# CSE 555 HOMEWORK ONE

DUE 05 FEBRUARY 2016, START OF CLASS

Let $\Sigma = \{\sigma_1, \ldots, \sigma_n\}$ be a finite alphabet, and suppose that the symbols have a total ordering $\sigma_1 \prec \sigma_2 \prec \cdots \prec \sigma_n$. We say that a string $w_1 \cdots w_m$ is *sorted* if $w_i \preceq w_{i+1}$ for all $1 \leq i < m$.

We say that a language $L \subseteq \Sigma^\star$ is *sorted* if every string in $L$ is sorted.

(1) Regular languages:
   (a) Let $\texttt{SORTED}_{\mathsf{NFA}} = \{\langle M \rangle : M \text{ is an NFA for which } L(M) \text{ is sorted}\}$. Prove or disprove: $\texttt{SORTED}_{\mathsf{NFA}}$ is Turing decidable.

   (b) Let $\texttt{MOSTSORT}_{\mathsf{NFA}} = \{\langle M \rangle : M \text{ is an NFA for which } L(M) \text{ contains a finite number of unsorted strings}\}$. Prove or disprove: $\texttt{MOSTSORT}_{\mathsf{NFA}}$ is Turing decidable.

(2) Context-free languages:
   (a) Let $\texttt{SORTED}_{\mathsf{PDA}} = \{\langle M \rangle : M \text{ is a PDA for which } L(M) \text{ is sorted}\}$. Prove or disprove: $\texttt{SORTED}_{\mathsf{PDA}}$ is Turing decidable.

   (b) Let $\texttt{MOSTSORT}_{\mathsf{PDA}} = \{\langle M \rangle : M \text{ is a PDA for which } L(M) \text{ contains a finite number of unsorted strings}\}$. Prove or disprove: $\texttt{MOSTSORT}_{\mathsf{PDA}}$ is Turing decidable.

(3) Suppose that $L$ is sorted.
   (a) Prove or disprove: $L$ must be Turing decidable.

   (b) Prove or disprove: $L$ must be Turing recognizable.

(4) Let $\texttt{SORTED}_{\mathsf{TM}} = \{\langle M \rangle : M \text{ is an TM for which } L(M) \text{ is sorted}\}$. Prove or disprove: $\texttt{SORTED}_{\mathsf{TM}}$ is Turing decidable.

(5) Let $\texttt{REVERSIBLE}_{\mathsf{CFG}} = \{\langle G \rangle : G \text{ is a context free grammar for which there exists a string } w \in L(G) \text{ for which } w = w^{\text{rev}}\}$. Prove or disprove that $\texttt{REVERSIBLE}_{\mathsf{CFG}}$ is Turing decidable.