will try methods 1 & 2

where $\sigma = 2$ & $\mu = 10$

method 1: when S.D is known & N is large:

will use for $n = 40, 80, 160, 1000$ at a 95% Confidence level.

Sample mean:

$$\bar{X} = \frac{\Sigma X_i}{n}$$

here we will assume $\bar{x} = \mu$

first, calculate standard error mean (SEM):

$$SEM = \frac{s}{\sqrt{n}} = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n}}$$

← where we will use the known S.D

To Calculate Confidence interval:

multiply the SEM by the z-score

z-score for multiple samples: $z = (x - \mu) / (\sigma / \sqrt{n})$ ← S.D

$z\text{-score} = 0.5 \times (1 - \text{Confidence interval})$ → 0.95

so 95% Confidence inverval:

$$[\bar{x} - SEM \times z\text{-score}, \bar{x} + SEM \times z\text{-score}]$$

(below) (above)

method 2: when n is small (less than 30 [n = 5, 10, 20]) & S.D is unknown

same as above, just multiply SEM by t-distribution

t-distribution:

$$t = \frac{(\bar{x} - \mu)}{(s / \sqrt{n})}$$

where $\bar{x}$ = sample mean

μ = population mean

s = S.D of μ sample

n = sample size

where $s = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n - 1}}$

Method 1:

For $n = 40$; $\sigma = 2$, $\mu = 10$

assume $\bar{x} = \mu$

$$SEM = \frac{s}{\sqrt{n}} = \frac{2}{\sqrt{40}} = 0.316$$

z-score: $0.5 \times (1 - 0.95) = 0.025 = 1.96$ ← from table

Confidence interval:

$10 \pm (0.316 \times \overset{1.96}{\cancel{0.025}})$

[9.38, 10.62]

For $n = 80$

$$SEM = \frac{2}{\sqrt{80}} = 0.224$$

$10 \pm (0.224 \times 1.96)$

[9.56, 10.44]

For $n = 160$

$$SEM = \frac{2}{\sqrt{160}} = 0.158$$

$10 \pm (0.158 \times 1.96)$

[9.69, 10.31]

For $n = 1000$

$$SEM = \frac{2}{\sqrt{1000}} = 0.0632$$

$10 \pm (0.0632 \times 1.96)$

[9.88, 10.12]

Method 2:

For n = 5

$$SEM = \frac{2}{\sqrt{5}} = 0.894$$

t - distribution from table

$0.5 \times [1 - 0.95] = 0.025 = 1.95$

Confidence interval:

$10 \pm (0.894 \times 1.95)$

$[8.26, 11.74]$

For n = 10

$$SEM = \frac{2}{\sqrt{10}} = 0.632$$

$10 \pm (0.632 \times 1.95)$

$[8.77, 11.23]$

For n = 20

$$SEM = \frac{2}{\sqrt{20}} = 0.447$$

$10 \pm (0.447 \times 1.95)$

$[9.13, 10.87]$