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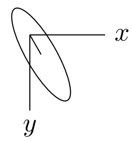
0.1 Simple Pendulum

Say we have some rigid body



Small oscillations, we have

$$\hat{r} = r(\hat{y}\cos(\theta+\varphi) + \hat{x}\sin(\theta+\varphi))$$

and

$$\vec{v} = \dot{\theta}r\left[-\hat{y}\sin(\theta+\varphi) + \hat{x}\cos(\theta+\varphi)\right]$$

then for U we write

$$U = mgd(1-\cos\theta)$$

with

$$E = \frac{1}{2}I\dot{\theta}^2 + mgd(1-\cos\theta)\dot{\theta} = \pm\sqrt{\frac{2}{I}(E - mgd(1-\cos\theta)}$$

So we think about $1-\cos\theta = 2\sin^2(\frac{\theta}{2})$. Let θ_0 be the max value of θ, i.e. where our approximation starts to fall apart.

To solve, we can evaluate

$$T = \frac{2r_0}{\sqrt{gd}}\int_0^{\theta_0}\frac{d\theta}{\sqrt{\sin^2\frac{\theta_0}{2} - \sin^2\frac{\theta}{2}}}$$

Let $k = \sin\frac{\theta_0}{2}$ and $k_z = \sin\frac{\theta}{2}$, so we write

$$dz = \frac{1}{2}\frac{\cos\theta/2}{k}d\theta$$

which makes

$$T = \frac{2r}{\sqrt{gd}}\int_{z=0}^{1}\frac{zkdz}{\sqrt{1-k^2z^2}}\frac{1}{k\sqrt{1-z^2}}$$

in the small angle approxmation, we have

$$\sqrt{1-k^2z^2} \approx 1 + \frac{1}{2}k^2z^2$$

which allows us to break up T into some shit that requires a lot of trig substitution, which ultimately yields

$$T = \frac{4r_0}{\sqrt{gd}}\left(\frac{\pi}{2} + \frac{1}{2}k^2\frac{\pi}{4}\right) = \frac{2\pi r_0}{\sqrt{gd}}\left(1 + \frac{1}{4}\sin^2\frac{\theta_0}{2} + \ldots\right)$$

which gives us a term T_0 that tells us how much variation we have from the simple harmonic oscillator

$$T_0 = \frac{2\pi r_0}{\sqrt{gd}}$$

For instance, if we plug in $\theta_0 = 23°$, we get about a 1% change.

Now, we do some

0.1.1 Pertubation Theory

and write down the ODE

$$\ddot{x} + \omega_0^2 x - \lambda x^2 = 0$$

which, looking for $x(\lambda, t)$

$$x(\lambda, t) = x_0(t) + \lambda x_1(t) + \lambda^2 x_2(t) + \dots$$

$$\begin{aligned}\dot{x} &= \dot{x}_0 + \lambda \dot{x}_1 \\ \ddot{x} &= \ddot{x}_0 + \lambda \ddot{x}_1\end{aligned}$$

which gives

$$\begin{aligned}\ddot{x} + \omega_0^2 x - \lambda x^2 &= 0 \\ \ddot{x}_0 + \lambda \ddot{x}_1 + \omega_0^2(x_0 + \lambda x_1) - \lambda(x_0 + \lambda x_1)^2 &= 0 \\ \ddot{x}_0 + \lambda \ddot{x}_1 + \lambda \ddot{x}_1 + \omega_0^2 \lambda x_1 - \lambda x_0^2 - 2\lambda^2 x_0 x_1 - \lambda^3 x_1 &= 0\end{aligned}$$

first order perturbation of the thing, so if we're forcing it

$$\ddot{x}_1 + \omega_0^2 = A^2 \cos^2 \omega_0 t$$

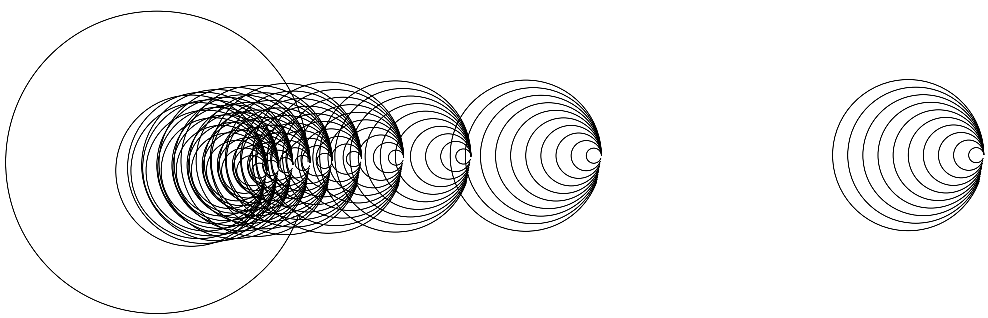
So, what I think he did

$$x(t) = x_0 + \lambda x_1 =_A \cos \omega_0 t - \lambda \frac{A^2}{6\omega_0^2}(\cos(2\omega_0 t) - 3)$$

3-wave coupling.

Kolomogorov (1941). No natural scale/boundary condition in hte problem

Imagine some eddy, that you've just broken up



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