University of Pittsburgh

Algorithms and Data Structures 2
CS 1501

Fall 2022

Sherif Khattab

ksm73@pitt.edu

(Slides are adapted from Dr. Ramirez's and Dr. Farnan's CS1501 slides.)

Announcements

- Upcoming Deadlines
  - Homework 5: this Friday @ 11:59 pm
  - Lab 3: tonight @ 11:59 pm
  - Lab 4: next Monday @ 11:59 pm
  - Assignment 1: Monday Oct 10th @ 11:59 pm
- **Live support session** for Assignment 1
  - Recording and slides on Canvas
- **Student Support Hours** of the teaching team are posted on the Syllabus page

Previous lecture

- R-way Radix Search Tries
- De La Briandais (DLB) Tries

This Lecture

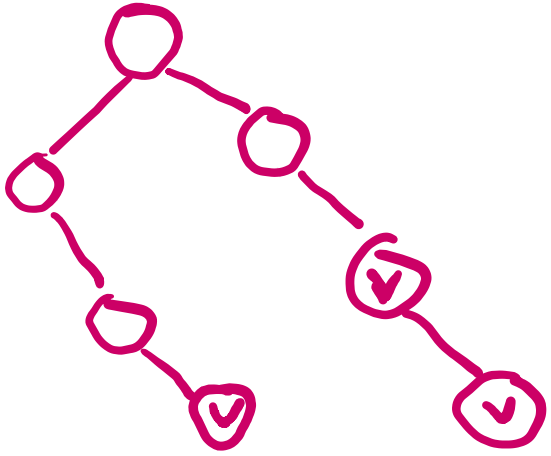
Compression

Muddiest Points

- **Q: When creating a DST, does it have to start handling keys starting with the leftmost bit, or can it also handle them by starting with the rightmost bit?**
- The algorithm can go either way on the bitstring of the key as long as the direction is the same for all operations

Muddiest Points

- **Q: Would a trie be able to contain a value with less bits than the root, and if so how?**
- In a trie, none of the nodes (including the root) contains any key
- If the questions is “can a trie contain keys of different bit lengths?”,
  - the answer is yes
  - Interior nodes have non-null values in that case
  - The trie here has three keys
    - 011
    - 111
    - 11



Muddiest Points

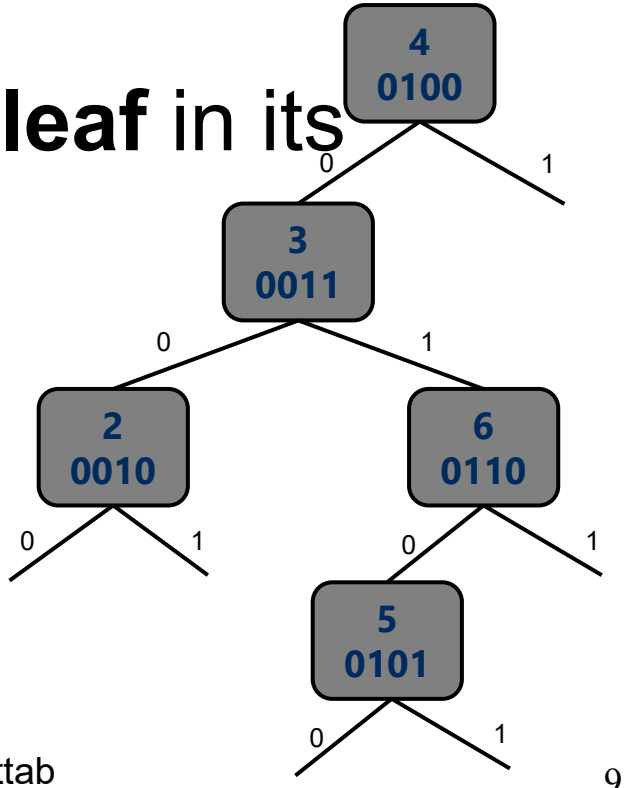
- **Q: How is a Trie different from a Red-Black BST?**
- a trie is different from a search tree because trie doesn't store the keys inside its nodes but a search tree does
- More in the next question

Muddiest Points

- **Q: when would you use a DST or an RST?**
- **Q: What's the application of RST?**
- DST and RST are efficient in checking if a target key is a prefix of any of the keys in the tree
  - e.g., making routing decisions in the Internet
- DSTs are preferred over BSTs when bits of keys are randomly distributed (i.e., the probability of each bit being zero is 0.5)
  - The DST will be balanced in this case without having to use the more complicated Red-Black BST
- RSTs are preferred over BSTs when bit lengths of keys are close to *log n*
  - The RST will be balanced in this case without having to use the more complicated Red-Black BST
- Note that DST and RST don't provide the extra operations (e.g., predecessor and successor) provided by BST

Muddiest Points

- **Q: how can any node in 3's subtree replace 3 in DST example**
- Because all nodes in 3's subtree share a common prefix with length 1 with 3
  - The node that replaces 3 will still be found using the DST search algorithm
- For simplicity, we replace 3 with any **leaf** in its subtree



Muddiest Points

- **Q: Could you spend more time going through new lecture**
- Sometimes, addressing the muddiest points takes up a large portion of class time
- Usually, new material is embedded between the muddiest points responses

Muddiest Points

- **Q: Is the insertion position for DST based on the first bit that is different from the last insert? Or is it based on the relative comparison to last insert?**
- DST Add Algorithm for adding a key *k* and a corresponding value
  - if root is null, add *k* at the root and return
  - current ← root
  - if k is equal to the current node's key, replace value and return
  - if current bit of k is 0,
    - if left child is null, add *k* as left child
    - else continue to left child
  - if current bit of k is 1,
    - if left child is null, add *k* as right child
    - else continue to right child

Muddiest Points

- **Q: When is DST preferable to radix search trie?**
- A: When bit lengths of keys are >> log n

Muddiest Points

- **Q: I don't understand the advantage of making another node in the DLB instead of the tree structure.**
- A: DLB saves space when the number of children per node in an R-way RST is small

Muddiest Points

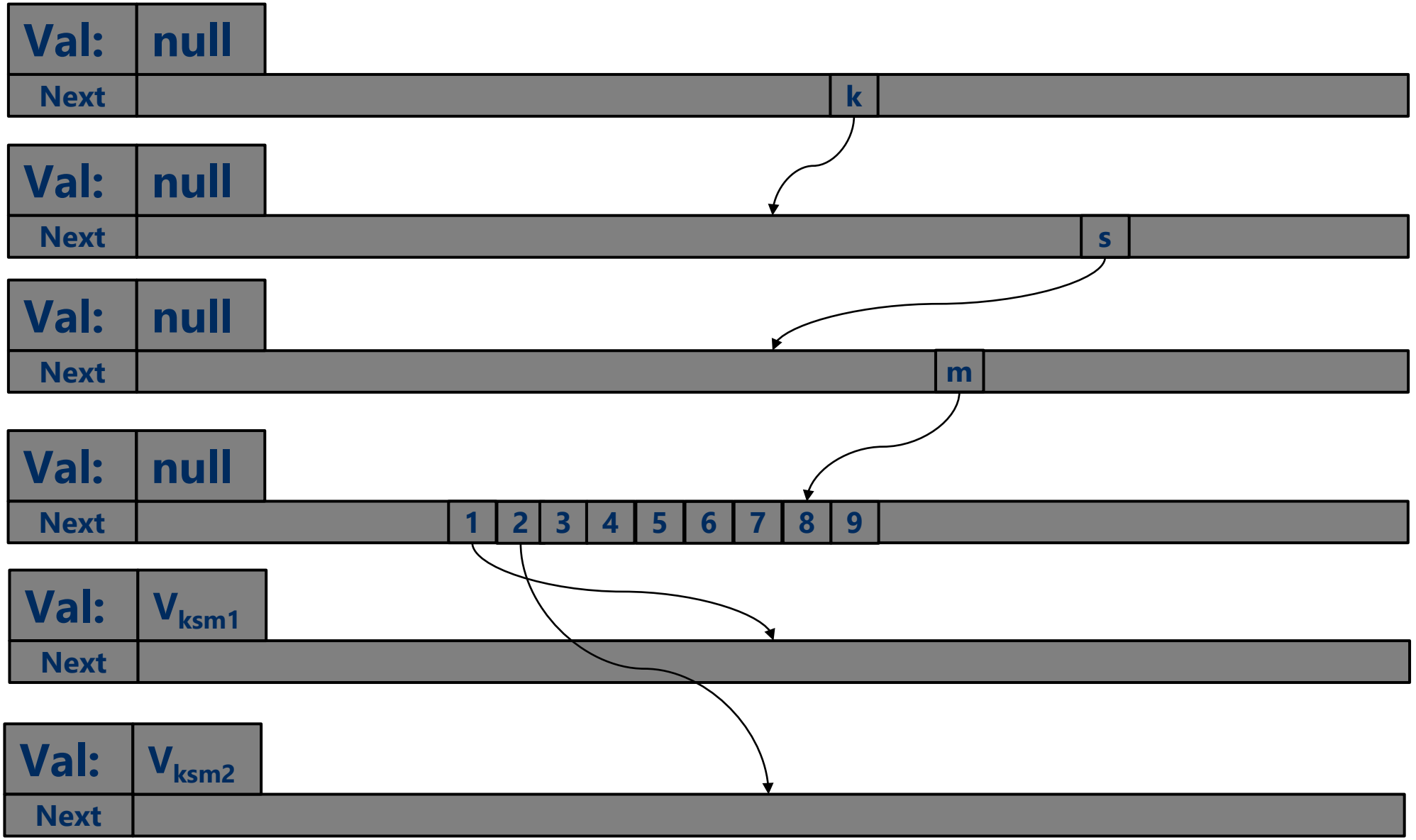
- **Q: How does DLB save space over r way trie? Example please?**
- Let's the set of keys:
  - ksm1 … ksm9
- How big does an 256-way RST take vs. a DLB trie?

R-way RST

```
private class Node {
    private Object val;
    private Node[] next;

    private Node(){

       next = new Node[R];

    }
}
```

Each node takes 4*(R+1) = 4*257 = 1028 bytes, assuming 4 bytes per reference variable

R-way RST


Val:
null
Next
k
Val:
null
Next
s
Val:
null
Next
m
Val:
null
Next
1
2
3
4
5
6
7
8
9
Val:
V_{ksm1}
Next
Val:
V_{ksm2}
Next

R-way RST

We will end up with 4 + 9 = 13 nodes

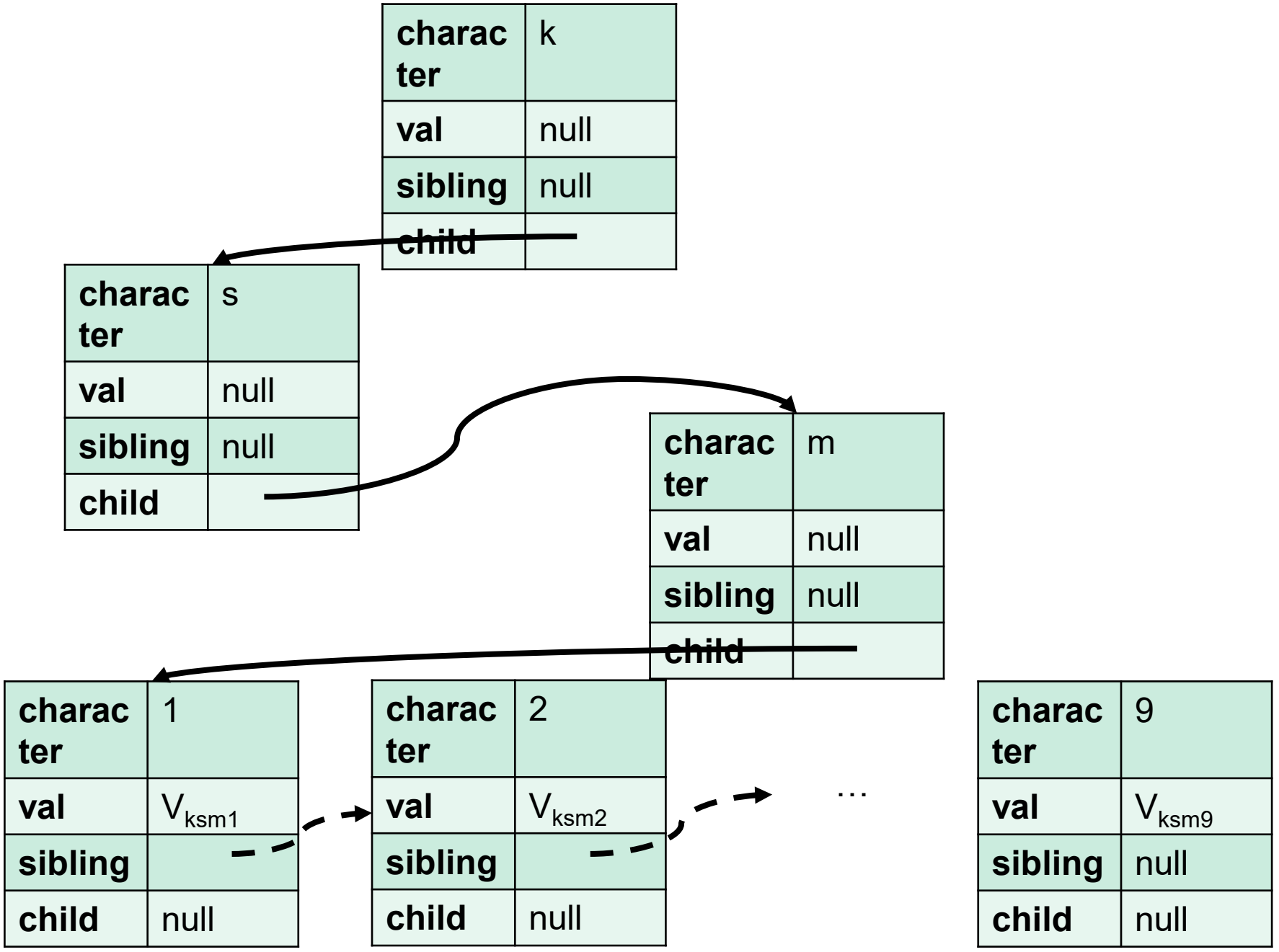
Total space is 13*1028 = 13,364 bytes

DLB Trie

```
private class DLBNode<T> {
    private Character character;
    private Object val;
    private Node sibling;
    private Node child;
}
```

Each node takes 4*4 = 16 bytes, assuming 4 bytes per reference variable

DLB Trie


character
k
val
null
sibling
null
child
character
s
val
null
sibling
null
child
character
m
val
null
sibling
null
child
character
1
val
V_{ksm1}
sibling
child
null
character
2
val
V_{ksm2}
sibling
child
null
…
character
9
val
V_{ksm9}
sibling
null
child
null

DLB Trie

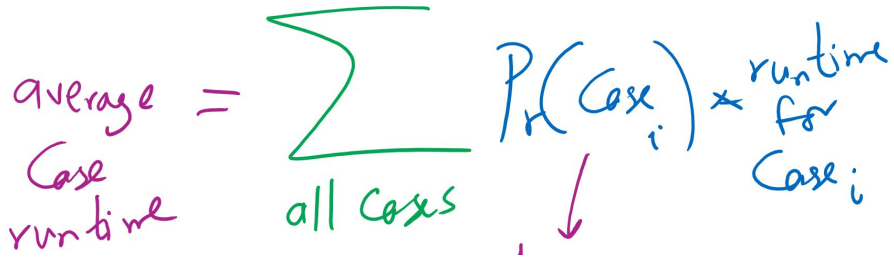
We will end up with 12 nodes

Total space is 12*16 = 192 bytes

Compare to 13,364 bytes with an R-way RST

Muddiest Points

- **Q: What determines the number of bits you use for the bit representation of a key in DSTs and RSTs?**
- Typically, the number of bits is determined by the application
  - e.g., keys are Pitt usernames, PeopleSoft IDs, English sentences, etc.
- It is better to re-encode the keys to have a bit length of *log n* bits each
  - Requires extra space to store the mapping from old keys to new keys
  - sometimes not possible: e.g., when *n* is not known in advance
- Better yet, we can assign bit lengths based on frequency of access:
  - Shorter bitstrings for more frequently accessed keys
    - Results in smaller ***average*** search time



Muddiest Points

- **Q: Not really a muddiest point, but it would be extremely helpful to see actual code (not pseudo code) next to some of these trees**
- You will see code in the recitations

Muddiest Points

- **Q: DLB do what?**
- De La Briandais (DLB) Trie
  - tree-like structure used for searching when keys are sequences of characters
  - each nodelet
    - stores one character,
    - points to a sibling (linked list of siblings), and
    - points to a child
  - worst-case running time is O(wR)
    - w: number of characters in the key
    - R: alphabet size
  - worst-case can be avoided by using DLB only when the sibling lists are short
  - check add algorithm in previous lecture

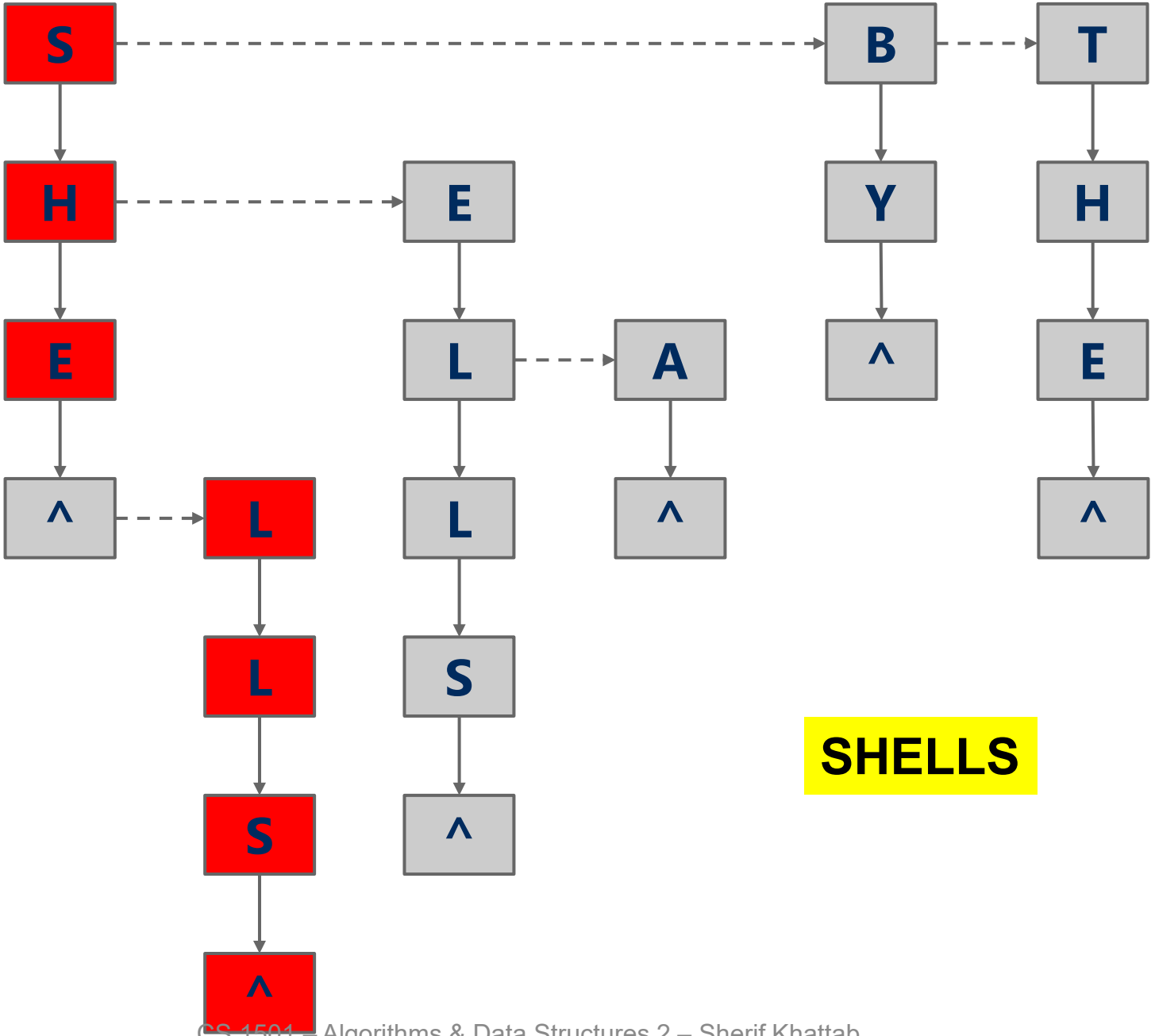
Muddiest Points

- **Q: Just wanted to quickly double check some details about DSTs: The max height of a DST is the number of key bits + 1**
- Correct
- **Q: and the max comparisons you can do is the key's bit length, correct?**
- No, it is also b + 1

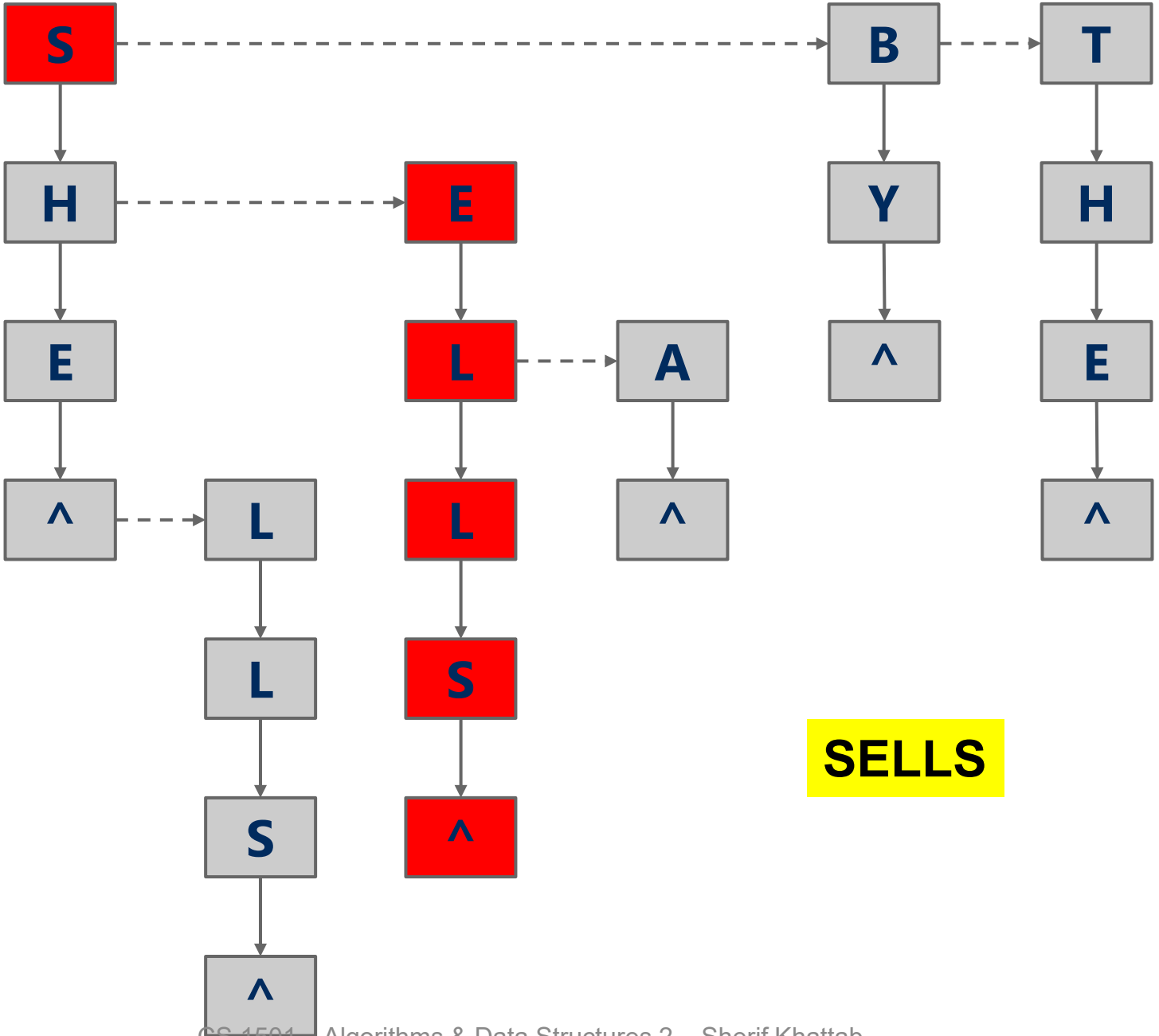
Muddiest Points

- **Q: why was there two paths for "shell" on the DLB example?**
- There was only one path!

DLB Example



DLB Example



Muddiest Points

- **Q: What is the point of the sentinel character? How does that implementation of the DLB differentiate it from the other DLB implementation?**
- If the DLB stores only keys (without corresponding values), we don't need the val field in the DLBNode
- But, val helped us determine if the node we stop at corresponds to a key or not
  - when val is not null, the node corresponds to a key
- Without val, we need a different method: using a sentinel
  - the sentinel is added to each key before adding and before searching
  - a key is found when the key with the sentinel is found
  - e.g., adding she results in adding she^
  - searching for she becomes searching for she^
  - if "she^" is found then she is a key
  - if only "she" is found (without the sentinel), she is not a key

Muddiest Points

- **Q: what sorting methods have the data explicitly and when is it implicit.**
- Tries store keys implicitly, whereas trees store keys explicitly inside tree nodes
- Check node structure for a tree vs. a trie

Muddiest Points

- **Q: Can you re-explain what you meant by w = b/ceiling(logR)?**
- The string “she” has 3 characters (*w=3)*
- If we look at the bit representation of “she”, assuming each character is an extended ASCII character (i.e., 8-bit character), the number of bits will be $b = 3*8 = 24$
- For extended ASCII, the alphabet size is $R = 2^8 = 256$
- w = b/8 = b/logR

Problem of the Day: Compression

- Input: A file containing a sequence of characters
  - *n* characters
  - each encoded as an 8-bit Extended ASCII
  - total file size = 8*n
- Output: A shorter bitstring
  - of length < 8*n
  - such that the original sequence can be fully restored from the bitstring

ASCII Encoding

A set of R symbols can be represented using fixed-size encoding of length *k* bits each

- iff $2^k >= R$
- that is, $k = \lceil \log R \rceil$