Abstract

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0.0.1 Second Order Energy Correction

$$\langle n^0| \left(H_0 |n^2\rangle + H_1 |n^1\rangle\right) = \langle n^0| \left(E_n^{(0)} |n^2\rangle + E_n^{(1)} |n^1\rangle + E_n^{(2)} |n^0\rangle\right)$$

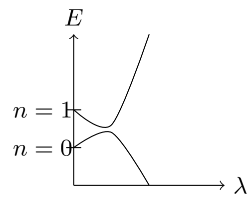
we get

$$E_n^0 \langle n^0|n^2\rangle + \langle n^0| H_1 |n^1\rangle = E_n^{(0)} \langle n^0|n^2\rangle + E_n^{(1)} \langle n^0|n^1\rangle + E_n^{(2)} \langle n^0|n^0\rangle$$

We have cancellation, and also $E_n^{(1)} \langle n^0|n^1\rangle \sim 0 + \mathcal{O}(\lambda^2)$, so we get

$$E_n^{(2)} = \langle n^0| H_1 |n^1\rangle = \sum_{m\neq n} \frac{\langle n^0| H_1 |m^0\rangle \langle m^0| H_1 |n^0\rangle}{E_n^{(0)} - E_m^{(0)}} = \sum_{m\neq n} \frac{|\langle m^0| H_1 |n^0\rangle|^2}{E_n^{(0)} - E_m^{(0)}}$$

An interesting property of this is level repulsion. Basically, higher order contributions cause repulsion.



Take some hamiltonian for the simple harmonic oscillator, however, in the presence of an external electric field.

$$H = H_0 + H_1 = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2 - \mathcal{E}qx$$

0.0.2 First order

First, we attempt the first order correction

$$E_n^{(1)} = \left\langle n^{(0)}\middle| /H_1 \middle|n^{(0)}\right\rangle = -\mathcal{E}q \left\langle n^{(0)}\middle| x \middle|n^{(0)}\right\rangle = 0$$

where the final equality holds by the homework problem where we showed that x can only have non-vanishing matrix elements between states of opposite parity.

0.0.3 Second order

This means we have to go to second order

$$|n\rangle = \left|n^{(0)}\right\rangle - q\mathcal{E} \sum_{m\neq n} \left|m^{(0)}\right\rangle \frac{\left\langle m^{(0}\middle| x \middle|n^{(0)}\right\rangle}{E_n^{(0)} - E_m^{(0)}}$$

rewriting $x = \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger)$, we take

$$\begin{aligned}|n\rangle &= \left|n^{(0)}\right\rangle - q\mathcal{E}\left(\left|n^{(0)} + 1\right\rangle \frac{\sqrt{n+1}}{-\hbar\omega}\sqrt{\frac{\hbar}{2m\omega}} + \left|n^{(0)} - 1\right\rangle \frac{\sqrt{n}}{\hbar\omega}\sqrt{\frac{\hbar}{2m\omega}}\right)\\ &= \left|n^{(0)}\right\rangle + \frac{q\mathcal{E}}{\hbar\omega}\sqrt{\frac{\hbar}{2m\omega}}\left(\sqrt{n+1}\left|n^{(0)} + 1\right\rangle - \sqrt{n}\left|n^{(0)} - 1\right\rangle\right)\end{aligned}$$

Then, we calculate the final energy correction as

$$E_n^{(2)} = \frac{\hbar q^2\mathcal{E}^2}{2m\omega}\left(\frac{n+1}{-\hbar\omega} + \frac{n}{\hbar\omega}\right) = -\frac{q^2\mathcal{E}^2}{2m\omega^2}$$

0.0.4 Complete the Squares

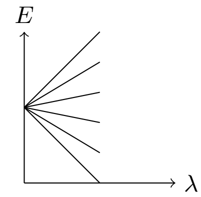
This is exactly the result that we get from completing the squares and writing

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega\left(x - \frac{q\mathcal{E}}{m\omega^2}\right)^2 - \frac{1}{2}\frac{q^2\mathcal{E}^2}{m\omega^2}$$

This gives us exactly the same result, which is a nice sanity check!

0.1 Degenerate Perturbation Theory

If we have some set of degenerate levels, and apply some perturbation, they might split, and be unique



Basically, we want to diagonalize our perturbation matrix within a basis where the degenerate states are no longer degenerate.