BROJNI REDOVI – ZADACI (II DEO)

Dalamberov kriterijum

Ako za red $\sum_{n=1}^{\infty} a_n$ postoji $\overline{\lim_{n\to\infty}}\left|\frac{a_{n+1}}{a_n}\right| = r$ onda važi:

- za r > 1 red divergira
- za r = 1 neodlučivo
- za r < 1 konvergira

Primer 1.

Ispitati konvergenciju reda $\sum_{n=1}^{\infty}\frac{1}{n!}$

Rešenje:

Najpre da odredimo a_n. Ovde je to $a_n = \frac{1}{n!}$ (znači uzimamo sve iza oznake za red). Dalje odredjujemo a_{n+1}.Kako ?

Gledamo a_n i umesto n stavimo $n+1$, pa je $a_{n+1} = \frac{1}{(n+1)!}$

Sada koristimo Dalamberov kriterijum:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{1}{(n+1)!}}{\frac{1}{n!}}\right| = \lim_{n\to\infty}\frac{n!}{(n+1)!} = \lim_{n\to\infty}\frac{\cancel{n!}}{(n+1)\cdot\cancel{n!}} = \lim_{n\to\infty}\frac{1}{(n+1)} = 0$$

Dakle, dobili smo da je r = 0 < 1, pa po ovom kriterijumu , red $\sum_{n=1}^{\infty}\frac{1}{n!}$ konvergira.

Primer 2.

Ispitati konvergenciju reda $\sum_{n=1}^{\infty}\frac{2^n}{n}$

Rešenje:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{2^{n+1}}{n+1}}{\frac{2^n}{n}}\right| = \lim_{n\to\infty}\frac{n\cdot 2^{n+1}}{2^n\cdot(n+1)} = \lim_{n\to\infty}\frac{n\cdot\cancel{2^n}\cdot 2}{\cancel{2^n}\cdot(n+1)} = 2\lim_{n\to\infty}\frac{n}{n+1} = 2\cdot 1 = 2$$

Ovde smo dobili da je r = 2, a to nam govori da je red $\sum_{n=1}^{\infty}\frac{2^n}{n}$ divergentan.

Primer 3.

Ispitati konvergenciju reda $\sum_{n=1}^{\infty} \frac{(2n-1)!!}{(2n)!!} \cdot \frac{1}{2^{n+1}}$

Rešenje:

Ovde je $a_n = \frac{(2n-1)!!}{(2n)!!} \frac{1}{2^{n+1}}$, a da se podsetimo šta znači ovaj dvostruki faktorijel.

$n! = n(n-1)(n-2)\cdot ....\cdot 3\cdot 2\cdot 1$
$n!! = n(n-2)\cdot (n-4)\cdot ....\cdot$

Zavisno da li je n paran ili neparan , kad ima !! stignemo do 2 ili 1. Recimo:

$10!! = 10\cdot 8\cdot 6\cdot 4\cdot 2$

$9!! = 9\cdot 7\cdot 5\cdot 3\cdot 1$

Ovde vam savetujemo da vodite računa o zagradama (recimo $(n!)! \neq n!!$)

Da se vratimo na zadatak:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{\frac{(2n+1)!!}{(2n+2)!!}\cdot\frac{1}{2^{n+2}}}{\frac{(2n-1)!!}{(2n)!!}\cdot\frac{1}{2^{n+1}}} = \lim_{n\to\infty} \frac{2^{n+1}}{2^{n+2}}\cdot\frac{(2n+1)!!}{(2n-1)!!}\cdot\frac{(2n)!!}{(2n+2)!!} =$$

$$= \lim_{n\to\infty}\frac{\cancel{2^{n+1}}}{\cancel{2^{n+1}}\cdot 2}\cdot\frac{(2n+1)\,\cancel{(2n-1)!!}}{\cancel{(2n-1)!!}}\cdot\frac{\cancel{(2n)!!}}{(2n+2)\,\cancel{(2n)!!}}$$

$$= \lim_{n\to\infty}\frac{1}{2}\cdot\frac{2n+1}{2n+2} = \boxed{\frac{1}{2}}$$

Dakle, r = ½, pa dati red konvergira.

Primer 4.

Ispitati konvergenciju reda $\sum_{n=1}^{\infty}\frac{n!}{n^n}$

Rešenje:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{\frac{(n+1)!}{(n+1)^{n+1}}}{\frac{n!}{n^n}} = \lim_{n\to\infty}\frac{(n+1)!}{n!}\cdot\frac{n^n}{(n+1)^{n+1}} = \lim_{n\to\infty}\frac{\cancel{(n+1)}\cdot\cancel{n!}}{\cancel{n!}}\cdot\frac{n^n}{\cancel{(n+1)}\,(n+1)^n} = \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n =$$

$$= \lim_{n\to\infty}\left(\frac{1}{\frac{n+1}{n}}\right)^n = \lim_{n\to\infty}\left(\frac{1}{1+\frac{1}{n}}\right)^n = \frac{1}{e}$$

Ajde da resimo ovo i na drugi način:
Upotrebićemo trikče koje se često koristi kad imamo $n!$. To je takozvana Stirlingova aproksimacija:

$$\boxed{n! \approx \sqrt{2n\pi} \cdot n^n \cdot e^{-n}}$$

Sada imamo:

$$a_n = \frac{n!}{n^n} \sim \frac{\sqrt{2n\pi} \cdot \cancel{n^n} \cdot e^{-n}}{\cancel{n^n}} = \frac{\sqrt{2n\pi}}{e^n}$$

Probamo opet Dalamberov kriterijum:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{\frac{\sqrt{2(n+1)\pi}}{e^{n+1}}}{\frac{\sqrt{2n\pi}}{e^n}} = \lim_{n\to\infty}\frac{e^n}{e^{n+1}} \cdot \frac{\sqrt{2(n+1)\pi}}{\sqrt{2n\pi}} = \lim_{n\to\infty}\frac{\cancel{e^n}}{\cancel{e^n} \cdot e} \cdot \sqrt{\frac{\cancel{2\pi}(n+1)}{\cancel{2\pi} n}} =$$

$$= \lim_{n\to\infty}\frac{1}{e} \cdot \underbrace{\boxed{\sqrt{\frac{(n+1)}{n}}}}_{teži\ 1} = \boxed{\frac{1}{e}}$$

Dakle r = 1/e , pa ovaj red konvergira…

<u>Primer 5.</u>

Ispitati konvergenciju reda $\sum_{n=1}^{\infty}\frac{n^p}{n!}$

<u>Rešenje:</u>

Ako probamo bez Stirlingove aproksimacije, imamo:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{\frac{(n+1)^{p+1}}{(n+1)!}}{\frac{n^p}{n!}} = \lim_{n\to\infty}\frac{(n+1)^{p+1}}{n^p}\frac{n!}{(n+1)!} = \lim_{n\to\infty}\frac{(n+1)^p\,\cancel{(n+1)}}{n^p}\frac{\cancel{n!}}{\cancel{(n+1)}\,\cancel{n!}} = \lim_{n\to\infty}\left(\frac{n+1}{n}\right)^p =$$

$$= \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^p = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n\frac{p}{n}} = e^{\lim\limits_{n\to\infty}\frac{p}{n}} = e^o = 1$$

Kriterijum je neodlučiv, primenimo aproksimaciju:

$$a_n = \frac{n^p}{n!} \sim \frac{n^p}{\sqrt{2n\pi} \cdot n^n \cdot e^{-n}} = \frac{n^p \cdot e^n}{\sqrt{2n\pi} \cdot n^n}$$

Sad ćemo opet probati isti kriterijum:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{\dfrac{(n+1)^p\cdot e^{n+1}}{\sqrt{2(n+1)\pi}\cdot(n+1)^{n+1}}}{\dfrac{n^p\cdot e^n}{\sqrt{2n\pi}\cdot n^n}} = \lim_{n\to\infty}\frac{\sqrt{2n\pi}}{\sqrt{2(n+1)\pi}}\frac{e^{n+1}}{e^n}\frac{n^n}{(n+1)^{n+1}}\frac{(n+1)^p}{n^p} =$$

$$\lim_{n\to\infty}\underbrace{\boxed{\frac{\sqrt{2n\pi}}{\sqrt{2(n+1)\pi}}}}_{teži\ 1}\frac{\cancel{e^n}\cdot e}{\cancel{e^n}}\frac{n^n}{(n+1)^n\cdot(n+1)}\frac{(n+1)^p}{n^p} =$$

$$\lim_{n\to\infty} e\cdot\left(\frac{n}{n+1}\right)^n\frac{1}{n+1}\underbrace{\boxed{\left(\frac{n+1}{n}\right)^p}}_{teži\ 1} = \lim_{n\to\infty} e\cdot\left(1+\frac{n}{n+1}-1\right)^n\frac{1}{n+1} = \lim_{n\to\infty} e\cdot\left(1+\frac{n-n-1}{n+1}\right)^n\frac{1}{n+1}$$

$$= \lim_{n\to\infty} e\cdot\left(1+\frac{-1}{n+1}\right)^n\frac{1}{n+1} = e\lim_{n\to\infty}\left(1+\frac{1}{-n-1}\right)^{n\cdot(-n-1)\cdot\frac{1}{(-n-1)}}\frac{1}{n+1} =$$

$$= e\boxed{\lim_{n\to\infty}\left(1+\frac{1}{-n-1}\right)^{(-n-1)\cdot\frac{n}{(-n-1)}}}\frac{1}{n+1} = e\cdot e^{-1}\lim_{n\to\infty}\frac{1}{n+1} = 0$$

Dakle, dati red konvergira!

<u>Košijev koreni kriterijum</u>:

Ako za red $\sum_{n=1}^{\infty} a_n$ postoji $\overline{\lim_{n\to\infty}}\sqrt[n]{|a_n|}$ = p onda važi :

- za p > 1 red divergira
- za p = 1 neodlučivo
- za p < 1 konvergira

<u>Primer 6.</u>

Ispitati konvergenciju reda $\sum_{n=1}^{\infty}\left(\frac{n-1}{n+1}\right)^{n(n-1)}$

Rešenje:

$$\lim_{n\to\infty}\sqrt[n]{\left(\frac{n-1}{n+1}\right)^{n(n-1)}}=\lim_{n\to\infty}\left(\frac{n-1}{n+1}\right)^{\frac{n(n-1)}{n}}=\lim_{n\to\infty}\left(\frac{n-1}{n+1}\right)^{n-1}=$$

$$\lim_{n\to\infty}\left(1+\frac{n-1}{n+1}-1\right)^{n-1}=\lim_{n\to\infty}\left(1+\frac{n-1-n-1}{n+1}\right)^{n-1}=\lim_{n\to\infty}\left(1+\frac{-2}{n+1}\right)^{n-1}=$$

$$=\lim_{n\to\infty}\left(1+\frac{1}{\frac{n+1}{-2}}\right)^{\frac{n+1}{-2}\cdot\frac{-2}{n+1}(n-1)}=\lim_{n\to\infty}\left(1+\frac{1}{\frac{n+1}{-2}}\right)^{\frac{n+1}{-2}\cdot\frac{-2}{n+1}(n-1)}=e^{\lim\limits_{n\to\infty}\frac{-2n+2}{n+1}}=e^{-2}=\boxed{\frac{1}{e^2}}$$

Kako je r = $\frac{1}{e^2}<1$, to znači da red $\sum\limits_{n=1}^{\infty}\left(\frac{n-1}{n+1}\right)^{n(n-1)}$ konvergira po Košijevom kriterijumu.

Primer 7.

Ispitati konvergenciju reda $\sum\limits_{n=1}^{\infty}\left(\frac{1+\cos n}{2+\cos n}\right)^{2n-\ln n}$

Rešenje:

$$\lim_{n\to\infty}\sqrt[n]{\left(\frac{1+\cos n}{2+\cos n}\right)^{2n-\ln n}}=\lim_{n\to\infty}\sqrt[n]{\left(\frac{1+\cos n}{2+\cos n}\right)^{n(2-\frac{\ln n}{n})}}=\lim_{n\to\infty}\left(\frac{1+\cos n}{2+\cos n}\right)^{2-\frac{\ln n}{n}}$$

Znamo da izraz $\frac{\ln n}{n}$ teži 0 kad n teži beskonačnosti, a da *cosn* ne može imati veću vrednost od 1. Onda je:

$$\lim_{n\to\infty}\left(\frac{1+\cos n}{2+\cos n}\right)^{2-\frac{\ln n}{n}}\le\lim_{n\to\infty}\left(\frac{1+1}{2+1}\right)^{2-0}=\lim_{n\to\infty}\left(\frac{2}{3}\right)^{2}=\frac{4}{9}<1$$

Dakle, ovaj red konvergira.

Rabelov kriterijum:

Ako za red $\sum\limits_{n=1}^{\infty}a_n$ postoji $\lim\limits_{n\to\infty}n(\frac{a_n}{a_{n+1}}-1)$ = t onda :

-za t > 1 konvergira
-za t = 1 neodlučiv
-za t < 1 divergira

Primer 8.

Ispitati konvergenciju reda $$\sum_{n=1}^{\infty}\frac{(2n-1)!!}{(2n)!!}\cdot\frac{1}{2n+1}$$

Rešenje:

$$\lim_{n\to\infty} n\left(\frac{a_n}{a_{n+1}}-1\right)=\lim_{n\to\infty} n\left(\frac{\frac{(2n-1)!!}{(2n)!!}\frac{1}{2n+1}}{\frac{(2n+1)!!}{(2n+2)!!}\frac{1}{2n+3}}-1\right)=\lim_{n\to\infty} n\left(\frac{(2n-1)!!}{(2n+1)!!}\frac{(2n+2)!!}{(2n)!!}\frac{2n+3}{2n+1}-1\right)$$

$$=\lim_{n\to\infty} n\left(\frac{\cancel{(2n-1)!!}}{(2n+1)\cancel{(2n-1)!!}}\frac{(2n+2)\cancel{(2n)!!}}{\cancel{(2n)!!}}\frac{2n+3}{2n+1}-1\right)=$$

$$=\lim_{n\to\infty} n\left(\frac{(2n+2)(2n+3)}{(2n+1)^2}-1\right)=\lim_{n\to\infty} n\left(\frac{(2n+2)(2n+3)-(2n+1)^2}{(2n+1)^2}\right)=$$

$$=\lim_{n\to\infty} n\left(\frac{4n^2+6n+4n+6-4n^2-4n-1}{(2n+1)^2}\right)=\lim_{n\to\infty} n\left(\frac{6n+5}{(2n+1)^2}\right)=$$

$$=\lim_{n\to\infty}\frac{6n^2+5n}{4n^2+4n+1}=\frac{6}{4}=\boxed{\frac{3}{2}>1}$$

Znači da ovaj red, po Rabelovom kriterijumu konvergira.

Primer 9.

Nadji vrednost parametra p tako da red $\sum_{n=1}^{\infty}\frac{n!e^n}{n^{n+p}}$ konvergira.

Rešenje:

Najpre ćemo srediti izraz $\frac{a_n}{a_{n+1}}$

$$\frac{a_n}{a_{n+1}}=\frac{\frac{n!e^n}{n^{n+p}}}{\frac{(n+1)!e^{n+1}}{(n+1)^{n+1+p}}}=\frac{n!}{(n+1)!}\frac{e^n}{e^{n+1}}\frac{(n+1)^{n+1+p}}{n^{n+p}}=\frac{n!}{(n+1)\cdot n!}\frac{e^n}{e^n\cdot e}\frac{(n+1)^{n+p}\cdot(n+1)}{n^{n+p}}=\frac{1}{e}\frac{(n+1)^{n+p}}{n^{n+p}}=$$

$$=\frac{1}{e}\left(\frac{n+1}{n}\right)^{n+p}=\frac{1}{e}\left(1+\frac{1}{n}\right)^{n+p}$$

Iskoristićemo trikče:

$e^{\ln\Theta} = \Theta,$ gde je $\Theta = \left(1+\frac{1}{n}\right)^{n+p}$

$$\left(1+\frac{1}{n}\right)^{n+p} = e^{\ln\left(1+\frac{1}{n}\right)^{n+p}} = e^{(n+p)\ln\left(1+\frac{1}{n}\right)}$$

Sada je

$$\frac{a_n}{a_{n+1}} = \frac{1}{e}\left(1+\frac{1}{n}\right)^{n+p} = \frac{1}{e}e^{(n+p)\ln\left(1+\frac{1}{n}\right)} = e^{-1}\cdot e^{(n+p)\ln\left(1+\frac{1}{n}\right)} = e^{-1+(n+p)\ln\left(1+\frac{1}{n}\right)}$$

$\ln\left(1+\frac{1}{n}\right)$ moramo razviti koristeći : $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1}\frac{x^n}{n}$, $-1<x<1$

$\ln(1+\frac{1}{n}) = \frac{1}{n} - \frac{1}{2n^2} + o(\frac{1}{n^2})$ sada je:

$$\frac{a_n}{a_{n+1}} = e^{-1+(n+p)\ln\left(1+\frac{1}{n}\right)} = e^{-1+(n+p)(\frac{1}{n}-\frac{1}{2n^2}+o(\frac{1}{n^2}))} = e^{-1+1-\frac{1}{2n}+\frac{p}{n}-\frac{p}{2n^2}+o(\frac{1}{n^2})} = e^{\frac{p}{n}-\frac{1}{2n}+o(\frac{1}{n})} = e^{\frac{p-\frac{1}{2}}{n}+o(\frac{1}{n})} = 1+\frac{p-\frac{1}{2}}{n}+o(\frac{1}{n}) \text{ kad } n\to\infty$$

Dalje ćemo iskoristiti Rabelov kriterijum:

$$\lim_{n\to\infty} n(\frac{a_n}{a_{n+1}}-1) = \lim_{n\to\infty} n(1+\frac{p-\frac{1}{2}}{n}-1) = \lim_{n\to\infty} n\frac{p-\frac{1}{2}}{n} = p-\frac{1}{2}$$

Sada, ako je:

$p-\frac{1}{2} > 1 \to p > \frac{3}{2}$ red konvergira

<u>Košijev integralni kriterijum:</u>

Ako funkcija f(x) opada, neprekidna je i pozitivna, tada red $\sum_{n=1}^{\infty} f(n)$ konvergira ili divergira istovremeno sa integralom $\int_1^{\infty} f(x)dx$

<u>Primer 10.</u>

Ispitati konvergenciju reda $\sum_{n=1}^{\infty}\frac{1}{n^\alpha}$

Rešenje:

Posmatramo integral: $\int_1^{\infty} \frac{1}{x^{\alpha}} dx$

$$\int_1^{\infty} \frac{1}{x^{\alpha}} dx = \lim_{A\to\infty} \int_1^{A} \frac{1}{x^{\alpha}} dx = \lim_{A\to\infty} \int_1^{A} x^{-\alpha} dx = \lim_{A\to\infty} \frac{x^{-\alpha+1}}{-\alpha+1} \Big/_1^A = \lim_{A\to\infty} \left(\frac{A^{-\alpha+1}}{-\alpha+1} - \frac{1}{-\alpha+1}\right) =$$

i) Ako je $\alpha>1$ onda je $\lim_{A\to\infty} \left(\frac{A^{-\alpha+1}}{-\alpha+1} - \frac{1}{-\alpha+1}\right) = 0 - \frac{1}{-\alpha+1} = \frac{1}{\alpha-1}$

ii) Ako je $\alpha \leq 1$ onda je $\lim_{A\to\infty} \left(\frac{A^{-\alpha+1}}{-\alpha+1} - \frac{1}{-\alpha+1}\right) = \infty$

Dakle, red konvergira za $\alpha>1$, a divergira za $\alpha \leq 1$.

Primer 11.

Ispitati konvergenciju reda sa opštim članom $a_n = \frac{1}{n \ln^p n}$ gde je n>1

Rešenje:

$$\int_2^{\infty} \frac{1}{x \ln^p x} dx = \lim_{A\to\infty} \int_2^{A} \frac{1}{x \ln^p x} dx$$

Rešimo najpre integral na stranu bez granica (da ne bi morali da menjamo granice jer moramo upotrebiti smenu)

$$\int \frac{1}{x \ln^p x} dx = \begin{vmatrix} \ln x = t \\ \frac{1}{x} dx = dt \end{vmatrix} = \int \frac{1}{t^p} dt = \int t^{-p} dt = \frac{t^{-p+1}}{-p+1} = \frac{t^{1-p}}{1-p}$$

$$\int_2^{\infty} \frac{1}{x \ln^p x} dx = \lim_{A\to\infty} \int_2^{A} \frac{1}{x \ln^p x} dx = \lim_{A\to\infty} \frac{(\ln x)^{1-p}}{1-p} \Big/_2^A = \lim_{A\to\infty} \frac{(\ln A)^{1-p}}{1-p} - \frac{(\ln 2)^{1-p}}{1-p} =$$

i) Ako je 1- p < 0 $\to$ p>1 konvergira $\frac{(\ln A)^{1-p}}{1-p} - \frac{(\ln 2)^{1-p}}{1-p} = 0 - \frac{(\ln 2)^{1-p}}{1-p} = \frac{(\ln 2)^{1-p}}{p-1}$

ii) Ako je p<1 divergira $\lim_{A\to\infty} \frac{(\ln A)^{1-p}}{1-p} - \frac{(\ln 2)^{1-p}}{1-p} = \infty$

Gausov kriterijum: Ako za red $\sum_{n=1}^{\infty} a_n$ sa pozitivnim članovima postoji:

$$\frac{a_n}{a_{n+1}} = \lambda + \frac{\mu}{n} + o(\frac{1}{n^{1+\varepsilon}}) \quad \text{za} \quad \forall \varepsilon > 0 \ \text{tada:}$$

i) Ako je $\lambda > 1$ red konvergira

ii) Ako je $\lambda < 1$ red divergira

iii) Ako je $\lambda = 1$ tada $\left\{ \begin{matrix} \text{za } \mu > 1 \text{ red konvergira} \\ \text{za } \mu < 1 \text{ red divergira} \end{matrix} \right\}$

<u>Primer 12.</u>

Ispitati konvergrenciju reda $\sum_{n=1}^{\infty} \left[\frac{(2n-1)!!}{(2n)!!} \right]^p$

<u>Rešenje:</u>

$$\frac{a_n}{a_{n+1}} = \frac{\left[\frac{(2n-1)!!}{(2n)!!} \right]^p}{\left[\frac{(2n+1)!!}{(2n+2)!!} \right]^p} = \left[\frac{(2n-1)!!}{(2n+1)!!} \frac{(2n+2)!!}{(2n)!!} \right]^p = \left[\frac{(2n-1)!!}{(2n+1)(2n-1)!!} \frac{(2n+2)(2n)!!}{(2n)!!} \right]^p = \left[\frac{2n+2}{2n+1} \right]^p$$

Sad spakujemo malo ovaj izraz i upotrebljavamo binomnu formulu:

$$\left[\frac{2n+2}{2n+1} \right]^p = \left[\frac{2n+1+1}{2n+1} \right]^p = \left[1 + \frac{1}{2n+1} \right]^p =$$

$$= \binom{p}{0} 1^p (\frac{1}{2n+1})^0 + \binom{p}{1} 1^{p-1} (\frac{1}{2n+1})^1 + \binom{p}{2} 1^{p-2} (\frac{1}{2n+1})^2 + \ldots$$

$$= 1 + \frac{p}{2n+1} + \frac{p(p+1)}{2(2n+1)^2} + o(\frac{1}{n^2})$$

$$= 1 + \frac{p}{2n+1} + o(\frac{1}{n^2})$$

$$= 1 + \frac{p}{2(n+\frac{1}{2})} + o(\frac{1}{n^2})$$

$$= 1 + \frac{p/2}{n+1/2} + o(\frac{1}{n^2}) \ \text{kad n} \to \infty$$

$$= 1 + \frac{p/2}{n} + o(\frac{1}{n^2})$$

Ovo uporedjujemo sa $\frac{a_n}{a_{n+1}} = \lambda + \frac{\mu}{n} + o(\frac{1}{n^{1+\varepsilon}})$

Jasno je da je $\lambda = 1$ pa nam treba $\mu = \frac{p}{2}$

i) Ako je $\mu = \frac{p}{2} > 1 \to p > 2$ red konvergira

ii) Ako je $\mu = \frac{p}{2} < 1 \to p < 2$ red divergira