Abstract

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1 Stationary Perturbation Theory

1.1 Formalism for non-degenerate eigenstates

We can try to solve unsolveable problems by dividing the hamiltonian in two, taking

$$\hat{H} = \hat{H}_0 + \lambda \hat{H}_1$$

with $\hat{H}_0$ being some hamiltonian we can solve exactly, with

$$H_0 \left|\varphi_n^{(0)}\right\rangle = E_n^{(0)} \left|\varphi_n^{(0)}\right\rangle$$

1.1.1 First order energy correction

Let's take our first order expansion of $|n\rangle = \left|n^{(0)}\right\rangle + \lambda \left|n^{(0)}\right\rangle ...$

$$(H_0 + \lambda H_1) \sum_{i=0}^{\infty} \lambda^i \left|n^{(i)}\right\rangle = \sum_{i=0}^{\infty} \lambda^n E_n^{(0)} \sum_{i=0}^{\infty} \lambda^i \left|n^{(i)}\right\rangle$$

The first order correction comes out to be that the first order correction is $E_n^{(1)} = \left\langle n^{(0)}\right| H_1 \left|n^{(0)}\right\rangle$. Stated more consicely, it is

$$E_n^{(1)} = \langle H_1 \rangle_{(0)}$$

1.1.2 First order state correction

If we want to find the first order correction to our state $|n\rangle$, we should take some $\sum_m c_m \left|m^{(0)}\right\rangle$, where $|m\rangle$ is the "natural basis" given by solving the eigenvalue equation of the unperturbed case. We can take

$$\left\langle m^{(0)}\right| H_0 \left|n^{(1)}\right\rangle + \left\langle m^{(0)}\right| H_1 \left|n^{(0)}\right\rangle = \left\langle m^{(0)} \middle| n^{(1)}\right\rangle E_n^{(0)}$$

Of course, $\left\langle m^{(0)} \middle| n^{(1)}\right\rangle$ is the c_1 correction we wanted from the sum above, so we can left multiply and get

$$E_0 \left\langle m^{(0)} \middle| n^{(1)}\right\rangle + \left\langle m^{(0)}\right| H_1 \left|n^{(0)}\right\rangle = \left\langle m^{(0)} \middle| n^{(1)}\right\rangle E_n^{(0)}$$

which gives

$$\left\langle m^{(0)} \middle| n^{(1)}\right\rangle E_n^{(0)} = \frac{\left\langle m^{(0)}\right| H_1 \left|n^0\right\rangle}{E_n^{(0)} - E_m^{(0)}}$$

We're still missing the $\left|n^{(0)}\right\rangle$ term, but if we use the fact that $|n\rangle$ is noramlized, we should have

$$\langle n|n\rangle = 1 = \left\langle n^{(0)} \middle| n^{(0)}\right\rangle + 2\lambda \operatorname{Re} \left\langle n^{(0)} \middle| n^{(1)}\right\rangle + O(\lambda^2)$$

This yields that $2\lambda \operatorname{Re} \left\langle n^{(0)} \middle| n^{(1)}\right\rangle \approx 0$ to order λ^2, which lets us write down

$$|n\rangle = \left|n^{(0)}\right\rangle + i\alpha \left|n^{(0)}\right\rangle + \lambda \sum_{m \neq n} \frac{\left\langle m^{(0)}\right| H_1 \left|n^0\right\rangle}{E_n^{(0)} - E_m^{(0)}}$$

If we want to correct the parallell component, we make a small angle approximation to write

$$|n\rangle = (1 + i\alpha) \left|n^{(0)}\right\rangle + ... = e^{i\alpha} \left|n^0\right\rangle + ...$$

which lets us simplify even further by writing

$$|n\rangle = \left|n^{(0)}\right\rangle + (1 - i\alpha)\lambda \sum_{m \neq n} \frac{\left\langle m^{(0)}\right| H_1 \left|n^0\right\rangle}{E_n^{(0)} - E_m^{(0)}}$$

and since α is of order λ, we can just drop it, since thier product is of order λ^2 Giving the final first order correction to the state is just (dropping λ by setting it equal to one, and letting H_1 be small compared to the energy difference)

$$|n\rangle = \left|n^{(0)}\right\rangle + \sum_{m \neq n} \left(\frac{\left\langle m^{(0)}\right| H_1 \left|n^0\right\rangle}{E_n^{(0)} - E_m^{(0)}} \right) \left|m^{(0)}\right\rangle$$

1.2 Example: Harmonic Oscillation + Perturbations

1.3 Degenerate Perturbation Theory

1.4 More Examples!