Derive the theoritical iteration relationship of Laplace equation

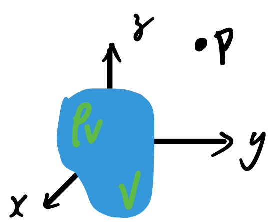
$$\nabla \cdot \vec{D} = \rho_v \Rightarrow \nabla \cdot \vec{E} = \frac{\rho_v}{\varepsilon_0} \Rightarrow -\nabla^2 \cdot V = \frac{\rho_v}{\varepsilon_0}$$

$$\boxed{D = \varepsilon_0 \cdot E} \qquad \boxed{\vec{E} = -\nabla V}$$

$$\nabla^2 V = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} \right)\left(\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\cdot\hat{y} \right)$$

$$= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -\frac{\rho_v}{\varepsilon_0}$$

↳ non-homogeneous equation



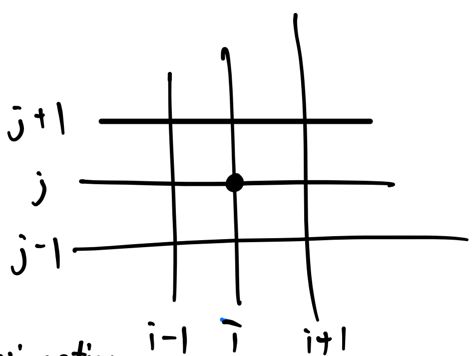
If we want to observe potential out of the ρ_v.

⇒ the whole question turns to homogeneous

Laplace equation, (A special case of Poisson's eq.)

$$\nabla^2 \cdot V = 0$$

$$\Rightarrow \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) V = 0$$



by central difference approximation

$$\left.\frac{\partial^2 V}{\partial x^2}\right|_{i,j} = \frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{\Delta x^2}$$

$$\left.\frac{\partial^2 V}{\partial y^2}\right|_{i,j} = \frac{V_{i,j+1} + V_{i,j-1} - 2V_{i,j}}{\Delta y^2}$$

$$\underline{\Delta x = \Delta y}$$

Laplace equation : $\nabla^2 \cdot V = 0$

$$\Rightarrow \quad \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

$$\Rightarrow \quad \frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{\Delta x^2} + \frac{V_{i,j+1} + V_{i,j-1} - 2 \cdot V_{i,j}}{\Delta y^2} = 0$$

$\times (\Delta x)^2$

$$\Rightarrow \quad \underline{V_{i+1,y} + V_{i-1,y} + V_{i,j+1} + V_{i,j-1} = 4 \cdot V_{i,j}} \quad \#$$