

# Assignment 6: Rigid Body Dynamics

Robot Kinematics and Dynamics

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# 1 Overview

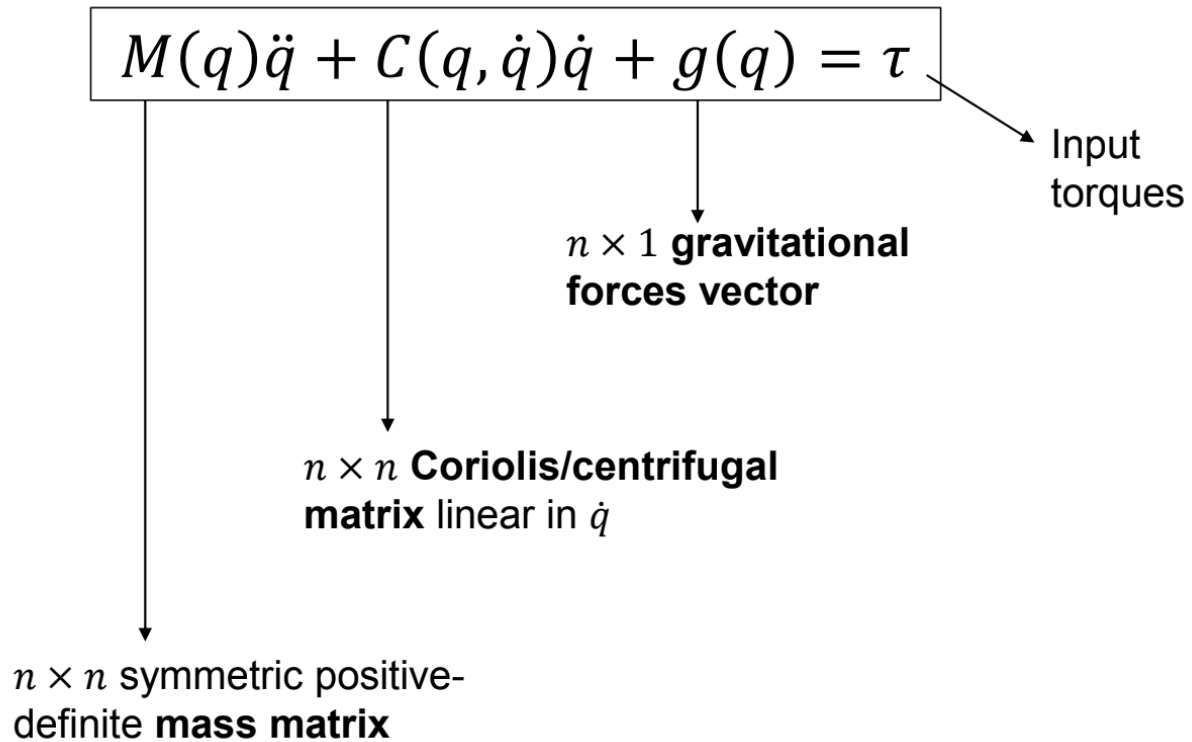
This assignment reinforces the following topics:

- Rigid Body Dynamics

## 2 Background

### 2.1 Standard Form

As a reminder, the standard form for writing equations of motion is as follows:

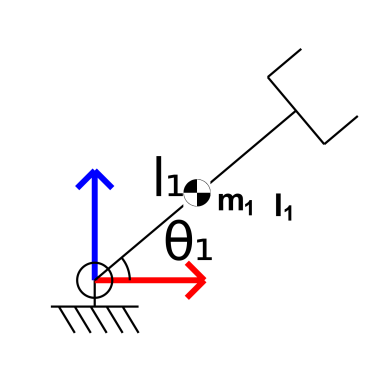


### 3 In Class Question

The following question will be done in class, as a part of a group. Your group's answer will still need to be turned in with the rest of your assignment, however unlike the rest of the work this is allowed to be done in groups.

#### 1) Rigid Body Dynamics

Please use the diagram of the arm below for the following questions:



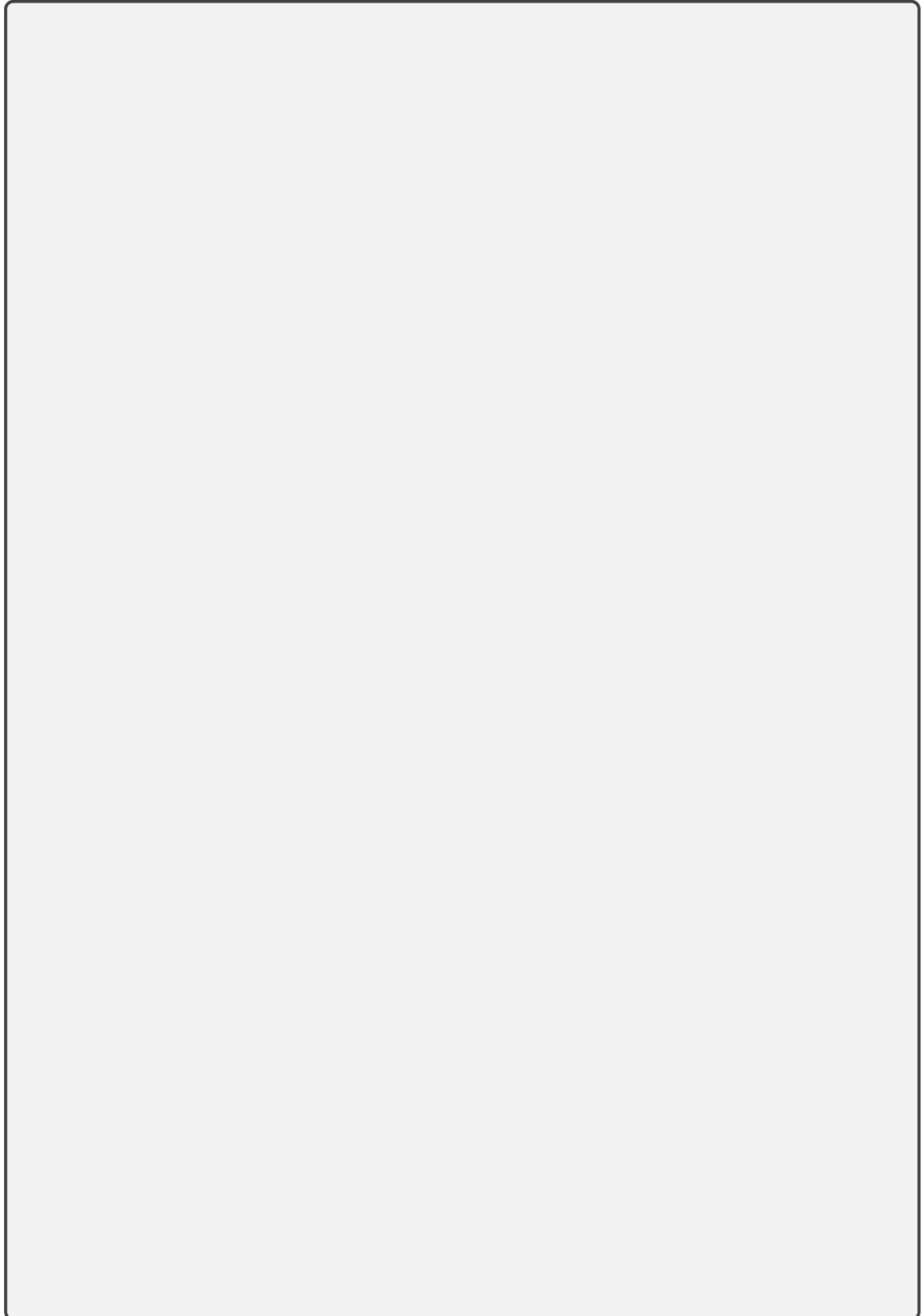
The arm has a link length of  $l_1$  where the center of mass is located in the center of the link at  $\frac{l_1}{2}$ . The link's mass is  $m_1$  and it has moment of inertia  $I_1$ .

(1) [5 points] Determine the Kinetic Energy for the R arm shown above.

- (2) [5 points] Determine the Potential Energy for the R arm shown above.

- (3) [5 points] Write the Lagrangian for the R arm shown above.

- (4) [5 points] Determine the Equations of Motion for the R arm shown above using the Lagrangian.



(5) [5 points] Rewrite the equations of motion into standard form.



## 4 Written Questions

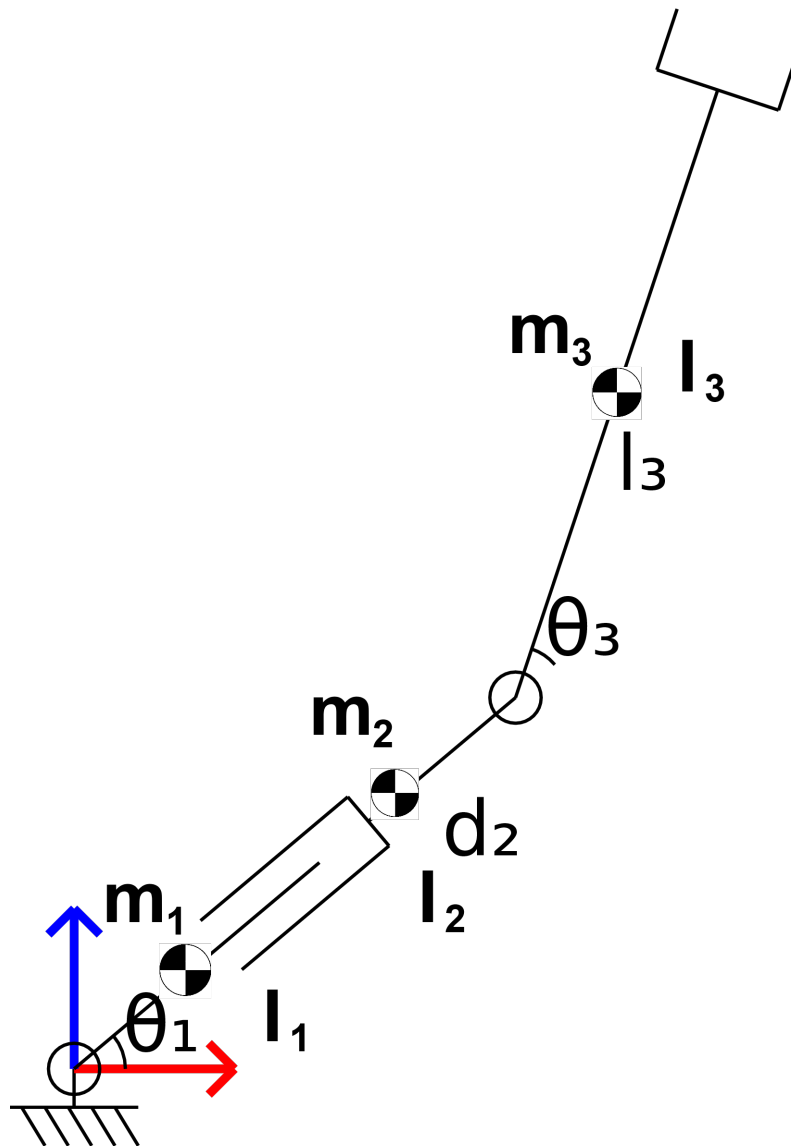
For the following problems, fully evaluate all answers unless otherwise specified.

Answers for written questions must be typed. We recommend L<sup>A</sup>T<sub>E</sub>X, Microsoft Word, OpenOffice, or similar. However, diagrams can be hand-drawn and scanned in.

Unless otherwise specified, **all units are in radians, meters, and seconds**, where appropriate.

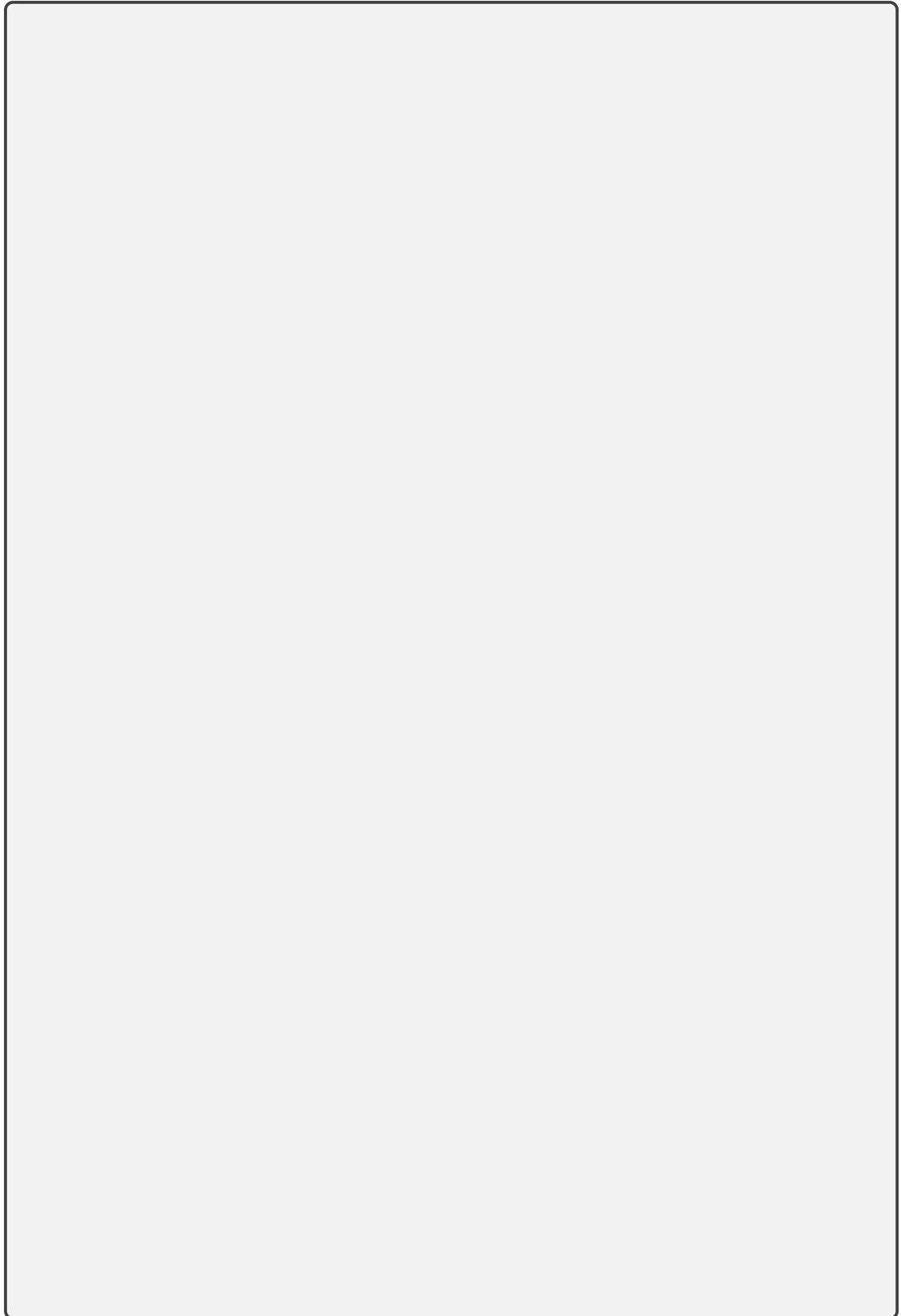
### 1) RPR robot

Consider the following robot

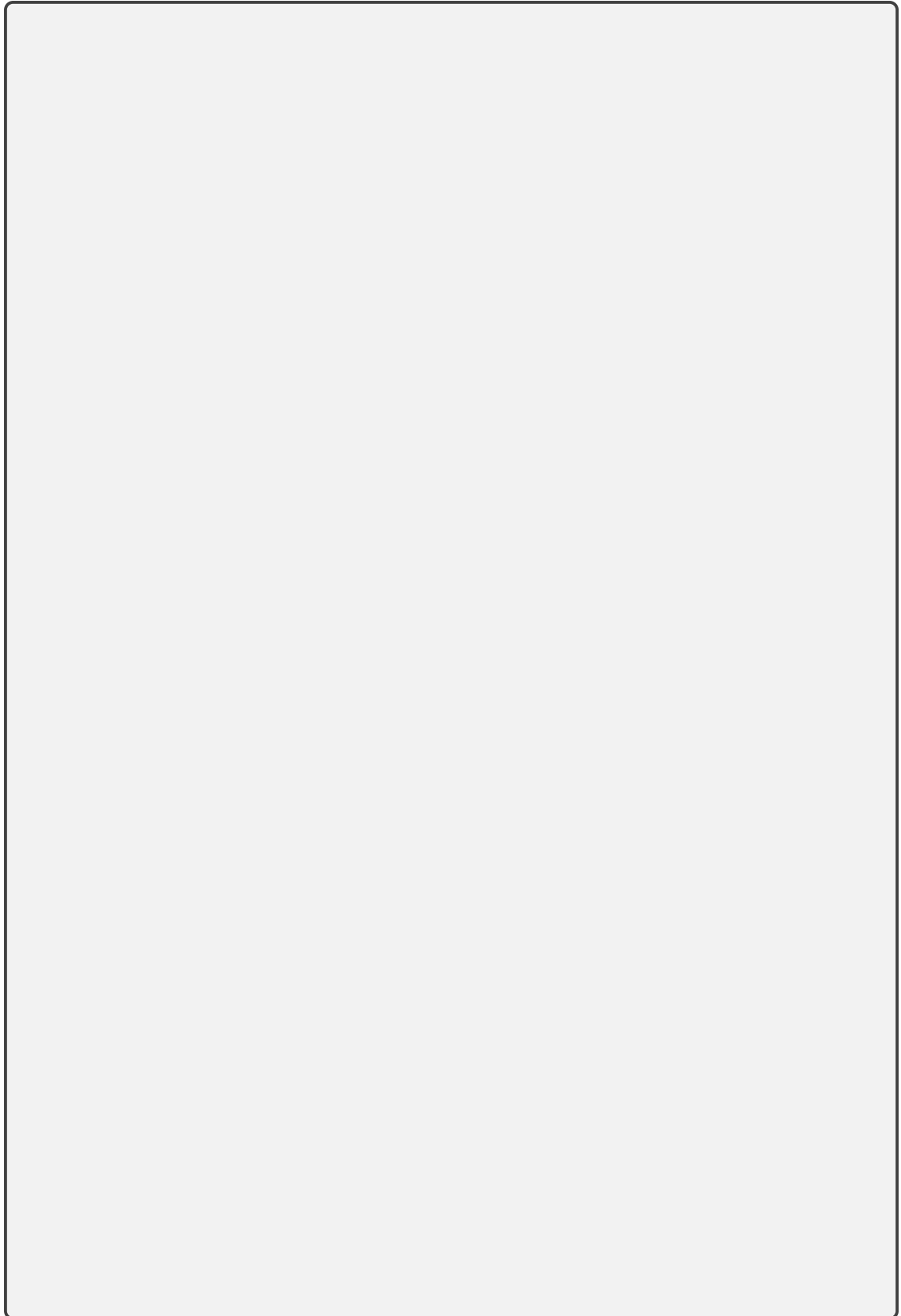


Define the entire distance between the revolute joints as  $d_2$ . The distance from the first revolute joint to  $m_1$  is  $r_1$ . The distance from the first revolute joint to  $m_2$  is  $\frac{3d_2}{4}$ . Finally, the distance from the second revolute joint to  $m_3$  is  $\frac{l_3}{2}$ .

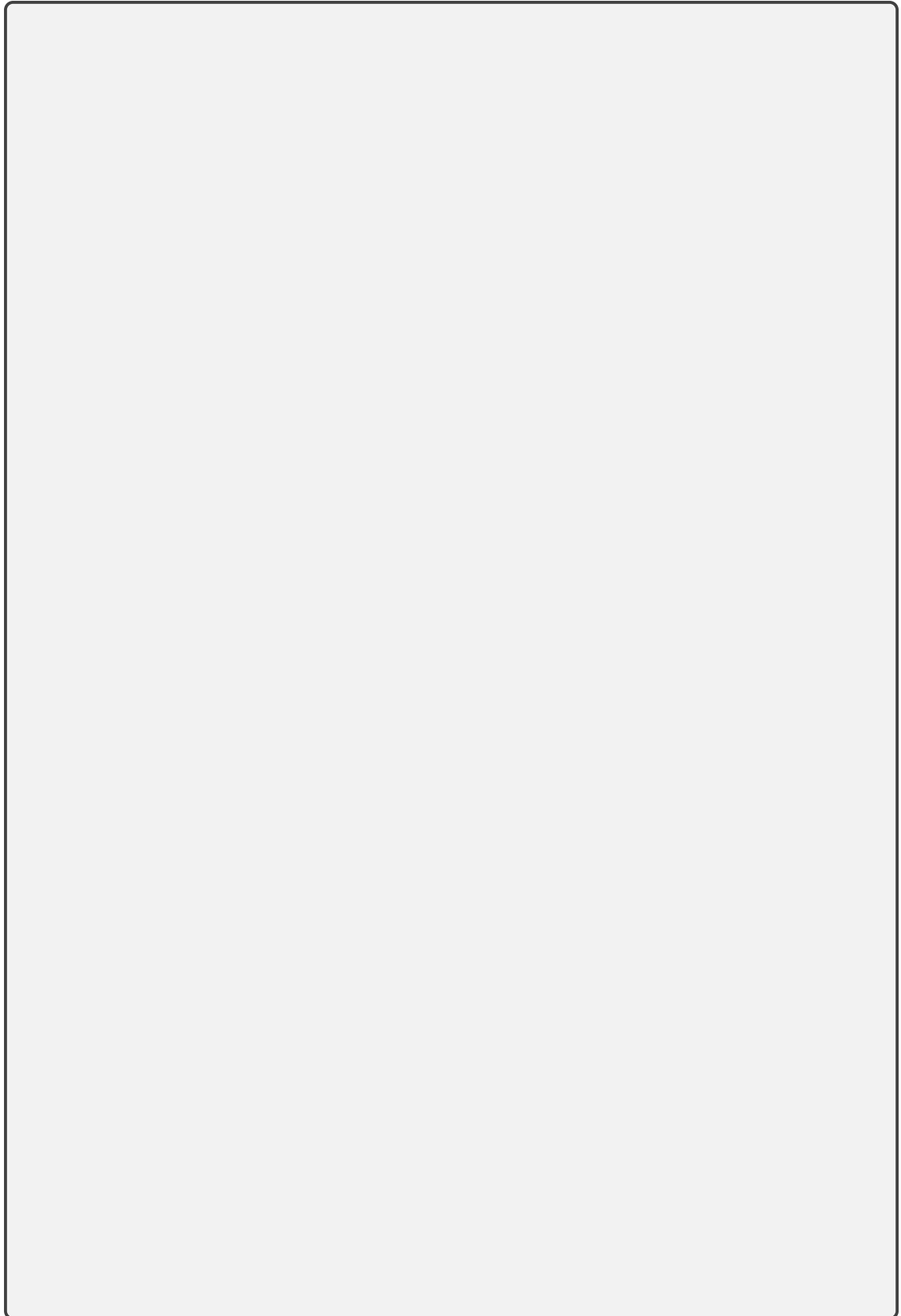
- (1) [10 points] Determine the Kinetic Energy for the RPR arm shown above.



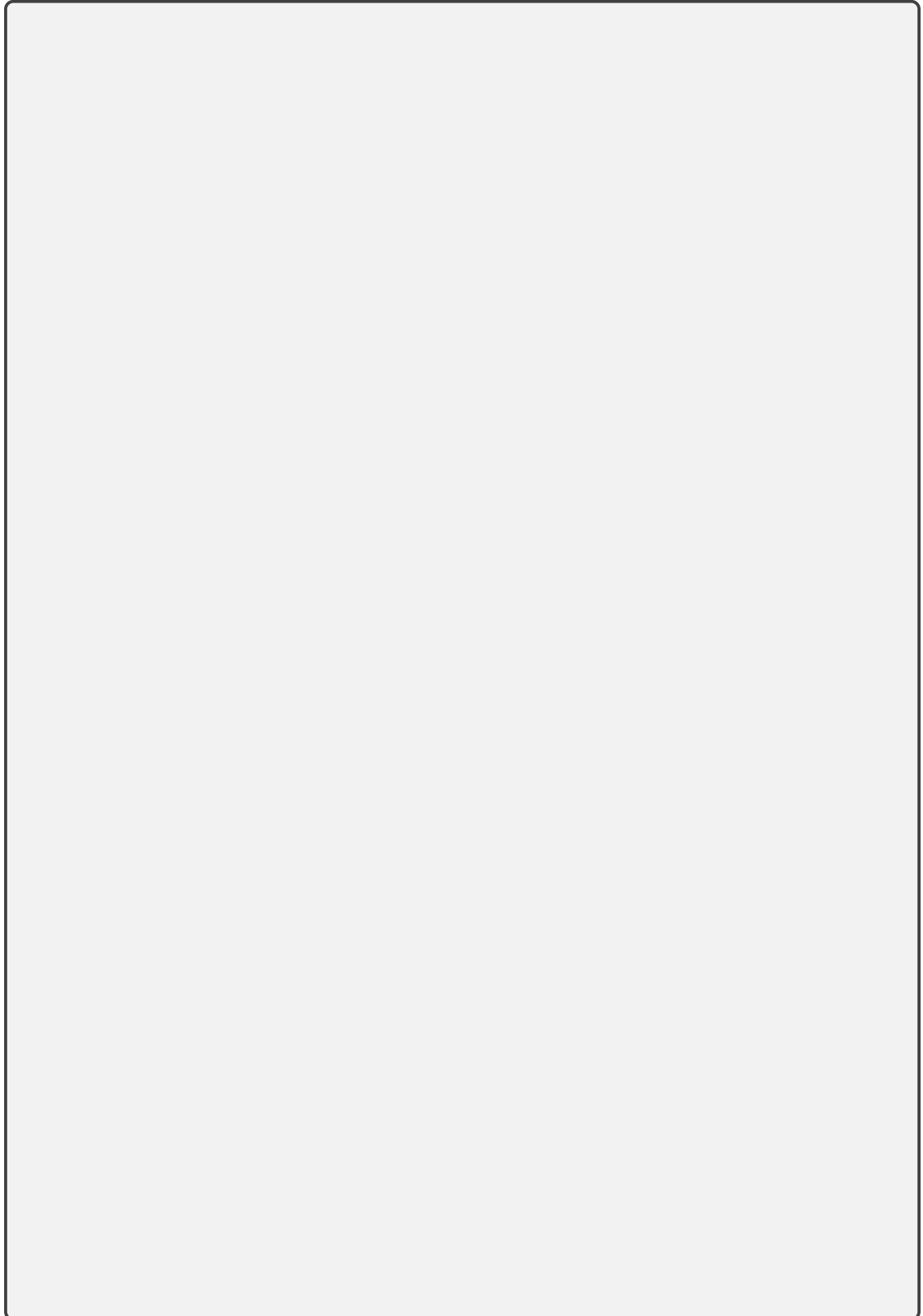
(2) [10 points] Determine the Potential Energy for the RPR arm shown above.



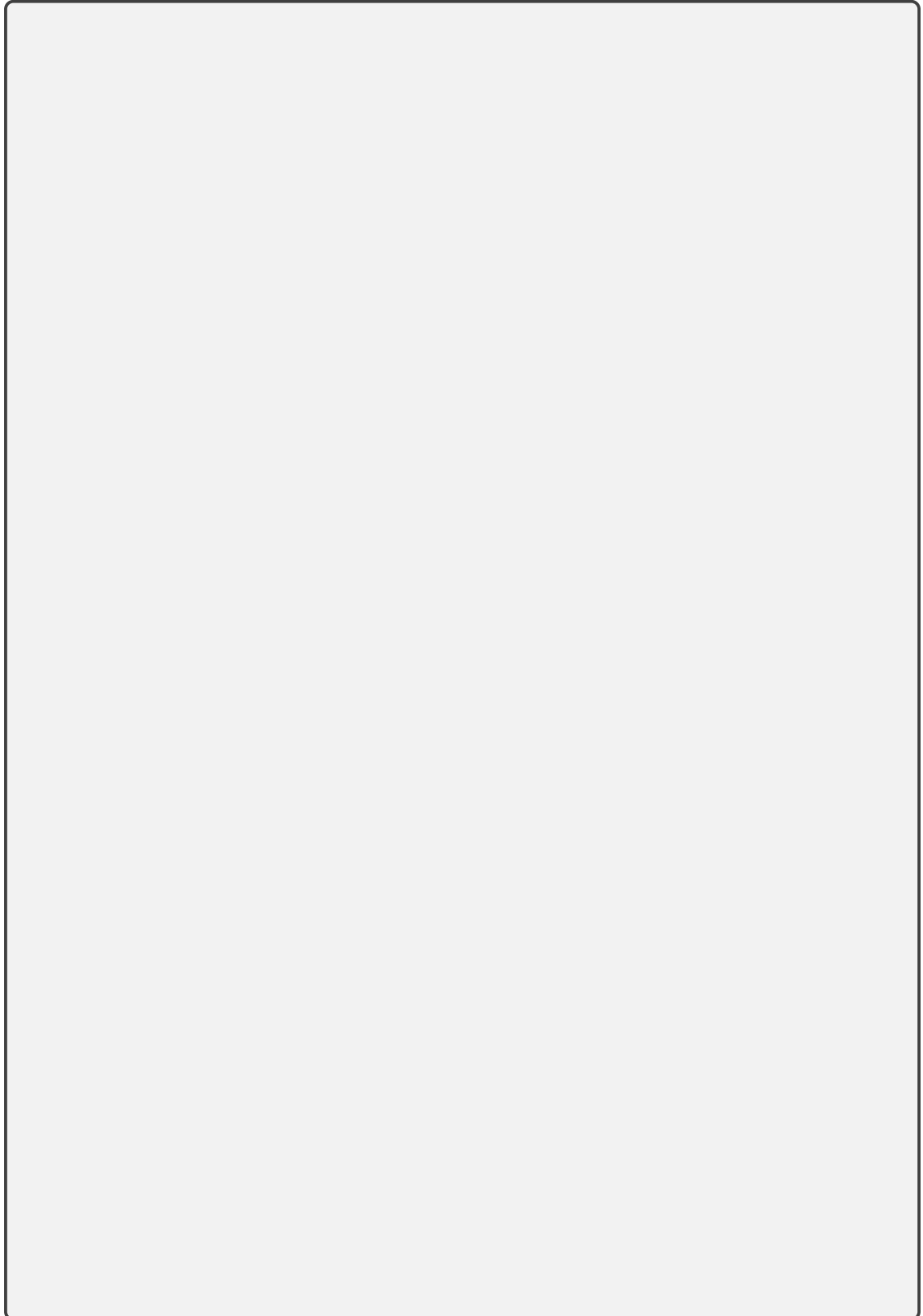
(3) [10 points] Write the Lagrangian for the RPR arm shown above.



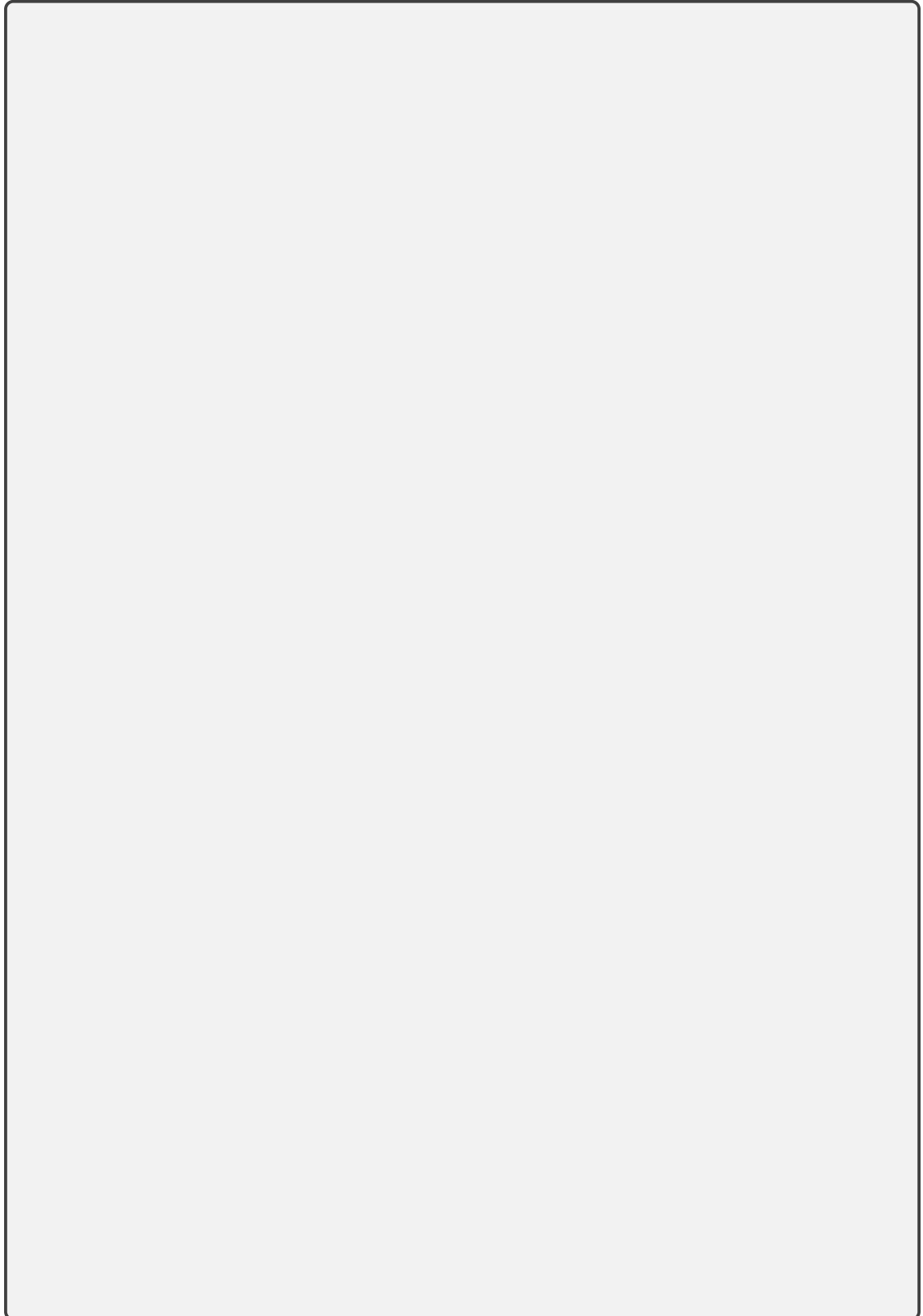
- (4) [10 points] Determine the Equations of Motion with respect to  $\theta_1$  for the RPR arm shown above using the Lagrangian.



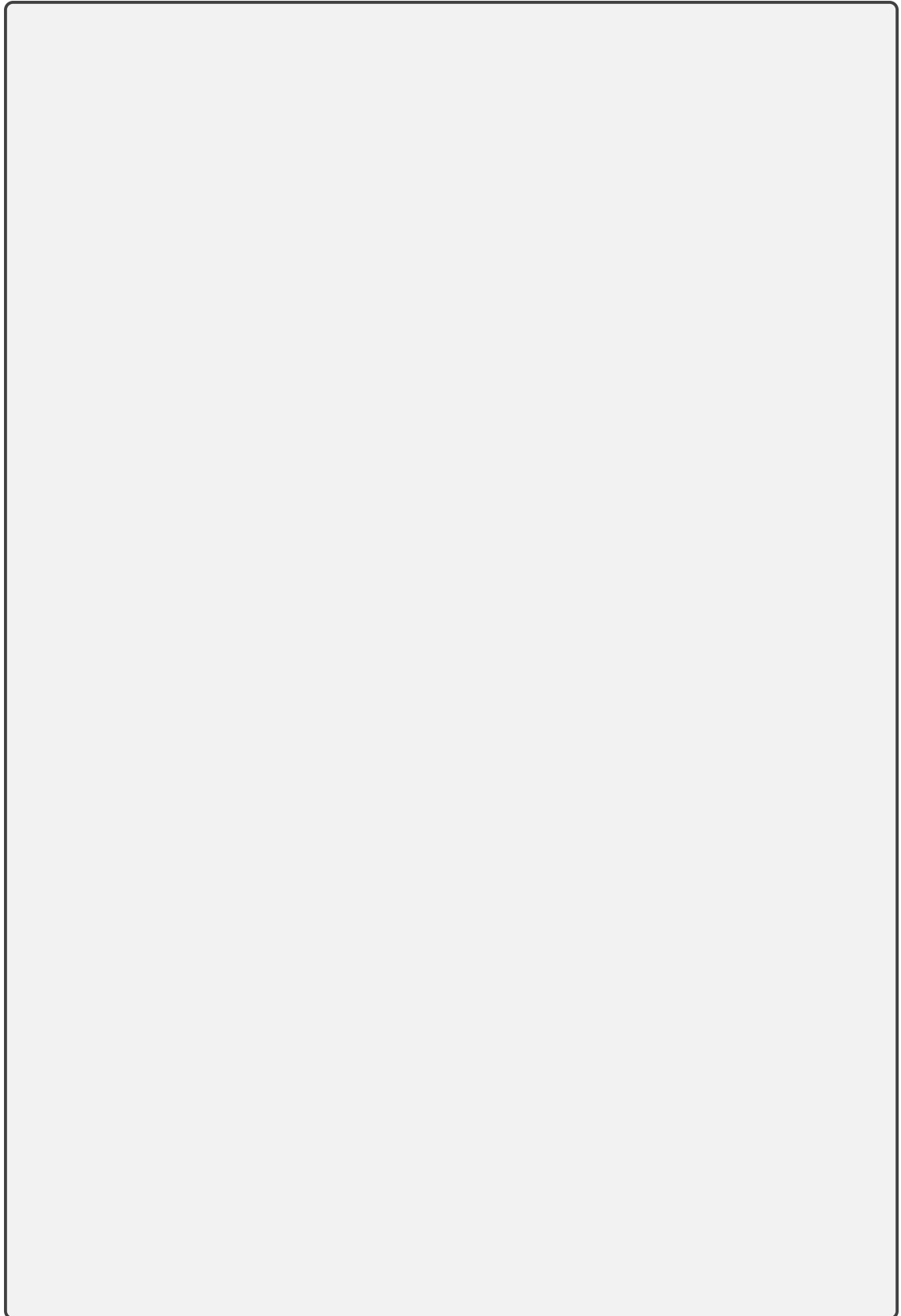
- (5) [10 points] Determine the Equations of Motion with respect to  $d_2$  for the RPR arm shown above using the Lagrangian.



- (6) [10 points] Determine the Equations of Motion with respect to  $\theta_3$  for the RPR arm shown above using the Lagrangian.



(7) [10 points] Rewrite the equations of motion into standard form.





## 5 Feedback

### 1) Feedback Form

*5 points*

We are always looking to improve the class! To that end, we're looking for your feedback on the assignments. When you've completed the assignment, please fill out the [feedback form](#).

## 6 Code Questions

There is no coding portion in this homework.

## 7 Submission Checklist

- ☐ Upload writeup.pdf to Gradescope and Canvas.
- ☐ After completing the entire assignment, fill out the feedback form<sup>1</sup>.

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<sup>1</sup><https://canvas.cmu.edu/courses/11823/quizzes/27788>