Abstract

A single document copy of these notes, as well as a mirror of every note, can be found at `connorduncan.xyz/notes`

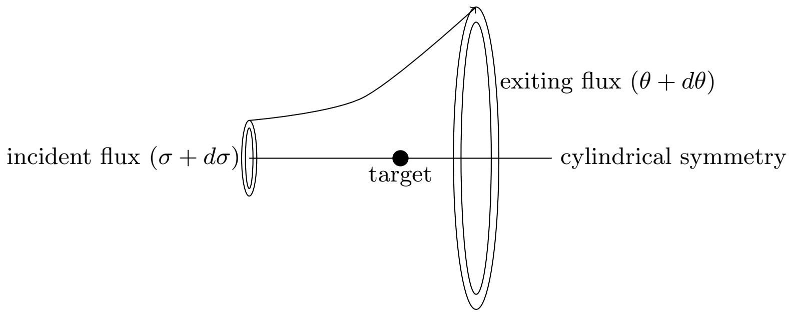
1 Quantum Scattering

1.1 Classical Scattering

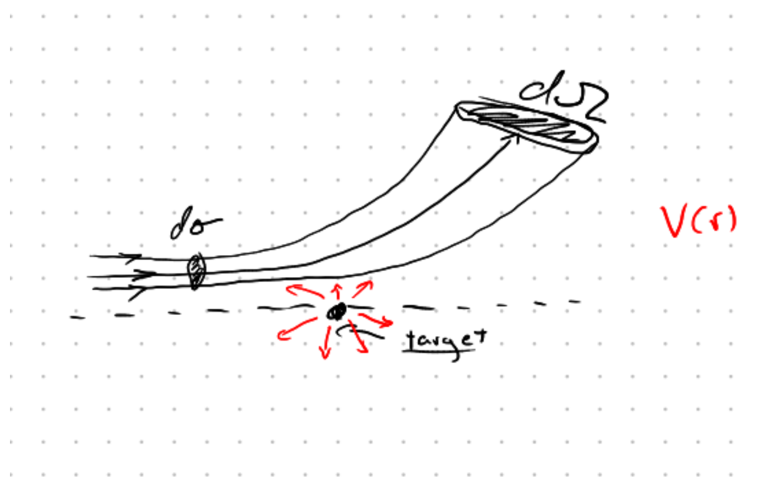
We want first to compare quantum with classical scattering. Consider the following setup, with some flux of particles coming into a sheet at a flux $j_{\text{in}} = \frac{\partial I}{\partial A}$. We have the outgoing flux per steradian, $\frac{\partial I}{\partial \Omega}$, and want t ocompute the ratio

$$\frac{\partial \sigma}{\partial \Omega} = \frac{\frac{\partial I}{\partial \Omega}}{\frac{\partial I}{\partial A}}$$

We will consier our target to exhibit some central potential $V(r)$. The classical model should project that it will behave as follows[1]



We're going to consider



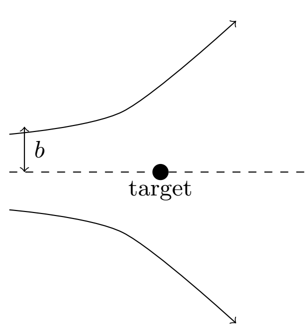
which has

$$dI = \frac{\partial n}{\partial t} = j_{\text{in}} d\sigma = j_{\text{in}} \frac{\partial \sigma}{\partial \Omega} d\Omega$$

where

$$\frac{\partial \sigma}{\partial \Omega} = \frac{\partial I / \partial \Omega}{j_{\text{in}}}$$

which gives our previous result! We can attempt to characterise this differential cross section by the distance b from the axis of symmetry of the incoming particle.



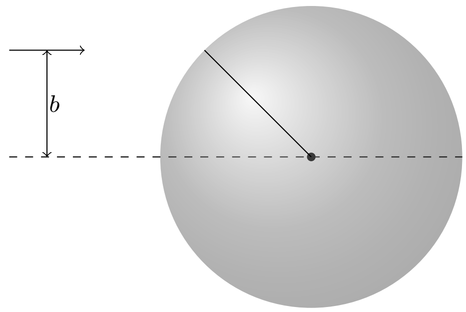
[1] this is just copy-pasted from my 105 notes, so the notation might not be exactly the same.

We can consider $d\sigma$ to be a small piece of area, $d\sigma = bdbd\phi$. This gives that $dI = jd\sigma = jdbd\phi = j\frac{\partial\sigma}{\partial\Omega}d\Omega$, which gives (after cancelling the $d\phi$ from $d\Omega = sin\theta d\phi d\theta$), so we have

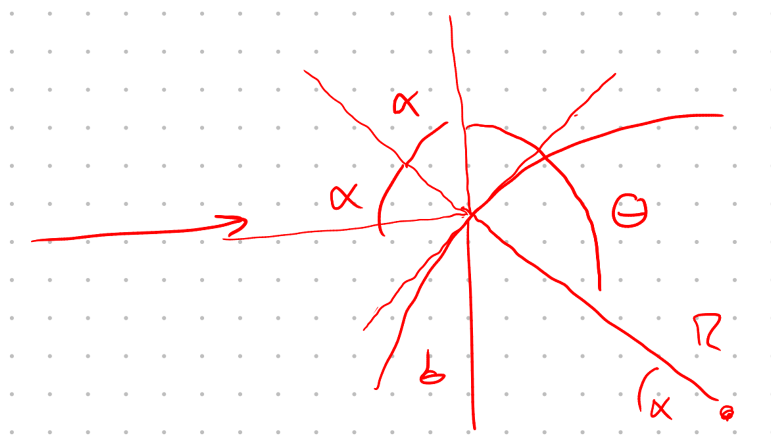
$$\frac{\partial\sigma}{\partial\Omega} = \frac{b}{\sin\theta}\frac{\partial b}{\partial\theta}$$

This is a pretty niche equation though. We have it in terms of b, the impact parameter, E, the precise energy that we shoot it at. We want a more general quantity, which we call σ_T, the total cross section. We're going to integrate over a sphere surrounding the target. We take

$$\sigma_T = \iint_V d\Omega \frac{\partial\sigma}{\partial\Omega}$$



I'll try and finish the pretty drawing later.



We can compute

$$\sigma_T = \int \frac{\partial\sigma}{\partial\Omega}d\Omega = \frac{R^2}{4}\int d\Omega = \frac{4\pi R^2}{4} = \pi R^2$$

For more on this topic, check out Chapter 14 of Taylor[2]. Altman is going to move on to quantum though.

1.2 Beginning Quantum Scattering

We're going to look for eigenvalues of the schroedinger equation, where we have

$$\left[\frac{-\hbar}{2m}\nabla^2 + V(r)\right]\psi(r) = E\psi(r)$$

We're going to begin by looking for solutions of the form

$$\psi(r) = \psi_{\text{inc}}(r) + \psi_{\text{sc}}(r)$$

where ψ_{inc} is an incident plane wave, and ψ_{sc} is the scattered component, where

$$\psi_{\text{inc}} = e^{ikz} \qquad\qquad \psi_{\text{sc}} = f(\theta,\phi)\frac{e^{ikr}}{r}$$

We make the switch to spherical coordinates, in the following manner

$$\vec{r} = (\sin\theta\cos\phi|r|, \sin\theta\sin\phi|r|, \cos\theta|r|)$$

Eventually, we're going to find for various cylindrical symmetries that ϕ will drop out of our function f.

[2]Not reccomended by altman, I just like the book lol. He said he would post a summary.