Physics-105-Lecture-Notes-03-19-2019

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Abstract

A single PDF with all lectures in a single document can be downloaded at `https://www.dropbox.com/sh/8sqzvxghvbjifco/AAC9LoSRnsRQDp7pYedgWpQMa?dl=0`. The password is 'analytic.mech.dsp'. This file was automatically generated using a script, so there might be some errors. If there are, you can contact me at `mailto:ctdunc@berkeley.edu`.

0.1 Inertial Tensor

$$\Omega_1 = \sqrt{\left(\frac{I_2 - I_1}{I_2}\right)\left(\frac{I_2 - I_1}{I_1}\right)}\omega_2$$

cyclic differential equation

$$\begin{aligned}
\dot{\omega}_1 + \left(\frac{I_3 - I_2}{I_1}\right)\omega_2\omega_3 &= 0 \\
\dot{\omega}_2 + \left(\frac{I_1 - I_3}{I_2}\right)\omega_1\omega_3 &= 0 \\
\dot{\omega}_3 + \left(\frac{I_2 - I_1}{I_3}\right)\omega_2\omega_1 &= 0
\end{aligned}$$

0.2 Spinning Top

If we want to be more rigorous, we should include gravity and torque. Let some coordinate system $\vec{r}' \equiv$ space coordinates $\equiv$ fixed in space. $\vec{r}$ is our body coordinates, rotating with some spinning top. These are related by $\vec{r}' = U\vec{r}$. Also θ, φ, ψ are the euler angles. We need to choose a convenient definition for these, so let θ be the angle between $z, \hat{x}_3'$, and φ the angle betwween $x, \hat{x}_2'$, and ψ the angle between the xy plane and $\hat{x}_2'$. So, step 1 is that if we rotate around $z - \hat{x}_3$ by some angle φ (in xy plane), we get new coordinate transformation $\alpha, \beta\gamma$, which gives

$$\begin{bmatrix}\alpha\\ \beta\\ \gamma\end{bmatrix} = \begin{bmatrix}\cos\varphi & -\sin\varphi & 0\\ -\sin\varphi & \cos\varphi & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}$$

0.2.1 Step 2

Rotate around α by angle θ. Su, we get

$$\begin{bmatrix}\alpha'\\ \beta'\\ \gamma'\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta & \sin\theta\\ 0 & -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\\ \gamma\end{bmatrix}$$

0.2.2 Step 3

Finally, wrotate by ψ about γ'

$$\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \begin{bmatrix}\cos\psi & \sin\psi & 0\\ -\sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix}$$

0.3 gettting the euler angles! (space 2 body)

$U^* = U_3^* U_2^* U_1^*$ transforms from space to body, with $U = U_1 U_2 U_3$ transforms from body to space. We get the big transpose form of U as

$$U^* = \begin{bmatrix} \cos\psi\cos\varphi - \sin\psi\sin\varphi\cos\theta & \cos\psi\sin\varphi\sin\psi\cos\varphi\cos\theta & \sin\theta\sin\psi \\ -\sin\psi\cos\varphi - \cos\psi\sin\varphi\cos\theta & -\sin\varphi\sin\psi + \cos\psi\cos\varphi\cos\theta & \sin\theta\cos\psi \\ \sin\theta\sin\varphi & -\cos\varphi\sin\theta & \cos\theta \end{bmatrix}$$

¡++¿