	$x_i-(\mu_0+K)+C_{i-1}^+$	C^+
1	1045	-17.5500	0.00
2	1055	-7.5500	0.00
3	1037	-25.5500	0.00
4	1064	1.4500	1.45
5	1095	33.9000	33.90
6	1008	-20.6500	0.00
7	1050	-12.5500	0.00
8	1087	24.4500	24.45
9	1125	86.9000	86.90
10	1146	170.3500	170.35
11	1139	246.8000	246.80
12	1169	353.2500	353.25
13	1151	441.7000	441.70
14	1128	507.1500	507.15
15	1138	582.6000	582.60
16	1125	645.0500	645.05
17	1163	745.5000	745.50
18	1188	870.9500	870.95
19	1146	954.4000	954.40
20	1167	1058.8500	1058.85

Sample	x	$(\mu_0-K)-x+C_{i-1}^-$	C^-
1	1045	-7.55	0.00
2	1055	-17.55	0.00
3	1037	0.45	0.45
4	1064	-26.10	0.00
5	1095	-57.55	0.00
6	1008	29.45	29.45
7	1050	16.90	16.90
8	1087	-32.65	0.00
9	1125	-87.55	0.00
10	1146	-108.55	0.00
11	1139	-101.55	0.00
12	1169	-131.55	0.00
13	1151	-113.55	0.00
14	1128	-90.55	0.00
15	1138	-100.55	0.00
16	1125	-87.55	0.00
17	1163	-125.55	0.00
18	1188	-150.55	0.00
19	1146	-108.55	0.00
20	1167	-129.55	0.00

Cumulative sum control chart - Example

Step 4: Plot C+, C^- & H on cusum chart



Note:- Except CUSUM chart all other charts have memory loss property.

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- **Exponentially Weighted Moving Average Control chart :-**
  - Very effective against small process shifts.
  - Uses all information in the sequence of ~~small~~ sample values.
  - Highly effective for subgroup size of $n = 1$.
  - Ensures the quality characteristic will always be on or close to the target.
  - Mostly used in chemical and process industries where sub-group size is often 1.
  - Highly suitable for modern industries with automated inspection or on line control.
  - EWMA is used extensively in time series modelling & forecasting.

EWMA Control Chart : Logic :-

Let x_i are independent random variables with variance σ^2, then exponentially weighted moving average z_i is

$$z_i = \lambda x_i + (1-\lambda) z_{i-1}.$$

where, $0 \le \lambda \le 1$, $z_0 = \mu_0$, the process target variance of z_i,

$$\sigma_{z_i}^2 = \sigma^2 \left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]$$

In EWMA charts, z_i is plotted against sample number i.

$$UCL = \mu_0 + L\sigma \sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}$$

$$CL = \mu_0$$

$$LCL = \mu_0 - L\sigma \sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}$$

Generally, $\lambda = 0.2$ and $L = 2.962 \simeq 3$.

- **Slopping Control Charts:-** Many process are subject to tool wear. As tool wears out, there will be a drift or trend in the process mean.
  As $\overline{X}$ chart will show the process out of control and assignable cause is tool wear.
  Tracking this assignable cause or replacing the tool very often is expensive.
  If the process highly capable ($C_p, C_{pk} > 1$) then slopping control chart can be used to detect other assignable causes and the tool can be used till its useful life or till it produce non-conforming units/products.

- **Assumptions:-** — The process is highly capable.
  — The tool wears out more or less at uniform rate.
  — Set the process such that the mean is close to LSL.
  — Collect sample data on quality characteristic at regular

- **Steps:-** — Compute $\overline{X}$ and R
  — Construct R chart and ensure the process variation is in control.
  — Estimate the process standard deviation σ.
  — Take the time (h) corresponding to middle sample as 0 so that there will be equal number of samples on either side of zero.
  — Plot $\overline{X}$ versus h and Fit a line.
  — Construct slopping control chart with fitted line as CL and fitted value $\pm A_2\overline{R}$ as control limits.
  — Set the process initially with $\overline{X} = LSL + L\sigma$
  Reset the process when $\overline{X}$ reaches $USL - L\sigma$.

- **Control Charts for short production runs:-**

Use deviations from nominal or target value instead of measured variable on the control chart.

Also known as DNOM (Deviations from Nominal) chart.

- **Steps:-** 1. Let m_i, $i=1,2,\dots,k$ be the values of quality characteristics with target values t_i.
  Compute $x_i = m_i - t_i$, $i=1,2,\dots,k$.

  2. Compute $\overline{X}$ and Range for x_i.

  3. Construct $\overline{X}$ and R chart.

Example of EWMA Control Chart:-

Exponentially Weighted Moving Average Control Chart

Example: Bath concentration are measured hourly in a chemical process. Data (in ppm) for the last 32 hours shown below. The process target is $\mu_0 = 175$ ppm

Sample No.	Data	Sample No.	Data	Sample No.	Data	Sample No.	Data
1	160	9	180	17	190	25	206
2	158	10	195	18	189	26	210
3	150	11	179	19	185	27	216
4	151	12	184	20	182	28	212
5	153	13	175	21	181	29	211
6	154	14	192	22	180	30	202
7	158	15	186	23	183	31	205
8	162	16	197	24	186	32	197

a. Estimate the process standard deviation?
b. Set up EWMA control chart

Set up MR chart & Estimate Process sigma:-

Sample No	Data	MR	Sample No	Data	MR
1	160		17	190	7
2	158	2	18	189	1
3	150	8	19	185	4
4	151	1	20	182	3
5	153	2	21	181	1
6	154	1	22	180	1
7	158	4	23	183	3
8	162	4	24	186	3
9	180	18	25	206	20
10	195	9	26	210	4
11	179	16	27	216	6
12	184	5	28	212	4
13	175	9	29	211	1
14	192	17	30	202	9
15	186	6	31	205	3
16	197	11	32	197	8

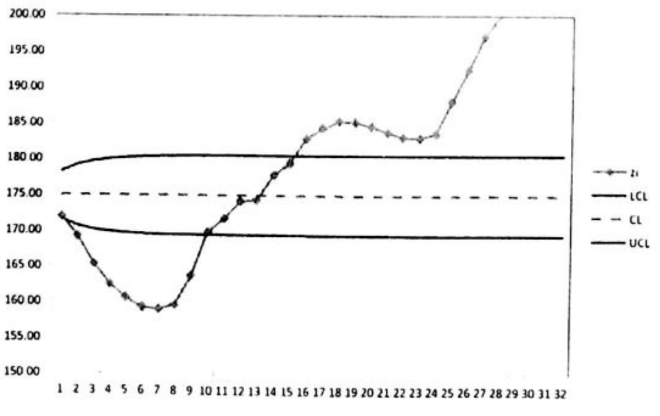
UCL = 20.761
CL = 6.355
LCL = 0.000
σ = 5.634

Choose λ and L, $\lambda = 0.2$, $\mu_0 = 175$ and $L = 2.962$

Compute z_i's and control limits

Sample No	Data	zi	SD	LCL	UCL	Sample No	Data	zi	SD	LCL	UCL
1	160	172.00	0.20	171.66	178.34	17	190	184.41	0.33	169.44	180.56
2	158	169.20	0.26	170.73	179.27	18	189	185.33	0.33	169.44	180.56
3	150	165.36	0.29	170.22	179.78	19	185	185.26	0.33	169.44	180.56
4	151	162.49	0.30	169.93	180.07	20	182	184.61	0.33	169.44	180.56
5	153	160.59	0.31	169.74	180.26	21	181	183.89	0.33	169.44	180.56
6	154	159.27	0.32	169.63	180.37	22	180	183.11	0.33	169.44	180.56
7	158	159.02	0.33	169.56	180.44	23	183	183.09	0.33	169.44	180.56
8	162	159.61	0.33	169.52	180.48	24	186	183.67	0.33	169.44	180.56
9	180	163.69	0.33	169.49	180.51	25	206	188.14	0.33	169.44	180.56
10	195	169.95	0.33	169.47	180.53	26	210	192.51	0.33	169.44	180.56
11	179	171.76	0.33	169.46	180.54	27	216	197.21	0.33	169.44	180.56
12	184	174.21	0.33	169.45	180.55	28	212	200.17	0.33	169.44	180.56
13	175	174.37	0.33	169.45	180.55	29	211	202.33	0.33	169.44	180.56
14	192	177.89	0.33	169.44	180.56	30	202	202.27	0.33	169.44	180.56
15	186	179.52	0.33	169.44	180.56	31	205	202.81	0.33	169.44	180.56
16	197	183.01	0.33	169.44	180.56	32	197	201.65	0.33	169.44	180.56

Step 4: Construct EWMA control chart



Example of Slopping Control Charts :-

The specifications on a critical dimension of a process subject to tool wear is 0.644 ± 0.004. Thirteen samples of subgroup size 5 are collected at every half an hour interval and the xbar and range computed. The data is given below.

1. Construct a slopping control chart to monitor the process
2. Estimate the duration and number of samples after which the process need to be reset?
3. How much should be the reset value?

Sample	xbar	R	Time
1	0.6417	0.0011	-6
2	0.6418	0.0016	-5
3	0.6424	0.001	-4
4	0.6431	0.0015	-3
5	0.6433	0.0009	-2
6	0.6437	0.001	-1
7	0.6433	0.0014	0
8	0.6436	0.0004	1
9	0.6441	0.0006	2
10	0.6444	0.0011	3
11	0.6456	0.0009	4
12	0.6457	0.0007	5
13	0.6457	0.0009	6

$\overline{\overline{X}} = 0.6437$

$\overline{R} = 0.0010$

R Chart :-

UCL = 0.0021

CL = 0.0010

LCL = 0.00

Capability Analysis:-

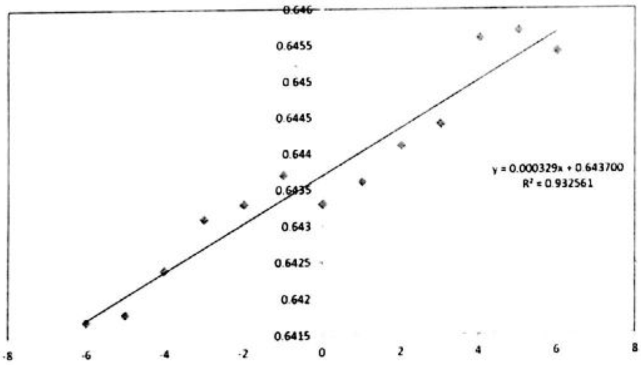
Mean = 0.6437

SD = 0.0004

USL = 0.6480

LSL = 0.6400

$C_p = 3.078 > 1.$



xbar = 0.6437+ 0.000329h

Initial set up point :- $\overline{X}_{initial} = LSL + L\sigma = LSL + 4\sigma = 0.6417$

Reset point:- $\overline{X}_{final} = USL - 4\sigma = 0.6463$

Interval between resets :- $(\overline{X}_{final} - \overline{X}_{initial}) / \text{slope}$

$$= \frac{0.6463 - 0.6417}{0.000329} = 14$$

Conclusion:- Reset the process after 14 subgroups to initial set up point of $\overline{X} = 0.6417$. Since sampling frequency is half an hour reset the process at every 7 hrs.

Time h correspond to initial set up point, $h = \frac{\overline{X}_{initial} - a}{b} = \frac{0.6417 - 0.6437}{0.000329}$

$= -6$

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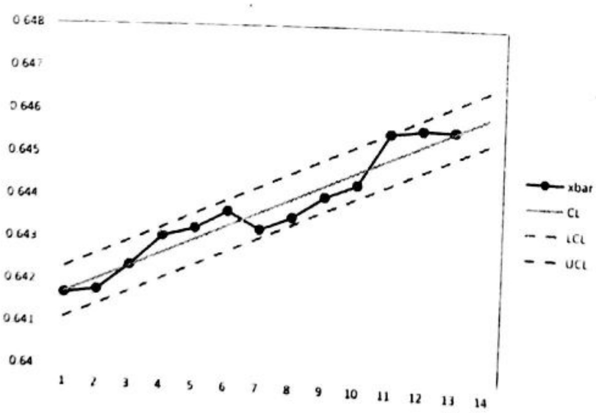
Time h correspond to Reset point

$$h = \frac{\bar{x}_{final} - a}{b} = \frac{0.6463 - 0.6437}{0.000329} = 7 .$$

Time	Sample	xbar	CL (Model)	LCL	UCL
-6	1	0.6417	0.6417	0.6411	0.6423
-5	2	0.6418	0.6421	0.6415	0.6427
-4	3	0.6424	0.6424	0.6418	0.6430
-3	4	0.6431	0.6427	0.6421	0.6433
-2	5	0.6433	0.6430	0.6424	0.6436
-1	6	0.6437	0.6434	0.6428	0.6440
0	7	0.6433	0.6437	0.6431	0.6443
1	8	0.6436	0.6440	0.6434	0.6446
2	9	0.6441	0.6444	0.6438	0.6450
3	10	0.6444	0.6447	0.6441	0.6453
4	11	0.6456	0.6450	0.6444	0.6456
5	12	0.6457	0.6453	0.6447	0.6459
6	13	0.6457	0.6457	0.6451	0.6463
7	14		0.6460	0.6454	0.6466



Example of Short Production Runs:-

Example

Use the following data to set up short run xbar and R charts using DNOM approach. The target dimensions for each part are $T_A = 100$, $T_B = 60$, $T_C = 75$ and $T_D = 50$

Sample	Part Type	m1	m2	m3	Sample	Part Type	m1	m2	m3
1	A	105	102	103	11	C	77	75	74
2	A	101	98	100	12	C	75	72	79
3	A	103	100	99	13	C	74	75	77
4	A	101	104	97	14	C	73	76	75
5	A	106	102	100	15	D	50	51	49
6	B	57	60	59	16	D	46	50	50
7	B	61	64	63	17	D	51	46	50
8	B	60	58	62	18	D	49	50	53
9	C	73	75	77	19	D	50	52	51
10	C	78	75	76	20	D	53	51	50

Compute $x_i = m_i - t_i$,

Sample	Part Type	m1	m2	m3	Sample	Part Type	m1	m2	m3
1	A	5	2	3	11	C	2	0	-1
2	A	1	-2	0	12	C	0	-3	4
3	A	3	0	-1	13	C	-1	0	2
4	A	1	4	-3	14	C	-2	1	0
5	A	6	2	0	15	D	0	1	-1
6	B	-3	0	-1	16	D	-4	0	0
7	B	1	4	3	17	D	1	-4	0
8	B	0	-2	2	18	D	-1	0	3
9	C	-2	0	2	19	D	0	2	1
10	C	3	0	1	20	D	3	1	0

Control charts for short production runs

Compute xbar and R

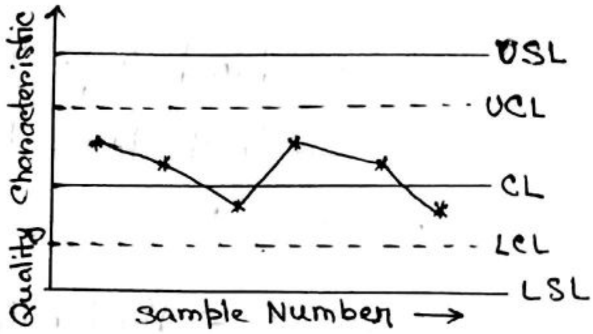
Sample	m1	m2	m3	xbar	Range	Sample	m1	m2	m3	xbar	Range
1	5	2	3	3.333	3	11	2	0	-1	0.333	3
2	1	-2	0	-0.333	3	12	0	-3	4	0.333	7
3	3	0	-1	0.667	4	13	-1	0	2	0.333	3
4	1	4	-3	0.667	7	14	-2	1	0	-0.333	3
5	6	2	0	2.667	6	15	0	1	-1	0.000	2
6	-3	0	-1	-1.333	3	16	-4	0	0	-1.333	4
7	1	4	3	2.667	3	17	1	-4	0	-1.000	5
8	0	-2	2	0.000	4	18	-1	0	3	0.667	4
9	-2	0	2	0.000	4	19	0	2	1	1.000	2
10	3	0	1	1.333	3	20	3	1	0	1.333	3

Statistical Process Control 2

— SPC used to maintain a process at a particular level of performance where the process will at least meet specifications.

— Control Chart is a technique for defect prevention.

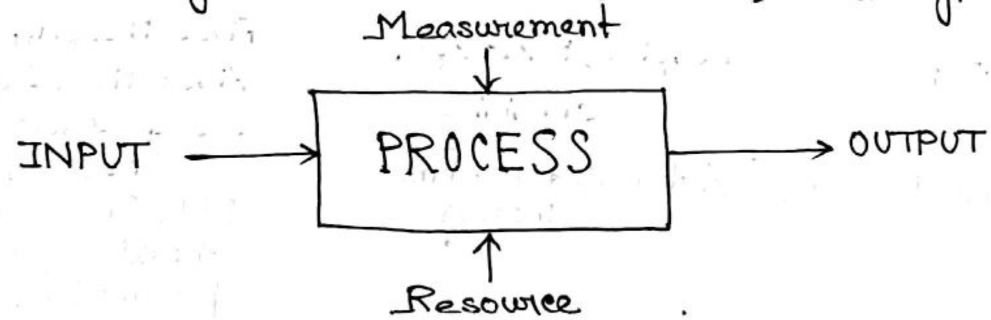
— Defect is a particular product characteristics not meeting specifications. Defects denote the points below LSL and above USL.



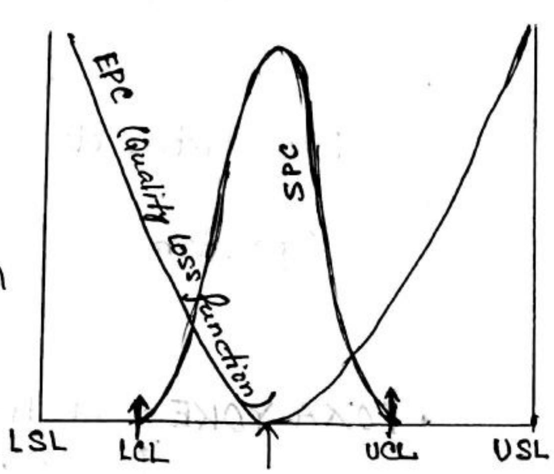
Inspection

Measurement
[Useful for discrete or variable type data]

Perception (Expert's advice)
[Useful for Attribute or Discrete type Data]



- A process is a sequence of activities converting input into output.
- SPC is implemented for stable and capable process where sometimes assignable causes may come and we may control it
- EPC : Engineering Process Control ; here no action is taken as long as parts are within specifications.
- Quality means hitting the target with minimum variability around it.
- Capability means how well the process is meeting the tolerance levels.
- Process Capability is the total variation due to chance causes.
- Assignable Cause has prob. of occurance is very low but it occurs.



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Schemes of Process Control:-

- Set up Approval / First piece Inspection
- In process Inspection
  - 100%
  - Sampling
  - SPC
- Final Inspection
- Last piece Inspection

Levels of Process Control?

Quality = Variability $\longrightarrow$ Causes

Causes $\uparrow$ Control

Causes → Prevent the cause from occuring. (POKA YOKE)

Causes → Detect the cause as and when it appears.

Dominance System - Process Control

Dominated by	Process	Control
Set up	Stamping, Photocopy, Machining	First Piece Approval (FPA) First Piece Inspection (FPI) Set up Control chart (Pre-control Chart)
Machine Parameter	Automated machine Process	FPI, Freq. check, checking of process parameter.
Raw material	Any product produced by using natural raw material	Incoming inspection of critical parameter
Machine	Machining Process — Turning, Press Parts	FPI, Control Chart
Process Qualification	Welding, Painting, Riveting, Plating, Heat treatment	Monitor the process parameter and control them (Operation Qualification)
Tools, Fixture, etc	Press Operation	FPA, LPI, Tool Maintainance, SPC
Operator	Manual (Assembly operations)	Operator training & Qualification, POKA YOKE (Mistake Proofing)

- **POKA YOKE:** At product design; At process design; During inspection in the same process; Design next process usage

- Implementing Control Chart :-

1. Calculate Process Capability
2. Process Monitoring (control)

Calculation of Process Capability :-

— Select the product characteristic

— collect data

→ collect continuous chronological data, then divide it into subgroups

→ collect data in subgroup format with adequate time interval between them.

— Check for normality by using Normality Probability Paper.

— Carry out Control limit calculation

— Check stability of the process.

— If found stable, calculate Process Capability (C_p, C_{pK})

$$C_p = \frac{USL - LSL}{6s}, \quad \hat{s} = \frac{\overline{R}}{d_2} \text{ or } \frac{\overline{MR}}{d_2}$$

$$C_{pK} = \min\left\{\frac{\overline{x} - LSL}{3s}, \frac{USL - \overline{x}}{3s}\right\}$$

- **Ex.** Show that $C_{pK} \le C_p$ and illustrate equality case.

Solution:-

$$C_p = \frac{USL - LSL}{6s} = \frac{USL - \overline{X} + \overline{X} - LSL}{6s}$$

$$= \frac{1}{2}\left[\frac{USL - \overline{X}}{3s} + \frac{\overline{X} - LSL}{3s}\right]$$

$$\geq \min\left\{\frac{USL - \overline{X}}{3s}, \frac{\overline{X} - LSL}{3s}\right\} = C_{pK}$$

When target is at centre, $\mu = \frac{USL + LSL}{2}$.

So, $C_{pK} = \min\left\{\frac{USL - \mu}{3s}, \frac{\mu - LSL}{3s}\right\}$

$$= \min\left\{\frac{USL - LSL}{6s}, \frac{USL - LSL}{6s}\right\}, \text{ putting value of } \mu.$$

$$= \frac{USL - LSL}{6s} = C_p.$$

Ex. Derive a strategy to achieve a C_{pk} value of 1.33.

Sol. To achieve $C_{pk} = 1.33$, we should have $C_p = 1.33$.

$$\frac{\text{Tolerance}}{6s} = 1.33$$

$$\frac{6s}{T} = 0.75$$

$T = 0.1$ mm, 75% of $T = 0.075$ mm

$$6s = 0.075 \text{ mm (max)}$$

$$\therefore s_{max} = 0.0125$$

then $\bar{x} = 5$

$$\frac{\bar{x} - LSL}{3s} = 1.33 \Rightarrow \bar{x} - LSL = 4s$$

$$\frac{USL - \bar{x}}{3s} = 1.33 \Rightarrow USL - \bar{x} = 4s$$

So, $\bar{x} - 4s > LSL$ and $\bar{x} + 4s < USL \ \forall (\bar{x}, s)$

Ex. Prove that $C_{pk} = (1-K)C_p$, where $K = \dfrac{\left|\frac{USL+LSL}{2} - \mu\right|}{\frac{USL-LSL}{2}}$;

Sol. For $T > \mu$; $T = \dfrac{USL+LSL}{2}$;

$|T - \mu| = T - \mu$.

$$K = \frac{\frac{USL+LSL}{2} - \mu}{\frac{USL-LSL}{2}}$$

$$\Rightarrow K = \frac{\left(\frac{USL+LSL-2\mu}{6\sigma}\right)}{\frac{USL-LSL}{6\sigma}}$$

$$\therefore K \cdot C_p = \frac{(USL-\mu) - (\mu - LSL)}{6\sigma} = \frac{|(USL-\mu) - (\mu - LSL)|}{6\sigma};$$

since $KC_p > 0$

Note:- Control chart always gives short term capability.

Q. Why rational subgrouping used in control chart?

$$\Rightarrow KC_p = \frac{|(USL-\mu)-(\mu-LSL)|}{6\sigma} + \left[\frac{(USL-\mu)}{6\sigma} + \frac{(\mu-LSL)}{6\sigma}\right] - \left[\frac{(USL-\mu)}{6\sigma} + \frac{(\mu-LSL)}{6\sigma}\right]$$

$$\Rightarrow \left\{ \left(\frac{USL-\mu}{6\sigma}\right) + \left(\frac{\mu-LSL}{6\sigma}\right) - \left|\frac{(USL-\mu)}{6\sigma} - \frac{(\mu-LSL)}{6\sigma}\right| \right\}$$

$$= \left(\frac{USL-\mu}{6\sigma} + \frac{\mu-LSL}{6\sigma}\right) - kC_p$$

$$\Rightarrow \frac{1}{2}\left\{\left(\frac{USL-\mu}{3\sigma}\right) + \left(\frac{\mu-LSL}{3\sigma}\right) - \left|\frac{(USL-\mu)}{3\sigma} - \frac{(\mu-LSL)}{3\sigma}\right|\right\}$$

$$= \frac{USL-LSL}{6\sigma} - kC_p$$

$$\Rightarrow \min\left\{\left(\frac{USL-\mu}{3\sigma}\right), \left(\frac{\mu-LSL}{3\sigma}\right)\right\} = C_p - kC_p$$

$$\Rightarrow C_{pk} = (1-k)C_p.$$

Same can be shown for $T < \mu$. Hence the proof.

- Use Control Chart for monitoring :-

1. Select the process.
2. Select the product/product characteristic.
3. Select the most appropriate control charts to implement.
4. Carry out brainstorming to identify the possible/likely assignable causes and their counter measures (OCAP).
5. Collect data.
6. Carry out initial study and capability study.
7. If process found both stable and capable, use the control limit for process control in future.

Questions:- 1. Difference between Process Capability and Machine capability and how to calculate.

2. Difference between long term and short term capability.

3. Find out different formulae of process capability index and present them with example when
(i) Target at centre
(ii) Target not at centre.

Taguchi Capability Index

The process capability ratio C_{pk} was initially developed because C_p does not adequately deal with the case of a process with mean μ that is not centered between the specification limits. However, C_{pk} alone is still an inadequate measure of centering. For any fixed value of μ in the interval from LSL to USL, C_{pk} depends inversely on σ and becomes large as σ approaches zero. This characteristic can make C_{pk} unsuitabe measure of centering.

$$C_p = \frac{USL-LSL}{6\sigma} = \frac{1}{2}\cdot\frac{USL-LSL}{3\sigma} = \frac{d}{3\sigma}$$

$$C_{pk} = \frac{\min\{USL-\mu, \mu-LSL\}}{3\sigma} = \frac{d-|\mu-T|}{3\sigma}$$

where, $d = \frac{USL-LSL}{2}$, $T = \frac{USL+LSL}{2}$.

$$\tau^2 = E(X-T)^2 = E(X-\mu)^2 + (\mu-T)^2 = \sigma^2 + (\mu-T)^2$$

Define $$C_{pm} = \frac{USL-LSL}{6\tau} = \frac{USL-LSL}{6\sqrt{\sigma^2+(\mu-T)^2}}$$

$$= \frac{C_p}{\sqrt{1+\xi^2}}, \quad \xi = \frac{\mu-T}{\sigma}$$

$C_{pk} = 0$ when $\mu \geq USL$ and $\mu \leq LSL$

$C_{pm} \xrightarrow{a} 0$ as $|\mu-T| \to \infty$.

$$\therefore C_{pm} < \frac{USL-LSL}{6|\mu-T|}$$

$C_{pm} = 1$, necessary condition is $|\mu-T| < \frac{USL-LSL}{6}$

$C_{pm} = 1 \Rightarrow \mu$ lies with in the middle third of specification range.

$C_{pm} = 4/3 \Rightarrow \mu$ lies within the middle fourth of the specification range.

These statements provide a concrete interpretation of C_{pm} as a measure of process centering.

Group Control Chart

Example:- A machine has four heads. Samples of $n=3$ units are selected from each head, and the R and $\bar{x}$ values for an important quality characteristic are computed. Set up group control chart for this process.

[Page:- 488 10.4. Montgomery]

Sample No	Head 1 $\bar{x}$	Head 1 R	Head 2 $\bar{x}$	Head 2 R	Head 3 $\bar{x}$	Head 3 R	Head 4 $\bar{x}$	Head 4 R
1	53	2	54	1	56	2	55	3
2	51	1	55	2	54	4	54	4
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								

* **Rational Subgrouping:-** It is a method of collecting data where variation within subgroups is minimum but variation between subgroups is maximum.

Solution:- Given $s = 4$.

R-Chart:- $\bar{R} = \dfrac{\sum\sum R_{ij}}{20 \times s}$; s = no. of stream = 4.
total sample number = 20
sample size = $n = 3$

$= \dfrac{187}{20 \times 4}$

$= 2.3375$

$UCL_{\bar{R}} = D_4 \bar{R} = 2.574 \times 2.3375$, for $n = 3$; $D_4 = 2.574$
$= 6.016$

$CL_{\bar{R}} = 2.3375 = \bar{R}$

$LCL_{\bar{R}} = 0 = D_3 \bar{R}$

The minimum and maximum of R for all the samples fall within the control limits, so process is in control.

$\bar{X}$ chart:- $\bar{\bar{X}} = \dfrac{\sum\sum \bar{X}_{ij}}{20 \times s} = \dfrac{4239}{80} = 52.9875$

$UCL = \bar{\bar{X}} + A_2 \bar{R} = 55.3788$

$CL = \bar{\bar{X}} = 52.9875$

$LCL = \bar{\bar{X}} - A_2 \bar{R} = 50.2962$

Now, 16 points go out of the control limits, so we have to find out assignable variation in the process.

From 1st machine, only one point goes out.

from other 3 machines 5 points go out from each 3.

So, machine 1 is at better condition.

Rule:- Maximum no. of points than can be removed is 20% from the subgroups. If more than 20% points come, then the process is not stable, no need of control chart.

Group Control Chart:- There is a group of M/c doing similar process. The characteristic of the product is same. Steps are : Data collection
Limit calculation
Plot & Monitor

[Used for Multiple Stream Processes]

Homogenization:- It is a process by which we can remove the assignable cause from control limits.

For R chart:- $$\overline{R} = \frac{\Sigma\Sigma R_{ij}}{\text{sample No} \times \text{No. of heads}}$$

$$= \frac{\Sigma\Sigma R_{ij}}{20 \times 8}$$

$LCL_{\overline{R}} = D_3 \overline{R}$, D_3 for n = sample size

$UCL_{\overline{R}} = D_4 \overline{R}$.

Plot only the maximum & minimum of a subgroup.

R chart is to detect within subgroup variation.

For $\overline{X}$ chart:- $$\overline{\overline{X}} = \frac{\Sigma\Sigma \overline{X}_{ij}}{20 \times 8}$$

$$LCL = \overline{\overline{X}} - A_2 \overline{R}$$
$$UCL = \overline{\overline{X}} + A_2 \overline{R}$$

Here also we plot the maximum and minimum value.

$\overline{X}$ chart is to detect between subgroup variation.

Assumption:- Chance cause variation is smaller than assignable cause of variation.

Chance cause of variation is many in number but have little variation.

Assignable cause of variation is small in number but have large variation.

Assumption for Group Control:- There is no significant difference between process capabilities of the machines.

variation less $\longrightarrow$ highly capable

variation high $\longrightarrow$ less capable

If we group then assignable causes for highly capable process comes in control.

<u>Note:-</u> $(\overline{x}_1, s_1)$; $(\overline{x}_2, s_2)$: if there is no significant difference between two samples, we can use pooled variance.

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}.$$

Test statistics $t = \dfrac{\overline{x}_1 - \overline{x}_2 - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \sim t_{n_1+n_2-2}$.

<u>Group Charts:-</u>

<u>Case-I:-</u> Target is different but tolerance is same.

M/c-1 (10 ± 0.5 mm)					M/c-2 (8 ± 0.5mm)		M/c-3 (7.5 ± 0.5mm)	
x_{11}	x_{12}	...... x_{15},	$\overline{x}_{11}-10$,	R_{11}	$\overline{x}_{12}-8$,	R_{12}	$\overline{x}_{13}-7.5$,	R_{13}
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$x_{20,1}$	$x_{20,2}$	... $x_{20,5}$	$\overline{x}_{201}-10$,	R_{201}	$\overline{x}_{20,2}-8$,	R_{202}	$\overline{x}_{20,3}-7.5$,	$R_{20,3}$

0 ± 0.5 is common tolerance, the target is transformed to 0.

So, changing the scale, in the place of $\overline{x}$ taking, $\overline{x}-10$, $\overline{x}-8$, $\overline{x}-7.5$.

Assumption:- All product target is same.

Objective:- Everything looks alike.

<u>Case-II:-</u> Target and tollerance both are different.

10 ± 0.5 mm $\qquad$ 8 ± 1 mm $\qquad$ 7.5 ± 0.5mm

$$\text{Transformation} = \frac{x_{ij} - \text{target}}{\text{tolerance}}$$ so, 0 ± 0.5 mm

This is called target $\overline{X}$ chart.

<u>Condition:-</u> Value of C_p, C_{pk} is very much similar between machines.

Short term & long term variability:-

Control chart only gives short term variability.

In short-term variability s is given by

$$\hat{s} = \frac{\overline{R}}{d_2}.$$

In long-term variability s is given by

$$\hat{s}' = \sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n-1}}.$$

Also, $\hat{s}' > \hat{s}$.

Q. Given specification: 800 ± 20; $\hat{s} = 4$, $n=4$

Design a control chart, if rejection is 1% then prob. of a point outside $\bar{x}$ chart will be 0.9? Draw the control limit.

→ UCL = 820

LCL = 780

$$\hat{s} = \frac{\overline{R}}{d_2} = \frac{\overline{R}}{2.059} \Rightarrow \overline{R} = 8.236$$

$$P(x > 820) = 0.005$$

$$\Rightarrow P\left(Z > \frac{820-\bar{\bar{x}}}{s/\sqrt{n}}\right) = 0.005$$

$$\Rightarrow P\left(Z < \frac{820-\bar{\bar{x}}}{4/\sqrt{4}}\right) = 0.995$$

So, $\frac{820-\bar{\bar{x}}}{2} = 2.58$ (from Normal table)

so, $\bar{\bar{x}} = 814.84$

Machine Capability:- Variation must be less than the process variation.

— variability of the machine.

— To study machine capability, all other factors should be constant

Two way:- 1. Dry run ; 2. With component

— Run the machine without manufacturing anything.

— Study of vibration, temperature.

— We can adjust a value by looking at statistics but not on a single value.

— Use control chart. That is adjust the process when there is some assignable causes are present.

Q. How to implement a control chart for small batch production?

→ – M/c is fixed.
Part No + Progg. change.

– Of cycle time is very high or no. of component produced is very less.

– then we can collect data individually or in subgroups.

– standardize the data because data on different type of products will be available.

– Check the normality.

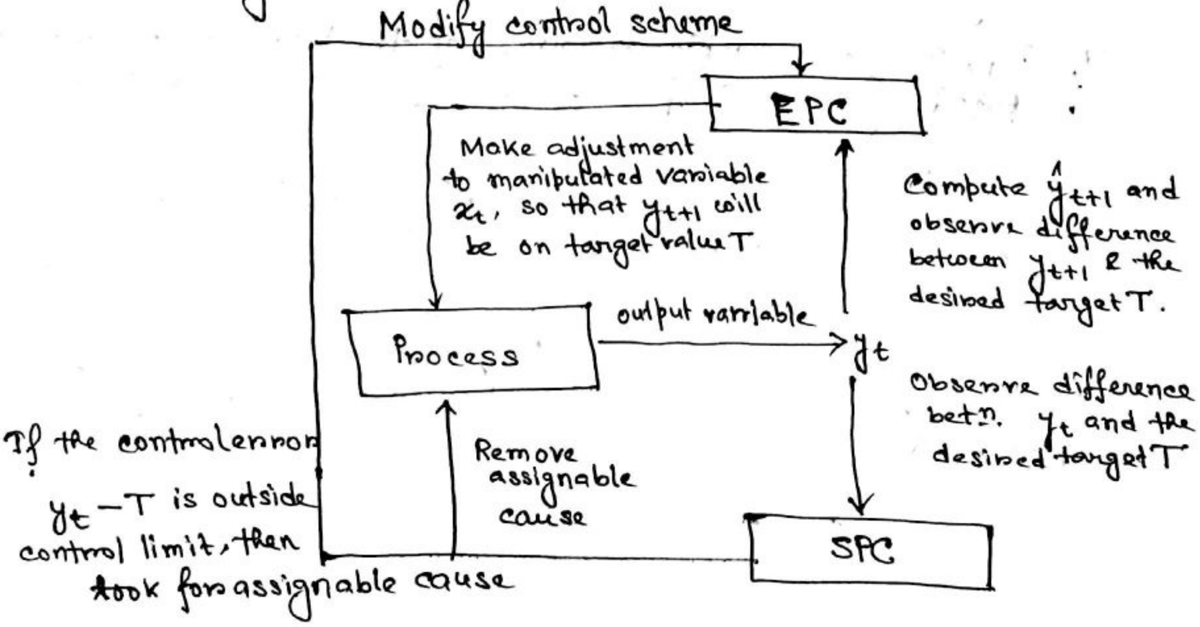
– IMR chart can be used for short term variation.

Combining SPC and EPC:- Engineering control theory is based on the idea that if we can

1. predict the next observation on the process,
2. have some other variable that we can manipulate in order to affect the process output,
3. know the effect of the manipulated variable.

Note that, this is in sharp contrast with SPC, where 'control action' or a process adjustment is taken only when there is statistical evidence that the process is out of control. On the other hand, EPC makes no attempt to identify an assignable cause that may impact the process. All EPC schemes do react to process upsets; they don't make any effort to remove the assignable causes.

Ex:- Consider the process of driving a car, with the objective of keeping it in the center of the right hand lane. The driver can adjust the process at any time without using statistical control chart.



Multivariate Control Chart :- Used where simultaneous monitoring or control of two or more related quality characteristics is necessary.

$P(\bar{x}_1 > 3\sigma) = P(\bar{x}_2 > 3\sigma) = 0.0027$

$P(\bar{x}_1 > 3\sigma, \bar{x}_2 > 3\sigma) = 0.0027 \times 0.0027 < 0.0027$

So, the use of two independent $\bar{x}$ charts has distorted the simultaneous monitoring of $\bar{x}_1$, and $\bar{x}_2$.

Normal distn :- $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \ -\infty < x < \infty$

Multivariate Normal :-

$$f(\underset{\sim}{x}) = \frac{1}{(2\pi)^{p/2} |\Sigma|^{1/2}} e^{-\frac{1}{2}(\underset{\sim}{x} - \underset{\sim}{\mu})' \Sigma^{-1} (\underset{\sim}{x} - \underset{\sim}{\mu})} ;$$

$-\infty < x_j < \infty$

$j = 1(1)p$.

$\underset{\sim}{x} = (x_1, \dots, x_p)$.

$\underset{\sim}{\mu} = (\mu_1, \dots, \mu_p)$

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_1\sigma_2 & \cdots \\ & \sigma_2^2 & \cdots \\ & & \ddots \\ & & & \sigma_p^2 \end{bmatrix}$$

$(x-\mu)(\sigma^2)^{-1}(x-\mu)$

$= (\underset{\sim}{x} - \underset{\sim}{\mu})' \Sigma^{-1} (\underset{\sim}{x} - \underset{\sim}{\mu})$

Σ : covariance matrix

The most familiar multivariate SPC procedure is the Hotelling T^2 control chart for monitoring the mean vector of the process.

Suppose two quality characteristic x_1, x_2 are jointly distd. according to the Bivariate Normal Distn.

$E(X_1) = \mu_1$, $E(X_2) = \mu_2$, $V(X_1) = \sigma_1^2$, $V(X_2) = \sigma_2^2$; $\mathrm{cov}(x_1, x_2) = \sigma_{12}$

Assuming that $\sigma_1, \sigma_2, \sigma_{12}$ are known.

If $\bar{x}_1, \bar{x}_2$ are sample averages of the two quality characteristic computed from a sample of size n, then the statistic

$$\chi_0^2 = \frac{n}{\sigma_1^2\sigma_2^2 - \sigma_{12}^2}\left[\sigma_2^2(\bar{x}_1 - \mu_1)^2 + \sigma_1^2(\bar{x}_2 - \mu_2)^2 - 2\sigma_{12}(\bar{x}_1 - \mu_1)(\bar{x}_2 - \mu_2)\right]$$

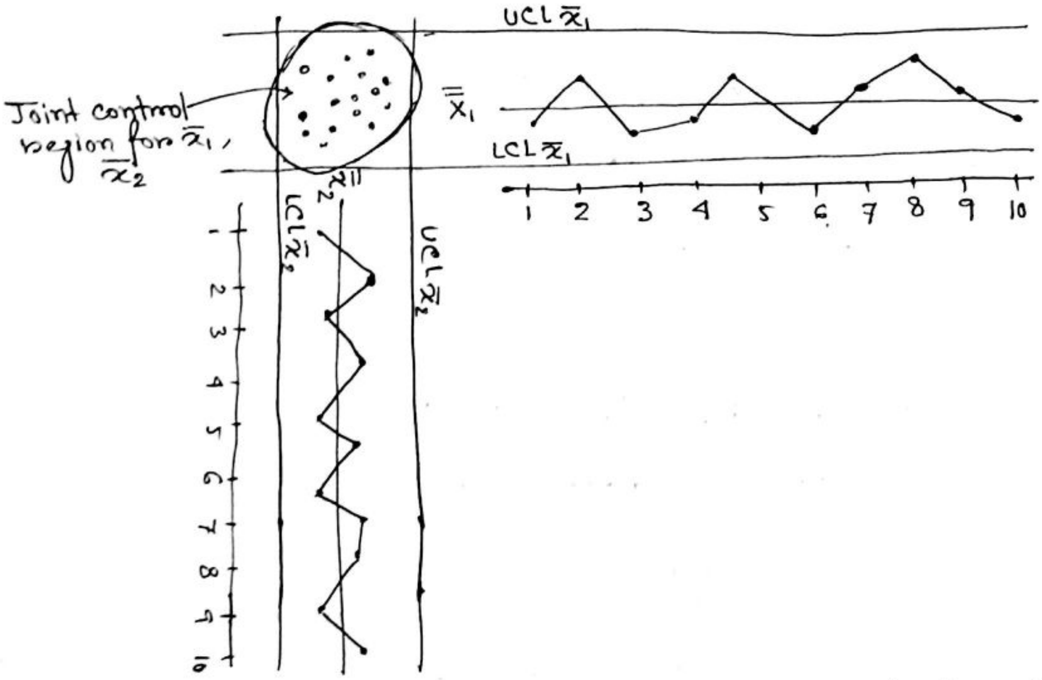
$\sim \chi_2^2$.

UCL = $\chi^2_{\alpha, 2}$ = upper α percentage point of χ_2^2.

LCL = 0

<u>Example:-</u> x_1 and x_2 are independent, i.e., $\rho_{12} = 0$.

If corresponding value of χ_0^2 plots outside the ellipse, the process is out of control.



<u>Modified Control Charts ($\bar{x}$ Chart)</u> :— It is used when the natural variability or spread of the process is considerably smaller than the spread in the specification limits; i.e. C_p or C_{pk} is much > 1.

Usually $C_{pk} \geq 2$.

process output is normally distributed.

$$\mu_L = LSL + Z_\delta \sigma$$

$$\mu_U = USL - Z_\delta \sigma$$

where, Z_δ is the upper $100(1-\delta)$ percentage point of $N(0,1)$.

If we specify type I error of α, then

$$UCL = \mu_U + \frac{3\sigma}{\sqrt{n}}$$

$$= USL - \left(Z_\delta - \frac{3}{\sqrt{n}}\right)\sigma$$

$$LCL = \mu_L - \frac{3\sigma}{\sqrt{n}}$$

$$= LSL + \left(Z_\delta - \frac{3}{\sqrt{n}}\right)\sigma$$

Note that the modified control chart is equivalent to testing the hypothesis that the process mean lies in $\mu_L \leq \mu \leq \mu_U$

Ex. Consider a normally dist^d. process with a target value $\mu = 20$, $\sigma = 2$. LSL = 8, USL = 32, $C_p = C_{pk} = 2$.
In this six-sigma process it is assumed that the mean may drift as much as 1.5 s.d.s off target without causing serious problems. Set up a control chart for monitoring the mean of this 6σ process with $n = 4$.

Solution:-

$$Z_\delta = 3\delta = 3 \times 1.5 = 4.5$$

$$UCL = USL - \left(4.5 - \frac{3}{\sqrt{4}}\right)\sigma \quad \text{and} \quad LCL = LSL + \left(4.5 - \frac{3}{\sqrt{4}}\right)\sigma$$

$$= 32 - (4.5 - 1.5)2 \qquad\qquad = 8 + (4.5 - 1.5)2$$

$$= 26 \qquad\qquad = 14.$$

$\bar{x}$ and R Charts for Short Production Runs:-

- Deviation from Normal (DNOM) Control Chart

$T_A = 50$ mm, $T_B = 25$ mm

M_i : i^{th} actual sample measurement in mm.

$x_i = M_i - T_A$ would be the deviation from Nominal

Sample No	Part No	Measurement M_1	M_2	M_3	DNOM x_1	x_2	x_3	$\bar{x}$	R
1	A	50	51	52	0	1	2	1	2
2	A								
3	A								
4	A								
5	A								
6	B	25	27	24	0	2	−1	0.33	3
7	B								
8	B								
9	B								
10	B								

$\bar{\bar{x}} = 0.17$ $\quad \bar{R} = 2.7$

1. An assumption is process s.d. is approx. same for all parts. If this assumption is invalid, then used standardized $\bar{x}$ & R chart.
2. This procedure works best when the sample size is constant for all part numbers.

Acceptance Sampling :-

Online Quality control tool : SPC
Offline Quality control tool : Acceptance sampling

- 100% inspection (either if the process is ~~stable~~/capable)
- Sampling (the process should be stable and capable)
- No inspection ($C_{pk} > 2$)
    - Part criticality
    - Capability of the process
    - Type of inspection (destructive or non-destructive)
    - Cost of inspection
    - Availability of Resources.

Variable Inspection :- Measurable, part dimension measured by an instrument. eg. length, power.

Attribute Inspection :- When check by visual inspections.

Two stategy :-
- Acceptance Rejection : The quality can be improved (customer)
- Acceptance Rectifying : The quality can be made better (manufacturer)

When lot quality is good $\rightarrow$ 100% inspection

risk : good lot quality product getting rejected (Producer's risk)
bad lot quality product getting accepted (Customer's risk)

When we can do sampling?

- When the lot is homogenious (all the parts in the lot is similar; i.e., from same batch, same machine)
- When the process is stable and capable.

Online Quality Control :- When we can take action back to the process.

Skip-lot Sampling Plan

One step ahead of Chain sampling plan.

→ When quality by vendor is very good.

→ and he demonstrated it for very long time.

→ lot by lot inspection plan supplied

→ if the manufacturing parts supplied are very good, we can skip inspecting few lots.

→ an extension of CSP from part to lot.

Start a reference sampling plan

↓

Start checking every lot (normal inspection)

↓

i consecutive lots are accepted under normal inspection

↓

switch to skip lot inspection ($0 < f < 1$)

↓

moment a lot is rejected go back to normal inspection

$$P_a(f, i) = \frac{f P_a + (1-f) P_a^i}{f + (1-f) P_a^i}$$; where P_a = Prob. of occurrence of reference plan

<u>Case I</u> let $f_2 < f_1$ for fixed i

$P_a(f_1, i) \le P(f_2, i)$

<u>Case II</u> When $i > j$ for a fixed f.

$P_a(f, j) \le P_a(f, i)$

$$\text{ASN}(S_K SP) = \text{ASN}(R) \times K = \text{ASN}(R) \times \frac{f}{(1-f)P_a^i + f}$$

Skip-lot Sampling plan ($S_K SP$)

Reference Sampling plan (R)

⇒ $\text{ASN}(S_K SP) < \text{ASN}(R)$.

Sequential Sampling Plan

Checking one item at a time and counting the no. of defective pieces we get.

Item by item Sequential sampling plan by Wald (1947)

Acceptance line, $X_A = -h_1 + sn$

Rejection line, $X_R = h_2 + sn$

where $h_1 = \left(\log \frac{1-\alpha}{\beta}\right) / K$

$h_2 = \left(\log \frac{1-\beta}{\alpha}\right) / K$

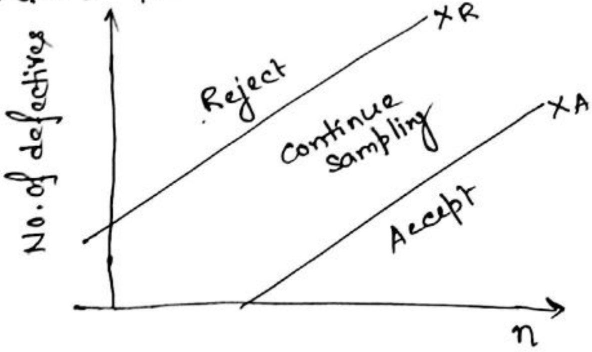
$K = \log \frac{p_2(1-p_1)}{p_1(1-p_2)}$

$s = \log\left[(1-p_1)/(1-p_2)\right] / K$

OC curve with prob. α, β;

α = producer's risk, β = consumer's risk;

p_1 = AOQL, p_2 = LTPD.



$$ASN = P_a\left(\frac{A}{c}\right) + (1-P_a)\frac{B}{c}$$

$A = \log \frac{\beta}{1-\alpha}$

$B = \log \frac{1-\beta}{\alpha}$

$c = p \log\left(\frac{p_2}{p_1}\right) + (1-p)\log\left(\frac{1-p_2}{1-p_1}\right)$, $p = s$.

Rectifying inspection:-

$ATI = P_a\left(\frac{A}{c}\right) + (1-P_a)N$

$AOQ = P_a \cdot P$.

Chain Sampling Plan

Condition:- 1. You take small sample size
2. May be test is destructive / lot quality is very good & consistent.

Draw a sample size n.

(i) $c = 0$ accept the lot.
(ii) $c = 1$ accept the lot if i preceeding lots were accepted.
(iii) $c > 1$ Reject the lot.

The points on the OC curve of a chain sampling plan are given by

$$P_a = P(0,n) + P(1,n)\left[P(0,n)\right]^i$$

where, $P(0,n)$ and $P(1,n)$ are the probabilities of obtaining 0 and 1 defectives, respectively, out of a random sample of size n.

Example:- Chain sampling plan with $n = 5$, $c = 0$, and $i = 3$.

For $p = 0.10$, we have

$$P(0,n) = \frac{n!}{d!\,(n-d)!}\, p^d (1-p)^{n-d}$$

$$= \frac{5!}{0!\,5!} (0.10)^0 (0.9)^5 = 0.590$$

$$P(1,n) = \frac{5!}{1!\,5!} (0.10)^1 (0.90)^4 = 0.328$$

$$Pa = P(0,n) + P(1,n)\left[P(0,n)\right]^i$$

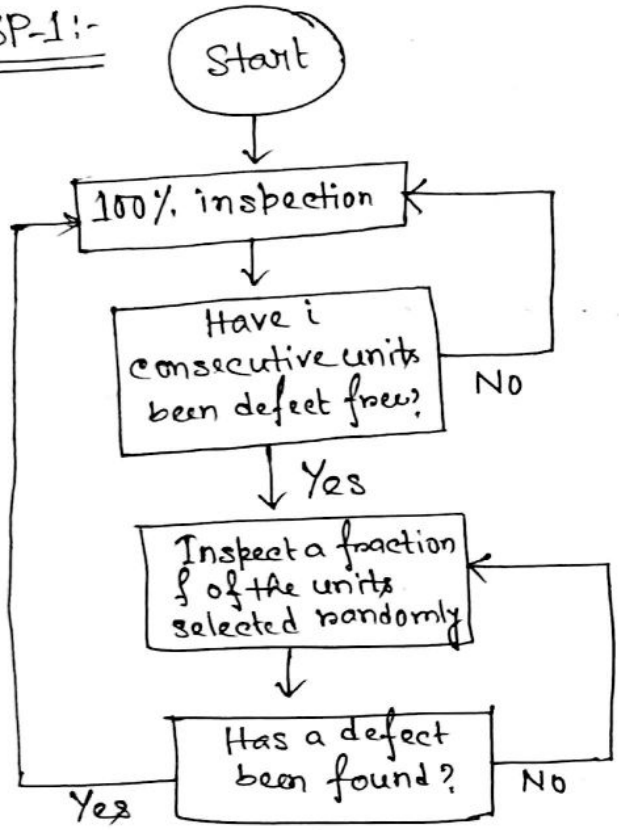
$$= 0.590 + (0.328)(0.590)^3$$

$$= 0.657.$$

Continuous Sampling Plan (CSP)

In this sampling plan first do 100% inspection. After a definite length if all items are good then go for sampling. If in sampling any bad item is detected then go for 100% inspection.

CSP-1:-



i = clearance number

f = fraction of inspection

AOQL = 0.143%

if $f = \frac{1}{25}$, i = 1147

$f = \frac{1}{7}$ if $i = 623$

So, if we decide to do sampling once in every 7 items, we should have 623 units defect free.

The average number of units inspected in a 100% screening sequence following occurrence of a defect is equal to

$$u = \frac{1 - q^i}{pq^i}$$

where, $q = 1 - p$, and p is the fraction defective produced when the process is operating in control.

The average number of units passed under the sampling inspection procedure before a defective unit is found is

$$v = \frac{1}{fp}$$

The average fraction of total manufactured units inspected in the long run is $AFI = \frac{u + fv}{u + v}$

The average fraction of manufactured units passed under the sampling procedure is

$$P_a = \frac{v}{u+v}.$$

β-conrection technique

When process suffer from adjustment problem then we apply β-conrection technique.

Data (x_i)	add cons 2	add cons 4	add next 2	Total sum
75	145			
70		285		
70	140		587	
70				
75	150			
75		302		1198
77	152			
75				
75	147			
72		300		
78	153		611	
75				
75	153			
78		311		
78	158			
80				

So, $\sum_{i=1}^{16} x_i = 1198$, $\sum^{16} x_i^2 = 89840$

Total sum of square $= \sum x_i^2 - CF$; $CF = \frac{(\sum x_i)^2}{16} = \frac{(89840)^2}{16}$

$= 139.75$ $\qquad = 89700.25$

$SS_{\text{between 8 obsn.}} = \frac{587^2 + 611^2}{8} - CF = 36$

$SS_{\text{between 4 obsn}} = \frac{285^2 + 302^2 + 300^2 + 311^2}{4} - CF - SS_8 = 51.25$

$SS_{\text{between 2 obsen.}} = \frac{145^2 + 140^2 + 150^2 + 152^2 + \cdots + 158^2}{2} - CF - SS_8 - SS_4$

$= 22.5$

ANOVA Table:-

Source of Variation	df	SS	MS	Fcal	F-tab
among 8 obs.	1	36	36		
among 4 obs.	2	81.25	25.625	$5.857 >$	$F_{0.05,2,12} = 3.89$
among 2 obs.	4	22.5	5.625	$1.5 <$	$F_{0.05,4,8} = 3.84$ Not significant
Error	8	30	3.75		
Total	15				

Now, $MSE^* = \dfrac{SSE + SS_2}{df_E + df_2} = \dfrac{30+22.5}{4+8} = 4.375$

So, $MS_4 = 25.625$

so, $F_{cal} = \dfrac{MS_4}{MSE^*} = 5.857$

So, $MSE^* = \text{Variance} = 4.375 = \sigma^2$

Now, $m = \text{target}$

$\hat{\mu} = \text{Estimator} = \text{population mean.}$

$D = \hat{\mu} - m = \text{off target}$

Adjustment $= -\beta D$

Define β as $\beta = \begin{cases} 0 & \text{if } \dfrac{D^2}{\sigma^2} < 1 \\ 1 - \dfrac{1}{F} & \text{ow} \end{cases}$

where $F = \dfrac{D^2}{\sigma^2} =$

Method:- Data on $\hat{\mu}$ & σ

Check $(\hat{\mu} - m) > \sigma$

Calculate $F = \dfrac{(\hat{\mu} - m)^2}{\sigma^2}$

then find $\beta = 1 - \dfrac{1}{F}$.

— : Taguchi Loss Function:-

Loss function is defined as deviation as the quantity proportional to the squared deviation from the target quantity characteristic. At zero deviation, the performance is at target and the loss is zero.

Y = Deviation from target

Y_0 = Target

$$L(Y) = K(Y - Y_0)^2$$

Derivation:- $L(Y) = L(Y_0) + L'(Y_0)(Y - Y_0) + \frac{1}{2!} L''(Y_0)(Y - Y_0)^2$

$$= \frac{1}{2!} L''(y_0)(Y - Y_0)^2$$

$$= K(Y - Y_0)^2$$

$L(Y_0) = K(Y_0 - Y_0)^2 = 0$

$L'(Y_0) = 2K(Y - Y_0)$

$= 0$

L_0 = Loss at $Y_0 + \Delta$

$$= K(Y_0 + \Delta - Y_0)^2$$

$$= K\Delta^2$$

$$\therefore K = \frac{L_0}{\Delta^2}$$

so, $L(y) = \frac{L_0}{\Delta^2}(y - y_0)^2$.

$$= K\left[\frac{(y_1 - y_0)^2 + \cdots + (y_n - y_0)^2}{n}\right]$$

$$= K \cdot MSD.$$

Ex.1. Target = 12

Tolerance = ± 0.35

L_0 = Rs. 20/-

Data: 11.80 12.30 12.20 12.40 12.10

12.20 11.90 11.80 11.85 12.15

Estimate $L(y) = ?$

Solution:- $\Delta = 0.35$

$$K = \frac{L_0}{\Delta^2} = \frac{20}{(0.35)^2} = 163.265$$

$$L(Y) = KMSD$$

$$= 7.10$$

Ex.2. The target value of a quality characteristic is 100. The loss to customer beyond 115 is Rs. 40. The internal loss is Rs. 15 for the same value. What should be the mfg. tolerance for this characteristic?

Sol.

$$L(y) = k(y - y_0)^2$$

$$40 = k(115 - 100)^2$$

$$\therefore k = \frac{40}{15^2} = 0.178$$

$$L_0 = k\Delta^2$$

$$15 = \frac{40}{15^2}\Delta^2$$

$$\Rightarrow \Delta^2 = \frac{15^3}{40} = 9.185$$

Note:- Taguchi said that external loss is much more higher than internal loss.

Quality talks mainly performance feature.

Performance characteristic

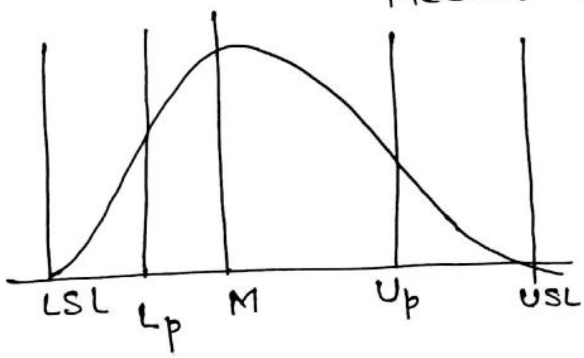
- Warranty
- Inhouse testing
- Accelerated life testing

Process Capability for Non-Normal Distributions

$$\text{Process capability index} = \frac{\text{Allowed Variation}}{\text{Actual Variation}} = \frac{T}{6s}$$

$$C_p = \frac{USL - Median}{Maximum - Median} \quad \text{(lower the better)}$$

$$= \frac{Median - LSL}{Median - Min} \quad \text{(higher the better)}$$



$$C_p = \frac{USL - LSL}{U_p - L_p}$$

$$C_{pU} = \frac{USL - M}{U_p - M}$$

$$C_{pL} = \frac{M - LSL}{M - L_p}$$

$$C_{pK} = \min\{C_{pU}, C_{pL}\}$$

$$L_p = \bar{X} - s.L_p'$$

$$U_p = \bar{X} + s.U_p'$$

$$M = \bar{X} + s.M'$$

L_p', U_p' & M' value is in table.

[Process capability Calculations for Non-normal distn.
(Quality Progress) by John. A. Clements]

Trouble Shoting & Problem Solving for Quality Improvement

CTQ : Critical to Quality Characteristics

COPQ : Cost of Poor Quality

Bias :-
1. Instant Bias : how much on an average deviated from target.
2. Linearity : What happened to the bias factor when measuring through its range.
3. Stability : Over a period of ~~time~~ bias should be stable.

Quality Cost (COPQ) – Cost of Poor Quality

- Cost incurred to achieve quality
  - Prevention cost (Preventive work before failing)
  - Appraisal cost (to know the status of quality).
- Cost incurred for not achieving quality
  - internal failure cost (Rework, Reprocessing, etc)
  - External failure costs (After going to customer, warranty, etc)

Invitro :- (within glass) refers to the technique of performing a given procedure in a control environment outside of a living organism (cellular biology environment) – fail to replicate the precise cellular condition of an organism. So, this may lead to results that don't correspond to the circumstances occurring around a living organism.

Invivo :- (within the living) refers to experimentation using a whole, living organism as opposed to a particular or dead organism. Animal studies & clinical trials are two forms of invivo research. This is suited for observing the overall effects of an experiment on a living subject.

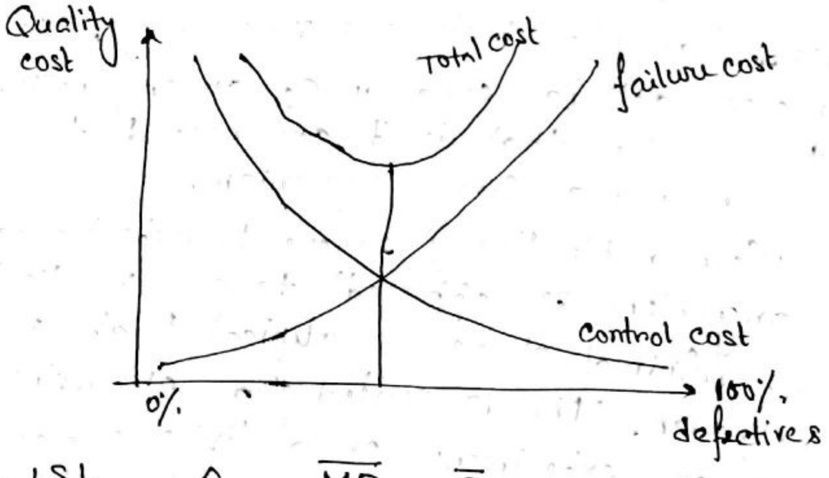
Invitro is better than Invivo :-

- reduce cost
- more directly assess product performance.
- offer benefits in terms of ethical consideration.
- Invivo is costly, tested on living, so error is high.

Note :- For World class Company, COPQ is less than 10%, for average company 10 – 30%, poor company > 30%.

Terms:-

1. Craftman: people develop some trades (individual come with some idea)
2. Inspection: An activity to segregate good from bad.
3. Quality Control:- prevents defects from occuring (in the process)
4. Quality Assurance: Set of all activity which ensure every activity associated is working as planned.
5. Total Quality Control: Associative activity (manufacturing) control throughout various dept. (production, design, etc)
6. Total Quality Management: 1. Don't think of product, think about the process producing it.
   2. Never think about profit, think about customers.
   3. Never think about the task, think about people doing the task.
7. Six Sigma :: achiving business excellence (balancing product & profit margin)
8. Lean Six Sigma: There are 8 waste due to manufacturing, how to minimize these waste.

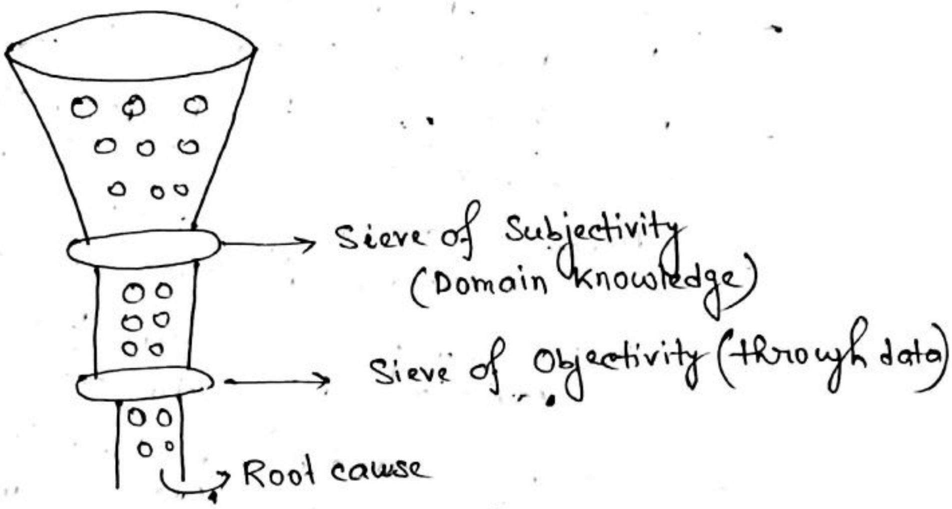


C_p : $\dfrac{USL - LSL}{6\sigma}$, $\hat{\sigma} = \dfrac{\overline{MR}}{d_2} = \dfrac{\overline{R}}{d_2}$

P_p : $\dfrac{USL - LSL}{6\sigma}$, $\hat{\sigma} = \dfrac{1}{n-1} \Sigma (x_i - \bar{x})^2$.

House of Quality:-
- Product Planning
- Parts Deployment
- - Process Deployment
  - Design Process
- Product deployment

Root-Cause Identification:- Funelling approach



Best model conditions:-
- VIF < 5
- R^2-adjusted > 0.6
- Residual Plot (Normal)
- Residual vs fitted value
- Histogram
- Residual vs. order

Seven QC Tools:-

1. Flow Diagram
2. Stratification and Check Sheet
3. Pareto Analysis
4. Graphs, Charts & Plots
5. Cause & Effect diagram
6. Histogram
7. Scatter Diagram

New 7M tools:-

1. Relation Diagram
2. Affinity Diagram (KJ)

} Planning stage, identifying problems

3. Systematic Diagram
4. Matrix Diagram

} matching goals with the means

5. Matrix Data Analysis (PCA)
6. Process Decision Program chart (PDPC) → Implementation stage.
7. Arrow Diagram (PERT/CPM)

Note:- Cause & Effect Diagram can't interconnect the various causes, but Relation diagram does.

Principal Component Analysis:-

- Describes the variation in a set of correlated variables (x's) by a set of uncorrelated variables.
- Each principal component is a linear combination of the x's.
- The new variables are derived in decreasing order of importance
- Hence y_1 account for maximum possible variation in x among all linear combinations of x.
- y_2 account for maximum possible of the remaining variation subject to being uncorrelated to y_1 & so on.
- Helps to understand the variability in large data sets with intercorrelated variables using a smaller number of uncorrelated factors.
- Explaining variability of a set of n variables using m factors, $m < n$

Objective:- 1. Reduces the complexity of a large set of variables by summarizing them in a smaller set of components / factors.

2. Tries to improve the interpretation of complex data through logical factors.

Relation Diagram:- When something achieved by intution in past, depending upon the past experience some logic is made.

Affinity Diagram:- This technique clarifies important but unresolved problems by collecting verbal data. One way to understand VOC.

Systematic Diagram:- This technique searches for the most appropriate & effective means of accomplishing given objectives.

Matrix Diagram:- When we have multiple solutions, finding the best. A techniques that clarifies problematic spots through multidimensional thinking.

Rate of Improvement (RI) $= \dfrac{\text{Stage what you want to achieve}}{\text{Current stage}}$

P DPC :- This technique helps to determine which process to use to obtain desired result by evaluating the progress of events and the variety of conceivable outcomes.

Looks difficulties in the process (diffence between flow chart)

FMECA → static

PDPC → Dynamic

Six-Sigma for Business Excellence

DMAIC: Define, Measure, Analyse, Improve, control.

Statistical Thinking:-
1. Variation is inevitable
2. Everything is executed as a process.
3. Understanding & reducing variation is key to success.

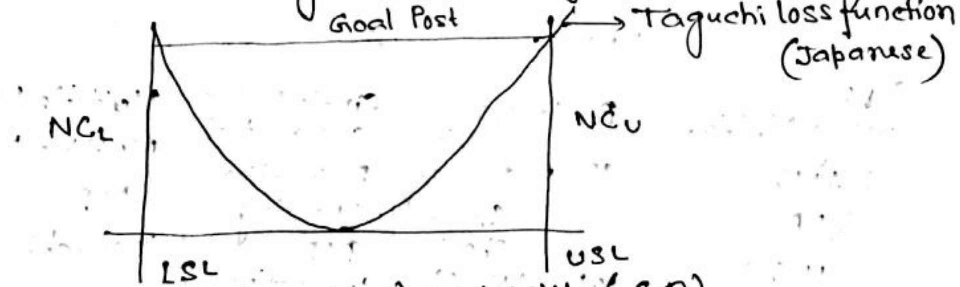
Six-sigma try to achieve as much as less variation possible.

- Cause & Effect relationship: $E = f(C)$.

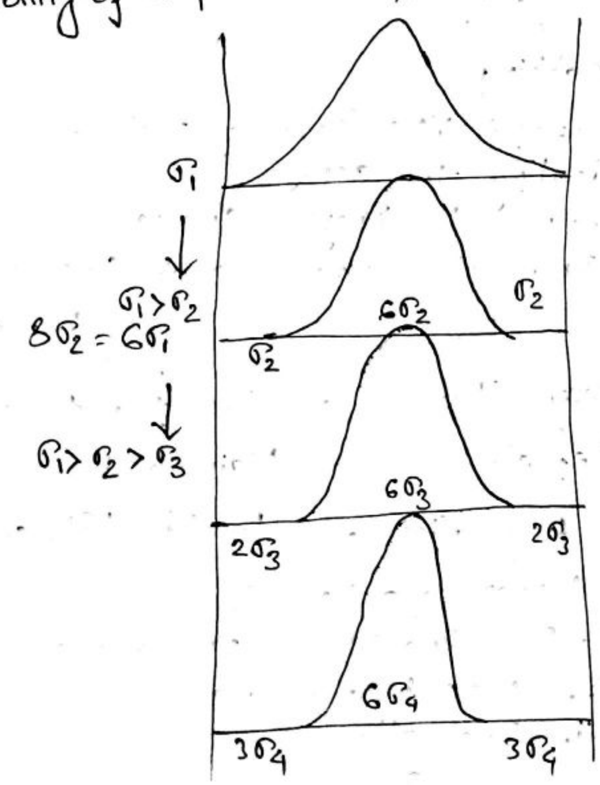
Variation

Assignable	Chance/Random
- small in number	- large in number
- contribution is large	- Each small contribution
- Easily identifiable	- Very difficult to identify individual.
- can be eliminated / should be eliminated.	- Difficult to eliminate for the individual.

American Approach to Quality (Goal post syndrome)



Capability of a process is its natural variability (6σ).



$C_p = 1$
$C_{pK} = 1$

$C_p = 1.33$

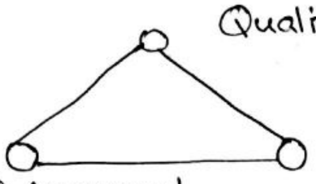
$8\sigma_2 = 6\sigma_1$
$= \frac{8}{6} = 1.33$

$10\sigma_3 = 6\sigma_1$
$C_p = 1.67$

$12\sigma_4 = 6\sigma_1$
$C_p = 2$
if moving by $1.5\sigma_4$, $C_{pK} = 1.5$

Stability Analysis:-
- Run chart
- Histogram (if # observation > 70)
- Control Chart (I-MR)
- N.P.P.
- Box Plot. — Six-pack

Juran Triology:-



PDCA Cycle:- Plan - Do - Check - Act.

R D M A I C S I

- R → Recognize
- D → Define
- M → Measure
- A → Analyse
- I → Improve
- C → control
- S → Standardize
- I → Integrate

Modelling :- DOE, Multiple Regression.

D	M	A	I	C
VOC Survey Method KANO QFD SIPOC Project Charter	Basic stat SPC — Stability / Capability / Control Chart $\bar{X}$-R, ANOVA Gauge R&R Kappa	Simple Problem solving tool Estimation Testing DOE Correlation Regression	Multiple Reg. Multiple Method DOE Tajuchi	MSA SPC Sign off

VSM: Value Stream Mapping

Define
1. Capture the VOC (Voice of customer)
2. Identify the CTQ
3. Prepare the project Charter
4. Praco the process map (SIPOC)

VOC	CTQ	Solution
Slow room service	Delivery Time	Deliver food to room in 20min
Quality is not good	Defects	Maintain it
Picture Quality is bad	Resolution	Provide 16M colour pics.

Measure
1. Performance Variable
2. Establish the performance variable
3. MSA performance
4. Evaluation

Kappa Analysis:-

	1	2	Total
1	a	b	a+b
2	c	d	c+d
Total	a+c	b+d	N

Expected prob. for cell (1,1)

$$= \frac{a+b}{N} \times \frac{a+c}{N} \times N$$

$$= \frac{(a+b)(a+c)}{N} \quad \text{(Marginal prob.)}$$

$Pr(0)$ = Prob. of obs. agreement $= \frac{a+d}{N}$

$Pr(e)$ = Prob. of expected agreement $= \frac{(a+b)(a+c)+(b+d)(c+d)}{N^2}$

$$K = \frac{Pr(0) - Pr(e)}{\underbrace{1 - Pr(e)}_{\text{disagreement}}}$$

∴ Kappa ranges from −1 to 1.

0 → agreeing by chance

−1 → complete disagreement

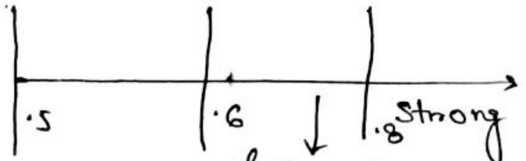
1 → Complete agreement

Sigma level → is Z-level (so it can be −ve) — point in Z-scale corresponding to area meeting our requirements

- SIPOC: Supplier + Input + Process + Output + Customer

DFSS: Design for Six-sigma

Thumb Rule for R^2-adjusted:-



if there is any other variable, try to add, or can go further.

EXAM:- 1. Six sigma Case Study

2. Need to identify a concept/objective, what's the six sigma stage on which we will be using what tool or technique.

7 STEP PROBLEM SOLVING METHODS:-

1. Select a Theme
2. Collect Data
3. Analyse Causes

} PLAN } Check Sheet, Graph, Histogram, Scatter diagram, Pareto, C&E diagram, Flowcharts

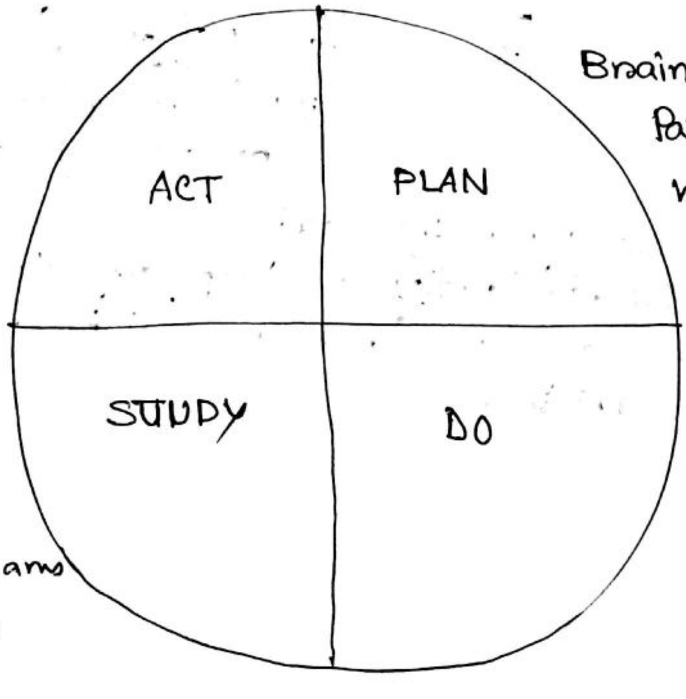
4. Plan and implement solution } Flowcharts } DO
5. Evaluate effects } CHECK } Check sheets, graphs, Histogram, scatter diagram, Pareto, C&E diagram, Run chart/control chart
6. Standardize
7. Reflect on Process

} ACT } Flowchart

" Problem Solving, the isolation and analysis of a problem and the development of a permanent solution, is an integral part of the quality improvement process.

Problem Solving Process:-

Symptom Recognition → Fact finding → Probem Identification → Idea generation → Solution Development → Plan Implementation → Follow up



PLAN: Brainstorming, Pareto Analysis, Why-Why diagram

DO: C&E diagram, Scatter diagram, Control Charts, Check Sheets, Run charts

STUDY: Run charts, Control Charts, Histograms, Check sheets, Scatter diagrams, Pareto charts

8D PROBLEM SOLVING TECHNIQUES:-

1. Define the Team
2. Define Problem/Failure
3. Choose and Verify Interim Containment Action (ICA)
4. Define and Verify root causes
5. Choose & Verify Permanent Corrective Action (PCA)
6. Implement & Validate PCA
7. System Prevent Actions to Prevent Recurrence
8. Team Recognition/Celebration

Deming Quality Principles:- (14 Point Management Philosophy)

1. Create constancy of purpose for continual improvement of products.
2. Adopt a commitment to seek continual ;
3. Switch from defect detection to defect prevention
4. In dealing with suppliers one should end the practice of awarding business on price. Move towards quality of product, reliability of delivery and willingness to cooperate and improve. Build partnerships.
5. Improvement is not confined to products and their direct processes but to all supporting services and activities.
6. Train a modern way
7. Supervision must change from chasing to coaching and support.
8. Drive out fear and encourage two way communication.
9. Remove barriers between departments.
10. Do not have unrealistic targets.
11. Eliminate quotas and numerical targets.
12. Remove barriers that prevent employees having pride in the work that they perform.
13. Encourage education and self-improvement for everyone.
14. Publish top management's permanent commitment to continuous improvement of quality & productivity.