Abstract

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0.1 Validity Criterion

This was an aside on the date 11-4-19. We ended up getting a sinc function from the last time. I made an error, last time. WE should actually have

$$F(t,\omega) = \frac{2\sin^2(\omega_{fi}t/2)}{\omega_{fi}^2 t}$$

where in the limit of very large times, $F(t,\omega) \to \pi\delta(\omega)$. From this[1], we can get the maximum probability

$$P_{fi}^{\max} = \frac{2|\langle f| H_1 |i\rangle|^2}{E_f^{(0)} - E_i^{(0)}}$$

The validity criterion here is the same roughly as it was previously, where the energy difference should be small. Also, resonant transitions are going to give a really weird case that we need to be extra mindful of. Even if one transition is resonant, it breaks validity of perturbation theory, the true criterion is

$$\sum_f P_{fi}^{\max} = \sum_f \frac{2|\langle f| H_1 |i\rangle|^2}{E_f^{(0)} - E_i^{(0)}} \ll 1$$

We're going to move on, we want to take

- Static Perturbations $\to$ Oscillatory Perturbation
- Transition into a Continuum of Levels

We're going to discuss this through the lense of **Fermi's Golden Rule**.

0.2 Periodic Perturbations

Consider

$$H_1(t) = \Theta(t)\left[\hat{V}e^{-i\omega t} + \hat{V}^\dagger e^{i\omega t}\right]$$

As an example of ramping up a perturbation suddenly. Or,

$$\bar{H}_1(t) = e^{\varepsilon t}\left[\hat{V}e^{-i\omega t} + \hat{V}^\dagger e^{i\omega t}\right]$$

here $t < 0, \varepsilon \ll \frac{2\pi}{t}$ as an example of ramping up slowly.

0.2.1 Sudden Step Into Periodicity

We can just use the first order formalism to get

$$d_f^{(1)}(t) = -\frac{i}{\hbar}\langle f| \hbar V |i\rangle \int_0^t e^{i(\omega_{fi}-\omega)t'}dt' - \frac{i}{\hbar}\langle f| \hat{V}^\dagger |i\rangle \int_0^t e^{i(\omega_{fi}+\omega)t'}dt'$$

if we let $V_{fi} = \langle f| \hat{V} |i\rangle$, we have

$$d_f^{(1)}(t) = V_{fi}\frac{1-\exp(i(\omega_{fi}-\omega)t)}{\hbar(\omega_{fi}-\omega)} + V_{fi}^\dagger\frac{1-\exp(i(\omega_{fi}+\omega)t)}{\hbar(\omega_{fi}+\omega)}$$

which gives

$$P_{fi} = \left|V_{fi}\frac{1-\exp(i(\omega_{fi}-\omega)t)}{\hbar(\omega_{fi}-\omega)} + V_{fi}^\dagger\frac{1-\exp(i(\omega_{fi}+\omega)t)}{\hbar(\omega_{fi}+\omega)}\right|^2$$

We have really strong resonances at $\omega = \omega_{fi}$, and decay over time otherwise. This gives us that it's a reasonable approximation to use

$$P_{fi} = \begin{cases} \frac{2}{\hbar^2}(V_{fi})^2 F(t,\omega_{fi}-\omega)t & \omega_{fi}\approx\omega \\ \frac{2}{\hbar^2}(V_{fi}^\dagger)^2 F(t,\omega_{fi}+\omega)t & \omega_{fi}\approx-\omega \end{cases} \xmapsto[t\to\infty]{} \begin{cases} \pi\delta(\omega_{fi}-\omega) & \omega_{fi}\approx\omega \\ \pi\delta(\omega_{fi}+\omega) & \omega_{fi}\approx-\omega \end{cases}$$

0.2.2 Slowly Ramped Perturbation

Let's consider as an example, the case where we are nearly resonant with the first term of our perturbation $\bar{H}$.

$$V(t) = \hat{V}e^{\varepsilon t}e^{-i\omega t}$$

Then we have

$$d_f^{(1)}(t) = -\frac{i}{\hbar}\int_{-\infty}^t \hat{V}_{fi}e^{i(\omega_{fi}-\omega-i\varepsilon)t'}dt' = -\frac{1}{\hbar}\frac{e^{i(\omega_{fi}-\omega-i\varepsilon)t}}{\omega_{fi}-\omega-i\varepsilon}$$

If we consider the probability, we're going to get

$$P_{fi}(t) = \frac{1}{\hbar^2}\frac{e^{2\varepsilon t}}{(\omega_{fi}-\omega)^2+\varepsilon^2}|V_{fi}|^2$$

[1] todo: wtf

we can ask what is the rate of transitions we make at long times,

$$W_{fi} = \frac{\partial P_{fi}}{\partial t} = \frac{2}{\hbar^2} \frac{\varepsilon}{(\omega_{fi} - \omega)^2 + \varepsilon^2} |V_{fi}|^2$$

this is a a Lorenztian! We have some peaked function