Assignment 0

Robot Kinematics and Dynamics

Prof. Howie Choset

Contents

1 Overview

Welcome to Robot Kinematics and Dynamics! This assignment serves to ensure you can get started with the tools we will be using this semester, as well as a brief overview of some of the prerequisite information we'll be assuming you already know. The topics and processes covered in this assignment will be heavily used throughout this class. Please meet with course staff if you have trouble with any part of the assignment, so we can make sure you are ready for assignment 1.

2 Background

In this section, we will give a brief review of the main concepts covered in lecture as they pertain to the labs.

2.1 Matrices

In this course, we will make extensive use of matrices and matrix multiplication. While we will not need the anything but the basics of linear algebra, you must absolutely be comfortable with the following:

- matrix multiplication
- transposition
- inversion
- rank
- vector norms
- determinants
- cross products
- dot products

2.2 Calculus

We expect you to know the following calculus concepts:

- Derivatives (including trigonometric functions)
- Partial Derivatives

2.3 Course Logistics (Specific to CMU)

In this class you would be using the following websites regularly:

- Course Website: This site will act as the central hub for the course. The course calendar, due dates, syllabus, etc. can be found on this site.
- Piazza: This is the best way to ask course staff questions, and see course announcements.
- Canvas: We'll be releasing all the assignments on Canvas, as well as keeping track of your grades.
- Gradescope: Create an account using your CMU email. You should have already been added to the course. If you haven't, please contact the course staff via a Piazza post.

3 Written Questions

For the following problems, fully evaluate all answers unless otherwise specified.

Answers for written questions must be typed. We recommend LaTeX, Microsoft Word, OpenOffice, or similar. However, diagrams can be hand-drawn and scanned in.

Unless otherwise specified, **all units are in radians, meters, and seconds**, where appropriate.

1) Piazza *5 points*

You should have received an email inviting you to the course Piazza instance. We'll make course announcements via Piazza. To ensure that you have been properly added, please go to the Assignment 0 release post. On this post there will be a code, submit that code as the answer to this question.

$i_{f}ound_{p}iazza$

2) Matrices and Matrix Operations

(1) [6 points] Which of the following are valid expressions? Give your answer as a list of numbers (for example: `1,2,4`).

1. $\begin{bmatrix}1&1&1\\1&1&1\end{bmatrix} \times \begin{bmatrix}1&1&1\\1&1&1\end{bmatrix}$
2. $\begin{bmatrix}1&1\\1&1\end{bmatrix} \times \begin{bmatrix}1\\1\end{bmatrix}$
3. $\begin{bmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\end{bmatrix} \times \begin{bmatrix}1&1&1\\1&1&1\\1&1&1\\1&1&1\end{bmatrix}$
4. $\begin{bmatrix}1\\1\end{bmatrix} \times \begin{bmatrix}1&1\\1&1\end{bmatrix}$
5. $\begin{bmatrix}1&1&1\\1&1&1\end{bmatrix} \times \begin{bmatrix}1\\1\end{bmatrix}$
6. $\begin{bmatrix}1&1&1\\1&1&1\end{bmatrix} \times \left(\begin{bmatrix}1&1&1\\1&1&1\end{bmatrix}\right)^{\top}$

2, 3, 6

(2) [15 points] Evaluate the following:

1. $\begin{bmatrix}a&b&c\\d&e&f\end{bmatrix} \times \begin{bmatrix}3\\1\\7\end{bmatrix}$

$\begin{bmatrix}3a+b+7c\\3d+e+7f\end{bmatrix}$

2. $\begin{bmatrix}9&3\\0&1\end{bmatrix} \times \begin{bmatrix}2&1\\4&3\end{bmatrix}$

$\begin{bmatrix}30&18\\4&3\end{bmatrix}$

3. $\begin{bmatrix}2&1\\4&3\end{bmatrix} \times \begin{bmatrix}9&3\\0&1\end{bmatrix}$

$\begin{bmatrix}18&7\\36&15\end{bmatrix}$

4. Given the rotation matrix $R = \begin{bmatrix} \cos(x) & -\sin(x) \\ \sin(x) & \cos(x) \end{bmatrix}$

   (a) Determine R^{-1}

   R is orthonormal, so $R^{-1} = R^T$
   $R^{-1} = \begin{bmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{bmatrix}$

   (b) Determine R^T

   $R^T = \begin{bmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{bmatrix}$

   (c) Do you see a relationship between R^{-1} and R^T

   They are equivalent

5. Given the matrices $A = \left[\begin{smallmatrix} 1 & 3 \\ 4 & 15 \end{smallmatrix}\right]$ and $B = \left[\begin{smallmatrix} 1 & 2 \\ 4 & 8 \end{smallmatrix}\right]$

   (a) Determine $\det(A)$

   $\det(A) = (1)(15) - (3)(4) = 3$

   (b) Determine $\text{rank}(A)$. Is A full rank?

   In RREF, $R_A = \left[\begin{smallmatrix} 1 & 3 \\ 0 & 1 \end{smallmatrix}\right]$, so A is rank 2 (Full rank)

(c) Determine $\det(B)$

$\det(B) = (1)(8) - (2)(4) = 0$

(d) Determine $\text{rank}(B)$. Is B full rank?

$R_B = \left[\begin{smallmatrix} 1 & 2 \\ 0 & 0 \end{smallmatrix}\right]$, so B is rank 1 (Not full rank)

(e) Do you notice a relationship between the rank and the determinant of these matrices?

If M is not full rank, its determinant is 0

6. Given the vectors $\mathbf{x} = [1, 2, 3]$ and $\mathbf{y} = [4, 5, 6]$

(a) Determine $\mathbf{x} \cdot \mathbf{y}$

$\mathbf{x} \cdot \mathbf{y} = 32$

(b) Determine $\mathbf{x} \times \mathbf{y}$

$\mathbf{x} \times \mathbf{y} = det(\left[\begin{smallmatrix} i & j & k \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{smallmatrix}\right])$
$= [(2)(6) - (3)(5), (3)(4) - (1)(6), (1)(5) - (2)(4)]$
$= [-3, 6, -3]$

(c) Determine $\|\mathbf{x}\|$ and $\|\mathbf{y}\|$

$\|\mathbf{x}\| = \sqrt{14}$
$\|\mathbf{y}\| = \sqrt{77}$

7. Invert $\begin{bmatrix} 1 & 1 \\ 2/3 & 1 \end{bmatrix}$.

$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 2/3 & 1 & 0 & 1 \end{array}\right]$
$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1/3 & -2/3 & 1 \end{array}\right]$
$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 3 \end{array}\right]$
$\left[\begin{array}{cc|cc} 1 & 0 & 3 & -3 \\ 0 & 1 & -2 & 3 \end{array}\right]$
$M^{-1} = \begin{bmatrix} 3 & -3 \\ -2 & 3 \end{bmatrix}$

3) Calculus

(1) [4 points] Find the derivative of $x\cos(x)$.

$\cos(x) - x\sin(x)$

(2) [10 points] Find the partial derivatives with respect to x and y of $f(x, y) = x\sin(y) + y^2\cos(x)$.

$f_x = \sin(y) - y^2\sin(x)$
$f_y = x\cos(y) + 2y\cos(x)$

4 Feedback

1) Feedback Form *5 points*

We are always looking to improve the class! To that end, we're looking for your feedback on the assignments. When you've completed the assignment, please fill out the feedback form.

5 Code Questions

In this section, we will re-enforce the written questions with practical examples. These questions, written in Matlab, will be the beginning of your code for the hands-on lab. To begin, you must install Matlab. A free copy is available to all CMU students at:

http://www.cmu.edu/computing/software/all/matlab/

Once you've installed Matlab on a personal computer, open Matlab and navigate to Code Handout folder. In this folder there will be a few exercises. These exercises will review the basics of Matlab syntax.

1) Exercise 01 *20 points*

For this exercise, edit the `ex_01.m` file. In the file is four variables.

- Set `A` to be the 2x2 matrix $\left[\begin{smallmatrix}1 & 2\\ 3 & 4\end{smallmatrix}\right]$.
- Set `B` to be the 2x3 matrix $\left[\begin{smallmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{smallmatrix}\right]$.
- Set `C` to be `A` times `B`.
- Set `D` to be $(\texttt{B})^\top$ times `A`.

2) Exercise 02 *10 points*

For this exercise, edit the `ex_02.m` file. In it are two variables.

- Set `a` to be an array containing values from 0 to 100 (inclusive), incrementing by `0.2`. Hint: look into the `:`[1] operator.
- Set `b` to be the `100x100` identity matrix. Hint: look at the `eye`[2] function.

3) Exercise 03 *20 points*

In this exercise, we'll review Matlab's function syntax. The file `ex_03.m` contains a function that outputs a matrix `R`. Fill it out so that the function multiplies its parameters `A`, `B` as `A` times `B` times `A` times `B`.

4) Submission

To submit, run `create_submission.m`. It will first check that your submitted files run without error, and perform a small sanity check. Note, this is not going to grade your submission! The function will create a file called `handin.tar.gz`. Upload it to Canvas to complete the submission.

[1] http://www.mathworks.com/help/matlab/ref/colon.html
[2] http://www.mathworks.com/help/matlab/ref/eye.html

6 Submission Checklist

- ☐ Create a PDF of your answers to the written questions with the name `writeup.pdf` and submit it to Gradescope.
- ☐ Make sure you have `writeup.pdf` in the same folder as the `create_submission.m` script.
- ☐ Run `create_submission.m` in Matlab.
- ☐ Upload `handin.tar.gz` to Canvas.
- ☐ After completing the entire assignment, fill out the feedback form[3].

[3] https://canvas.cmu.edu/courses/18336/quizzes/39692