Physics-105-Lecture-Notes-01-24-2019

Connor Duncan

January 24, 2019

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Abstract

A single PDF with all lectures in a single document can be downloaded at `https://www.dropbox.com/sh/8sqzvxghvbjifco/AAC9LoSRnsRQDp7pYedgWpQMa?dl=0`. The password is 'analytic.mech.dsp'. This file was automatically generated using a script, so there might be some errors. If there are, you can contact me at `mailto:ctdunc@berkeley.edu`.

0.1 More orthogonal transformations

0.1.1 Group!

Orthogonal transformation Λ from coordinate system $S \to S'$, form a group, so that $\forall \Lambda, W$,

$$\Lambda W \neq W\Lambda$$
$$\Lambda\Lambda^\dagger = I_n$$
$$\Lambda_{ij}W_{jk} \neq W_{ij}\Lambda jk$$

the reason that we care it's a group is because it's closed under multiplication, i.e. for any orthogonal transformation Λ, W, their product $W\Lambda$ is also an orthogonal transformation.

0.1.2 $\det \Lambda = 1 \Leftrightarrow \Lambda$ has eigenvalues=1

$$(\Lambda - I)\Lambda^\dagger = 1 - \Lambda^\dagger = (1-\Lambda)^\dagger$$

Now, solve $||\Lambda_{ij} - a\delta_i j|| = 0$

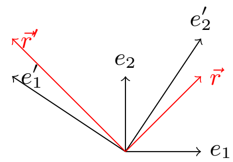
$$||\Lambda - 1|| \cdot ||\Lambda^\dagger|| = ||(1-\Lambda)^\dagger|| = ||1-\Lambda||$$
$$\therefore$$
$$||\Lambda - 1|| = ||1-\Lambda|| \to ||\Lambda - 1|| = 0$$

Where here, $1, I$ are used interchangeably to represent identity Consider operator $P|P_{ij} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, it's the inversion operator, takes any vector $\vec{r} = (x,y,z) \to (-x,-y,-z)$. Determinant is -1, which allows P to be unitary (i.e. P^2=1) We can now write any transformation Λ as a combination of rotation and inversion. Take $W = P\Lambda$, where W is a rotation matrix, since $||W|| = ||\Lambda||||P|| = 1$, then $PW = PP\Lambda = \Lambda$.

0.1.3 Eigenvectors

Consider the transformation

$$\begin{bmatrix} -1.5 & 1 \\ 1 & -1.5 \end{bmatrix}$$



We can consider either transformations of coordinate systems (i.e. basis vectors e_1, e_2) or of individual vectors ($\vec{r}$).

0.2 Rigid Body Motion

0.2.1 Vector Product (Cross)

Take vectors a_1, a_2 in the coordinate system defined with basis vectors e_1, e_2, e_3, so that $a_1 = (a_{11}, a_{12}, a_{13})$ and a_2 defined similarly.

$$|\vec{a_1} \times \vec{a_2}| = |\vec{a_1}| \cdot |\vec{a_2}| \sin(\theta)$$

Where θ is the angle between the two vectors. When $S = \text{span}(\{e_1, e_2, e_3\})$, and is orthogonal basis.

$$e_1 \times e_2 = e_3 \qquad e_2 \times e_3 = e_1 \qquad e_3 \times e_1 = e_2$$

Levi-Civita tensor density defined by $[e_i \times e_j] = \epsilon_{ijk} e_k$, we can write the cyclic permutations of 1,2 and 3 to get the above identities regarding S. We can also find the area of a parallelogram formed by two vectors a_1, a_2, it will be the square of the magnitude of the cross of these two vectors: $A = |a_1 \times a_2|^2$. We can also calculate this using

$$(a_1 \times a_2) = \left|\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}\right| = \det(a) e_3$$

0.2.2 Scalar Triple Product?

This man lectures very rapidly with lots of subscripts. Where's the professor? I'm pretty sure he's talking about the scalar triple product rn. Want to prove that $\epsilon_{\alpha\beta\gamma} = \epsilon_{ijk} \Lambda_{\alpha i} \Lambda_{\beta j} \Lambda_{\gamma k}$. Alternatively we can show that $|| \det A || \epsilon_{\alpha\beta\gamma} = \epsilon_{ijk} A_{\alpha i} A_{\beta j} A_{\gamma k}$.

$$a_3 \cdot (a_1 \times a_2) = V = ||a_{ij}|| = \left|\begin{vmatrix} a_{11} & a_{12} & 0 \\ a_{12} & a_{22} & 0 \\ 0 & 0 & 0 \end{vmatrix}\right|$$

double check that those zeroes are there. His handwriting was kind of scratchy here. Here's probably a better example `https://en.wikipedia.org/wiki/Triple_product` Also, $a \times (b \times c) = b(a \cdot c) - c(a \cdot b)$, to be proven at home.

1 Newtonian Physics

1.1 Angular Velocity

Reintroduce angular velocity. Consider $\vec{r}$, with $\Lambda_{ij} = \delta_{ij} + \delta\varphi_{ij}$, with $\delta\varphi << 1$. We still want Λ to be unitary ($\Lambda\Lambda^\dagger = 1$), so

$$(1 + \delta\varphi)(1 + \delta\varphi^\dagger) = 11 + \delta\varphi + \delta\varphi^\dagger + \delta\varphi\delta\varphi^\dagger = 1$$

We know that $\delta\varphi = -\delta\varphi^\dagger$, since $\delta\varphi\delta\varphi^\dagger$ is very very small. Now consider $x' = (1 + \delta\varphi)x$. We can take

$$x' - x = \delta\varphi x$$

$$\delta x = \begin{bmatrix} 0 & -\delta\varphi_3 & \delta\varphi_2 \\ 0 & 0 & -\delta\varphi_1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\delta x = (\delta\varphi \times x)$$

In other words, $\delta r = [\delta\varphi \times r]$. Then, $\frac{\partial x}{\partial t} = (\frac{\partial \phi}{\partial t} \times x \Rightarrow \frac{\mathrm{d}r}{\mathrm{d}t} = [\Omega \times r]$ where Ω is the *Angular Velocity*.

1.2 Linear Velocity

It's the time derivative of position. In arbitrary coordinates it's expressed simply

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \lim_{\Delta t \to 0} \frac{r(t + \Delta t) - r(t)}{\Delta t}$$

In cartesian coordinates it simplifies to the sum of the componentwise time derivatives.

1.3 Coordinate Transform w/ AV

Relations of coordinate transformation with some from S that has a very complicated motion compared to frame S'.

$$\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)_S = \left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)_{S'} + (\Omega \times r)$$