Midterm 1

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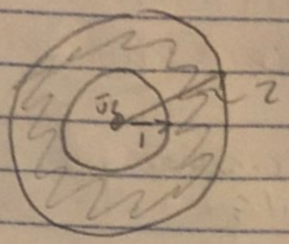
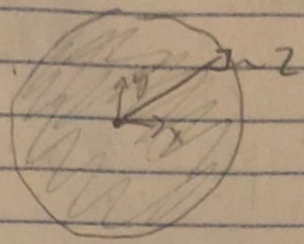
1. a. 5 DOF 2 lengths constrain all
   b. 7 DOF 4 lengths constrain all
   c. 0 DOF no rot
   d. 3 DOF no joints constrain others

2. a. x - z



x - y



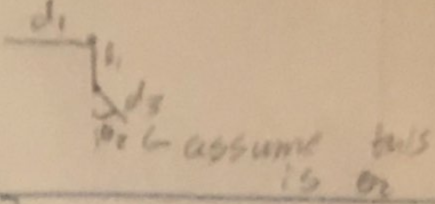
3. a. $$\begin{bmatrix} x \\ y \\ \varphi \end{bmatrix} = \begin{bmatrix} l_1 \cos\theta_1 \\ l_1 \sin\theta_1 \\ \theta_1 \end{bmatrix}$$

b. $$\begin{bmatrix} x \\ y \\ \varphi \end{bmatrix} = \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) \\ l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) \\ \theta_1+\theta_2 \end{bmatrix}$$

c. $$\begin{bmatrix} x \\ y \\ \varphi \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} l_i c_{1,\dots i} \\ \sum_{i=1}^{n} l_1 s_{1,\dots i} \\ \sum_{i=1}^{n} \theta_i \end{bmatrix}$$ (ex) $l_1c_1 + l_2c_{12} + \dots l_n c_{12\dots n}$)

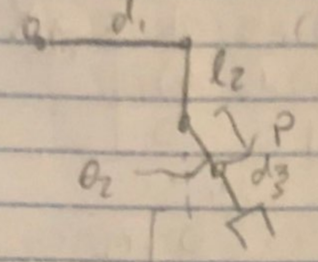
d. $$J_{*i} = \begin{bmatrix} \partial x/\partial\theta_i \\ \partial y/\partial\theta_i \\ \partial\varphi/\partial\theta_i \end{bmatrix} = \begin{bmatrix} \sum_{j=i}^{n} -l_j s_{j\dots n} \\ \sum_{j=i}^{n} l_j c_{j,\dots n} \\ 1 \end{bmatrix}$$

where n is num joints



4. a. $\begin{bmatrix} x \\ y \\ \phi \end{bmatrix} = \begin{bmatrix} d_1 + d_3 \sin\theta_2 \\ -l_1 - d_3 \cos\theta_2 \\ \theta_2 - \pi/2 \end{bmatrix}$

b.



$$X_p = \begin{bmatrix} d_1 + d_3/2 \sin\theta_2 \\ -l_1 - d_3/2 \cos\theta_2 \\ \theta_2 - \pi/2 \end{bmatrix}$$

$$J_p = \begin{bmatrix} 1 & d_3/2 \cos\theta_2 & \frac{1}{2}\sin\theta_2 \\ 0 & d_3/2 \sin\theta_2 & -\frac{1}{2}\cos\theta_2 \\ 0 & 1 & 0 \end{bmatrix}$$

$$F_p = -m \begin{bmatrix} 0 \\ -g \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ mg \\ 0 \end{bmatrix}$$

$$\tau_p = J^T F_p = \begin{bmatrix} 0 \\ \frac{1}{2} mg d_3 \sin\theta_2 \\ -\frac{1}{2} mg \cos\theta_2 \end{bmatrix}$$

Forces due to m_{d_1} & m_{l_2} are zero b/c only prior joints in serial chain are d_1 which only exerts F in the x dir

$$\boxed{\tau = \begin{bmatrix} 0 \\ \frac{1}{2} mg d_3 \sin\theta_2 \\ -\frac{1}{2} mg \cos\theta_2 \end{bmatrix}}$$

4.c. $\begin{bmatrix} x \\ y \\ \varphi \end{bmatrix} = \begin{bmatrix} d_1 + d_3 \sin\theta_2 \\ -l_1 - d_3 \cos\theta_2 \\ \theta_2 - \pi/2 \end{bmatrix}$

$\boxed{\theta_2 = \theta_e + \pi/2}$

$y_e = -l_1 - d_3 \cos\theta_2$

$\boxed{d_3 = (-y_e - l_1) / \cos(\theta_e + \pi/2}$

$x_e = d_1 + d_3 \sin\theta_2$

$d_1 = x_e - d_3 \sin\theta_e$

$\boxed{d_1 = x_e - (y_e - l_1)\tan(\theta_e + \pi/2}$

$$\boxed{\begin{bmatrix} d_1 \\ \theta_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} x_e - (y_e - l_1)\tan(\theta_e + \pi/2) \\ \theta_e + \pi/2 \\ (-y_e - l_1)/\cos(\theta_e + \pi/2) \end{bmatrix}}$$

d. Choose: $X_{ee} = \begin{bmatrix} D - r \\ -F + r \\ -\pi/2 \end{bmatrix}$

$$\begin{bmatrix} d_1 \\ \theta_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} (D-r) - (-F + r - l_1)\tan(-\pi/2 + \pi/2) \\ -\pi/2 + \pi/2 \\ (-(-F + r) - l_1)/\cos(-\pi/2 + \pi/2) \end{bmatrix}$$

$\begin{bmatrix} d_1 \\ \theta_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} D - r \\ 0 \\ F - r - l_1 \end{bmatrix}$ $\quad d_1 = D - r < D$

$\theta_2 = 0 \quad -\pi/2 < 0 < \pi/2$

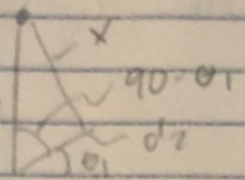
$d_3 = F - r - l_1 \rightarrow l_1 + d_3 < F$

✓

Possible

5. a. $\boxed{T = \frac{1}{2}m\dot{d}_2^2 + \frac{1}{2}md_2\dot{\theta}_1^2}$

b.



$x^2 = h^2 + d_2^2 - 2hd_2\cos(90-\theta_1)$

$x^2 = h^2 + d_2^2 - 2hd_2\sin(\theta_1)$

$$\boxed{V = mgd_2\sin\theta_1 + \frac{1}{2}k(h^2 + d_2^2 - 2hd_2\sin\theta_1)}$$

c. $L = T - V$

$$\boxed{L = \frac{1}{2}m\dot{d}_2^2 + \frac{1}{2}md_2^2\dot{\theta}_1^2 - mgd_2\sin\theta_1 - \frac{1}{2}k(h^2 + d_2^2 - 2hd_2\sin\theta_1)}$$

$-\frac{1}{2}kd_2^2$
$-\frac{1}{2}kh^2$
$+khd_2\sin\theta_1$

d. $\partial L/\partial\theta_1 = -mgd_2\cos\theta_1 + khd_2\cos\theta_1$

$\partial L/\partial\dot{\theta}_1 = md_2^2\dot{\theta}_1$

$md_2^2 \quad \dot{d}_2 \cdot 2md_2$

$\frac{d}{dt}(\partial L/\partial\dot{\theta}_1) = md_2^2\ddot{\theta}_1 + 2md_2\dot{d}_2\dot{\theta}_1$

$\tau_{\theta_1} = \frac{d}{dt}(\partial L/\partial\dot{\theta}_1) - \partial L/\partial\theta_1$

$$\boxed{\tau_{\theta_1} = md_2^2\ddot{\theta}_1 + 2md_2\dot{d}_2\dot{\theta}_1 + mgd_2\cos\theta_1 - khd_2\cos\theta_1}$$

e. $\partial L/\partial d_2 = md_2\dot{\theta}_1^2 - mg\sin\theta_1 - kd_2 + kh\sin\theta_1$

$\partial L/\partial\dot{d}_2 = m\dot{d}_2$

$\frac{d}{dt}(\partial L/\partial\dot{d}_2) = m\ddot{d}_2$

$\tau_{d_2} = \frac{d}{dt}(\partial L/\partial\dot{d}_2) - \partial L/\partial d_2$

$$\boxed{\tau_{d_2} = m\ddot{d}_2 - md_2\dot{\theta}_1^2 + mg\sin\theta_1 + kd - kh\sin\theta_1}$$