Physics-105-Lecture-Notes-04-02-2019

Connor Duncan

April 2, 2019

Contents

Abstract

A single PDF with all lectures in a single document can be downloaded at `https://www.dropbox.com/sh/8sqzvxghvbjifco/AAC9LoSRnsRQDp7pYedgWpQMa?dl=0`. The password is 'analytic.mech.dsp'. This file was automatically generated using a script, so there might be some errors. If there are, you can contact me at `mailto:ctdunc@berkeley.edu`.

0.1 Euler Angles, Body/Space Frames

The beginning of lecture was some complicated example using the euler angles to transform into body coordinates. Here's another version of this problem.

0.1.1 Symmetric Top

We have $I_1 = I_2 = I$. It can be a cube, or really something arbitrary. I_3 might be different. KE can be written $T = \frac{1}{2}\vec{\omega}(I \cdot \vec{\omega}) = \frac{1}{2}(I(\omega_1^2 + \omega_2^2) + I_3\omega_3^2)$. Using this, and the formulation of the euler angles, we can write down htat

$$\omega_1^2 = \dot{\theta}^2 \cos^2\chi + \dot{\varphi}^2 \sin^2\chi \sin^2\theta + 2\dot{\theta}\dot{\varphi}\cos\chi \sin\chi \sin\theta$$

with omega 2 defined in a similar manner, which gives

$$\begin{aligned} \omega_1^2 + \omega_2^2 &= \dot{\theta}^2 + \dot{\varphi}^2 \sin^2\theta \\ \omega_3^2 &= (\dot{\chi} + \dot{\varphi}\cos\theta)^2 \end{aligned}$$

In gravity, there's also potential $V = mgl\cos\theta$, so we can write down the lagrangian

$$\mathcal{L} = \frac{I}{2}(\dot{\theta}^2 + \dot{\varphi}^2\sin^2\theta) + \frac{I_3}{2}(\dot{\chi} + \dot{\varphi}\cos\theta)^2 - mgl\cos\theta$$

From our work on canonical variables, we can note that

$$\begin{aligned} p_\varphi &= \text{constant} \\ p_\chi &= \text{constant} \end{aligned}$$

we can calculate these then, as

$$p_\chi = \frac{\partial \mathcal{L}}{\partial \dot{\chi}} = I_3(\dot{\chi} + \dot{\varphi}\cos\theta) = I_3\omega_3$$

Which is angular momentum around x_3. Also, define $a = \frac{I_3\omega_3}{I} \equiv$const. Now, we can calculate

$$p_\varphi = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} = I\dot{\varphi}\sin^2\theta + I_3(\dot{\chi} + \dot{\varphi}\cos\theta)\cos\theta \equiv \text{constant}$$

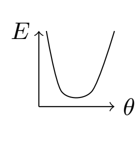
Define $b = \frac{p_\varphi}{I}$. We can calculate the total energy then as

$$E = T + U = \frac{I_2}{2}(\dot{\theta}^2 + \dot{\varphi}^2\sin^2\theta) + \frac{I_3}{2}\omega_3^2 + Mgl\cos\theta$$

This whole equation simplifies down to a nice form, which is $b = \dot{\varphi}\sin^2\theta + a\cos\theta$. If we wirte $E' = E - \frac{I_3\omega_3^2}{2}$, then we can have a one-dimensoinal equation

$$E' = \frac{I}{2}\dot{\theta}^2 + \frac{I}{2}\frac{(b - a\cos\theta)^2}{\sin^2\theta} + mgl\cos\theta$$

We can look at this like an energy equation



If we define $\dot{\theta}^2(1-u^2) = \dot{u}^2$, we can do some fancy substitution. As $u \to \infty$, everythign goes as u^3. Likewise for $-\infty$. If we now consider $\dot{\varphi} = \frac{b - a\cos\theta}{\sin^2\theta}$, we can see that it will not have zero average. In fact, it will look like some curly path between these two lines on a sphere.