Physics-105-Lecture-Notes-05-02-2019

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Abstract

A single PDF with all lectures in a single document can be downloaded at `https://www.dropbox.com/sh/8sqzvxghvbjifco/AAC9LoSRnsRQDp7pYedgWpQMa?dl=0`. The password is 'analytic.mech.dsp'. This file was automatically generated using a script, so there might be some errors. If there are, you can contact me at `mailto:ctdunc@berkeley.edu`.

1 Infinitesimal Canonical Transformations

(this stuff is not examinable) Consider some

$$\tilde{H} = H + \frac{\partial F(g, Q, p, P, t}{\partial t}$$

such that $p_1 = \frac{\partial F}{\partial q}$, $P_1 = -\frac{\partial F}{\partial Q_1}$ where F is some generating function. Maybe we let $S = qP$, then it's just the identity transformation. If we taek

$$S = qP + \epsilon G(q, p, t)$$

then

$$\tilde{H} = H + \frac{\partial S}{\partial t}$$

and $p = \frac{\partial S}{\partial q}$, and $Q = \frac{\partial S}{\partial P}$, so we want to fund $P(\epsilon) \approx P(\epsilon = 0) + \epsilon \frac{\partial P}{\partial \epsilon} \ldots$, and same for Q which gives us

$$0 = \frac{\partial p}{\partial \epsilon} + \frac{\partial G}{\partial q}$$

together, these give

$$P = p - \epsilon \frac{\partial G}{\partial q}$$
$$Q = q + \epsilon \frac{\partial G}{\partial p}$$

Now, if we choose our G to be the hamiltonian, i.e. $G(q, p, t) = H(q, p, t)$, then

$$p \approx p - \epsilon \frac{\partial H}{\partial q} = p + \epsilon \dot{p}$$
$$q \approx q + \epsilon \frac{\partial H}{\partial p} = q + \epsilon \dot{q}$$

If we take ϵ to be dt, then we can think of the hamiltonian as being the propagator of time translation, i.e. using the hamiltonian as a generating function of canonical transformations gives out time translation. If we take $G = \hat{z} \cdot \hat{L}_z$, we get out that $xp_{-}yp_x$, which takes

$$X \approx x + \epsilon \frac{\partial G}{\partial p_x} = x - \epsilon y$$
$$Y = y + \epsilon \frac{\partial G}{\partial p_y} = y + \epsilon x$$

which shows angular momentum is the generator of rotation!! (Shoutout to my 137a peeps) What if we want to be more general, examining what happens to a function $u(Q, P, t)$ under such transformations?

$$\left.\frac{\mathrm{d}u}{\mathrm{d}t}\right|_{t=0} = \left.\left(\frac{\partial u}{\partial Q} \frac{\partial Q}{\partial \epsilon} + \frac{\partial u}{\partial P} \frac{\partial P}{\partial \epsilon} \right)\right|_{\epsilon=0}$$

which is approximately

$$\frac{\partial u}{\partial \epsilon} = \left(\frac{\partial u}{\partial q} \frac{\partial G}{\partial p} - \frac{\partial u}{\partial p} \frac{\partial G}{\partial q} \right)$$

which takes

$$u(t) = u(q,p,t) + \epsilon\left(\frac{\partial u}{\partial q}\frac{\partial G}{\partial p} - \frac{\partial u}{\partial p}\frac{\partial G}{\partial q}\right)$$

where the term on the left is called the Poisson Bracket of $\{u, G\}$

$$\{u, G\} = \frac{\partial u}{\partial q}\frac{\partial G}{\partial p} - \frac{\partial u}{\partial p}\frac{\partial G}{\partial q}$$

Generally, if we take

$$\begin{aligned}\frac{\mathrm{d}u}{\mathrm{d}t} &= \frac{\partial u}{\partial q}\frac{\partial}{\partial q, t} + \frac{\partial u}{\partial p}\frac{\partial p}{\partial t} + \frac{\partial u}{\partial t}\\ &= \frac{\partial u}{\partial q}\frac{\partial H}{\partial p} - \frac{\partial u}{\partial p}\frac{\partial H}{\partial q} + \frac{\partial u}{\partial t}\\ &= \{u, H\} + \frac{\partial u}{\partial t}\end{aligned}$$

some more properties of poisson brackets Bale isnt' going to derive.

- $[u, v] = -[v, u]$
- $[u, u] = 0$
- $[(u_1 + u_2), v] = [u_1, v] + [u_2, v]$
- $[u_1 u_2, v] = u_1[u_2, v] + [u_1, v]u_2$
- (also jacobi identity)

and if we have

$$\frac{\partial u}{\partial t} = [u, H] = 0$$

then u is a constant of motion. Also, you can take poisson brackets and generate more conserved quantities (i.e.

$$\frac{\mathrm{d}}{\mathrm{d}t}[u, v] = 0$$