

1.

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} + O(h^2)$$

$$f(x+h) = f(x) + \frac{f'(x)}{1!} \cdot h + \frac{f''(x)}{2!} \cdot h^2 + \frac{f'''(x)}{3!} \cdot h^3 + \frac{f^{(4)}(x)}{4!} \cdot h^4 + \dots \quad \text{--- ①}$$

$$f(x-h) = f(x) - \frac{f'(x)}{1!} \cdot h + \frac{f''(x)}{2!} \cdot h^2 - \frac{f'''(x)}{3!} \cdot h^3 + \frac{f^{(4)}(x)}{4!} \cdot h^4 + \dots \quad \text{--- ②}$$

$$f(x+h) - f(x-h) = 2 \cdot f'(x) \cdot h + 2 \cdot f'''(x) \cdot \frac{1}{3!} \cdot h^3 + 2 \cdot f^{(5)}(x) \cdot \frac{1}{5!} \cdot h^5 + \dots$$

$$\frac{f(x+h) - f(x-h)}{2h} = f'(x) + f'''(x) \cdot \frac{1}{3!} \cdot h^2 + f^{(5)}(x) \cdot \frac{1}{5!} \cdot h^4 + \dots$$

$$\Rightarrow \frac{f(x+h) - f(x-h)}{2h} = f'(x) + f'''(x) \cdot \frac{1}{3!} \cdot h^2 + f^{(5)}(x) \cdot \frac{1}{5!} \cdot (h^2)^2 + \dots$$

$$\Rightarrow f'(x) = \frac{f(x+h) - f(x-h)}{2h} + O(h^2)$$

2.

$$f''(x) = \frac{f(x+h) - 2h + f(x-h)}{h^2} + O(h^2)$$

$$f(x+h) = f(x) + \frac{f'(x)}{1!} \cdot h + \frac{f''(x)}{2!} \cdot h^2 + \frac{f'''(x)}{3!} \cdot h^3 + \frac{f^{(4)}(x)}{4!} \cdot h^4 + \dots \quad \text{--- ①}$$

$$f(x-h) = f(x) - \frac{f'(x)}{1!} \cdot h + \frac{f''(x)}{2!} \cdot h^2 - \frac{f'''(x)}{3!} \cdot h^3 + \frac{f^{(4)}(x)}{4!} \cdot h^4 + \dots \quad \text{--- ②}$$

$$f(x+h) + f(x-h) = 2 \cdot f(x) + f''(x) \cdot h^2 + 2 \cdot \frac{f^{(4)}(x)}{4!} \cdot h^4 + \dots$$

$$\Rightarrow f(x+h) - 2f(x) + f(x-h) = f''(x) \cdot h^2 + 2 \cdot \frac{f^{(4)}(x)}{4!} \cdot h^4 + \dots$$

$$\Rightarrow \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} = f''(x) + 2 \cdot \frac{f^{(4)}(x)}{4!} \cdot h^2 + \dots$$

$$\Rightarrow f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} + O(h^2)$$