Physics-105-Lecture-Notes-02-05-2019

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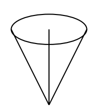
Abstract

A single PDF with all lectures in a single document can be downloaded at `https://www.dropbox.com/sh/8sqzvxghvbjifco/AAC9LoSRnsRQDp7pYedgWpQMa?dl=0`. The password is 'analytic.mech.dsp'. This file was automatically generated using a script, so there might be some errors. If there are, you can contact me at `mailto:ctdunc@berkeley.edu`.

0.1 Examples of Lagrangian Mechanics

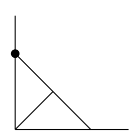
0.1.1 Cone?

Missed the first one, but we know that *angular momentum is conserved.* Basically, just a whole lot of algebran happening here, with something rotating in a conic shape, or on the surface of a cone (maybe like throwing a coin into one of those things at McDonalds). Stable solutions can be given by $\ddot{r} = 0$. I.e. the coin doesn't ever go into the money receptacle. Gives that $\dot{\theta}^2 \tan\alpha = \frac{g}{r_0} \Rightarrow \dot{\theta}^2 = \frac{\omega_0}{\tan\alpha}$ Which implies that in order to have some stable orbit in a cone at a certain angle α, you have explicit angular momentum dependence.



0.1.2 Mass/Spring on a T

Imagine some T on a tabletop, that looks a bit like this



Where the dot is connected to the T by a spring that's hooked up at the juncture. Let ωt be the angle between the x-axis and the T. then we can write $\vec{r} = (l\cos\omega t - \rho\sin\omega t)\hat{x} + (l\sin\omega t + \rho\cos\omega t)\hat{y}$

$$T = \frac{1}{2}m(\dot{r}\dot{r}) = \frac{1}{2}m(\omega^2(l^2+\rho^2) + \dot{\rho}^2 + 2\omega l\dot{\rho})$$

put in to the euler lagrange equation

$$\frac{\partial L}{\partial \rho} = m\omega^2\rho - k\rho$$
$$\frac{\partial L}{\partial \dot{\rho}} = m\dot{\rho} + \omega l$$

So, equating these two things, gives us that

$$\ddot{\rho} + (\frac{k}{m} - \omega^2)\rho = 0$$

which yields 3 sort of 'classes' of solutions. First is where $\omega < \sqrt{\frac{k}{m}}$, which yields a simple harmonic oscillator, very fun! We also could have $\omega > \sqrt{\frac{k}{m}}$, which gives us that $\rho(t) = Be^{\alpha t} + Ce^{-\alpha t}$. There's also the case of equality, which gives us resonant oscillation, or just a growth term $\rho(t) \sim t$.

0.1.3 Now with Gravity!

Take the previous problem, and just add gravity into the mix, since we all like to have fun. Now we have $V = mgy$, and we have y from the previous problem, so the lagrangian becomes some really long wild thing, that I cannot see (Bale didn't do the whole thing out, but the principle of the problem is similar to what we did above).

1 Symmetries (Formally)

Considering changes to L (the lagrangian) when we perturb one of the coordinates. Say it's $q_i \to \tilde{q}_i = q_i + k_i$

1.1 Linear Momentum

consider

$$\begin{aligned} \tilde{x} &= x + \epsilon & \dot{\tilde{x}} &= \dot{x} \\ T &= \frac{1}{2}m\dot{x}^2 = \frac{1}{2}m\dot{\tilde{x}}^2 & & \\ L &= \frac{1}{2}m\dot{x}^2 - V(x) & \tilde{L} &= \frac{1}{2}m\dot{\tilde{x}}^2 - \tilde{V}(x) \end{aligned}$$

If we apply the constraint that $L(x, \dot{x}) = \tilde{L}(\tilde{x}, \dot{\tilde{x}})$, then $V(x)$ has to be invariant to spatial pertubation, which implies that $F_x = 0$, since $-F = \frac{\partial V}{\partial x}$. This is just a meme'd way of writing conservation of linear momentum, since it boils down to

$$\frac{\mathrm{d}}{\mathrm{d}t}(m\dot{x}) = 0$$

1.2 Noethers Theorem (intro)

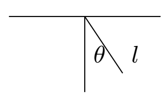
$$\begin{aligned} L(q, \dot{q}) &= L(q + \epsilon k, \dot{q} + \epsilon \dot{k}) \\ L(q, \dot{q}) = L(q + \epsilon k, \dot{q} + \epsilon \dot{k}) &= L(q, \dot{q}) + \epsilon \sum_i \dot{k}_i \frac{\partial L}{\partial \dot{q}_i} + \epsilon \sum_i k \frac{\partial L}{\partial q_i} .... \end{aligned}$$

This just applies the constraint that the sum of the first n taylor expanded terms has to be zero, which is *Noethers Theorem.* Another, simpler way of writing this is that

$$\sum K \frac{\partial L}{\partial \dot{q}} = C$$

where C is constant.

Example



we just apply

$$L = \frac{1}{2}m\dot{l}^2 + \frac{1}{2}ml^2\dot{\theta}^2 - mgl\cos\theta$$

And derive properties from this, like conservation of angular momentum.

1.3 Probably the Hamiltonian

consider

$$\begin{aligned} H &= \dot{q}\frac{\partial L}{\partial \dot{q}} - L \\ \frac{\mathrm{d}H}{\mathrm{d}t} &= -\frac{dL}{dt} \end{aligned}$$

some long thing using the chain rule. has it simplify down to the above form, which implies htat H is a conserved quantity with respect to time. Take

$$\begin{aligned} L &= \frac{1}{2}m\dot{x}^2 - V(x) \\ H = \dot{x}\frac{dL}{d\dot{x}} - L = m\dot{x}^2 - (\frac{1}{2}m\dot{x}^2 + V(x)) &= \text{total energy} \end{aligned}$$

which lets us say H is a total energy, the *hamiltonian.*