Abstract

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0.0.1 Ex: Soft Sphere Scattering (Altman Returns)

We have some radial potential

$$V(r) = \begin{cases} -V_0 & r < R \\ 0 & r \geq R \end{cases}$$

where

$$U(r) = \frac{2mV(r)}{\hbar^2}$$

What's going to happen when we send a low-energy wavevector into this spherical potential? We can write down the radial equation for $\ell = 0$ (an s-wave).

$$\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} - U(r) + k^2\right)\psi$$

Take ξ such that

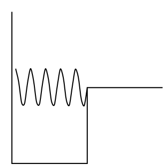
$$\xi(r) = r\psi(r)$$

where the new wave equation becomes

$$\left(\frac{\partial^2}{\partial r^2} - U(r) + k^2\right)\xi(r) = 0$$

This is going to give us a plane wave solution to the schroedinger equation, so we're going to get

$$\xi(r) = \begin{cases} c\sin(kr + \delta_0) & r > R \\ A\sin(Qr) & r < R \end{cases}$$



We then apply continuity at $R = r$, where $Q = \sqrt{k^2 + r^2}$, to get[1]

$$A\sin Qr = c\sin(kR + \delta_0) \qquad AQ\cos Qr = ck\cos(kR + \delta_0)$$

From this, we're going to end up dividing the two equations by each other to find that

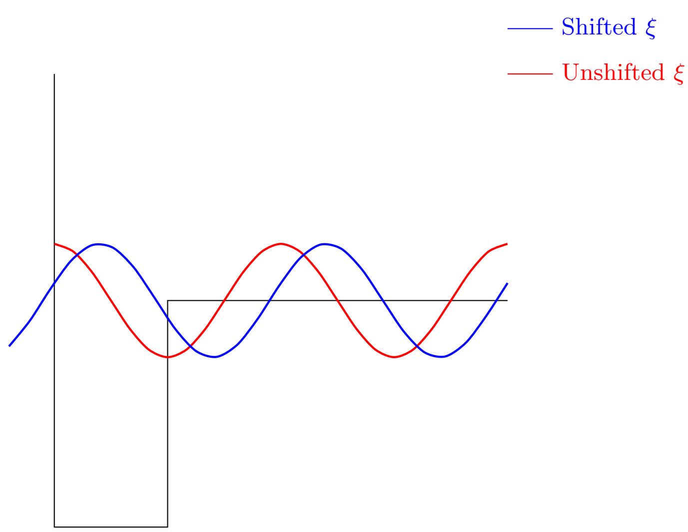
$$\tan(kR + \delta_0) = \frac{kR}{QR}\tan(QR)$$

Tangent has a lot of divergences though, which makes this equation somewhat challenging to solve. If we assume the argument of tan is small however, we can taylor expand this to find a close solution to our problem. We'll take $\tan(QR) \sim 1 \Rightarrow kR + \delta_0 \cong \frac{kR}{QR}\tan(QR)$, so

$$\delta_0 \cong kR\left(\frac{\tan(QR)}{QR} 01\right)$$

We can find our scattering length as

$$a_s = -\lim_{k\to 0}\frac{\tan(\delta_0)}{k} = -R(\frac{\tan(\gamma R)}{\gamma R} - 1)$$



[1] note that on the homework, we have a δ function potential which has different discontinuous boundary conditions at the boundary

Scattering Resonances When $\tan(QR)$ diverges, we're going to have

$$\tan(kR + \delta_0) + \frac{k}{\gamma}\tan(\gamma R)$$

where $\gamma R \approx \frac{\pi}{2}$. We then have the limit where $k \to 0$, we have that $\delta_0 = \pi/2$, so we have a condition. We end up with

$$\sigma_0 = \frac{4\pi}{k^2}\sin^2\delta_0 = \frac{4\pi}{k^2}$$

Now, if we want to tune across the resonances, we assume phase shift goes through $\pi/2$ at a certain energy