DEMAND ANALYSIS

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Introduction :- Government formulates economic policies in order to have a balanced economic structure in the country.
The demand analysis deals with the following two important aspects of economic statistics:

(i) Demand analysis studies the relationship between market price and demand on the basis of market data (time series data).

(ii) How is the demand affected by gradual changes in 'income level', on the basis of family budget data (also called cross-sectional data)?

Necessities & Luxuries :- The goods or commodities which satisfy our primary needs (e.g., food, cloth, housing, etc.) are called necessities. The goods which satisfy luxuries (e.g., ornaments, liquor, cigars, etc) are called luxuries.
There is an intermediate class of wants, viz.; those wants which are not primary wants but are required by convention or habits without which we may not live happily, those goods are termed as conventional necessities.

Demand and Supply :- In economics, mere desire for a commodity does not mean demand, unless one can pay and is willing to pay the necessary amount for it. Thus the increase in demand for a particular commodity does not merely mean an increase in the desire for that commodity but it implies that the desire to have it at a given price has increased. Demand for any commodity depends on a number of factors such as price, income, time, price of other commodities, etc.

The functional relationship between consumption of commodity and the factors responsible for the changes in consumption, is defined as the demand function of the commodity. From the demand function, we can find different quantities of a commodity demanded at different prices.

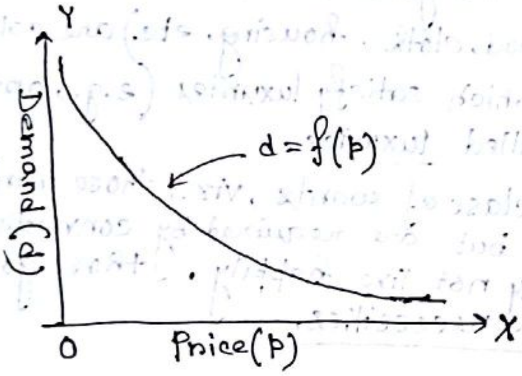
In economics, supply means the amount of commodity available at a given price, which in turn depends on the amount produced which further depends on the price. Supply is a function of price at which commodity is sold in the market.

Laws of supply and demand :-

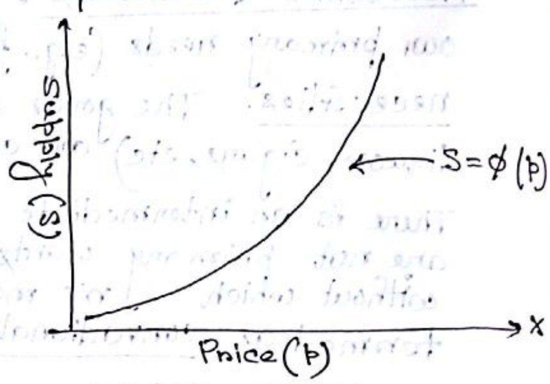
" Demand for a commodity, in general, varies in the direction opposite to that of price whereas supply in general varies in the same direction as price".

According to Cournot, demand is a function of price, i.e., $d = f(p)$, the only assumption regarding $f(p)$ is that it is a diminishing function of p, i.e. $f'(p) < 0$. And also supply is a function of p, i.e., $s = \phi(p)$, where $\phi(p)$ is an increasing function on p, i.e., $\phi'(p) > 0$.

Here is the graphical representation of supply and demand functions:

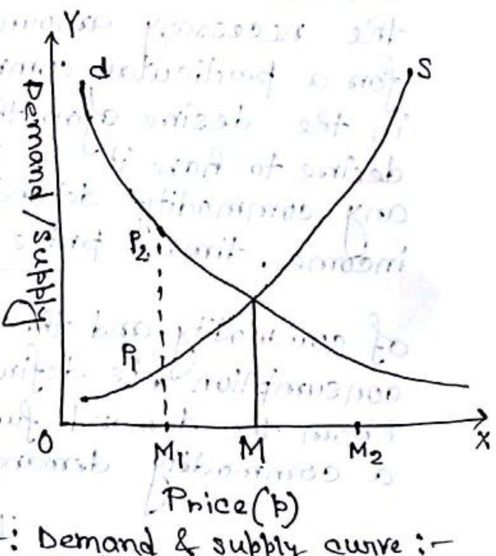


-: Demand curve :-



-: Supply curve :-

These laws further states that the market price settles at a level at which supply and demand are equal and is determined by the point of intersection of the two curves $f(p) = d$ and $\phi(p) = s$. The price OM determined by the point of intersection of the curves $d = f(p)$ and $s = \phi(p)$ is termed as equilibrium price. Thus, the equilibrium price is the solution of the equation $f(p) = \phi(p)$.



-: Demand & supply curve :-

Remark:-

There are some exceptional demand curves which instead of sloping downwards rise upwards. However, Robbert Giffen one such situation in Ireland during 19th century, when as a result of serious fall in the real wage of the workers, even after a sharp rise in the prices of bread, its consumption was on the increase since bread was still cheapest food as compared others. This phenomenon is termed as Giffen's paradox. The demand for Giffens goods, i.e., inferior goods, rises with a rise in price and falls with a fall in price.

WORKED OUT EXAMPLES :-

Ques:- The demand curve and the supply curve of a commodity are given by $D = 19 - 3p - p^2$ and $S = 5p - 1$. Find the equilibrium price and the quantity exchanged.

Solution:- For equilibrium, we have $D = S$.

$$\Rightarrow 19 - 3p - p^2 = 5p - 1$$
$$\Rightarrow p^2 + 8p - 20 = 0$$
$$\Rightarrow (p+10)(p-2) = 0$$
$$\Rightarrow p = 2,\ p = -10 \text{ (rejected)}$$

So, equilibrium price is $p = 2$.

Substituting it in the demand or supply curve, we get $D = S = 9$.

Ques:- The demand function of two commodities A and B are

$$D_A = 10 - p_A - 2p_B,\quad D_B = 6 - p_A - p_B$$

and the corresponding supply functions are

$$S_A = -3 + p_A + p_B\ ;\ S_B = -2 + p_B$$

where p_A and p_B denote the prices of A and B respectively. Find (i) equilibrium prices (ii) equilibrium quantities exchanged in the market.

Solution:- For equilibrium, we have $D_A = S_A$, $D_B = S_B$

$$\Rightarrow 10 - p_A - 2p_B = -3 + p_A + p_B \quad \& \quad 6 - p_A - p_B = -2 + p_B$$
$$\Rightarrow 2p_A + 3p_B - 13 = 0 \quad \text{and} \quad p_A + 2p_B - 8 = 0$$

solving, we have $p_A = 2$, $p_B = 3$ as equilibrium prices.

Substituting in demand function or supply function, the equilibrium quantities are given by $D_A = S_A = 2$ and $D_B = S_B = 1$.

Ques:- Demand and supply functions for tea are given by

$$x_d = 120 - 2p + 5\frac{dp}{dt} \text{ kgms. per week}$$

$$x_s = 3p - 30 + 50\frac{dp}{dt} \text{ kgms. per week}$$

where p is the price at time t. Find the time path of p for dynamic equilibrium if the initial price is given to be 36 paise per kg.

Solution:- For equilibrium condition, we have $x_d = x_s$.

$$\Rightarrow 120 - 2p + 5\frac{dp}{dt} = 3p - 30 + 50\frac{dp}{dt}$$
$$\Rightarrow \frac{dp}{dt} + \frac{p}{9} = \frac{10}{3}, \text{ which is a linear differential equation}$$

Integrating factor: $e^{\int \frac{1}{9} dt} = e^{t/9}$.

Solution is given by $p \cdot e^{t/9} = \frac{10}{3} \cdot \frac{e^{t/9}}{1/9} + c \Rightarrow p = 30 + ce^{-t/9}$.

Initial price is 36 p/kg, so $c = 6$, when $t = 0$, $p = 36$.

So, $p(t) = 30 + 6e^{-t/9}$.

Price Elasticity of Demand:— One of the most important characteristics of demand function is what is known as its 'elasticity'—according to the law of demand, the changes in price and demand are in opposite direction and it is a common experience that price changes affect the demand for different commodities in different degrees. The quality of demand by virtue of which it extends or contracts with a fall or rise in price is known as 'price elasticity of demand', a term introduced by Marshall.

Definition:- Price elasticity of demand is defined as the value of the ratio of the relative (or proportionate) change in the demand to the relative (or proportionate) change in price.

Mathematically, let x be the quantity demanded of a commodity 'A' such that the demand function of A is $x = f(p)$, where $f(\cdot)$ is a continuous function. Let the increment in demand x, corresponding to an increment δp in p, be δx. Then elasticity of demand (η_D) is given by

$$\frac{\text{Proportionate change in demand}}{\text{Proportionate change in price}} = \frac{(\delta x)/x}{(\delta p)/p} = \frac{p}{x}\cdot\frac{\delta x}{\delta p}$$

This is the average elasticity of demand over the price change $(p, p+\delta p)$.

The elasticity of demand (η_p) at a particular price level p is given by

$$\eta_p = \lim_{\delta p \to 0} \frac{p}{x}\cdot\frac{\delta x}{\delta p} = \frac{p}{x}\lim_{\delta p\to 0}\frac{\delta x}{\delta p}$$

$$= -\frac{p}{x}\cdot\frac{dx}{dp}$$ [negative sign being taken for the case of demand & price move in opposite direction]

$$= -\frac{p}{f(p)}\cdot\frac{df}{dp}$$

$$= -\frac{d\log f}{d\log p}.$$

Remark:- 1. Since $d = f(p)$ is a decreasing function of p,

$$\frac{df}{dp} < 0 \Rightarrow \eta_p = -\frac{p}{f}\cdot\frac{df}{dp} > 0$$

i.e., price elasticity of demand is always positive.

For $\eta = 1$, elasticity is called normal.

For $\eta > 1$, demand is called overrealistic.

For $\eta < 1$, demand is called underrealistic.

2. Significance of Elasticity of Demand :– In order to understand the significance of the price elasticity of demand of a commodity for market analysis, we need to consider the total outlay or market turn over for the commodity, viz.,

$F(p) = p \times d = p \times f(p)$ is the total expenditure of the population for the purchase of the given commodity.

$$\frac{d}{dp}\left[F(p)\right] = 1 \cdot f(p) + p \cdot f'(p)$$

$$= f(p)\left[1 + p \cdot \frac{f'(p)}{f(p)}\right]$$

$$= f(p)\left[1 - \eta_p\right]$$

(i) If $\eta_p = 1$, then $F(p)$ is constant.

i.e., the money value of the turnover is constant independent of variations in prices of the commodity.

$$F(p) = p \cdot f(p) = c \Rightarrow d = f(p) = \frac{c}{p}.$$

Hence, if $\eta_p = 1$, the demand d is inversely proportional to the price p.

(ii) If $\eta_p < 1$, then $\frac{d}{dp}\left[F(p)\right] > 0$,

$\Rightarrow F(p)$ is an increasing function of p, i.e., the money value of the turnover increases as the prices rise.

(iii) If $\eta_p > 1$, then $\frac{d}{dp}\left[F(p)\right] < 0$,

$\Rightarrow F(p)$ is a decreasing function of p, i.e., the money value of the turnover decreases as the prices rise.

A knowledge of η_p for a given commodity will enable us to determine if the increase (or fall) in the price results in an increase in the turnover and the profit of the monopolist and if so, its extent also.

3. Demand function with constant price elasticity :–

The demand function $d = f(p)$ with constant price elasticity of demand, say, $\eta = \alpha \ (>0)$ at all the points of the curve, is given by

$$-\frac{p}{f} \cdot \frac{df}{dp} = \alpha \Rightarrow \frac{df}{f} = -\frac{dp}{p} \cdot \alpha$$

After integration, $\log f = -\alpha \log p + \log k = \log\left(k p^{-\alpha}\right)$

$$\Rightarrow d = f(p) = kp^{-\alpha} \ ; \ \alpha > 0, \ k > 0.$$

$$f'(p) = -k\alpha p^{-\alpha-1}.$$

Hence, the price elasticity of demand is given by

$$\eta_p = -\frac{p}{f} \cdot f'(p) = -\frac{p}{kp^{-\alpha}} \cdot \left(-k\alpha p^{-\alpha-1}\right)$$

Thus, we conclude that the curve of the constant elasticity of demand is a hyperbola whose shape depends on the value of the parameter α

In particular, if we take $\alpha = 1$, the demand curve of constant elasticity becomes $d = f(p) = kp^{-1}$

$\Rightarrow d.p = k$, which is the equation of a rectangular hyperbola ($xy = c$).

Ques :- If the demand function is $p = 4 - 5x^2$, for what values of x the elasticity of demand will be unitary ?

Solution:- $p = 4 - 5x^2$

Differentiating w.r.t. p, we have

$$1 = -10x \cdot \frac{dx}{dp}$$

$$\Rightarrow \frac{dx}{dp} = -\frac{1}{10x}$$

$$\therefore \eta_p = -\frac{p}{x} \cdot \frac{dx}{dp} = \frac{4 - 5x^2}{10x^2}$$

Elasticity of demand will be unitary if $\frac{4 - 5x^2}{10x^2} = 1 \Rightarrow x = \frac{2}{\sqrt{15}}$

Ques:- If the demand curve is of the form

$$p = ae^{-kx}$$

where, p is the price and x is the demand, prove that the elasticity of demand is $\frac{1}{kx}$. Hence deduce the elasticity of demand on the curve $p = 10e^{-x/2}$.

Solution:- $p = ae^{-kx}$

Differentiating w.r.t. p, we get

$$1 = -ake^{-kx} \cdot \frac{dx}{dp}$$

$$\therefore \eta_p = -\frac{p}{x} \cdot \frac{dx}{dp} = \frac{ae^{-kx}}{x} \cdot \frac{1}{ake^{-kx}} = \frac{1}{kx}$$

(Proved)

Comparing $p = ae^{-kx}$ & $p = 10e^{-x/2}$, we have

$a = 10$, $k = 1/2$.

So, $\eta_p = \frac{2}{x}$.

Ques:- The price elasticity of demand curve $x = f(p)$ is of the form $(a - bp)$, where a and b are given constants. Find the demand curve.

Solution:-

$$\eta_D = -\frac{p}{x}\cdot\frac{dx}{dp} = a - bp$$

$$\therefore \left(\frac{a - bp}{p}\right)dp + \frac{dx}{x} = 0$$

$$\therefore \left(\frac{a}{p} - b\right)dp + \frac{dx}{x} = 0$$

Integrating, we get

$$(a\log p - bp) + \log x = \log c$$

$$\Rightarrow \log\left(p^{\alpha} e^{-bp}\right) + \log x = \log c$$

$$\Rightarrow x p^{\alpha} e^{-bp} = c$$

$$\Rightarrow x = c p^{-\alpha} e^{bp}.$$

Price Elasticity of Supply:- If $s = \phi(p)$ is the supply curve for a commodity 'A' then the price elasticity of supply of A at price p is given by

$$\varepsilon_p = \frac{\text{Relative change in supply of 'A'}}{\text{Relative change in price of 'A'}} = \frac{\delta s/s}{\delta p/p} = \frac{p}{s}\cdot\frac{\delta s}{\delta p}$$

The price elasticity of supply (ε_p) at a particular price level p is given by

$$\varepsilon_p = \lim_{\delta p\to 0}\frac{p}{s}\cdot\frac{\delta s}{\delta p}$$

$$= \frac{p}{s}\cdot\lim_{\delta p\to 0}\frac{\delta s}{\delta p}$$

$$= \frac{p}{s}\cdot\frac{ds}{dp}$$

$$= \frac{d\log s}{d\log p}.$$

[positive sign is taken since price & supply change in the same direction]

Taking $s = \phi(p)$, we get $\varepsilon_p = \frac{p}{s}\cdot\frac{d}{dp}[\phi(p)] = \frac{p}{\phi(p)}\cdot\phi'(p)$

Since, $\phi(p)$ is an increasing function of p, so $\phi'(p) > 0$ & $\varepsilon_p > 0$.

Supply curve with the Constant Price Elasticity :-

The supply function $s=\phi(p)$ with constant price elasticity of supply, $\varepsilon_p=\alpha>0$, at all points of the curve is given by

$$\frac{p}{\phi(p)}\cdot\phi'(p)=\alpha$$

$$\therefore \quad \frac{\phi'(p)}{\phi(p)}=\frac{\alpha}{p}$$

On integration $\quad \log\phi(p)=\alpha\cdot\log p+\log c=\log(c\cdot p^{\alpha})$

$$\therefore \quad s=\phi(p)=c\cdot p^{\alpha}\ ;\ c>0,\ \alpha>0.$$

Hence $\log s=\alpha\log p+\log c$ represents the curve of constant elasticity of supply which can be graphically drawn on a double logarithmic scale is a straight line.

Income Elasticity of Demand :- In general, the demand function x for any commodity 'A' can be written as

$$x=f(\mu, p_1, p_2, p_3, \dots, p_n),$$

where μ is income of the people, p is the price of commodity A, $p_1, p_2, \dots, p_n$ are the prices of related commodities, say, $A_1, A_2, \dots, A_n$.

Income Elasticity of Demand — Suppose that all prices are assumed to remain constant while income is variable. As income changes prices remaining the same, there will be an income effect and under the influence of the income effect the quantities demanded will change. The quantity demanded will therefore be a function of income only. Let q represent the quantity demanded and y represent income. Then the demand function can be written as $q=q(y)$. It is also known as Engel curve or the income demand curve.

The elasticity of this function is given by $\eta_e=\dfrac{dq/q}{dy/y}=\dfrac{y}{q}\cdot\dfrac{dq}{dy}$

since $y>0$, $q>0$, it is clear that if $\dfrac{dq}{dy}>0$, then $\eta_e>0$.

The concept of income elasticity of demand can be used for classifying commodities.

Value of income elasticity	Type of goods
$\eta_e>1$	Normal luxury
$0<\eta_e<1$	Normal necessity
$\eta_e<0$	Inferior necessity

Question:- The demand function for a commodity 'X' is given by

$$x = 300 - 0.5 p_x^2 + 0.02 p_o + 0.05 y$$

where x is the quantity demanded of 'x', p_x is the price of x, p_o is the price of a related commodity and y is the constant income. Compute

(i) The price elasticity of demand for x.

(ii) The income elasticity of demand for x.

Solution:- (i) $\eta p_x = -\frac{p_x}{x} \cdot \frac{\partial x}{\partial p_x}$

$$= \frac{-p_x}{300 - 0.5 p_x^2 + 0.02 p_o + 0.05 y} \times \{0.05(-2p_x)\}$$

$$= \frac{p_x^2}{300 - 0.5 p_x^2 + 0.02 p_o + 0.05 y}$$

When $p_x = 12$; $p_o = 10$, $y = 200$,

$$\eta_{p_x} = 0.60 \text{ (approx.)}$$

(ii) $\eta_e = \frac{y}{x} \cdot \frac{\partial x}{\partial y} = \frac{0.05 y}{300 - 0.5 p_x^2 + 0.2 p_o + 0.05 y}$

when $p_x = 12$, $p_o = 10$, $y = 200$,

$$\eta_e = \frac{10}{237.8} = 0.04 \text{ (approx.)}$$

Theorem:- If the proportion of income spent on any commodity remains constant as income increases than the income elasticity of demand for this commodity is equal to unity.

Solution:- Let us suppose that y is the income of the consumer and q is the quantity demanded of any commodity whose price per unit is p. Then total expenditure on this commodity equal to $p.q$ and the proportion of income spent on this commodity $= \frac{p.q}{y}$. Let us suppose that the price of the commodity remains constant. If the proportion remains constant as income increases,

$$\frac{d\left(\frac{p.q}{y}\right)}{dy} = 0 \quad \text{or,} \quad \frac{p\frac{dq}{dy} y - p.q}{y^2} = 0$$

$$\text{or, } p\frac{dq}{dy} \cdot y = p.q$$

$$\text{or, } \frac{dq}{dy} \cdot \frac{y}{q} = 1.$$

or, Income elasticity $= 1$.

Remark:- The theorem provides us with a convenient way of considering whether the income elasticity of demand for any commodity is greater than or less than unity. It has been empirically founded by Engel that an income increases the proportion of income spent on food grains decreases. This is known as Engel's Law.

Engel's Law and Engel's Curve :–

A german statistician Ernst Engel after a detailed and systematic study of the family budget, has given the following law :

"As the income grows, the share of income spent on food decreases" In other words, "The proportion of expenditure on food decreases as household expenditure increases". — Engel's law.

Hence, as income increases the expenditures on different items have changing proportions, and the proportions devoted to urgent needs decrease while for luxuries and semi-luxuries increase.

The graphic representation of the basic relationship between household income and its expenditure on a particular item of consumption is known as Engel's curve.

In general, the demand for any commodity among a class of people may be regarded as depending on the price of the commodity and the income of the people, the two factors not necessarily summing up. Thus,

$$d = f(\mu, p),$$

where d is the demand for a commodity, p is its price and μ, the national income.

Since demand is, in general, an increasing function of income μ and decreasing function of price p, we have

$$\frac{\partial}{\partial \mu}(d) > 0 \text{ and } \frac{\partial}{\partial p}(d) < 0$$

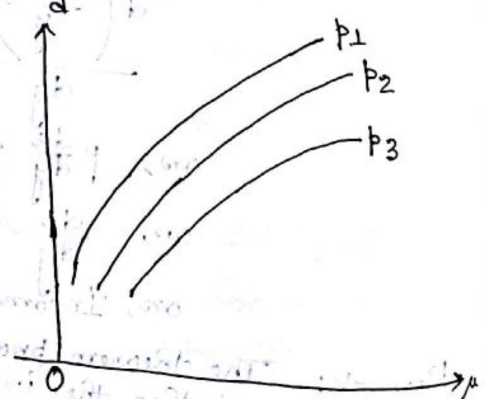
Regarding demand function d as a two parameter function of price (p) and income (μ) it can be represented graphically by a certain surface D in the three-dimensional space, taking the three variables d, p and μ along three rectangular coordinate axes Od, Op and $O\mu$, O being the origin.

Engel's curves for constant prices :– In particular, if we regard price as fixed constant, $p = p^*$, (say), then the demand function becomes

$$d = f(p^*, \mu) = f_1(\mu),$$

i.e., d becomes a single parameter function of μ. These curves are called Engel's curves for constant prices.

For constant price p, the Engel's curve is concave downwards, i.e., $\frac{\partial^2}{\partial \mu^2}(d) < 0$. This means that as income increases, the rise in demand is slower and slower; a conclusion contained in Engel's law.



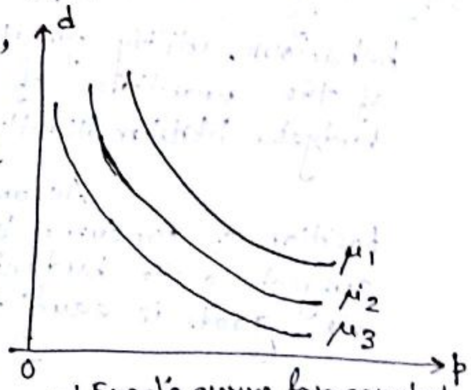
Engel's curve for constant prices :–

Engel's curves for constant income :- If we regard income as constant, $\mu = \mu^*$, (say), then the demand function becomes

$$d = f(p; \mu^*) = f_2(p),$$

i.e., d becomes a function of the single parameter p (price).

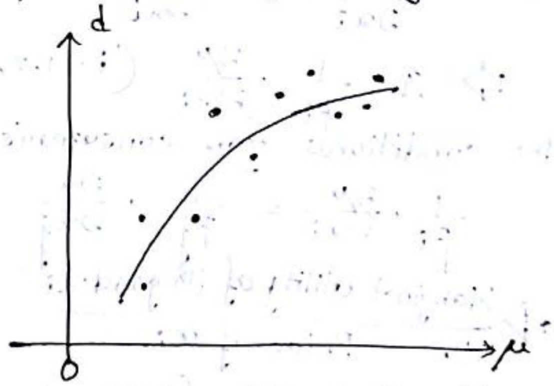
These are called Engel's curve for constant income.



-: Engel's curve for constant incomes: -

Methods of drawing Engel's curves :— (For constant prices)

METHOD 1. The method consists in simultaneously study of the budgets of different families with different income levels. Let d_i be the demand for a commodity at income level μ_i, $i = 1, 2, 3, \dots, n$. The Engel's curve $d = f(p^*, \mu) = f_1(\mu)$; $p = \text{constant} = p^*$, it then obtained by using the principle of least squares.



This method assumes that the consumption pattern of families at different levels of income is same, an assumption which is far from reality. This drawback may be overcome by stratifying the given population into relative homogeneous groups.

METHOD 2. It consists in comparing the budgets of the same family during different periods of time and studying their consumption patterns on different items of consumption as a consequence of variation in income. The obvious drawback of this method is that the assumption $p = \text{constant} = p^*$ during the periods under consideration is not generally true. Hence this method can be recommended only in the situations when the prices of given commodity and the substitution and complementary goods remain more or less constant during the given period of investigation.

Utility Function:- According to the classical theory of consumer behaviour, utility function (u) can be regarded as a function of the quantities of goods, x_i ($i=1,2,3,\ldots,n$) in the consumer's budget. Mathematically, $u = \phi(x_1, x_2, \ldots, x_n)$ ---------(*)

According to the principle of substitution, the position of consumer's equilibrium is obtained on maximising (*) subject to the budget constraint that aggregate expenditure on all goods is equal to the income, i.e.,

$$\sum_{i=1}^{n} p_i x_i = y \qquad \text{——— (1)}$$

where, p_i is the price of the quantity consumed of i^{th} good i.e., x_i and y is the income.

In other words, for consumer's equilibrium, we have to maximise

$$Z = u - \lambda\left[\sum_{i=1}^{n} p_i x_i - y\right]$$

unconditionally, where λ is Lagrange's multiplier.

For extremum, $\frac{\partial Z}{\partial x_i} = 0 = \frac{\partial u}{\partial x_i} - \lambda p_i$

$$\Rightarrow \lambda = \frac{1}{p_i} \cdot \frac{\partial u}{\partial x_i} \quad (i = 1,2,3,\ldots,n)$$

This gives the conditions for consumer's equilibrium as:

$$\frac{1}{p_i} \cdot \frac{\partial u}{\partial x_i} = \frac{1}{p_j} \cdot \frac{\partial u}{\partial x_j} \quad (i \neq j = 1,2,\ldots,n)$$

$$\Rightarrow \frac{\text{Marginal utility of } i^{th} \text{ good } x_i}{\text{Price of } x_i} = \frac{\text{Marginal utility of } j^{th} \text{ good } x_j}{\text{Price of } x_j}$$

$$\Rightarrow \frac{\text{Marginal utility of ith good}}{\text{Marginal utility of jth good}} = \frac{p_i}{p_j} \qquad \text{——— (2)}$$

There are $(n-1)$ such independent marginal utility ratios which provide $(n-1)$ relationships between the quantities of the goods consumed and the relative prices or price ratios. These $(n-1)$ equations along with the budget constraint enable us to solve for x_i's ($i = 1,2,\ldots,n$) in terms of the prices p_i's and income y.

Thus, we see that marginal utility equations (2) which are derived from the total utility functions, are used together with the budget equation (1) to get demand functions. Hence the parameters of the utility function will determine the parametric structure of the demand functions.

Ex:- If $u = cx^{\alpha}y^{\beta}$ is an individual's utility function of two goods, show that his demand for the goods is

$$x = \frac{\alpha}{\alpha+\beta}\cdot\frac{\mu}{p_x}, \quad y = \frac{\beta}{\alpha+\beta}\cdot\frac{\mu}{p_y}.$$

where, p_x and p_y are fixed prices and μ is individual's fixed income. Deduce that the elasticity of demand for either good with respect to income or price is equal to unity in absolute value.

Solution:-

$$\frac{1}{p_x}\cdot\frac{\partial u}{\partial x} = \frac{1}{p_y}\cdot\frac{\partial u}{\partial y}$$

$$\Rightarrow \frac{1}{p_x}\cdot c\alpha x^{\alpha-1}y^{\beta} = \frac{1}{p_y}\cdot c\beta x^{\alpha}y^{\beta-1}$$

$$\Rightarrow \frac{\alpha y}{p_x} = \frac{\beta x}{p_y}$$

$$\Rightarrow y \cdot p_y = \frac{\beta x}{\alpha}\cdot p_x \quad \text{......} (1)$$

Since individual's income is fixed, we have

$$xp_x + yp_y = \mu$$

$$\Rightarrow xp_x + \frac{\beta}{\alpha}\cdot xp_x = \mu$$

$$\Rightarrow x = \frac{\alpha}{\alpha+\beta}\cdot\frac{\mu}{p_x}$$

Hence, $y = \frac{\beta}{\alpha+\beta}\cdot\frac{\mu}{p_y}$ [substituting x in (1)]

Income and price elasticities of demand for 'x' are given by

$$\eta_{\mu}(x) = \frac{\mu}{x}\cdot\frac{\partial x}{\partial \mu}$$

$$= \frac{\mu}{x}\cdot\frac{\alpha}{\alpha+\beta}\cdot\frac{1}{p_x}$$

$$= 1.$$

$$\eta_D(x) = -\frac{p_x}{x}\cdot\frac{\partial x}{\partial p_x} = \frac{p_x}{x}\cdot\frac{\alpha}{\alpha+\beta}\cdot\frac{\mu}{p_x^2} = 1$$

$$\therefore \eta_{\mu}(x) = \eta_D(x) = 1.$$

Similarly, it can be shown that $\eta_{\mu}(y) = \eta_D(y) = 1.$

Types of Data Required for Estimating Elasticities :-

The empirical demand analysis is based on the data obtained from two main sources of statistical observation.

(a) Family Budget Data:- Family budget data are collected through sample surveys covering sample of households which are representative of different classes of people w.r.t. income, family size, social class etc., and their expenditure on budget items during a period of a year or over is recorded. In order to study the influence of income level on the expenditure habits of the people, we carry out an experiment consisting of the following main steps:

1. The first step is to select a group of households which are as homogeneous as possible w.r.t. regional environments, social and economic characteristics, family size and other factors that affect the demand, without making any reference to their family income.

2. The next step consists in regulating the family income by allotting the households to different income levels at random, randomisation being resorted to neutralise the effect of factors other than income.

3. Finally a detailed account of the expenditure of each household during a period of a year on various budget items is compiled.

(b) Market Statistics or Time Series Data:— By market statistics we understand time series data relating to the prices of the commodities and their quantities bought or sold at that price at different points of time. The treatment of such data is quite analogous to that of family budget data except that demand is now primarily regarded as a function of price and not of income. The market price of any commodity settles at a level known as the 'equilibrium price' p_1 (say), which is the intersection of the supply and demand curves, $d = f(p)$ and $s = \phi(p)$. A variation in the price of a commodity over time means a shift in either or both of the demand and supply curves. If both the curves $d = f(p)$, $S = \phi(p)$ remain fixed, then the market data remains more or less static and doesn't provide enough number of points for their determination. If both the supply and demand curves shift their positions, then it is unlikely to trace either the supply or the demand functions closely. However, if one of the two curves remains fixed and the other changes its position then the family budget data provide a number of points on the fixed curve and hence the curve, is determined.

Again the demand for any commodity doesnot depend only on its price but also on a number of factors such as income, the price of the substitution (i.e., price of related commodities), etc. Hence for sound statistical analysis of demand, we should either take into account those factors explicitly or eliminate their effect on the demand and the price.

Remark:- (1) We see that both the methods, viz., the family budget (cross-sectional) data and the time series (market statistics) data serve to single out the effect of just one factor, viz., income and price respectively by nutralising the simultaneous effect of other factors that influence the demand.

(2) Taking demand function $d = f(x_1, x_2, \ldots, x_n)$, We know demand function is negatively sloped. Some useful assumptions are:

(i) The shape of the demand curve should remain fixed as the supply curve shifts its position from time to time. e.g. – cournot and Marshall demand curves.

(ii) The demand curve is of constant elasticity.

(iii) Demand functions are of the following forms:

$$d = a_0 + a_1 x_1 + a_2 x_2 + \cdots + a_n x_n$$

or, $d = a_0 x_1^{a_1} \cdots x_n^{a_n}$; where a_i $(i = 1(1)n)$ are constants.

Methods of Estimating Demand functions :–

(1) Leontief's Method (From Time Series Data) :–

Assumptions:– (i) Each market transaction represents the intersection of instantaneous demand and supply curves which change their position from time to time. This implies that in addition to determining the elasticities of demand and supply we must also study the extent to which the curves have shifted from time to time.

(ii) The shifting of supply and demand curves are independent of each other and donot affect the shape of the curves. This means that a shift of demand curve to the right is just as likely to be associated with a shift of the supply curve to the left as to the right.

(iii) Each of the supply and demand curves is of constant elasticity i.e., the demand and supply curves when plotted on a double logarithmic scale should be straight lines.

If Y_t and X_t are the logarithms of the consumption and price of a commodity at time t, $(t = 1, 2, \ldots, n)$, then we have

$$\text{Demand Curve: } Y_t = \eta_1 X_t + U_t \quad \text{——— (1)}$$

$$\text{Supply Curve: } Y_t = \eta_2 X_t + V_t \quad \text{——— (2)}$$

where, η_1 and η_2 are the elasticities of demand and supply respectively and U_t and V_t are independently distributed with

$$E(U_t) = E(V_t) = 0.$$

Note that in ① and ②, we have taken Y_t for consumption as well as supply, since for market equilibrium, we have

$$d = S \Rightarrow \log d = \log S = Y_t \text{ (say)}.$$

① and ② can be written as

$$Y_t - \eta_1 X_t = U_t \quad \text{——— ③}$$

$$Y_t - \eta_2 X_t = V_t \quad \text{——— ④}$$

Multiplying ③ and ④, we get

$$Y_t^2 + \eta_1\eta_2 X_t^2 - (\eta_1+\eta_2)X_t Y_t = U_t V_t$$

U_t and V_t are independently distributed with $E(U_t) = 0 = E(V_t) = 0$, such that $Cov(U_t V_t) = 0 \Rightarrow E(U_t V_t) = 0$

$$\text{So, } \sum_t U_t V_t = 0.$$

$$\text{We get, } \sum Y_t^2 + \eta_1\eta_2 \sum X_t^2 - (\eta_1+\eta_2)\sum X_t Y_t = 0$$

Time range $t : [1, n]$ is divided into two equal halves:

$$t_1 : [1, \tfrac{n}{2}] \text{ and } t_2 : [\tfrac{n}{2}+1, n]$$

$$\left.\begin{aligned} &\sum_{t=1}^{n/2} Y_t^2 + \eta_1\eta_2 \sum_{t=1}^{n/2} X_t^2 - (\eta_1+\eta_2)\sum_{t=1}^{n/2} X_t Y_t = 0 \\ \text{and } &\sum_{t=\frac{n}{2}+1}^{n} Y_t^2 + \eta_1\eta_2 \sum_{t=\frac{n}{2}+1}^{n} X_t^2 - (\eta_1+\eta_2)\sum_{t=\frac{n}{2}+1}^{n} X_t Y_t = 0 \end{aligned}\right\} \text{——— (*)}$$

$\sum Y_t^2$, $\sum X_t^2$ and $\sum X_t Y_t$ can be calculated from the time series data and (*) can be solved simultaneously for η_1 & η_2.

Limitations :- (i) The assumption that demand and supply curves can shift in any direction, independent of each other violates the fundamental principle of general theory of equilibrium, viz., "the demand for any one commodity is a function not only of its price but of all the prices".

(ii) Leontief's assumption that the supply and demand curves shift simultaneously is not a reasonable assumption for agricultural commodity.

(iii) There is no way of testing the validity of the assumption (iii) viz., the elasticities of demand and supply curves are constant.

Criticism:- Apart from a number of economic objections inherent in the assumptions made above, the procedure adopted for estimating η_1 and η_2 is faulty from statistical point of view. The mathematical solution of (*) leads to two curves only when the ellipses of the two scatters into which Leontief breaks up his series are not similar and the corresponding axes are not parallel to one another. But a significant difference between the scatters of the first half and the second half periods indicates that the data are not homogeneous and as such each period needs to be studied separately.

(2) Pigous's Method (From Time Series Data):-

Assumptions:- (i) The demand curve is likely to have a smooth appearance in each interval i.e., the demand curve, for each interval of time, is a curve of constant elasticity, given by

$$d = cp^{-\alpha}$$

$$\Rightarrow \log d = \log c - \alpha \log p$$

$$\Rightarrow \log d = \log c + a \log p \quad (\because a = -\alpha)$$

$$\Rightarrow Y = aX + b;$$

(ii) The demand curve shifts steadily over different periods of time, the rate of shifts being equal in two successive intervals. In other words, Pigou assumed that the rate of shift is such that distance between the ith and (i+1)th position on a logarithmic scale is same as the distance between (i+1)th and (i+2)th position.

Hence according to the assumptions, we have

$$Y_1 = aX_1 + b$$

$$Y_2 = aX_2 + b + r$$

$$\vdots$$

$$Y_i = aX_i + b + (i-1)r$$

$$Y_{i+1} = aX_{i+1} + b + ir$$

$$Y_{i+2} = aX_{i+2} + b + i(r+1)$$

Accordingly, we have

$$(Y_{i+1} - Y_i) - a(X_{i+1} - X_i) = (Y_{i+2} - Y_{i+1}) - a(X_{i+2} - X_{i+1})$$

$$\Rightarrow a = \frac{Y_{i+2} - 2Y_{i+1} + Y_i}{X_{i+2} - 2X_{i+1} + X_i}, \ (i = 1, 2, \ldots) \quad \text{———} (*)$$

Pigou's method involves the following steps:

1. Prepare a table of logarithms of time-series values of consumption (Y) and price (X), i.e., log d and log p.

2. Compute a_i from (*). Since demand is diminishing function of price, if for any interval the value of 'a' comes out to be positive it can't be taken as a measure of elasticity of demand for the relevant set of times. On the other hand, if it comes out to be negative it may be regarded as a measure of the elasticity.

3. If negative a_i's exceed the positive a_i's and if all the a_i's are grouped fairly closely about a given value and further if the data are not suspected otherwise then each a_i can be regarded as an observation on the unknown elasticity of demand curve. The mean value of a_i's then taken as a measure of unknown elasticity.

Limitations :- 1. Pigou's method is based on the assumption that the demand curve for the commodity is given by $x = f(p, t)$, where, x is the quantity of the commodity that is demanded, p is the price of the commodity demanded and t is time. This implicitly assumes that the prices of all other related commodities have only a negligible or no effect upon the commodity in question or all the influencing factors are conceived as frozen while studying the variation in x as a result of variations in y. However, in practice it is impossible to freeze all other factors without first taking them into account.

2. Pigou's method breaks down if in the three successive sets of observations, the three price-quantity points are collinear. However, the method can be applied to non-linear functions and the functions which change in directions.

(3) Pigous's Method (From Family Budget Data) :-

This method differs from Pigou's method (from Time Series Data) in that it is derived from the theory of utility and makes use of family budget data which gives us the expenditure of a group of people classified according to their income (wages). Let

$$U_1 = U_1(x) \text{ and } U_2 = U_2(x)$$

be the marginal degree of utility functions (i.e., the rate of change of utility function) for a quantity x of a given commodity 'A' for two neighbouring income groups I and II.

It is assumed that the functions U_1 and U_2 are independent of the quantities of other commodities and hence of the degree of utility of money. If μ_1 and μ_2 denote the degree of utility of money to the two groups respectively then

$$\mu_1 = \frac{U_1(x_1)}{p} \; ; \; \mu_2 = \frac{U_2(x_2)}{p} \quad \cdots\cdots (1)$$

where x_1 and x_2 are the equilibrium quantities of 'A' consumed by the two groups and p is the price of commodity 'A' which must be same for both the groups.

Hence the two income groups are neighbouring ones i.e., the wage grouping can be taken to be small, Pigou assumed that the tastes and temperament of the people in any two adjacent income groups are approximately the same so that

$$u_1(x) = u_2(x) = u(x), \text{ say} \quad \cdots\cdots (2)$$

Hence from (1) and (2), we get

$$p = \frac{u(x_1)}{\mu_1} = \frac{u(x_2)}{\mu_2}$$

We have $u(x_2) = u[x_1 + (x_2 - x_1)]$

$$= u x_1 + (x_2 - x_1)\, u'(x_1) \quad \text{[Taylor's expansion]}$$

$$\Rightarrow u'(x_1) = \frac{u(x_2) - u(x_1)}{x_2 - x_1} = \frac{p(\mu_2 - \mu_1)}{x_2 - x_1}$$

$$= \frac{1}{x_2 - x_1} \cdot \frac{\mu_2 - \mu_1}{\mu_1} \cdot u(x_1) \quad \cdots\cdots (3)$$

By definition, the elasticity of demand (consumption) x_1 w.r.t. utility $u(x_1)$ is

$$\eta_{x_1,u} = \frac{u(x_1)}{x_1} \cdot \frac{dx_1}{d[u(x_1)]} = \frac{u(x_1)}{x_1} \cdot \frac{1}{\frac{d}{dx_1}[u(x_1)]}$$

$$= \frac{u(x_1)}{x_1 u'(x_1)}.$$

Substituting the value of $u'(x_1)$ from (3), we have

$$\eta_{x_1,u} = \frac{x_2 - x_1}{x_1} \cdot \frac{\mu_1}{\mu_2 - \mu_1} \quad \cdots\cdots (**)$$

Pigou further assumed that " a small change in the consumption of any ordinary commodity on which a small proportion of man's total income is spent, cannot involve any appreciable change in the marginal utility of money". In other words, he assumed that "the price elasticity of demand curve in respect of any consumption x_1 is equal to the elasticity of the utility curve in respect of that consumption" .... (*)

The elasticity of demand w.r.t. price for the commodity in question viz., 'A' in the lowest wage group when x_1 units of it are consumed is given by

$$\eta_{x_1,p} = \frac{p}{x_1} \cdot \frac{d(x_1)}{dp}$$

$$\therefore \frac{dp}{dx_1} = \frac{u'(x_1)}{\mu_1} . \quad \left[\text{From } \mu_1 = \frac{u(x_1)}{p} \right]$$

So, we get $\eta_{x_1,p} = \frac{p}{x_1} \cdot \frac{\mu_1}{u'(x_1)} = \frac{u(x_1)}{x_1\, u'(x_1)} = \eta_{x_1,u}$

$$= \frac{x_2 - x_1}{x_1} \cdot \frac{\mu_1}{\mu_2 - \mu_1} .$$

Thus Pigou concluded that the price elasticity of demand for the commodity in question in the lowest wage group when x_1 units of it are consumed is equal to the elasticity of the consumption x_1 w.r.t. the utility $u(x_1)$.

Similarly, if the lowest income group consumes y_1 units of commodity 'B', say, then

$$\eta_{y_1,p} = \frac{y_2 - y_1}{y_1} \cdot \frac{\mu_1}{\mu_2 - \mu_1}$$, where notations have usual meanings.

So, $$\frac{\eta_{y_1,p}}{\eta_{x_1,p}} = \frac{y_2 - y_1}{y_1} \cdot \frac{x_1}{x_2 - x_1}$$

$$\Rightarrow \eta_{y_1,p} = \left[\frac{y_2 - y_1}{y_1} \cdot \frac{x_1}{x_2 - x_1} \right] \cdot \eta_{x_1,p} .$$

If any one of these elasticities of demand η_{y_1} or η_{x_1} is given then other can be obtained from above without any reference to the incomes μ_1 and μ_2.

Limitations :- 1. Even if Pigou's assumptions are granted, his method gives only the ratios of the elasticities of demand of two commodities and not the absolute values of elasticities.

2. However, Pigou's most important assumption viz., " Since a small change .... in respect of that consumption in (*) can't be granted because of basic contradiction in his development. Though the assumption of " constancy of utility of money " is reasonable when income remains fixed and the price of one of the commodities varies, it is not valid when prices are fixed and income varies. This latter assumption implies $\mu_1 = \mu_2$ which, on substituting in (**) gives $\eta_{x_1,u} = \infty$, thus contradicting the assumption of diminishing degree of utility.