Introduction

Most people believe that the game section (analytical reasoning) is the most difficult part of the LSAT. They're right. The games are disproportionately hard for two reasons: First, the questions are complex, convoluted, and subtle. Second, it is the most highly "timed" part of the test. That is, you are intentionally not given enough time to finish.

The game section is also the most contrived part of the test. The possible arrangements of a group of people around a circular table has little correlation with the daily activities of a lawyer. The writers of the LSAT use the games as a litmus test, to see whether you have the intellectual ability to make it in law school.

Although this may sound intimidating, everyone taking the LSAT is in the same situation. You probably have never seen this type of problem during you academic career. No specific college course will prepare you for them, except perhaps mathematics.

Furthermore, games are the easiest problems on which to improve, for two reasons: (1) The process of solving a game is very systematic. (2) There are only a few different kinds of games: just three major categories, plus a couple that don't quite fit. This chapter is dedicated to classifying the major types of games and then mastering their systematic solution.

The Three Major Types of Games

ORDERING GAMES

These games require you to order elements, either in a line or around a circle. The criteria used to determine order can include size, time, rank, etc. Ordering games are the easiest games on the LSAT. Luckily, they are also the most common.

Example: *Ordering Game*

Six people—Rick, Steve, Terry, Ulrika, Vivian, and Will—are standing in line for tickets to an upcoming concert.

Rick is fifth in line and is not next to Steve.
Ulrika is immediately behind Terry.
Will is not last.

Which one of the following people must be last in line?

(A) Steve
(B) Terry
(C) Ulrika
(D) Vivian
(E) Rick

Clearly, Rick cannot be last since he is already fifth. So eliminate choice (E). Next, neither Terry nor Ulrika can be last since Ulrika must stand immediately behind Terry. So eliminate choices (B) and (C). Finally, Steve cannot be last, either. If he were, then he would be next to Rick, which would violate the first condition. So eliminate choice (A). Hence, by process of elimination, Vivian must be last in line. The answer is (D).

This method of elimination is the most efficient way to solve many, if not most, game questions. We will discuss this method in more detail later in the chapter.

GROUPING GAMES

Grouping games, as the term implies, require you to separate elements—typically people—into groups. Some conditions of the game can apply to entire groups only, some to elements within a group only, and some to both. This added complexity makes grouping games, in general, harder than ordering games.

Example: *Grouping Game*

Eight people—A, B, C, D, E, F, G, and H—ride to work in three cars. Two cars each take three people, and one car takes only two people.

B rides with H.
G rides with only one other person.
F rides with two other people.

If C rides with B, all of the following are groups of people that can ride together EXCEPT:

(A) A and G
(B) G and E
(C) A, D, and E
(D) A, D, and F
(E) B, C, and H

Combining the conditions *"C rides with B"* and *"B rides with H"* gives the completed car-pool CBH. Now F and G must ride in different car-pools because G rides in a pool of two and F rides in a pool of three. However, the group ADE, in choice (C), could not ride with either F or G because in either case they would form a group of four. Hence (C) is the answer.

ASSIGNMENT GAMES

These games involve assigning characteristics to the elements, typically people. The most common task in these games is to assign a schedule. You probably have had some experience with schedules; you may have written the weekly work-schedule for a business. If so, you know how difficult the task can become, even when only a few conditions are placed on the employees: Bob will work Monday, Tuesday, or Friday only. Susan will work evenings only. Steve will not work with Bob. Add to this that the company must have a full staff weekdays, but only three people can work weekends. Scheduling games on the LSAT are similar to this. Because the conditions can apply to individuals separately, to groups of individuals, to times, to places, etc., scheduling games tend to be the most difficult—save them for last.

Example: *Assignment Game*

The Mom & Pop liquor store employs five cashiers—Adams, Bates, Cox, Drake, and Edwards—each of whom works alone on exactly one day, Monday through Friday.

Adams will work only on Tuesday or Thursday.
Bates will not work on Monday or Wednesday.
Cox works on Friday.
Drake and Edwards do not work on consecutive days.

Which one of the following is a possible work schedule?

(A) Edwards, Bates, Adams, Drake, Cox
(B) Drake, Adams, Bates, Edwards, Cox
(C) Edwards, Adams, Cox, Bates, Drake
(D) Edwards, Adams, Drake, Bates, Cox
(E) Drake, Edwards, Bates, Adams, Cox

Begin by eliminating (A); it is not a possible work schedule since the first condition states that Adams works only on Tuesday or Thursday. Next, (B) is not a possible work schedule since the second condition states that Bates will not work on Monday or Wednesday. Next, (C) is not a possible work schedule since the third condition states that Cox works on Friday. Finally, (E) is not a possible work schedule since the last condition states that Drake and Edwards do not work on consecutive days. Thus, by process of elimination, we have learned the answer is (D).

We will thoroughly analyze each of the three major types of games. Additionally, we will study some games that don't fit neatly into this classification.

Format of the Game Section

The game section of the LSAT consists of four games, each of which has about six questions—sometimes 5 and sometimes 7. Thus, there are usually twenty-four questions. The section is 35 minutes long.

FORMAT
Game #1 (6 questions ±1)
Game #2 (6 questions ±1)
Game #3 (6 questions ±1)
Game #4 (6 questions ±1)

This means that you have a little less than nine minutes for each game. Or if you skip the most difficult game, as most people should, then you have a little less than 12 minutes for each game. If this sounds fast-paced, you're right. The LSAT is a highly "timed" test, and the game section is the most highly "timed" part.

Lawyers typically have to think quickly on their feet. This section is testing how quickly you can understand the structure of a set of relationships and how swiftly you can draw conclusions from the implications of those relationships.

Skipping a Game

Because games are difficult and time consuming, you should consider skipping the hardest one. **Do not, however, skip parts of each game; rather, skip one entire game.**

The time you save by skipping the hardest game can be used to read and solve the other games more carefully. Reading and solving the problems too quickly is the major cause of errors in this section of the test. The questions come in blocks of 6, so misreading the setup to a game can cause you to miss an entire block of 6 questions.

If you decide to skip the hardest game, you will need, of course, some criteria to determine which game is the hardest. You might think that a game with many conditions is harder than one with only a few conditions. This is not necessarily true. In fact, most often, the fewer the conditions the harder the game. Think of the conditions as clues. If you were a detective investigating a case, the more clues you had the easier the case would be to solve. The same is true for games. Furthermore, some of the conditions are often superfluous or are needed only for the last couple of questions (see Obfuscation, page 42). As mentioned before, ordering games generally are the easiest, grouping games are harder, and assignment games are the hardest.

Order of Difficulty

Unlike most standardized tests, the questions on the LSAT are not listed from the easiest to the hardest. If they were, then deciding which game to skip would be easy—skip the last game. However, this much can be said: the first game will not be the hardest and the last game will not be the easiest; do not, therefore, skip the first game. This is also true of the question-set to a game.

Reading with Precision

We are not accustomed to reading and thinking with the degree of precision required for the games. If I ask you to count from one to four, you will probably respond with: one, two, three, four. But is this the correct response? I said to count from one to four, not from one through four. So the correct response actually is one, two, three. This may seem like "splitting hairs", but it is precisely this degree of precision which must be exercised when reading the conditions of a game.

Typically, when reading, we skim the words looking for the gist of what the writer is trying to convey. However, the conditions of a game *cannot* be read in that manner. They must be read slowly and carefully, taking each word for its literal meaning—and making no unwarranted assumptions.

It may seem at times that the wording of a game is designed to trick you. It is not (except for what will be discussed later in the section *Obfuscation*). A game is a logic puzzle that must be solved by applying the fundamental principles of logic—that is, common sense—to the given conditions. If a word has two different meanings, then two different answers may be possible. For this reason, ambiguity in a game cannot be tolerated.

To avoid ambiguity, the LSAT writers always use the literal or dictionary meaning of a word.

To illustrate, take the little, nondescript word "or." We have little trouble using it in our day-to-day speech. However, it actually has two meanings, one inclusive and one exclusive. In the sentence "Susan or John may come to the party" we understand that both may come. This is the *inclusive* meaning of "or." On the other hand, in the phrase "your money or your life" we hope that the mugger intends the *exclusive* meaning of "or." That is, he does not take both our money and our life.

Unless otherwise stated, the meaning of "or" on the LSAT is inclusive.

Unwarranted Assumptions

When analyzing a game, you must not only take the meaning of the words literally but you must also take the meaning of the sentences literally. **That is, do not make any unwarranted assumptions about what a condition implies.** For example, if the setup to a problem states that five people are standing in a line and a condition states that Craig is standing behind Jane, then you *cannot* assume he is immediately behind her. There may be one or more people between them. For another example, take the word "never" in the sentence "Jane never arrives at work before Kelly." Based on this statement we cannot assume that Kelly always arrives before Jane: they may on occasion arrive at the same time. For a more subtle example, take the sentence "John will go only if Steve goes." We cannot assume that if Steve goes, then John will go too. The sentence implies only that if John left, then Steve must also have left, not necessarily vice versa.

The last two sections are not intended to imply that you should dwell on minutia. Instead, they should alert you to pay attention to the exact meanings of words and sentences. Words are used in their literal sense, not to make the conditions "tricky," but to avoid ambiguity.

Reading With Precision/Unwarranted Assumption Drill

<u>Directions:</u> Each condition in this exercise is followed by a series of deductions. Determine whether each deduction is valid or invalid. Answers and solutions begin on page 45.

1. A and B are two consecutive elements in a line.
   a. A is next to B.
   b. A is to the left of B.
   c. C is between A and B.

2. Of four elements—A, B, C, D—in a line, B sits between A and C.
   a. B is next to A.
   b. B is next to either A or C.

3. Only people over eighteen years of age can enter.
   a. A person eighteen years of age can enter.

4. John is older than Mary who is younger than Betty, and Robert is not older than Mary.
   a. Mary is younger than John.
   b. Mary is older than Robert.
   c. Betty is older than Robert.

5. To be in the theater, each child (a person less than 13 years old) must be accompanied by an adult.
   a. Johnny who is ten could be in the theater by himself.
   b. John who is thirty-five could be in the theater by himself.

6. If it is cloudy, then Biff is not at the beach.
   a. If it is sunny, Biff is at the beach.
   b. If Biff is not at the beach, it is cloudy.
   c. If Biff is at the beach, it is not cloudy.

7. I will take the LSAT unless I am not prepared.
   a. I prepared thoroughly, so I took the LSAT.
   b. I prepared thoroughly, but I did not take the LSAT.

8. On a line A and B are two spaces apart.
   a. Two people sit between A and B.

9. Only Bob works evenings.
   a. Bob works only evenings.

Logical Connectives

While no training in formal logic is required for the LSAT, essentially it is a logic test. So some knowledge of formal logic will give you a definite advantage.

To begin, consider the seemingly innocuous connective "if..., then...." Its meaning has perplexed both the philosopher and the layman through the ages.

The statement "if A, then B" means by definition "if A is true, then B must be true as well," and nothing more.

We often need to rephrase a statement when it's worded in a way that obscures the information it contains. The following formulas are very useful for rewording and simplifying the conditions of a game.

On the LSAT, as in everyday speech, two negatives make a positive—they cancel each other out.

not(not A) = A

Example:

"It is not the case that John did not pass the LSAT"

means the same thing as

"John did pass the LSAT." ■

The statement "if A, then B; and if B, then A" is logically equivalent to "A if and only if B." Think of "if and only if" as an equal sign: if one side is true, then the other side must be true, and if one side is false, then the other side must be false.

(If A, then B; and if B, then A) = (A if and only if B)

A if and only if B	
A	B
True	True
False	False

Example:

"If it is sunny, then Biff is at the beach; and if Biff is at the beach, then it is sunny"

is logically equivalent to

"It is sunny if and only if Biff is at the beach." ■

"A only if B" means that when A occurs, B must also occur. That is, "if A, then B."

A only if B = if A, then B

Example:

"John will do well on the LSAT only if he studies hard"

is logically equivalent to

"If John did well on the LSAT, then he studied hard."

> (Note: Students often wrongly interpret this statement to mean *"if John studies hard, then he will do well on the LSAT."* There is no such guarantee. The only guarantee is that if he does not study hard, then he will not do well.) ■

The statement "A unless B" means that A is true in all cases, except when B is true. In other words if B is false, then A must be true. That is, if not B, then A.

A unless B = if not B, then A

Example:

> *"John did well on the LSAT unless he partied the night before"*

is logically equivalent to

> *"If John did not party the night before, then he did well on the LSAT."* ■

The two statements "if A, then B" and "if B, then C" can be combined to give "if A, then C." This is called the transitive property.

("if A, then B" and "if B, then C") = ("if A, then C")

Example:

From the two statements

> *"if John did well on the LSAT, then he studied hard"* and *"if John studied hard, then he did not party the night before the test"*

you can conclude that

> *"if John did well on the LSAT, then he did not party the night before the test."* ■

These fundamental principles of logic are never violated in either the games or the arguments.* Hence by using the above logical connectives, you can safely reword any statement on the LSAT.

* It is *conceivable* that these principles of logic might be violated in a reading passage because rhetoric is often quoted there; however, I've yet to see it occur.

Diagramming

Virtually every game can be solved more easily and efficiently by using a diagram. Unless you have a remarkable memory and can process reams of information in your head, you *must* draw a diagram. Because of the effectiveness of diagrams, games are the best candidates for improvement. A well-constructed diagram can change a convoluted, unwieldy mass of information into an easily read list. In fact, from a well-constructed diagram, you can often read-off the answers without any additional thought. Before we begin studying how to construct diagrams, we need to develop some facility for creating and manipulating symbols.

Symbols

The ability to symbolize sentences is one of the most important skills you need to develop for the LSAT.

A good symbol is complete; it summarizes all the relevant information in the sentence. It is succinct. And it is functional, easy to use. The last condition makes creating symbols an art. A good symbol helps you organize your thoughts and frees your mind from the fetters of indecision.

Five basic symbols are used throughout this book. They are

Symbol	Meaning
&	and
or	or
~	not
—>	If..., then...
( )	parentheses

I. The ampersand symbol, &, connects two statements of equal rank. The two statements

"Rob sits next to Jane" and *"Susan sits next to Adam"*

can be translated as

(RJ) & (SA).

Two statements joined by "&" will be true as a group only when both are true.

II. The symbol "or" also connects two statements of equal rank. The above statement can be symbolized using "or" as

(RJ) or (SA).

For an *or*-statement to be true, only one of the two statements need be true, though both can be. For example, the statement *"it is raining" or "it is not raining"* is true even though one of the statements must be false. This makes an *or*-statement much weaker than an &-statement.

III. Placing ~ in front of any true statement makes the statement false, and vice versa. The symbol ~ can be read as "it is not the case that." For example, the symbol ~(RJ) translates as "it is not the case that R sits next to J."

IV. The *if..., then...* symbol (—>) causes much consternation, even though we are rarely confused by its meaning in everyday speech. It is true in all cases, <u>except</u> when the statement on the left side is true and the statement on the right side is false.* As mentioned in the section Logical Connectives "if P, then Q" is logically equivalent to "if not Q, then not P"; the latter is the contrapositive.

For example, the statement

"if it is sunny, then Biff is at the beach"

is logically equivalent to

"if Biff is not at the beach, then it is not sunny."

Sometimes you can use this equivalency to simplify a convoluted condition. For example, the condition "if Jane does not go, then Steve will not go" can be simplified to "if Steve goes, then Jane goes," or in symbols

S—>J.

V. Parentheses clarify a symbol statement's meaning in the same way that commas clarify sentences. The symbol statement A&B—>C is ambiguous; we don't know whether it means

(A&B)—>C

or

A&(B—>C).

Sometimes parentheses are used even when they are not truly needed. For example, the symbol statement ~A&B is not technically ambiguous; however, it is less likely to be misread when written (~A)&B. Now clearly the negation applies *only* to the A.

* For a thorough discussion of "if..., then..." see Logical Connectives, page 34.

Symbol Exercise

Directions: Translate the given statements into symbols. Note: There is no "best" symbol. However, you should choose one that is short yet clear. Solutions begin on page 46.

1. On a line, B is to the immediate right of A. Symbol:	8. Four people are standing in a line. If A is last, then A must be next to either B or C. Symbol:
2. On a line, B is to the right of A. Symbol:	9. Four people are standing in a line. A is next to B if and only if C is last. Symbol:
3. On a line, B is to the right of either A or C, but not both. Symbol:	10. Of three people, Bob is older than Susan but younger than Ted. Symbol:
4. On a line, A is not next to B. Symbol:	11. A and B are before C, and C is before X and Y. Symbol:
5. On a line, B and A are two spaces apart. Symbol:	12. At a table, if Bob sits next to Ted, Alice does not sit next to Carol. Symbol:
6. If A, then B. Symbol:	13. At a table, Mr. Williams sits directly opposite from Mrs. Adams. Symbol:
7. A only if B. Symbol:	

Advanced Problems

14. A and B cannot serve on a panel unless they serve together. Symbol:	18. Stan will only fly in either plane 1 or plane 2. Symbol:
15. John lives two floors above Craig. Symbol:	19. No one but union members can work. Symbol:
16. Nancy and Tom do not both enter the race. Symbol:	20. There are exactly two full working days between the day Mike works and the day Jane works, and Mike always works before Jane during a single workweek. Symbol:
17. No one drives without a driver's license. Symbol:	

Diagramming Continued

As stated before, diagramming is the way to solve nearly every game. This section covers how to construct an efficient and functional diagram. The process is very mechanical.

We begin with the setup to a game. The setup gives the context or background for a game. The elements, often people, are named here, too. We abbreviate names by using the first letter of the name. Kindly, the LSAT writers use names that begin with different letters.

For example, we may be told that six people—Jack, Kathy, Larry, Mary, Nick, and Olivia—are seated, evenly spaced, around a circular table. We let J, K, L, M, N, and O stand for their names.

Next, take the conditions and turn them into symbols. For example, suppose for the above setup we are told that

Jack sits at the "top" of the table.
Jack sits directly opposite Larry.
Mary does not sit next to Larry.

The first condition, "Jack sits at the 'top' of the table," is naturally described as **J = top**. The second condition, "Jack sits directly opposite Larry," can be symbolized as J<—>L (where the *arrow* means "sits directly opposite"). Finally, the third condition, "Mary does not sit next to Larry," can be written as **M ≠ L**. [Another possible symbol is ~(ML). You may prefer to use this symbol or to create one of your own. The symbol you choose is irrelevant so long as it is short and functional.]

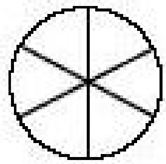
Adding these symbol statements to our previous work yields the following schematic:

J K L M N O
J = top
J<—>L
M ≠ L

Note, the elements O, K, and N are "independent" because no conditions refer directly to them. In general, independent elements can be placed in more positions on a diagram than dependent elements. Think of independent elements as "wild cards".

Now, we need a diagram to complete our scheme. To this end, draw a circle with three spokes inside, and place it below the schematic:

J K L M N O (K, N, O are "wild")
J = top
J<—>L
M ≠ L



Next, we come to the all-important decision—in what order do we place the conditions on the diagram.

The following rules should guide your decision.

1. **First, place conditions which fix the position of an element.**
   Examples: Allison is second from the left.
   Steve works on Monday only.
2. **Next, place conditions which limit the positions an element may have.**
   Examples: Allison is either the second or third person from the left.
   Steve works only the night shift.
3. **Then, place conditions which connect two or more elements.**
   Examples: Allison sits directly opposite Jane.
   Steve works only when Bob works.
4. **Finally, on the diagram designate any place an element *cannot* be.**
   (This is the negative counterpart of rule 2, and it is much weaker.)
   Examples: Allison cannot sit in an even numbered chair.
   Steve does not work when Bob works.

Short Drill on Order of Placement

<u>Directions:</u> Using the guidelines on this page, determine the order in which each of the following sets of conditions should be placed on a diagram. Answers and solutions are on page 48.

1. Six students—John, Kelly, Laura, Mick, Nina, Sean—took a test. The following is known about their grades:
   (A) Neither John nor Laura received a B.
   (B) Mick received either an A or a B.
   (C) Kelly received the lowest grade.

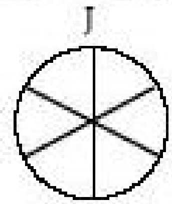
2. Commuters P, Q, R, S, and T board a bus. The bus makes six subsequent stops. Each commuter gets off at a different stop, and at one of the stops no one gets off.
   (A) P always gets off at an even numbered stop.
   (B) Q always gets off second.
   (C) S always gets off after Q, and none of the other commuters gets off the bus at a stop that comes after Q's stop but before S's stop.

3. On one side of a street there are five houses, each of which is home to exactly one of five families: the Howards, Ingrams, Jones, Kilpatricks, Leoffs. The street runs west to east.
   - (A) The Leoffs do not live in the last house.
   - (B) The Howards live in the second house from the west end of the street.
   - (C) The Kilpatricks live east of the Howards but west of the Ingrams.

4. Six people—M, N, O, P, Q, R—are seated around a circular table.
   - (A) M sits directly opposite N.
   - (B) Neither O nor P sits next to M.

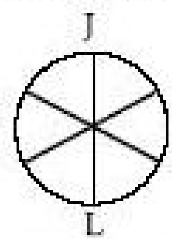
Diagramming Continued

Now let's apply these rules to the conditions of our game. J = **top** is the only condition that fixes the position of an element, so place it on the diagram first:



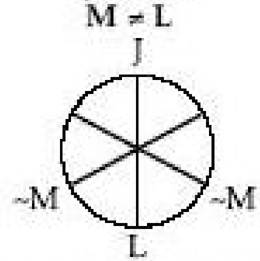
Next, scan the conditions for one that limits the placement of an element. There are none. So the second rule does not apply.

Then, scan the conditions for one that connects two or more elements. The condition **J<—>L** connects J and L. Place it on the diagram as follows:



Finally, scan the conditions for one that tells where an element can <u>not</u> be. The last condition, **M ≠ L** states that M cannot be next to L. Thus, our final diagram looks like this:

J K L M N O (K, N, O "wild")
J = top
J<—>L
M ≠ L



This diagram is self-contained. There is no need to refer to the original problem. Looking back should be avoided whenever possible.

Readily Derived Conditions

Before turning to the questions, check whether any further conditions can be easily derived from the given conditions. Suppose in addition to the conditions given in the game above, you were told that K and O do not sit next to each other. You could then deduce that K and O sit on opposite sides of the base axis J<—>L since there are only two seats on either side of this axis. Place the derived conditions on the diagram before turning to the questions. Do not, however, spend an inordinate amount of time on this step.

Do not erase previously derived diagrams; they are often useful for later questions.

Keep your symbols and diagrams as simple and functional as possible. **Warning**: Some books suggest diagrams which appear elegant but soon prove too complicated to actually use. A complex "elimination grid" may be very effective in answering the questions, but it probably cannot be constructed and applied to all the questions in less than nine minutes. Your diagrams should be simple and able to evolve with changing conditions.

Obfuscation

To make a game harder, the LSAT writers have two methods available. One is to make the relationships between the elements more complex and subtle. This is the hard way. Working out all the connections possible for a complex condition can be very time consuming, and thinking up a subtle condition can require much creative inspiration. The other way is to obscure the conditions and answers. This is the easy way.

The LSAT writers do not resort to obfuscation as often with games as they do with arguments because games are inherently difficult, whereas arguments are inherently easy. Still, five tactics are occasionally used.

With games, the most common obfuscation ploy is the **leading question**. The question "Which one of the following must be false?" *leads* you to look directly for the *false* answer. However, it is often easier if you reword this type of question as "All of the following could be true EXCEPT?" Then search for and eliminate the true answer-choices.

It is much harder to make a difficult game than it is to solve one. The easiest way to make a game harder is to **convolute the wording of a condition**. For example, the complex condition

"If O is off, then N is off; if O is on, then N is on"

positive. In these cases, use the contrapositive to rephrase the statement. For example, the condition "If Carl Lewis enters the 100-meter dash, then he will not enter the long jump" may release more relevant information when reworded as "If Carl Lewis enters the long jump, then he will not enter the 100-meter dash."

Adding many conditions to a game can obscure the more important ones. Typically, a game consists of two or three core conditions from which nearly all the questions can be answered. Master these few conditions and you've mastered the game. To obscure this fact, the LSAT writers sometimes **surround the core conditions with other conditions that relate to only one or two questions, if any.**

Another way to make a game harder, or at least longer, is **to word a question so that you must check every answer-choice** (see Indirect Proof, page 58). For example, questions such as "All of the following could be true EXCEPT" often require you to check every answer-choice. Unfortunately, there is no effective countermeasure to this tactic.* If pressed for time, you should skip this type of question. Remember, whether a question is short and easy or long and difficult, it is worth the same number of points.

<u>Advanced Concepts:</u>

The last and most pernicious obfuscating tactic is **to apply subtle changes to the standard wording of a question.** We have already seen an example of a question with the wording "Which one of the following is a complete and accurate list of . . . ?" In this case, the correct answer must include *all* the possibilities. But sometimes (though rarely) the verb "is" is replaced with "could be": "Which one of the following *could be* a complete and accurate list of . . . ?" In this case, the correct answer could include all, some, or even none of the possibilities. Mercifully, this tactic is not often used. But it can occur, so be alert to it.

As you work through the examples and exercises in this chapter, notice how the five tactics of obfuscation are used.

* Rewording the question as "Which one of the following is false?" usually does not help. In fact, the latter type of question is often reworded as the former.

Points to Remember

1. Unless you are one of the few who have a knack for games, you should skip the hardest one. This will leave you with about twelve minutes per game, instead of only nine.

2. The three major types of games are

   Ordering Games
   Grouping Games
   Assignment Games

3. Although the games are not presented in ascending order of difficulty, the first game will not be the hardest and the last game will not be the easiest.

4. Do not assume that a game with many conditions is harder than one with only a few.

5. In general, ordering games are the easiest, and assignment games are the hardest.

6. Read the conditions to a game very carefully, and avoid making any unwarranted assumptions about what they imply.

7. A—>B is logically equivalent to its contrapositive ~B—>~A. It is not logically equivalent to the following fallacies:

   ~A—>~B (Invalid)
   B—>A (Invalid)

8. The following logical connectives are equivalent:

   ~(~A) = A
   (A—>B and B—>A) = (A<—>B)
   (A only if B) = (A—>B)
   (A—>B and B—>C) = (A—>C)
   (A unless B) = (~B—>A)

9. On the LSAT, the meaning of "or" is inclusive, unless stated otherwise.

10. Reword convoluted questions.

11. The questions to a game are independent of one another.

12. Virtually every game should be solved with a diagram.

13. When deciding the order in which to place elements on a diagram, use the following guidelines.

    First: Place any element whose position is fixed.
    Second: Place any element whose possible positions are limited.
    Third: Place any element whose position is connected to one or more other elements.
    Last: Note any place an element cannot be.

14. In general, independent elements can be placed in more positions than dependent elements. Think of independent elements as "wild cards".

15. When answering the questions, refer to the diagram. Avoid returning to the original problem.
16. Before turning to the questions, note any readily derived conditions.
17. Keep your diagrams simple and functional.
18. Do not erase previously derived diagrams.
19. The LSAT writers use 5 methods to obfuscate a game:
    a. The leading question.
    b. Convoluting the wording of a condition.
    c. Surrounding the core conditions with superfluous conditions.
    d. Wording a question so that you must check every answer-choice.
    e. Applying subtle changes to a question's standard wording.

Solutions to Reading with Precision Drill

1. a. **Valid**
   Saying that A is next to B is equivalent to saying that A and B are consecutive.
   b. **Invalid**
   Although A is written to the left of B, we cannot assume that its position on the line is left of B.
   c. **Invalid**
   If C were between A and B, then A and B would not be consecutive.

2. a. **Invalid**
   We know only that B is between A and C, not whether B is next to A. (The ordering could be ADBC.)
   b. **Valid**
   D is the only element which can separate B from either A or C. Therefore, B must be next to either A or C—though not necessarily both.

3. a. **Invalid**
   A person eighteen years of age is not over eighteen.

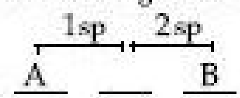
4. a. **Valid**
   "John is older than Mary" is logically equivalent to "Mary is younger than John."
   b. **Invalid**
   We know only that Robert is not older than Mary; they may be the same age.
   c. **Valid**
   Since Betty is older than Mary who is older than or the same age as Robert, Betty must be older than Robert.

5. a. **Invalid**
   According to the statement, he must be with an adult.
   b. **Valid**
   We know only that children must be accompanied by an adult; the reciprocal relation that adults be accompanied by children need not be true.

6. a. **Invalid**
   We know only that if it is in fact cloudy, then Biff is not at the beach. We know nothing about where he is when it is sunny.
   b. **Invalid**
   Again, we know only that if it is cloudy then Biff is not at the beach. It may be sunny, and Biff may decide not to go to the beach.
   c. **Valid**
   If it were in fact cloudy, this would contradict the premise, "if it is cloudy, then Biff is not at the beach."

7. a. **Valid**
   Generally, "unless" means "if...not." So the sentence can be recast as "I will take the LSAT if I am prepared." (Note two negatives make a positive, so the phrase "if I am *not not* prepared" was simplified to "if I am prepared.") With this interpretation of the sentence, we see that the two statements are consistent. In fact, the second statement necessarily follows from the first.
   b. **Invalid**
   This contradicts the premise of the statement.

8. a. **Invalid**
   Many students confuse these two conditions. They do not mean the same thing. "A and B are two spaces apart" means that <u>only one</u> spot separates them.



9. a. **Invalid**
   "Only Bob works evenings" means only that no one else works evenings. Bob could still work an afternoon shift in addition to his evening shift.

Answers to Symbol Exercise

Statement	Symbol
1. On a line, B is to the immediate right of A.	**AB**
2. On a line, B is to the right of A.	**A...B**
3. On a line, B is to the right of either A or C, but not both.	**A...B...C**

Explanation: The statement is equivalent to "B is between A and C," which can be symbolized as A...B...C. The line linking A with C, the "flip-flop" symbol, indicates that A and C can be interchanged. The symbol ABC would be misleading because it implies that B is next to A and C, which cannot be assumed.

4. On a line, A is not next to B.	**~(AB)**

Explanation: Once you become accustomed to the fact that the order could be reversed, you won't need to write the flip-flop symbol.

5. On a line, B and A are two spaces apart.	**A __ B**

Explanation: Many people incorrectly symbolize this as A __ __ B because they read "two spaces apart" as "separated by two spaces" (see Reading with Precision, page 31).

6. If A, then B.	**A—>B**

7. A only if B.	**A—>B**

Explanation: See Logical Connectives, page 33.

8. Four people are standing in a line. If A is last, then A must be next to either B or C.	**(A=last)—>(AB or AC)**

9. Four people are standing in a line. A is next to B if and only if C is last.	**(AB)<—>(C=last)**

Explanation: This is a double implication. That is, if A is next to B, then C is last; and if C is last, then A is next to B. Remember the symbol AB means that A is next to B, though not necessarily in that order.

10. Of three people, Bob is older than Susan but younger than Ted.	**TBS**

11. A and B are before C, and C is before X and Y.	**(A&B) > C > (X&Y)**

12. At a table, if Bob sits next to Ted, Alice does not sit next to Carol.	**(BT)—>~(AC)**

13. At a table, Mr. Williams sits directly opposite from Mrs. Adams.	**W<—>A** (Here, <—> means "opposite to")

14. A and B cannot serve on a panel unless they serve together.	**(A=P)<—>(B=P),** (where P stands for "is on the panel")

15. John lives two floors above Craig.	**J** □ **C** Note only one floor separates J and C—not two floors.

16.	Nancy and Tom do not both enter the race.	**N—>~T** (T—>~N would also work.)
17.	No one drives without a driver's license.	**(A=D)—>(A=DL)** (where D stands for "drives" and DL stands for " has a driver's license")
18.	Stan will only fly in either plane 1 or plane 2.	**(S=F)—>(S=1 or S=2)** (where F stands for "flying")
19.	No one but union members can work.	**(A=W)—>(A=U)** (where W means "works" and U means "is a member of the union")
20.	There are exactly two full working days between the day Mike works and the day Jane works, and Mike always works before Jane during a single workweek.	**M __ __ J**

Solutions to Order of Placement Drill

1. (C) first. (B) second. (A) last.

   Condition (C) fixes Kelly's grade; place it first. Next, condition (B) limits the possible grades that Mick received; place it second. Next, none of the conditions connects two or more elements, so Guideline 3 does not apply. Finally, condition (A) states what grade John and Laura did not receive; place it last.

2. (B) first. (A) second. (C) third and fourth.

   Because Q may get off before or after the stop at which no one gets off, two diagrams must be drawn. On each diagram, the elements should be placed as follows: Place condition (B) first since it fixes the position of Q. Next, place condition (A) since it limits the possible stops at which P can get off. Then, place the first part of condition (C) since it connects the two elements S and Q. Finally, place the second part of condition (C) since it states where the elements P, R, and T cannot get off.

3. (B) first. (C) second. (A) last.

   Place condition (B) first; it fixes the home of the Howards. No condition limits where a family lives, e.g., the Jones live in either the first or second house from the west end of the street. So Guideline 2 does not apply, and we turn to Guideline 3. Place condition (C) next; it connects the Kilpatricks with the Howard and the Ingrams. Finally, place condition (A); it states where the Leoffs do not live—the last house.

4. (A) first. (B) second.

   Neither condition fixes the position of any person, nor does either limit the places where any person may sit. So Guidelines 1 and 2 do not apply. Condition (A) connects M and N, so place it first. Condition (B) states where O and P may not be—next to M, so place it last.

LINEAR ORDERING

Introduction

Linear ordering games are the easiest games, and fortunately they also appear the most often. They can be classified according to whether they order elements spatially or sequentially.

- ❑ Spatial games
- ❑ Hybrid games
- ❑ Sequential games

We will study in turn each of the three types of linear ordering games. At the end of most lessons, you will find a warm-up drill. The drills are not remedial. In fact, some are quite challenging. They are designed to bring to the fore some of the subtle issues with which you will have to contend when solving a game, but in a more tractable form. Unlike LSAT games, the drills are presented in ascending order of difficulty. Following the warm-up drill is a "mentor" exercise. A mentor exercise is a full length game which offers hints, partial solutions, and insight in the right hand column. You should work through each mentor exercise slowly, giving yourself time to study the game carefully. Finally, several full length games are presented for you to solve on your own. If you intend to skip a game when you take the LSAT, give yourself 12 minutes to complete the exercise; otherwise you have only 9 minutes.

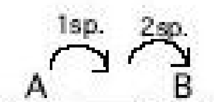
It is essential that you time yourself during this exercise, because time is the greatest obstacle when solving games. After studying games for a short while, students often develop an unwarranted self-confidence. Given sufficient time to solve them, many games appear simple. Be forewarned that you must not only master how to solve games—you must master how to solve them rapidly. It is one matter to solve a game with ample time in the quiet and comfort of your home; it is quite another to solve it in a room filled with 100 other people—all racing against the clock.

Spatial Ordering

THE LINE-UP

As the term "line-up" suggests, these games involve ordering elements in a line, from left to right or from front to back.

Before we begin, we need to study the vocabulary peculiar to these games.

Condition	Meaning
A sits next to B. A sits immediately next to B.	Both conditions mean that no one sits between A and B. Although A is to the left of B in the text, that order cannot be assumed on the line. The phrase "immediately next to" is redundant; however, that style is often used on the LSAT.
B sits immediately between A and C. B sits between and next to A and C. B sits directly between A and C.	All three conditions mean that no element separates B from A, nor B from C.
Two spaces separate A and B. A and B are two spaces apart.	Students often confuse these two conditions. They do **not** mean the same thing. "A and B are two spaces apart" means that *only one* spot separates them. 

It is essential that you master the similarities and distinctions described above. In addition to testing analytical skills, games measure your ability to notice subtle distinctions. Further, since game questions typically appear in blocks of six, misinterpreting even one condition can cost you six questions! Following is a common ordering game; one of this type—or a close variant—has occurred on every recent LSAT.

Line-up Game

There are five people—Bugsy, Nelson, Dutch, Clyde, and Gotti—in a police line-up standing in spaces numbered 1 through 6, from left to right. The following conditions apply:

There is always one empty space.
Clyde is not standing in space 1, 3, or 5.
Gotti is the third person from the left.
Bugsy is standing to the immediate left of Nelson.

1. Nelson CANNOT stand in which one of the following spaces?
   (A) 2
   (B) 3
   (C) 4
   (D) 5
   (E) 6

2. Which one of the following is a possible ordering of the 5 people from left to right?
   (A) Clyde, empty, Dutch, Gotti, Bugsy, Nelson
   (B) Bugsy, Clyde, Nelson, Gotti, Dutch, empty
   (C) Dutch, Bugsy, Gotti, Nelson, empty, Clyde
   (D) Dutch, Clyde, Gotti, empty, Nelson, Bugsy
   (E) Bugsy, Nelson, Gotti, Clyde, Dutch, empty

3. If space 6 is empty, which one of the following must be false?
   (A) Clyde stands in space 4.
   (B) Dutch stands in space 4.
   (C) Clyde is to the left of Nelson.
   (D) Clyde is to the right of Dutch.
   (E) Nelson stands in space 2.

4. Which one of the following spaces CANNOT be empty?
   (A) 1
   (B) 2
   (C) 3
   (D) 4
   (E) 5

5. If Clyde stands in space 6, Dutch must stand in space
   (A) 3 or 4
   (B) 5 or 6
   (C) 1 or 2
   (D) 2 or 3
   (E) 4 or 5

Following the strategies developed earlier, we abbreviate the names by using the first letter of the name and then symbolize the conditions. *"Clyde is not standing in space 1, 3, or 5"* is naturally symbolized as **C ≠ 1, 3, 5**. *"Gotti is the third person from the left"* is naturally symbolized as **G = 3rd**. Note: the fact that Gotti is third does <u>not</u> force him into space 3—he could stand in spaces 3 or 4. *"Bugsy is standing to the immediate left of Nelson"* is symbolized as **BN**.

Our diagram will consist of six dashed lines, numbered 1 through 6 from left to right. Summarizing this information gives the following schematic:

B N D C G
C ≠ 1, 3, 5
G = 3rd
BN

_1_ _2_ _3_ _4_ _5_ _6_

Now, we decide the most effective order for placing the elements on the diagram. Following the guidelines on page 40, we look for a condition that fixes the position of an element. There is none. Next, we look for a condition that limits the position of an element. The second condition, *"Gotti is the third person from the left,"* limits Gotti to spaces 3 and 4. This condition, as often happens with ordering games, generates two diagrams: one with the empty space to Gotti's left and one with the empty space to his right:

Diagram I ___ ___ ___ _G_ ___ ___

Diagram II ___ ___ _G_ ___ ___ ___

Next, we look for a condition that connects two or more people. The last condition, BN, connects B with N. However, at this stage we cannot place it on the diagram. Finally, we look for a condition that states where a person cannot be standing. The first condition states that Clyde cannot be standing in space 1, 3, or 5. Noting this on the diagram yields

BNDCG (D "wild")
C ≠ 1, 3, 5
G = 3rd
BN

Diagram I _~C_ ___ _~C_ _G_ _~C_ ___

1. **Nelson CANNOT stand in which one of the following spaces?**

   (A) 2
   (B) 3
   (C) 4
   (D) 5
   (E) 6

The method of solution to this problem is rather mechanical: We merely place Nelson in one of the spaces offered. Then check whether it is possible to place the other people in the line-up without violating any initial condition. If so, then we eliminate that answer-choice. Then place Nelson in another space offered, and repeat the process.

To that end, place Nelson in space 2 in Diagram II:

_~C_ _N_ _G_ ___ _~C_ ___

From the condition BN, we know that B must be in space 1:

_B_ _N_ _G_ ___ _~C_ ___

Now D could stand in space 4, and C could stand in space 6—both without violating any initial condition:

_B_ _N_ _G_ _D_ _X_ _C_ (Where X means "empty.")

This diagram is consistent with the initial conditions. So N *could* stand in space 2. This eliminates choice (A).

Next, place Nelson in space 4. Then Diagram I is violated since G is already in space 4, and Diagram II is also violated since there is no room for the condition BN:

B?
___ ___ _G_ _N_ ___ ___

The answer is (C).

As you read the remaining solutions, note the determining power of the condition BN.

2. **Which one of the following is a possible ordering of the 5 people from left to right?**

   (A) Clyde, empty, Dutch, Gotti, Bugsy, Nelson
   (B) Bugsy, Clyde, Nelson, Gotti, Dutch, empty
   (C) Dutch, Bugsy, Gotti, Nelson, empty, Clyde
   (D) Dutch, Clyde, Gotti, empty, Nelson, Bugsy
   (E) Bugsy, Nelson, Gotti, Clyde, Dutch, empty

This problem is best solved by the method of elimination. To apply this method take a condition; test it against each answer-choice, eliminating any that violate it. Then take another condition; test it against the remaining answer-choices, eliminating any that violate it. Continue until only one answer-choice remains. Many students apply every condition to the first answer-choice, then every condition to the second answer-choice, and so on. This should be avoided since it's inefficient; however, sometimes there is no other option. Because this question type is relatively easy, it often is the first or second question asked.

The first condition contradicts choice (A) since Clyde cannot be first. It does not contradict the other choices. So eliminate (A) only. The second condition contradicts choice (B) since Gotti must be 3rd. It does not contradict the remaining choices. So eliminate (B) only. The third condition contradicts choices (C) and (D) since in neither choice is Bugsy to the immediate left of Nelson. It does not contradict the remaining choice. So eliminate (C) and (D) only. Thus, by process of elimination, we have learned the answer is (E).

To answer this question, we had to test all the conditions; often, however, we will find the answer before testing the last condition.

3. **If space 6 is empty, which one of the following must be false?**

   (A) **Clyde stands in space 4.**
   (B) **Dutch stands in space 4.**
   (C) **Clyde is to the left of Nelson.**
   (D) **Clyde is to the right of Dutch.**
   (E) **Nelson stands in space 2.**

The structure of this question is awkward—the correct answer will always make a false statement! The question is more tractable when rephrased as "All of the following could be true EXCEPT." Now, merely test each answer-choice against the initial conditions until you find the choice that violates one or more conditions.

Adding the supplementary condition, "space 6 is empty," to the original diagrams gives

Diagram I _~C_ ___ _~C_ _G_ _~C_ _X_

Diagram II _~C_ ___ _G_ ___ _~C_ _X_

In Diagram I, the only space open for C is space 2:

___ _C_ ___ _G_ ___ _X_

Clearly, this diagram does not leave room for the condition BN. So we eliminate Diagram I.

Next, test each answer-choice against Diagram II, starting with (A). Place Clyde in space 4 as follows:

___ ___ _G_ _C_ ___ _X_

Now the condition **BN** forces B and N into spaces 1 and 2, respectively, which in turn forces D into space 5. So our uniquely determined diagram is

_B_ _N_ _G_ _C_ _D_ _X_

This diagram does not violate any initial condition. Hence Clyde could stand in space 4. So eliminate choice (A).

Next, turning to choice (B), place Dutch in space 4:

_~C_ ___ _G_ _D_ _~C_ _X_

The condition **BN** forces B and N into spaces 1 and 2, respectively:

_B_ _N_ _G_ _D_ ___ _X_

But this diagram forces C into space 5, violating the condition $\mathbf{C \neq 1, 3, 5}$. Hence D cannot stand in space 4, and the answer is (B).

The above method of analysis is what mathematicians and logicians call an *"indirect proof"*. To apply the method, assume an answer-choice is possible. Then check whether that leads to the desired result. If not, eliminate it. Then choose another answer-choice and repeat the process. Continue until either the choice with the desired result is found or until only one remains. This method of elimination is not as efficient as the previous one, because typically every condition must be tested against one answer-choice before considering the next answer-choice. Sometimes, however, this is the only method available.

4. **Which one of the following spaces CANNOT be empty?**

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Assume that space 1 is empty. Then in Diagram I, the condition **BN** can be placed in spaces 2 and 3, D can be placed in space 5, and C can be placed in space 6—all without violating any initial condition:

_X_ _B_ _N_ _G_ _D_ _C_

Thus space 1 *could be* empty. This eliminates (A).

Next, assume that space 2 is empty. In Diagram I, this forces **BN** into spaces 5 and 6:

___ _X_ ___ _G_ _B_ _N_

However, this diagram does not leave room for C (recall $\mathbf{C \neq 1, 3, 5}$). Diagram I is thus impossible when space 2 is empty. Turning to Diagram II, we see immediately that space 2 cannot be empty, for this would make G second, violating the condition G = 3rd. Hence Diagram II is also impossible when space 2 is empty. Thus space 2 *cannot* be empty, and the answer is (B).

5. If Clyde stands in space 6, Dutch must stand in space

(A) 3 or 4
(B) 5 or 6
(C) 1 or 2
(D) 2 or 3
(E) 4 or 5

Adding the new condition, C = 6th, to the original diagrams yields

Diagram I ___ ___ ___ _G_ ___ _C_

Diagram II ___ ___ _G_ ___ ___ _C_

In both diagrams, BN must come before G, and D must come after G, to insure that G is 3rd. This forces D into space 5 in Diagram I and into either space 4 or 5 in Diagram II. In either position, D does not violate any initial condition. Hence the answer is (E).

Points to Remember

1. To apply the method of elimination, take a condition. Then test it against each answer-choice, eliminating any that violate it. Then take another condition; test it against the remaining answer-choices, eliminating any that violate it, and so on, until only one answer-choice remains.

   Do not, as many students do, apply all the conditions to the first answer-choice; then, all the conditions to the second answer-choice, and so on. Such a procedure is inefficient.

2. To apply the method of "indirect proof," assume that a particular answer-choice is possible. Then check whether that leads to the desired result. If not, eliminate it. Then assume that another answer-choice is possible and repeat the process. Continue until the answer-choice with the desired result is found or until only one remains.

   This method of elimination is not as efficient as Point #1, because usually every condition must be tested against one answer-choice before testing the next answer-choice. However, sometimes this is the only method available.

WARM-UP DRILLS

Answers and solutions begin on page 80.

Drill A

Four people—A, B, C, and D—are standing in a line. For each of the following conditions count the number of orderings possible.

1. A and D are at either end of the line.
2. B is immediately between A and C.
3. B is between A and C.
4. There are no conditions on the people.

Drill B

Four books are arranged on a book shelf. The positions of the books are numbered 1 through 4, from left to right. For each of the following pairs of conditions, count the number of orderings possible.

1. A is left of B.
   B is between C and D.
2. A is first.
   If C is left of B, then C is next to A.
3. C sits to the right of B.
   If B sits next to C, then B sits next to D.

MENTOR EXERCISE

<u>Directions:</u> Each group of questions is based on a set of conditions. In answering some of the questions, it may be useful to draw a rough diagram. Choose the response that most accurately and completely answers each question. Hints, insights, partial solutions, and answers are provided in the right-hand column.

<u>Questions 1-6</u>

On Auto Row there are seven dealerships: Audi, Chrysler, Ford, Hyundai, Mazda, Toyota, Volkswagen. All the dealerships are on the same side of the street, which runs from west to east.

Ford is not next to Mazda.
Audi is the fourth dealership from the west end of the street.
Ford is next to Audi.
Toyota is west of both Audi and Ford but east of Chrysler.

Begin by symbolizing the conditions. *"Ford is not next to Mazda"* is naturally symbolized as ~(FM). *"Toyota is west of both Audi and Ford but east of Chrysler"* can be symbolized as C—>T—>AF, where the arrow points from west to east. The remaining conditions can be symbolized in like manner, which gives the following schematic:

ACFHMTV (V, H "wild")
~(FM)
A = 4th
FA
C—>T—>AF

The diagram will consist of seven dashed lines. To place the elements on the diagram, follow the guidelines on page 40. First, look for a condition that fixes the position of an element. It is A = 4th. This gives

_1_	_2_	_3_	_4_	_5_	_6_	_7_
			A			

Next, look for a condition that limits the position of a dealership. There is none. Now, look for a condition connecting two or more dealerships. The third condition, *"Ford is next to Audi,"* forces the Ford dealership into either space 3 or 5. This generates two alternate diagrams:

Diagram I

_1_ _2_ _3_ _4_ _5_ _6_ _7_
F A

Diagram II

_1_ _2_ _3_ _4_ _5_ _6_ _7_
A F

Finally, look for a condition that states where a dealership cannot be. There is none. No significant properties can be derived from the initial conditions, so we turn to the questions.

1. Which one of the following dealerships CANNOT be next to Chrysler?

(A) Toyota
(B) Ford
(C) Volkswagen
(D) Hyundai
(E) Mazda

1. Hint: Use the condition

C—>T—>AF.

The answer is (B).

2. If Ford is east of Audi, then Hyundai CANNOT be next to both

(A) Toyota and Ford
(B) Chrysler and Toyota
(C) Ford and Mazda
(D) Ford and Volkswagen
(E) Toyota and Audi

2. Since Ford is east of Audi, use Diagram II and again the condition C—>T—>AF.

The answer is (A).

3. If Volkswagen is west of Audi, then which one of the following must be false?

(A) Ford is east of Audi.
(B) Volkswagen is west of Toyota.
(C) Volkswagen is east of Toyota.
(D) Hyundai is west of Mazda.
(E) Hyundai is east of Mazda.

3. If Volkswagen is west of Audi, then, from the condition **C—>T—>AF**, we know that Ford must be east of Audi—otherwise Ford, Chrysler, Toyota, and Volkswagen would all be west of Audi, which would violate the condition **A = 4th**. Finally, use the condition **~(FM)**.

The answer is (E).

4. Which one of the following is a possible arrangement of the dealerships from west to east?

(A) C, F, T, A, H, M, V
(B) C, T, F, H, A, M, V
(C) V, C, T, A, F, M, H
(D) C, V, F, A, H, T, M
(E) H, C, T, A, F, V, M

4. This is a straightforward elimination problem: Take a condition. Test it against each answer-choice, eliminating any that violate it. Then take another condition; test it against the remaining answer-choices, eliminating any that violate it. Continue until only one answer-choice remains.

The answer is (E).

5. If Hyundai is west of Ford, which one of the following pairs of dealerships must be next to each other?

(A) Chrysler and Hyundai
(B) Volkswagen and Mazda
(C) Ford and Mazda
(D) Toyota and Audi
(E) Hyundai and Mazda

5. **Hint:** If Hyundai is west of Ford, then, from the condition **C—>T—>AF**, Ford must be east of Audi—otherwise Chrysler, Toyota, Hyundai, and Ford would all be west of Audi, which would violate the condition **A = 4th.**

The answer is (B).

6. If the Volkswagen dealership is on the east end of the street, then which one of the following must be false?

(A) Chrysler is second from the west end of the street.
(B) Ford is east of Audi.
(C) Hyundai is on the west end of the street.
(D) Ford is west of Audi.
(E) Hyundai is fifth from the west end of the street.

6. This question is hard because it does not give us much information to work with. Volkswagen was a "wild card". That is, its position on the street was independent of the other dealerships—except for Audi. So knowing where the Volkswagen dealership is located will probably tell us little, if anything, about where the other dealerships are located. Furthermore, the question leads us astray by asking *"Which one of the following must be false?"* This prompts us to look directly for the false answer. In problems of this type, however, it is often better to reword the question as *"All of the following could be true except."* Then look for and eliminate the true answer-choices.

We'll use an indirect proof to solve this problem. That is, for each answer-choice, we attempt to construct a possible ordering of the dealerships along the street. The one for which this is not possible will be the answer. Clearly you should save questions like this for last, or skip them all together.

Begin with choice (A). In Diagram II, place Chrysler second from the west end of the street:

_1_	_2_	_3_	_4_	_5_	_6_	_7_
	C		A	F		V

Next, the condition **C—>T—>AF** forces T into space 3:

_1_	_2_	_3_	_4_	_5_	_6_	_7_
	C	T	A	F		V

We can place M in space 1 without violating the initial conditions:

_1_	_2_	_3_	_4_	_5_	_6_	_7_
M	C	T	A	F		V

Finally, this forces the "wild card", H, into space 6:

_1_	_2_	_3_	_4_	_5_	_6_	_7_
M	C	T	A	F	H	V

This diagram does not violate any initial condition, so (A) <u>could be</u> true. This eliminates (A). Now apply this method to the remaining answer-choices until you find the one that violates one or more of the conditions or until you have eliminated four of the five choices.

The answer is (C).

Exercise

Directions: The following group of questions is based on a set of conditions. Choose the response that most accurately and completely answers each question. Answers and solutions begin on page 68.

Questions 1-6

A shelf contains six books on six different subjects: Art, Chemistry, Math, History, Physics, and Zoology. The positions of the books are numbered 1 through 6, from left to right.

The zoology book is not next to the math book.
The math book and the history book are exactly two spaces apart.
At most one other book separates the art book from the chemistry book.
The physics book cannot be on either end of the shelf.

1. If the math book is second from the left, then in which one of the following positions could the art book be located?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

2. The books located in positions 1, 2, and 3, respectively, could be

I. chemistry, math, and art
II. zoology, art, and math
III. art, chemistry, and history

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I, II, and III

3. What is the highest numbered position in which the history book can be located, if the zoology and math books are both to the right of it?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

4. Which one of the following is a possible arrangement of the six books on the shelf, from left to right?

(A) art, chemistry, physics, history, zoology, math
(B) history, art, math, chemistry, zoology, physics
(C) zoology, history, art, math, physics, chemistry
(D) zoology, chemistry, history, physics, math, art
(E) art, chemistry, math, physics, history, zoology

5. If the physics book is in position 3, then which one of the following must be true?

(A) The chemistry book is in position 6.
(B) The zoology book is in position 1.
(C) The art book is in position 1.
(D) The math book is in position 6.
(E) The zoology book is in position 2.

6. If the history and the math books are both to the left of the chemistry book, then which one of the following must be false?

(A) The art book is in position 3.
(B) The zoology book is in position 4.
(C) The history book is in position 2.
(D) The art book is in position 5.
(E) The chemistry book is in position 6.

Questions 7–10

Seven disks—G, H, L, O, P, S, U—are being inserted in a CD player. The order in which the disks are played is subject to the following restrictions:

L must be played before both O and U.
Exactly two disks must be played between G and P one of which must be L.
H cannot be played first.

7. If G is played third, which one of the following must be played second?

(A) G
(B) H
(C) L
(D) O
(E) N

8. If L and O are played consecutively, which one of the following cannot be true?

(A) S is played second
(B) G is played second
(C) L is played third
(D) O is played forth
(E) H is played sixth

9. What is the maximum number of disks that can separate S from U?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

10. If S is played second, which one of the following cannot be true?

(A) G is played sixth
(B) L is played third
(C) U is played seventh
(D) U is played fifth
(E) H is played fifth

The following games appeared on recent LSATs.

Questions 11–12

John receives one grade for each of the following six courses: economics, geology, history, Italian, physics, and Russian. From highest to lowest, the possible grades are A, B, C, D, and E. E is the only failing grade. Two letter grades are consecutive if and only if they are adjacent in the alphabet.

John's grades in geology and physics are consecutive.
His grades in Italian and Russian are consecutive.
He receives a higher grade in economics than in history.
He receives a higher grade in geology than in physics.

11. If John receives the same grade in economics and Italian, and if he fails Russian, which one of the following must be true?

(A) John's geology grade is a B.
(B) John's history grade is a D.
(C) John's history grade is an E.
(D) John's physics grade is a B.
(E) John's physics grade is a C.

12. If John passes all his courses and receives a higher grade in geology than in either language, which one of the following must be true?

(A) He receives exactly one A.
(B) He receives exactly one B.
(C) He receives exactly two Bs.
(D) He receives at least one B and at least one C.
(E) He receives at least one C and at least one D.

Questions 13–17

Seven children are to be seated in seven chairs arranged in a row that runs from west to east. All seven children will face north. Four of the children are boys: Frank, Harry, Ivan, and Joel. Three are girls: Ruby, Sylvia, and Thelma. The children are assigned to chairs according to the following conditions:

Exactly one child sits in each chair.
No boy sits next to another boy.
Ivan sits next to and east of the fourth child in the row.
Sylvia sits east of Ivan.
Frank sits next to Ruby.

13. What is the maximum possible number of different pairs of chairs in which Frank and Ruby could sit?
    (A) one
    (B) two
    (C) three
    (D) four
    (E) five

14. Which one of the following statements must be false?
    (A) Both Harry and Joel sit east of Frank.
    (B) Both Harry and Ruby sit east of Frank.
    (C) Both Harry and Joel sit west of Frank.
    (D) Both Harry and Ruby sit west of Frank.
    (E) Both Joel and Ruby sit east of Frank.

15. If Thelma sits next to Ivan, and if Frank sits next to Thelma, which one of the following statements could be false?
    (A) Both Frank and Ivan sit east of Ruby.
    (B) Both Frank and Ruby sit west of Thelma.
    (C) Both Frank and Sylvia sit east of Ruby.
    (D) Both Frank and Thelma sit west of Sylvia.
    (E) Both Frank and Ruby sit west of Joel.

16. If Frank does not sit next to any child who sits next to Ivan, which one of the following statements could be true?
    (A) Harry sits west of Frank.
    (B) Joel sits west of Ivan.
    (C) Ruby sits west of Frank.
    (D) Thelma sits west of Frank.
    (E) Thelma sits west of Ruby.

17. If Frank sits east of Ruby, which one of the following pairs of children CANNOT sit next to each other?
    (A) Frank and Thelma
    (B) Harry and Ruby
    (C) Harry and Sylvia
    (D) Ivan and Ruby
    (E) Joel and Ruby

Answers and Solutions to Exercise

Questions 1–6

This is a spatial ordering problem of above average difficulty. Begin by turning the conditions into symbols. The condition *"The zoology book is not next to the math book"* can be symbolized as ~(ZM). Note: although Z is written to the left of M in this symbol, that cannot be assumed on the diagram. We could just as easily have written ~(MZ). The flip-flop symbol could be used to remind us that the order is not fixed, but it would tend to clutter up the conditions. Just remember that the order in all the conditions of this game can be reversed.

Next, the condition *"The math book and the history book are exactly two spaces apart"* can be symbolized as M □ H. Don't make the mistake of symbolizing this condition as M □ □ H. The statement "A and B are two spaces apart" means that only one spot separates them. The symbol A □ □ B, on the other hand, reads "A and B are three spaces apart."

Next, the condition *"At most one other book separates the art book from the chemistry book"* can be symbolized as $\underset{\text{max 1}}{A \;\square\; C}$.

Finally, the condition *"The physics book cannot be on either end of the shelf"* yields **P ≠ 1st, last**.

Our diagram will consist of six compartments—one for each book:

ACMHPZ
~(ZM)
M □ H
$\underset{\text{max 1}}{A \;\square\; C}$
P ≠ 1st, last

~P					~P

There are no independent elements, no readily derived conditions, and no elements that can be placed on the diagram, so we turn to the questions.

1. **If the math book is second from the left, then in which one of the following positions could the art book be located?**

   (A) 2
   (B) 3
   (C) 4
   (D) 5
   (E) 6

This question asks *"Which one of the following could be true?"* This type of question is usually harder to answer than those that ask *"Which one of the following must be true?"* In general, the more information you have, the more likely it is that the order will be fully determined. In such cases, we only need to find one ordering.

On the other hand, the less information you have the less likely it will be that only one order is possible. In these cases there may be many orderings possible, but only one will be listed as an answer-choice. You may spend considerable time working out a possible order, only to be disappointed because it is not listed as an answer-choice. So if you are pressed for time, attempt the remaining *must*-questions before the *could*-questions. You may even want to preview the questions before you begin and then answer all the *must*-questions before tackling the *could*-questions.

I don't use this method myself, though. I find that I tend to lose concentration as I flash from one question to the next, trying to decide which ones are easier. Previewing can also waste precious time. Nonetheless, it may work for you, so experiment with it.

To begin, place the supplementary condition, *"the math book is second from the left,"* on the diagram as follows:

	M				
~P					~P

Next, the condition $M \square H$ forces H into position 4:

	M		H		
~P					~P

Now, the condition ~(ZM) forces Z into position 5 or 6. At this point, many students try to juggle the possible positions for Z in their heads. Unless you have a very strong memory, don't do it! Instead, write down a separate diagram for each of the two possible positions:

Diagram I

	M		H	Z	
~P					~P

Diagram II

	M		H		Z
~P					

In Diagram I, P must be in position 3, as it cannot be at either end of the shelf:

Diagram I

	M	P	H	Z	

This, however, leaves no room for the condition $A \underset{\text{max } 1}{\square} C$. Hence Diagram I is invalid. In Diagram II, P could be placed in position 5:

Diagram II

	M		H	P	Z

Then C and A could be placed in positions 1 and 3, respectively:

Diagram II

C	M	A	H	P	Z

This diagram satisfies all the initial conditions, so it represents a possible ordering. The answer is (B).

2. The books located in positions 1, 2, and 3, respectively, could be

I. chemistry, math, and art
II. zoology, art, and math
III. art, chemistry, and history

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I, II, and III

This is a triple-multiple-choice question. Many students skip these questions, assuming that the LSAT writers have conspired against them. However, a modified elimination method works quite well on these questions.

This question illustrates why you should not erase your previous diagrams. Begin with Statement I. From the final diagram in Question 1, we see that the chemistry, math, and art books could be in positions 1, 2, and 3, respectively. Applying the modified elimination method, we eliminate answer-choices (B) and (C), since they don't contain Statement I. At this point, even if we can't solve this problem, we have significantly increased our chances of correctly guessing the answer.

Turning to Statement II, we try to construct a valid ordering for it. To this end, place Z, A, and M on the diagram as follows:

Z	A	M			
					~P

Next, the condition M □ H forces H into position 5:

Z	A	M		H	
					~P

Now, since P cannot be last, it must be in position 4:

Z	A	M	P	H	

This diagram does not leave any room for the condition $A \underset{\text{max } 1}{\square} C$. Hence Statement II is not possible. This eliminates (D) and (E), since both contain Statement II. Therefore, by process of elimination, we have learned that the answer is (A), without having to check Statement III.

3. What is the highest numbered position in which the history book can be located, if the zoology and math books are both to the right of it?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

In problems such as this one, start with the largest number and work your way up the list. The first choice for which you are able to construct a valid order is the answer.

We can quickly dismiss (E) since there are only six spaces and two books are to the right of H.

We can also quickly dismiss (D). If H were in position 4, then the zoology and math books would have to be in spaces 5 and 6. This, however, would violate the condition ~(ZM).

Next, if H is in position 3, then—from the condition M □ H, and the fact that M is to the right of H—we know that M must be in position 5:

		H		M	
~P					~P

This diagram leaves only positions 4 and 6 for Z. In either case, however, Z would be next to M, which again would violate the condition ~(ZM). This dismisses (C).

Next, if H is in position 2, then M must be in position 4:

	H		M		
~P					~P

In this case, it is possible to place Z in space 6 as follows:

	H		M		Z
~P					

Then, place P in position 5:

	H		M	P	Z

Finally, place A and C in positions 1 and 3, respectively:

A	H	C	M	P	Z

This diagram does not violate any initial condition. Hence the largest numbered position that H could occupy is 2. The answer is (B).

4. Which one of the following is a possible arrangement of the six books on the shelf, from left to right?

(A) art, chemistry, physics, history, zoology, math

(B) history, art, math, chemistry, zoology, physics

(C) zoology, history, art, math, physics, chemistry

(D) zoology, chemistry, history, physics, math, art

(E) art, chemistry, math, physics, history, zoology

Never skip problems like this one; they are rarely difficult. We'll use elimination. (A) is not a possible arrangement since it has M next to Z. (B) is not a possible arrangement since it has P last. Neither (C) nor (D) is a possible arrangement since in each case more than one book separates A and C. Hence, by process of elimination, we have learned that the answer is (E).

5. If the physics book is in position 3, then which one of the following must be true?

(A) The chemistry book is in position 6.

(B) The zoology book is in position 1.

(C) The art book is in position 1.

(D) The math book is in position 6.

(E) The zoology book is in position 2.

To start, place P in position 3 on the diagram:

		P			

This yields two positions for the condition M $\square$ H—2 and 4, or 4 and 6. This generates two diagrams:

Diagram I

	M	P	H		

Diagram II

		P	M		H

(Note: M and H can exchange places in each diagram.)

Now in Diagram II, the condition $A \underset{\text{max } 1}{\square} C$ must be in positions 1 and 2:

Diagram II

A	C	P	M		H

This forces Z into position 5, which violates the condition ~(ZM). Hence Diagram II is invalid. Turning to Diagram I, we see that the condition $A \underset{\text{max } 1}{\square} C$ forces A and C into positions 5 and 6, though not necessarily in that order:

Diagram I

	M	P	H	A	C

This forces Z into position 1, which violates the condition ~(ZM). Don't forget, however, that we can switch M and H:

Diagram I

Z	H	P	M	A	C

This diagram satisfies all the initial conditions plus the supplementary condition. It also displays Z in its only possible position. Hence the answer is (B).

Watch out for choice (A). Although C can be in position 6, it need not be: A and C can be flip flopped in the final diagram.

6. **If the history and the math books are both to the left of the chemistry book, then which one of the following must be false?**

   (A) **The art book is in position 3.**
   (B) **The zoology book is in position 4.**
   (C) **The history book is in position 2.**
   (D) **The art book is in position 5.**
   (E) **The chemistry book is in position 6.**

In this problem, we have no choice but to apply an indirect proof. Start with (A). If the art book is in position 3, then, from the condition $A \underset{\max 1}{\square} C$, there are four positions in which C can be placed—1, 2, 4, 5. We consider each in turn. Position 1 can be quickly ruled out. If C were in position 1, then clearly neither H nor M could be to its left. Similarly position 2 can be ruled out. Next, place C in position 4 on the diagram:

		A	C		
~P					~P

Clearly in this diagram, there is no room to place the condition M □ H. Hence C cannot be in position 4. Next place C in position 5:

		A		C	
~P					~P

From the new condition, *"both H and M are left of C,"* and the condition M □ H, we see that M and H must occupy positions 2 and 4, though not necessarily in that order:

	M	A	H	C	
~P					~P

This diagram, however, forces P to be either first or last, which violates the condition **P ≠ 1st, last.** This shows C cannot be in position 5. Hence the art book cannot be in position 3, and the answer is (A).

Questions 7–10

The conditions can be symbolized as follows:

L—>O & U
G __ __ P
L between G & P
H ≠ first

_1_ _2_ _3_ _4_ _5_ _6_ _7_

Note, the flip-flop symbol will not be used in the symbol statement **G __ __ P**; just remember that G and P can be interchanged.

7. If G is played third, which one of the following disks must be played second?

(A) G
(B) H
(C) L
(D) O
(E) N

Since G is played third, the condition **G __ __ P** forces P into space 6:

_1_	_2_	_3_	_4_	_5_	_6_	_7_
		G			P	

The condition **L between G & P** forces L into spaces 4 or 5. But L cannot be in space 5 since that would leave no room for the condition **L—>O & U**. Thus, L must be in space 4:

_1_	_2_	_3_	_4_	_5_	_6_	_7_
		G	L	O/U	P	O/U

Finally, since **H ≠ first**, H must be played second. The answer is (B).

8. If L and O are played consecutively, which one of the following cannot be true?

(A) S is played second
(B) G is played second
(C) L is played third
(D) O is played forth
(E) H is played sixth

Suppose S is played second.

_1_	_2_	_3_	_4_	_5_	_6_	_7_
	S					

Since "L and O are played consecutively" and L is between G & P, the disks G, L, O, and P must be played consecutively. Now, G, L, O, and P cannot be placed in spaces 4, 5, 6, and 7 since that would violate the condition **L—>O & U**. So G, L, O, and P must be place in spaces 3, 4, 5, and 6:

_1_	_2_	_3_	_4_	_5_	_6_	_7_
	S	G	L	O	P	U

(Note, P and G can be interchanged in the diagram.) However, this diagram leaves no room for H (H ≠ first). Hence, S cannot be played second and the answer is (A).

9. What is the maximum number of disks that can separate S from U?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

The following diagram satisfies all the conditions and has S in space 1 and U in space 7:

_1_	_2_	_3_	_4_	_5_	_6_	_7_
S	G	L	H	P	O	U

Hence, a maximum of 5 disks can separate S from the U. The answer is (E).

10. If S is played second, which one of the following cannot be true?

(A) G is played sixth
(B) L is played third
(C) U is played seventh
(D) U is played fifth
(E) H is played fifth

Suppose G is played sixth:

_1_	_2_	_3_	_4_	_5_	_6_	_7_
	S				G	

Then the condition **G __ __ P** forces P into space 3:

_1_	_2_	_3_	_4_	_5_	_6_	_7_
	S	P			G	

The conditions **L—>O & U** and **L between G & P** yield

_1_	_2_	_3_	_4_	_5_	_6_	_7_
	S	P	L	O/U	G	O/U

However, this diagram does not leave any room for H (H ≠ first). The answer is (A).

Questions 11–12

This is a linear ordering game of medium difficulty. The condition *"John's grades in geology and physics are consecutive"* can be symbolized as $\widehat{\mathbf{G}\ \mathbf{P}}$, where the flip-flop symbol reminds us that the positions of G and P can be interchanged. The condition *"He receives a higher grade in economics than in history"* can be symbolized as e > H. Note, a lower case "e" is used to distinguish it from the letter grade E. The last condition *"He receives a higher grade in geology than in physics"* allows us to drop the flip-flop symbol in the condition $\widehat{\mathbf{G}\ \mathbf{P}}$. The remaining condition is symbolized in like manner, which yields the following schematic:

$$\mathbf{GP}$$

$$\widehat{\mathbf{I}\ \mathbf{R}}$$

$$\mathbf{e > H}$$

11. If John receives the same grade in economics and Italian, and if he fails Russian, which one of the following must be true?

    (A) John's geology grade is a B.
    (B) John's history grade is a D.
    (C) John's history grade is an E.
    (D) John's physics grade is a B.
    (E) John's physics grade is a C.

Since John received the same grade in economics and Italian, the condition $\widehat{\mathbf{I}\ \mathbf{R}}$ becomes $\widehat{\mathbf{e}\ \mathbf{R}}$. Since he failed Russian, this condition shows that he received a D in economics. The condition e > H forces him to have received an E in history. The answer is (C).

12. If John passes all his courses and receives a higher grade in geology than in either language, which one of the following must be true?

    (A) He receives exactly one A.
    (B) He receives exactly one B.
    (C) He receives exactly two Bs.
    (D) He receives at least one B and at least one C.
    (E) He receives at least one C and at least one D.

Since John receives a higher grade in geology than in either Italian or Russian, the condition $\widehat{\mathbf{I}\ \mathbf{R}}$ becomes $\mathbf{G} > \widehat{\mathbf{I}\ \mathbf{R}}$. This shows that his geology grade is either an A or a B (remember, he passes all his courses). If his geology grade is A, then from the condition **GP** his physics grade is B. This leaves two places for the condition $\widehat{\mathbf{I}\ \mathbf{R}}$:

A	B	C	D
G	P		
	I	R	

A	B	C	D
G	P		
		I	R

In each diagram he receives at least one B and at least one C. Next, if his geology grade is B, then we get the following diagram:

A	B	C	D
	G	P	
		I	R

This diagram also has him receiving at least one B and at least one C. Thus, the answer is (D).

Questions 13–17

The condition *"No boy sits next to another boy"* means that the arrangement will be boy/girl/boy/girl . . . , which is naturally symbolized as **Boy/Girl**. The condition *"Ivan sits next to and east of the fourth child in the row"* simply means that Ivan is 5th, which can be symbolized as **I = 5th**. The condition *"Sylvia sits east of Ivan"* forces Sylvia into space 6 or 7. However, since the arrangement is **Boy/Girl**, Sylvia must be in space 6, **S = 6th**. The final condition, *"Frank sits next Ruby,"* is naturally symbolized as **FR**, where F and R can be flip-flopped.

Boys FHIJ   Girls RST

Boy/Girl
I = 5th
S = 6th
FR

(West) _1_ _2_ _3_ _4_ _5_ _6_ _7_ (East)
I S

13. **What is the maximum possible number of different pairs of chairs in which Frank and Ruby could sit?**

    (A) one
    (B) two
    (C) three
    (D) four
    (E) five

(A) No. See answer-choice (C).
(B) No. See answer-choice (C).

(C) Yes. As the following diagrams illustrate, Frank and Ruby can sit in chairs 1&2, or 2&3, or 3&4:

1	2	3	4	5	6	7
F	**R**	**H**	**T**	**I**	**S**	**J**

15. If Thelma sits next to Ivan, and Frank sits next to Thelma, which one of the following statements could be false?

(A) Both Frank and Ivan sit east of Ruby.
(B) Both Frank and Ruby sit west of Thelma.
(C) Both Frank and Sylvia sit east of Ruby.
(D) Both Frank and Thelma sit west of Sylvia.
(E) Both Frank and Ruby sit west of Joel.

Placing Thelma next to Ivan and Frank next to Thelma yields

_1_	_2_	_3_	_4_	_5_	_6_	_7_
		F	**T**	**I**	**S**	

Since Ruby sits next to Frank, Ruby must sit in space 2:

_1_	_2_	_3_	_4_	_5_	_6_	_7_
	R	**F**	**T**	**I**	**S**	

(A) No. See above diagram.
(B) No. See above diagram.
(C) No. See above diagram.
(D) No. See above diagram.
(E) Yes. Suppose in the above diagram that Joel sits in space 1. Then Harry would sit in space 7:

_1_	_2_	_3_	_4_	_5_	_6_	_7_
J	**R**	**F**	**T**	**I**	**S**	**H**

This diagram satisfies all the conditions and has both Frank and Ruby seated <u>east</u> of Joel.

16. If Frank does not sit next to any child who sits next to Ivan, which one of the following statements could be true?

(A) Harry sits west of Frank.
(B) Joel sits west of Ivan.
(C) Ruby sits west of Frank.
(D) Thelma sits west of Frank.
(E) Thelma sits west of Ruby.

The condition *"Frank does not sit next to any child who sits next to Ivan"* means that Frank does not sit in space 3 or 7. Since the order is **Boy/Girl**, Frank must be in space 1. This in turn forces Ruby into space 2, **FR**:

Diagram I

_1_	_2_	_3_	_4_	_5_	_6_	_7_
F	**R**			**I**	**S**	

(A) No. See Diagram I.

(B) Yes. Suppose Joel sits in space 3. Then since the order is **Boy/Girl**, Harry and Thelma would be forced into spaces 7 and 4, respectively:

_1_	_2_	_3_	_4_	_5_	_6_	_7_
F	**R**	**J**	**T**	**I**	**S**	**H**

This diagram satisfies all the conditions and has Joel seated west of Ivan.
(C) No. See Diagram I.
(D) No. See Diagram I.
(E) No. See Diagram I.

17. **If Frank sits east of Ruby, which one of the following pairs of children CANNOT sit next to each other?**

    (A) Frank and Thelma
    (B) Harry and Ruby
    (C) Harry and Sylvia
    (D) Ivan and Ruby
    (E) Joel and Ruby

(A) No. The following diagram satisfies all the conditions and has Frank seated next to Thelma:

_1_	_2_	_3_	_4_	_5_	_6_	_7_
J	**R**	**F**	**T**	**I**	**S**	**H**

(B) No. The following diagram satisfies all the conditions and has Harry seated next to Ruby:

_1_	_2_	_3_	_4_	_5_	_6_	_7_
H	**R**	**F**	**T**	**I**	**S**	**J**

(C) No. See diagram to answer-choice (A).
(D) Yes. Since Frank now sits east of Ruby, the condition **FR** becomes **RF**, where R and F cannot be flip-flopped. From our previous work, we know that Ruby and Frank must sit west of Ivan. Hence, Frank will always be seated between Ruby and Ivan. Thus, Ruby and Ivan cannot sit next to each other.
(E) No. See diagram to answer-choice (A).

Answers and Solutions to Warm-up Drills

Drill A

1. **4 orderings.**

   Two diagrams are possible, one with A first and D fourth, and one with D first and A fourth:

   _A_ _C_ _B_ _D_

   _D_ _C_ _B_ _A_

   Interchanging the positions of C and B in each diagram above results in two more orderings. So there are 4 orderings possible.

2. **4 orderings.**

   Viewed as a group, ABC can be placed either at the beginning or at the end of the line:

   _A_ _B_ _C_ _D_

   _D_ _C_ _B_ _A_

   (The circle around D indicates that it was forced into that position by the conditions.) Two additional diagrams result from interchanging A and C in each diagram. So there are 4 possible orderings.

3. **8 orderings.**

   Now B is merely between A and C, not necessarily immediately between them. Thus D may come between A and B or between B and C:

   _A_ _D_ _B_ _C_

   _A_ _B_ _D_ _C_

   Interchanging A and C in each diagram gives 2 more orderings for a total of 4. Combining this with the result in drill 2 gives a total of 8 orderings.

4. **24 orderings.**

Drill B

1. **4 orderings.**

Since B is between C and D, it must be in position 2 or 3:

___ _B_ ___ ___
___ ___ _B_ ___

Next, adding the condition "A is left of B" yields the following diagrams:

Diagram I _A_ _B_ ___ ___
Diagram II _A_ ___ _B_ ___
Diagram III ___ _A_ _B_ ___

Eliminate Diagram I since it clearly violates the condition CBD. Next, placing C and D in the remaining diagrams gives the following valid diagrams:

_A_ _C_ _B_ _D_
_C_ _A_ _B_ _D_

Finally, interchanging C and D in each diagram gives two additional diagrams. So there is a total of 4 possible orderings.

2. **5 orderings.**

With A first, there are three possible positions for C:

Diagram I _A_ _C_ ___ ___
Diagram II _A_ ___ _C_ ___
Diagram III _A_ ___ ___ _C_

In Diagram I, B or D may be in space 3 or 4. This generates 2 diagrams. In Diagram II, B must be in position 2. If B were in position 4, then from the second condition C would have to be next to A. This gives only one diagram. Finally, in Diagram III, B or D could be in position 2 or 3. This generates two diagrams. So there is a total of 5 possible orderings.

3. **8 orderings.**

There are three possible positions for C:

Diagram I ___ ___ ___ _C_
Diagram II ___ ___ _C_ ___
Diagram III ___ _C_ ___ ___

In Diagram I, place B in each of the possible positions and count the number of diagrams thereby generated. If B is in position 1, then two diagrams are possible. If B is in position 2, then again two diagrams are possible. If B is in position 3, then from the second condition D must be in position 2. So in this case only one diagram is generated.

Next, in Diagram II, if B is in position 1, then two diagrams are possible. If B is in position 2, then by the second condition D must be in position 1. So in this case only one diagram results.

Finally, Diagram III is not valid. (Why?) Counting the above diagrams gives a total of 8 possible orderings.

Hybrid Games

In the ordering games we have studied so far, only one element at a time could occupy a particular position. However, in many ordering games two or more elements can occupy the same position at the same time. These games order elements as groups, rather than as individuals. (Grouping games will be presented later.) As you would expect, this added complication makes hybrid games harder than line-up games. Hybrid games are presented here because their ordering nature is more significant than their grouping nature. Some typical setups to these games are

- ❑ Seven books are placed on five shelves.
- ❑ A four-story apartment building has four apartments, one on each floor, and seven tenants.
- ❑ There are two lines of couples waiting to buy tickets to a play.

When analyzing hybrid games, pay close attention to the number of positions versus the number of elements. Also, pay close attention to the maximum or minimum number of elements that can occupy a particular position.

Hybrid Game

A cupboard has five shelves numbered 1 through 5, from bottom to top; each shelf has two compartments. There are eight items—A, B, C, D, E, F, G, H—in the cupboard, no two of which are in the same compartment.

Items D and E are on the same shelf.
B is on the shelf directly below G.
If a shelf contains only one item, it cannot be directly above or directly below another shelf that contains only one item.
C is the only item on one of the shelves.
There is only one item on the fourth shelf.

1. If H is on the fourth shelf, which one of the following CANNOT be true?
   (A) A is on the second shelf.
   (B) D and E are on the second shelf.
   (C) D and E are on the top shelf.
   (D) C is on the first shelf.
   (E) A is on the third shelf.

2. Which one of the following is a complete and accurate list of the items any one of which could be on the top shelf?
   (A) D
   (B) D, E, G, C
   (C) D, E, G, B
   (D) D, E, G, C, F
   (E) D, E, G, H, F, A

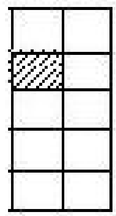
3. Which one of the following must be true?
   (A) If A is on the third shelf, then E is not on the top shelf.
   (B) If E is on the second shelf, then C is not on the bottom shelf.
   (C) If H is on the fourth shelf, then D and E are not on the second shelf.
   (D) If B is on the fourth shelf, then D is not on the third shelf.
   (E) If G is on the top shelf, then H is not on the bottom shelf.

4. If G is on the top shelf and A is on the third shelf, then which one of the following must be true?
   (A) D is on the first shelf.
   (B) E is on the second shelf.
   (C) C is on the fourth shelf.
   (D) Either F or H must be on the same shelf as A.
   (E) F is on the same shelf as G.

5. If A and B are on the second shelf, which one of the following must be true?
   (A) D and E are on the top shelf.
   (B) F is on the same shelf as H.
   (C) A is directly above F.
   (D) C is on the fourth shelf.
   (E) C is on the first shelf.

As in the previous examples, we construct a diagram to help answer the questions. The condition *"D and E are on the same shelf"* is naturally symbolized as **D = E**. The condition *"B is on a shelf directly below G"* can be symbolized as **G/B**. The condition *"C is the only item on one of the shelves"* can be symbolized as **C = alone**. The condition *"There is only one item on the fourth shelf"* can be symbolized as **4th = alone**. Finally, the condition *"If a shelf contains only one item, it* *cannot be directly above or directly below another shelf that contains only one item"* can be symbolized **not 1/1**. This yields the following diagram:

D = E
G/B
C = alone
4th = alone
not 1/1

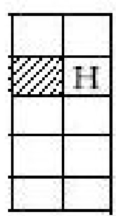


Two readily derived conditions should be noted: There are 10 compartments, 8 items, and C is the only item on its shelf. So two shelves must have only one item each, and no shelf can be empty. Neither of these conditions can be placed on the diagram, so we turn to the questions.

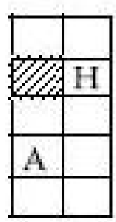
1. **If H is on the fourth shelf, which one of the following CANNOT be true?**

   (A) A is on the second shelf.
   (B) D and E are on the second shelf.
   (C) D and E are on the top shelf.
   (D) C is on the first shelf.
   (E) A is on the third shelf.

Add the new condition, "H is on the fourth shelf," to the diagram:



Now we attack the answer-choices, attempting to construct a diagram for each one. The answer-choice for which a valid diagram cannot be constructed will be the answer. Start with choice (A). Place A on the second shelf:



Next, place the condition **G/B** on shelves 2 and 3:

	H
	G
A	B

Then, place **D = E** on the top shelf:

D	E
	H
	G
A	B

Finally, place C on the bottom shelf and F on the third shelf:

D	E
	H
F	G
A	B
	C

This diagram does not violate any initial condition. Hence A *can* be on the second shelf. This eliminates choice (A).

Next, attack choice (B). Place the condition **D = E** on the second shelf as follows:

	H
D	E

Clearly this diagram leaves no room to place the condition **G/B**. Hence the answer is (B).

As we work through the remaining questions, note the determining power of the condition of **G/B**.

2. **Which one of the following is a complete and accurate list of the items any one of which could be on the top shelf?**

   (A) D
   (B) D, E, G, C
   (C) D, E, G, B
   (D) D, E, G, C, F
   (E) D, E, G, H, F, A

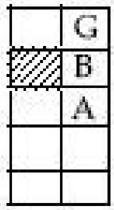
The first thing to note about the answer-choices is that they all contain D. So there is no need to check whether D can be on the top—it can. Next, since D and E must be on the same shelf, we eliminate (A). Next, since all remaining choices contain G, there is no need to check whether G can be on the top shelf. Next, since G must be above B, B clearly cannot be on the top shelf. This eliminates choice (C). Finally, C cannot be on the top shelf; if it were, then one shelf with only one item would be directly above another shelf with only one item. This eliminates both (B) and (D). Hence, by process of elimination, the answer is (E).

3. **Which one of the following must be true?**

   (A) If A is on the third shelf, then E is not on the top shelf.
   (B) If E is on the second shelf, then C is not on the bottom shelf.
   (C) If H is on the fourth shelf, then D and E are not on the second shelf.
   (D) If B is on the fourth shelf, then D is not on the third shelf.
   (E) If G is on the top shelf, then H is not on the bottom shelf.

This question is long because it actually contains five distinct questions. During the test you should save such a question for last. However, there is a shortcut

Next, add the condition **G/B** to the diagram:



Now the condition **D = E** can be placed on either the first or second shelf. We construct a separate diagram for each case:

Diagram 1

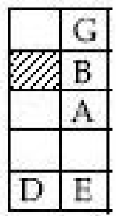
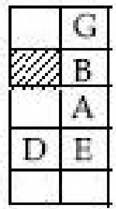


Diagram 2



Next, since C must be alone, it must be on the second shelf in Diagram 1 and on the bottom shelf in Diagram 2:

Diagram 1

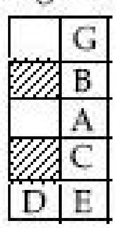
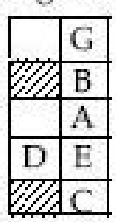


Diagram 2

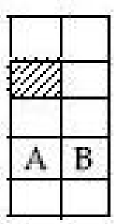


Clearly in both diagrams, either F or H must be next A. Hence the answer is (D).

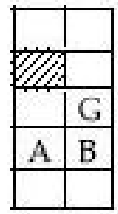
5. **If A and B are on the second shelf, which one of the following must be true?**

   (A) D and E are on the top shelf.
   (B) F is on the same shelf as H.
   (C) A is directly above F.
   (D) C is on the fourth shelf.
   (E) C is on the first shelf.

Adding the new condition to the diagram yields



Next, adding the condition **G/B** to the diagram gives



There are two places left for the condition **D = E**, the bottom shelf or the top shelf. We construct a separate diagram for each case.

Diagram 1

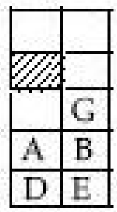
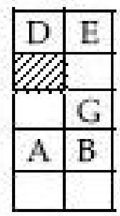


Diagram 2



In Diagram 1, the condition **C = alone** must be placed either on the top shelf or the fourth shelf. But in either case this violates the condition that a shelf with only one item cannot be either directly above or directly below another shelf with only one item. This eliminates Diagram 1. In Diagram 2, D and E are on the top shelf. Hence the answer is (A).

Points to Remember

1. Hybrid games order elements as groups, rather than as individuals.

2. When analyzing a hybrid game, pay close attention to the number of positions versus the number of elements. Also pay close attention to the maximum or minimum number of elements that can occupy a particular position.

3. It is not uncommon for the LSAT writers to repeat a question in a different form. You can save time by watching out for this.

MENTOR EXERCISE

Directions: Each group of questions is based on a set of conditions. In answering some of the questions, it may be useful to draw a rough diagram. Choose the response that most accurately and completely answers each question. Hints, insights, partial solutions, and answers are provided in the right-hand column.

Questions 1–4

Six people—Roger, Susan, Tim, Ulrika, Vic, and Walt—are competing for a job at Consolidated Conglomerate. They have been evaluated on a letter scale A, B, C, D, or E, with A the highest possible evaluation.

> **Exactly two people received Bs.**
> **Only one person received a D, and only one person received a C.**
> **Neither Roger nor Tim received a B.**
> **Susan's evaluation was lower than everyone else's.**

This is a moderately hard hybrid game. Half of the elements are "wild", so the situation is very fluid. This makes the game difficult: throughout the problem we will be groping for something concrete.

Begin by symbolizing the conditions. The condition *"Neither Roger nor Tim received a B"* is naturally symbolized as **(R&T)≠B**. The condition *"Susan's evaluation was lower than everyone else's"* can be symbolized as **S=lowest**. The condition *"Exactly two people received Bs"* can be symbolized as **2Bs**. The remaining conditions can be symbolized in like manner. This yields

R, S, T, (U, V, W "wild")
(R&T)≠B
S=lowest
2Bs, 1C, 1D

The diagram will consist of five boxes in a row—with the lettered evaluations listed at the top, the number of each evaluation listed in each box, and restrictions listed at the bottom:

A	B	C	D	E
	2	1	1	

~R
~T

One further condition should be drawn before turning to the questions. Since one person is assigned a D and everyone is evaluated above Susan, she must have received either a D or an E. Note this as follows:

A	B	C	D	E
	2	1	1	

~R
~T

S

1. **Which one of the following CANNOT be determined based on the information given?**

   (A) Ulrika did not receive an E.
   (B) At most one person received an E.
   (C) At least one person received an E.
   (D) Roger did not receive an E.
   (E) Tim did not receive a B.

1. Since this question asks for the answer-choice that cannot be determined, we attempt to construct a valid counter-example for each choice. The one for which this is not possible will be the answer.

Choice (A) can be determined from the initial conditions since Susan received the lowest evaluation. Next, choice (B) necessarily follows from the given conditions. (Why?) This eliminates (B). As to (C), suppose that S received a D:

A	B	C	D	E
	2	1	S	
	~R ~T			

Then both U and V could receive Bs, without violating any conditions:

A	B	C	D	E
	U V	1	S	

Finally, both R and T could receive As and W could receive a C—all without violating any condition:

A	B	C	D	E
R T	U V	W	S	

This is a valid counter-example. Hence the answer is (C).

2. If Vic and Walt received the same evaluation, which one of the following could be true?

   (A) Vic did not receive a B.
   (B) Walt did not receive a B.
   (C) Susan received a C.
   (D) Roger received a B.
   (E) Roger received a D.

2. Again, this problem requires an indirect proof. With problems like these, don't necessarily start with choice (A). Instead, scan the choices for a likely candidate.

As to (A), it is a poor candidate: we may have to construct a different diagram for each of Vic's four possible positions. The same holds for choice (B). Next, choices (C) and (D) violate the original diagram—eliminate. Finally, (E) is a good candidate because it fixes the position of an element.

Now try to construct a valid diagram, with Roger assigned a D.

The answer is (E).

3. If Vic and Walt received the same lettered evaluation, then which of following must be true?

   I. Both Vic and Walt received an A.
   II. Both Vic and Walt received a B.
   III. Ulrika received an A.

   (A) I only
   (B) II only
   (C) III only
   (D) I and III only
   (E) II and III only

3. Start with I. Place Vic and Walt on the diagram as follows:

A	B	C	D	E
V W	2	1	1	
	~R ~T		S	

Now since Roger, Tim, and Susan cannot receive a B, only Ulrika can be assigned a B. But this violates the first condition, *"Exactly two people received Bs."* So I is false, which eliminates both (A) and (D); they both contain I.

As to II, since Vic and Walt cannot both be assigned either a C or a D (not enough room), they must both receive a B. So II is true, which eliminates (C).

Unfortunately, we have to check III. Given the fluidness of the diagram and the fact that Ulrika is "wild", it is unlikely that III must be true. Nonetheless, you should construct a diagram to check this.

The answer is (B).

4. **If only Vic received an A and Roger received a score higher than Tim, which one of following must be true?**

   (A) Susan received an E.
   (B) Roger received a D.
   (C) Ulrika received a C.
   (D) Tim received a C.
   (E) Susan received a D.

4. To start, place Vic in box A:

A	B	C	D	E
V	2	1	1	
	~R			
	~T		S	

Next, since neither Roger nor Tim can be in box B, one must be in box C and one in box D:

A	B	C	D	E
V	2	R	T	
			S	

But this forces Susan into box E. Hence the answer is (A).

Notice: To solve this problem, we did not need the obfuscating condition *"Roger received a higher score than Tim."* It is not uncommon for the LSAT writers to introduce superfluous conditions. So don't become alarmed if you don't use all the conditions when solving a game. This may indicate an oversight on your part—it may not. Many students, upon discovering that they did not use all the conditions, will fruitlessly check and recheck their work, wasting precious time. If you don't use all the conditions, make a cursory inspection of your work. If no mistakes are found, cut your losses and move on—taking solace in the hope that the unused conditions were extraneous.

EXERCISE

Directions: The following group of questions is based on a set of conditions. Choose the response that most accurately and completely answers each question. Answers and solutions begin on page 97.

Questions 1–5

Four couples—JJ, KK, LL, MM—are standing in a line. Their positions are numbered consecutively from 1 to 8, and each person is holding hands with the persons on either side of him or her.

J and J are holding hands.
K and K are not holding hands.
L and L are holding hands.
One of the Ls is at one end of the line, and one of the Ms is at the other end.

1. If M is at position 2 and K is at position 3, then a J must be at position

(A) 1
(B) 4
(C) 6
(D) 7
(E) 8

2. If J is in position 2, it must be true that

(A) M is in position 3.
(B) J is in position 4.
(C) K is in position 7.
(D) L is in position 5.
(E) M is in position 5.

3. If an M is in position 8, which of the following CANNOT be true?

I. The other M is in position 5.
II. The Ks can be in positions 3, 5, or 7.
III. One of the Js is in position 3.

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III

4. If the Ks are separated by at most one other person, then which one of the following groups could be standing in the four even-numbered positions in one arrangement?

(A) The two Js and the two Ls.
(B) The two Ks and the two Ls.
(C) The two Js and the two Ms.
(D) One J, one K, one L, and one M.
(E) One J, the two Ks, and one L.

5. Which one of the following must be true?

(A) At least one J is holding hands with a K.
(B) At least one L is holding hands with a J.
(C) At least one L is holding hands with a K.
(D) At least one M is holding hands with an L.
(E) At least one J is holding hands with an M.

Questions 6–11

Five friends are playing chess. Three are women—Laura, Mary, and Naomi—and two are men—Oliver and Paul. There are three chessboards in a row.

> Naomi does not sit next to either Mary or Oliver.
> Laura does not play Naomi.
> The middle board always has two players.

6. If Mary plays Paul on the middle board and Paul does not sit between two other players, which one of the following is a complete and accurate list of those who might not have an opponent?

    (A) Laura
    (B) Naomi
    (C) Laura and Naomi
    (D) Laura and Oliver
    (E) Laura, Naomi, and Oliver

7. If Paul does not have an opponent, which one of the following must be false?

    (A) Mary plays Naomi.
    (B) Mary plays Oliver.
    (C) Mary plays Laura.
    (D) Laura does not sit between two other people.
    (E) Laura sits between Naomi and Paul.

8. If players of the same sex do not play each other and Mary sits between two other players, which one of the following is a complete and accurate list of those players who might be Oliver's opponent?

    (A) Mary and Laura
    (B) Mary and Naomi
    (C) Naomi
    (D) Mary, Naomi, and Laura
    (E) Mary

9. If players of the same sex do not play each other, which one of the following must be false?

    (A) Naomi plays Paul.
    (B) Naomi plays Oliver.
    (C) Laura plays Oliver.
    (D) Paul plays Mary.
    (E) Paul does not have an opponent.

10. If the women always play each other, which one of the following must be true?

    (A) Laura plays Mary.
    (B) Mary plays Naomi.
    (C) Laura has no opponent.
    (D) One of the women does not have an opponent.
    (E) Paul does not have an opponent.

11. How many different people can Naomi play against?

    (A) 0
    (B) 1
    (C) 2
    (D) 3
    (E) 4

The following game appeared on a recent LSAT.

Questions 12–16

A gymnastics instructor is planning a weekly schedule, Monday through Friday, of individual coaching sessions for each of six students—H, I, K, O, U, and Z. The instructor will coach exactly one student each day, except for one day when the instructor will coach two students in separate but consecutive sessions. The following restrictions apply:

H's session must take place at some time before Z's session.
I's session is on Thursday.
K's session is always scheduled for the day immediately before or the day immediately after the day for which O's session is scheduled.
Neither Monday nor Wednesday can be a day for which two students are scheduled.

12. Which one of the following is a pair of students whose sessions can both be scheduled for Tuesday, not necessarily in the order given?

    (A) H and U
    (B) H and Z
    (C) K and O
    (D) O and U
    (E) U and Z

13. If K's session is scheduled for Tuesday, then which one of the following is the earliest day for which Z's session can be scheduled?

    (A) Monday
    (B) Tuesday
    (C) Wednesday
    (D) Thursday
    (E) Friday

14. Which one of the following must be true?

    (A) If U's session is scheduled for Monday, H's session is scheduled for Tuesday.
    (B) If U's session is scheduled for Tuesday, O's session is scheduled for Wednesday.
    (C) If U's session is scheduled for Wednesday, Z's session is scheduled for Tuesday.
    (D) If U's session is scheduled for Thursday, Z's session is scheduled for Friday.
    (E) If U's session is scheduled for Friday, Z's session is scheduled for Thursday.

15. Scheduling Z's session for which one of the following days determines the day for which U's session must be scheduled?

    (A) Monday
    (B) Tuesday
    (C) Wednesday
    (D) Thursday
    (E) Friday

16. If H's session is scheduled as the next session after U's session, which one of the following could be true about H's session and U's session?

    (A) U's session is scheduled for Monday, and H's session is scheduled for Tuesday.
    (B) U's session is scheduled for Thursday, and H's session is scheduled for Friday.
    (C) They are both scheduled for Tuesday.
    (D) They are both scheduled for Thursday.
    (E) They are both scheduled for Friday.

Answers and Solutions to Exercise

Questions 1–5

This is a relatively easy hybrid game. Begin by symbolizing the conditions. The conditions *"J and J are holding hands"* and *"L and L are holding hands"* are naturally symbolized as JJ and LL, respectively. The condition *"K and K are not holding hands"* can be symbolized as K...K. [The symbol ~(KK) would also work well, but the symbol K...K is more descriptive because it shows the space between the Ks.] The condition *"One of the Ls is at one end of the line, and one of the Ms is at the other end"* can be symbolized as L=1st/last and M=1st/last. The diagram will consist of eight dashed lines numbered 1 through 8, from left to right:

JJ
LL
K...K
L=1st/last
M=1st/last

1	2	3	4	5	6	7	8

No significant conditions can be derived from the given information, and no conditions can be placed on the diagram. So we attack the questions.

1. **If M is at position 2 and K is at position 3, then a J must be at position**

   (A) 1
   (B) 4
   (C) 6
   (D) 7
   (E) 8

To start, place M and K at positions 2 and 3, respectively:

1	2	3	4	5	6	7	8
	M	K					

From the conditions LL and L=1st/last, we see that the Ls must be in positions 7 and 8, since M is in position 2:

1	2	3	4	5	6	7	8
	M	K				L	L

Now the condition M=1st/last forces the other M into position 1:

1	2	3	4	5	6	7	8
M	M	K				L	L

This yields two possible positions for the condition JJ:

Diagram I

1	2	3	4	5	6	7	8
M	M	K	J	J		L	L

Diagram II

1	2	3	4	5	6	7	8
M	M	K		J	J	L	L

Diagram II is impossible, though, because it forces the Ks next to each other, violating the condition K...K. Thus Diagram I, which has the Js in positions 4 and 5, is uniquely determined by the conditions. The answer is (B).

2. **If a J is in position 2, it must be true that**

   (A) M is in position 3.
   (B) J is in position 4.
   (C) K is in position 7.
   (D) L is in position 5.
   (E) M is in position 5.

Place a J in position 2:

1	2	3	4	5	6	7	8
	J						

Then, from the condition JJ, the other J must be in position 3, since only L or M can be first.

1	2	3	4	5	6	7	8
	J	J					

Next, since the Ls must be together and one of them must be either first or last, they must be in positions 7 and 8. This in turn forces M to be first:

1	2	3	4	5	6	7	8
M	J	J				L	L

Next, from the condition K...K, we see that the Ks must be in positions 4 and 6, which in turn forces the other M into position 5. Thus our uniquely determined diagram is

1	2	3	4	5	6	7	8
M	J	J	K	M	K	L	L

The answer is (E).

3. **If an M is in position 8, which of the following CANNOT be true?**

   I. The other M is in position 5.
   II. The Ks can be in positions 3, 5, or 7.
   III. One of the Js is in position 3.

   (A) I only
   (B) II only
   (C) III only
   (D) II and III only
   (E) I, II, and III

This question requires an indirect proof. That is, take a sub-statement; then try to construct a valid diagram for it. If this cannot be done, it is an answer—otherwise it is not.

Start with Statement I. If M is in position 5, then, from the supplemental condition *"M is in position 8"* and the condition **L=1st/last**, we know that the Ls must be in positions 1 and 2:

1	2	3	4	5	6	7	8
L	L			M			M

Now, the condition JJ forces the Js into either positions 3 and 4, or 6 and 7:

Diagram I

1	2	3	4	5	6	7	8
L	L	J	J	M			M

Diagram II

1	2	3	4	5	6	7	8
L	L			M	J	J	M

In both diagrams the Ks are forced next to each other, which violates the condition K...K. Hence Statement I cannot be true. This eliminates (B), (C), and (D), as they don't contain I.

Next, Statement II is time consuming because we will have to check all three positions for the Ks. So skip to Statement III. Place J in position 3:

1	2	3	4	5	6	7	8
L	L	J					M

Then from the condition JJ, we know that the other J must be in position 4:

1	2	3	4	5	6	7	8
L	L	J	J				M

Then the Ks can be placed in positions 5 and 7 and the M in position 6—all without violating any conditions:

1	2	3	4	5	6	7	8
L	L	J	J	K	M	K	M

This is a valid diagram with J in position 3. Hence Statement III *can* be true. This eliminates (E). Therefore, by process of elimination, we have learned that the answer is (A), without having to check Statement II.

4. If the Ks are separated by at most one other person, then which one of the following groups could be standing in the four even-numbered positions in one arrangement?

(A) The two Js and the two Ls.
(B) The two Ks and the two Ls.
(C) The two Js and the two Ms.
(D) One J, one K, one L, and one M.
(E) One J, the two Ks, and one L.

This is a moderately hard problem. The condition *"the Ks are separated by at most one other person"* is somewhat obscure. It is more clearly expressed as *"exactly one person separates the Ks."*

Since the two Js must be next to each other, they cannot both be in even-numbered positions. This eliminates both (A) and (C). The same is true for the Ls, which eliminates (B).

Since the Ks are separated by exactly one person, they must either both occupy even-numbered positions or both occupy odd-numbered positions. But choice (D) places only one K in an even-numbered position. This eliminates (D).

As a matter of test-taking strategy, this is sufficient analysis of the question to mark the answer (E). However, it is instructive to work out a valid order for (E).

To this end, place an M in position 1 and the Ls in positions 7 and 8:

1	2	3	4	5	6	7	8
M						L	L

Next, place the Ks in positions 2 and 4, with M separating them:

1	2	3	4	5	6	7	8
M	K	M	K			L	L

Finally, place the Js in positions 5 and 6:

1	2	3	4	5	6	7	8
M	K	M	K	J	J	L	L

This diagram satisfies all the conditions, which verifies that (E) is the answer.

5. Which one of the following must be true?

1	2	3	4	5	6	7	8
M	J	J	K			L	L

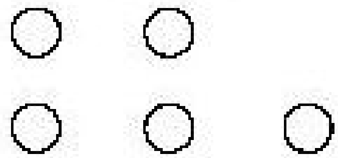
(A) At least one J is holding hands

<u>Questions 6–11</u>

This is a hybrid ordering game of medium difficulty. The game is quite fluid since there are no fixed elements. The condition *"Naomi does not sit next to either Mary or Oliver"* is naturally symbolized as ~(NM) and ~(NO). The condition *"Laura does not play Naomi"* can be symbolized as L<-/->N. Adding a diagram gives the following:

Women Men
L, M, N O, P
(P is "wild")

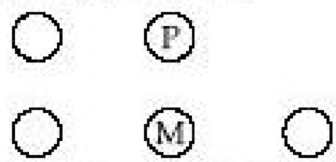
~(NM) and ~(NO)
L<-/->N



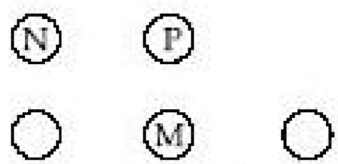
6. **If Mary plays Paul on the middle board and Paul does not sit between two other players, which one of the following is a complete and accurate list of those who might not have an opponent?**

   (A) Laura
   (B) Naomi
   (C) Laura and Naomi
   (D) Laura and Oliver
   (E) Laura, Naomi, and Oliver

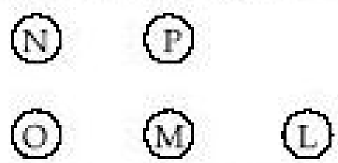
Adding the condition *"Mary plays Paul on the middle board and Paul does not sit between two other players"* to the diagram yields



Now N must sit next to P since she cannot sit next to M, [~(NM) and ~(NO)]. This yields



Next, the condition L<-/->N forces L to the right of M:

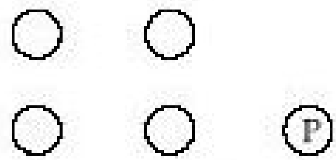


This diagram is uniquely determined by the conditions. Hence, L and only L will not have an opponent. The answer is (A).

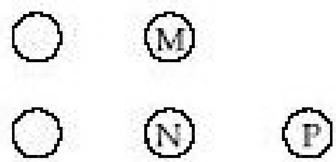
7. **If Paul does not have an opponent, which one of the following must be false?**

   (A) Mary plays Naomi.
   (B) Mary plays Oliver.
   (C) Mary plays Laura.
   (D) Laura does not sit between two other people.
   (E) Laura sits between Naomi and Paul.

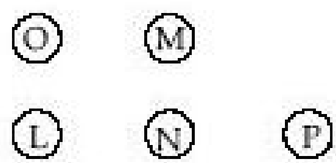
Adding the condition *"Paul does not have an opponent"* to the diagram yields



Now, we try to construct a diagram for each answer-choice. The one for which this cannot be done will be the answer. Begin with (A). Place M and N on the diagram as follows:

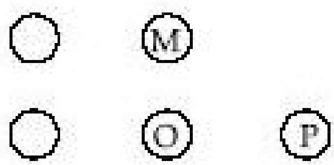


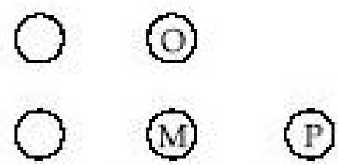
Next, place O and L on the diagram as follows:



This diagram satisfies every condition. Hence, Mary can play Naomi. This eliminates (A).

Choice (B) yields two possible diagrams:



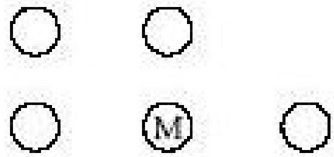


Clearly, in either diagram N must sit next to either M or O. However, this violates the condition ~(NM) and ~(NO). Hence, Mary cannot play Oliver, and the answer is (B).

8. **If players of the same sex do not play each other and Mary sits between two other players, which one of the following is a complete and accurate list of those players who might be Oliver's opponent?**

   (A) Mary and Laura
   (B) Mary and Naomi
   (C) Naomi
   (D) Mary, Naomi, and Laura
   (E) Mary

Adding the condition *"Mary sits between two other players"* to the diagram yields



Since players of the same sex do not play each other, M must play either O or P. This generates two diagrams:

Diagram I

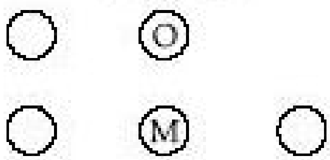


Diagram II

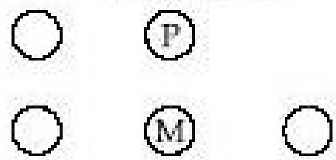
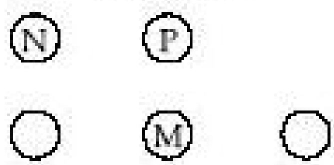


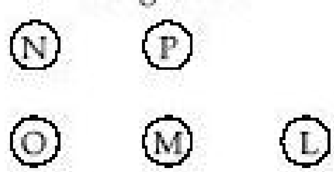
Diagram I is impossible since it forces N next to either M or O, violating the condition ~(NM) and ~(NO). In Diagram II, N must sit next to P since she cannot sit next to M:

Diagram II



Next, the condition L<-/->N forces L to the right of M and O to the left of M:

Diagram II

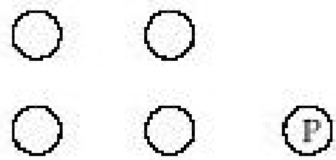


This diagram is uniquely determined by the conditions. Hence, only Naomi can play Oliver. The answer is (C).

9. **If players of the same sex do not play each other, which one of the following must be false?**

   (A) Naomi plays Paul.
   (B) Naomi plays Oliver.
   (C) Laura plays Oliver.
   (D) Paul plays Mary.
   (E) Paul does not have an opponent.

This type of problem can be time consuming because we may have to construct a separate diagram for each wrong answer. In these cases, you should quickly survey the answer-choices for a likely candidate and check whether any previous diagrams will help. Now, the final diagram in Question 8 shows that both (B) and (D) are possible. This eliminates (B) and (D). Turning to (E), place P on the diagram:



Now, there are three women but only four open seats in the diagram. Hence, two of the women must play each other, contradicting the condition *"players of the same sex do not play each other."* The answer is (E).

10. **If the women always play each other, which one of the following must be true?**

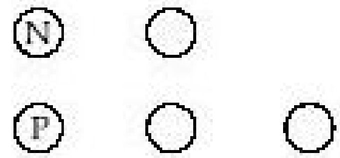
   (A) Laura plays Mary.
   (B) Mary plays Naomi.
   (C) Laura has no opponent.
   (D) One of the women does not have an opponent.
   (E) Paul does not have an opponent.

Since the women must play each other and there are three women, one woman will not have an opponent. The answer is (D).

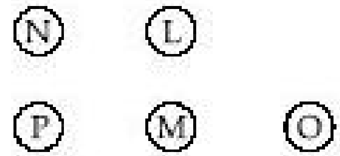
11. **How many different people can Naomi play against?**

   (A) 0
   (B) 1
   (C) 2
   (D) 3
   (E) 4

From the condition L<-/->N, we know that Naomi cannot play Laura. This eliminates (E). The final diagram in Question 6 has Naomi playing Oliver, and the third diagram in Question 7 has Naomi playing Mary. Next, suppose Naomi plays Paul:



Now, Laura could play Mary, which leaves Oliver without an opponent:



This diagram satisfies every condition. Hence, Naomi can play Paul. So Naomi can play three different people—Oliver, Mary, and Paul. The answer is (D).

<u>**Questions 12–16**</u>

Although this game is about scheduling, its structure is actually linear ordering. The condition *"H's session must take place at some time before Z's session"* can be symbolized as H—>Z. The condition *"K's session is always scheduled for the day immediately before or the day immediately after the day for which O's session is scheduled"* simply means that K and O must be scheduled on consecutive days; it can be symbolized as **KO** (note, we won't use the flip-flop symbol, just remember that K and O can be interchanged). Symbolizing the remaining conditions yields

H, I, K, O, U, Z
1 day = 2 students
H—>Z
I = Th
KO
M ≠ 2 & W ≠ 2

~2		~2		
M	Tu	W	Th	F
			I	

12. **Which one of the following is pair of students whose sessions can both be scheduled for Tuesday, not necessarily in the order given?**

    (A) H and U
    (B) H and Z
    (C) K and O
    (D) O and U
    (E) U and Z

(A) No. Adding H and U to the diagram yields

1	2	1	1	1
M	Tu	W	Th	F
	H		I	
	U			

This diagram shows that there is no room to place the condition KO.

(B) No. Just as in choice (A), there is no room to place the condition KO.

(C) No. K and O must be scheduled on consecutive days.

(D) Yes. The following is one of two scenarios that satisfy all the conditions:

M	Tu	W	Th	F
H	O	K	I	Z
	U			

(E) No. Just as in choice (A), there is no room to place the condition KO.

13. If K's session is scheduled for Tuesday, then which one of the following is the earliest day for which Z's session can be scheduled?

(A) Monday
(B) Tuesday
(C) Wednesday
(D) Thursday
(E) Friday

(A) No. Z's session must be scheduled after H's session, H—>Z, and only one student can be scheduled for Monday.

(B) Yes. With K and Z scheduled for Tuesday, we get the following unique ordering:

M	Tu	W	Th	F
H	K	O	I	U
	Z			

(C) No. See explanation for choice (B).

(D) No. See explanation for choice (B).

(E) No. See explanation for choice (B).

14. **Which one of the following must be true?**

    (A) If U's session is scheduled for Monday, H's session is scheduled for Tuesday.
    (B) If U's session is scheduled for Tuesday, O's session is scheduled for Wednesday.
    (C) If U's session is scheduled for Wednesday, Z's session is scheduled for Tuesday.
    (D) If U's session is scheduled for Thursday, Z's session is scheduled for Friday.
    (E) If U's session is scheduled for Friday, Z's session is scheduled for Thursday.

(A) No. Following is one of several counterexamples:

M	Tu	W	Th	F
U	K	O	I	Z
			H	

(B) No. Following is one of several counterexamples:

M	Tu	W	Th	F
K	U	H	I	Z
	O			

(C) No. Following is one of several counterexamples:

M	Tu	W	Th	F
K	O	U	I	Z
			H	

(D) **Yes.** Place U on the diagram:

M	Tu	W	Th	F
			I	
			U	

This diagram shows that the condition KO must be placed on Monday/Tuesday or Tuesday/Wednesday. Hence, one of the days Monday, Tuesday, or Wednesday is left for H and Z. But since H must be scheduled before Z, Z must be scheduled on Friday.

(E) No. Following is one of several counterexamples:

M	Tu	W	Th	F
K	O	H	I	U
				Z

15. Scheduling Z's session for which one of the following days determines the day for which U's session must be scheduled?

(A) Monday
(B) Tuesday
(C) Wednesday
(D) Thursday
(E) Friday

(A) No. From the conditions H—>Z and M ≠ 2, we know that Z cannot be scheduled on Monday.
(B) No. Following are two valid scenarios with U scheduled on different days:

M	Tu	W	Th	F
H	Z	K	I	U
			O	

M	Tu	W	Th	F
H	Z	U	I	K
			O	

(C) No. Following are two valid scenarios with U scheduled on different days:

M	Tu	W	Th	F
K	O	Z	I	U
	H			

M	Tu	W	Th	F
U	H	Z	I	O
			K	

(D) Yes. Place Z on the diagram:

M	Tu	W	Th	F
			I	
			Z	

This diagram shows that the condition **KO** must be placed on Monday/Tuesday or Tuesday/Wednesday. Since H must be scheduled before Z, H must be scheduled on Monday or Wednesday, which forces U to be scheduled on Friday.
(E) No. Following are two valid scenarios with U scheduled on different days:

M	Tu	W	Th	F
K	O	U	I	Z
			H	

M	Tu	W	Th	F
U	K	O	I	Z
			H	

16. If H's session is scheduled as the next session after U's session, which one of the following could be true about H's session and U's session?

    (A) U's session is scheduled for Monday, and H's session is scheduled for Tuesday.
    (B) U's session is scheduled for Thursday, and H's session is scheduled for Friday.
    (C) They are both scheduled for Tuesday.
    (D) They are both scheduled for Thursday.
    (E) They are both scheduled for Friday.

(A) Yes. Scheduling U on Monday and H on Tuesday yields the following diagram:

M	Tu	W	Th	F
U	H		I	

The condition KO can be placed on the diagram as follows:

M	Tu	W	Th	F
U	H	O	I	
	K			

Finally, placing Z on Friday yields the following valid scenario:

M	Tu	W	Th	F
U	H	O	I	Z
	K			

Note, the supplemental condition *"H's session is scheduled as the next session after U's session"* is not needed for this or any other answer-choice. It is not uncommon for the LSAT writers to introduce superfluous conditions.
(B) No. H cannot be scheduled on Friday since H must be scheduled before Z. Note, Z cannot also be scheduled on Friday since there are already two people—I and U—scheduled on Thursday.
(C) No. Place U and H on the diagram:

M	Tu	W	Th	F
	U		I	
	H			

This diagram leaves no room to place the condition KO.
(D) No. This would schedule three people—I, U, and H—on Thursday. But the setup to the game states that exactly one person is scheduled for each day, except for one day when two people are scheduled.
(E) No. H cannot be scheduled on Friday with U since H must be scheduled before Z.

Sequential Games

Unlike spatial and hybrid games, sequential games do not order elements in space. Sequential games can be classified according to the criteria used to order the elements:

- Chronological (before, after, etc.)
- Quantifiable (size, height, etc.)
- Ranking (first, second, etc.)

CHRONOLOGICAL GAMES

Chronological games order elements in a time-sequence. For example, James was born before George who was born before Kim who was born before Sara. In the line-up games that we studied earlier, the elements were ordered spatially. In chronological ordering games, the elements are ordered sequentially. This is true of many of the games that we will study in this chapter. Because these games are sequential in nature, their diagrams can be quite different from those used to solve spatial and hybrid games.

One of the most common and efficient types of diagrams is the flow chart. In these diagrams the elements are connected by arrows.*

Now that we have a second way to diagram linear ordering games, we need, of course, some means of deciding which method to use.

In general, a game with no fixed elements should be solved using a flow chart.

In constructing a flow chart, follow these guidelines:

1. **Look for a condition that starts the "flow".**
2. **Build the chart around the element that occurs in the greatest number of conditions.**
3. **Keep the chart flexible; it will probably have to evolve with the changing conditions.**

An example will illustrate the flow chart method of diagramming.

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* See Paths and Flow Charts for a treatment of flow charts for non-linear games.

Chronological Game

Eight people—S, T, U, V, W, X, Y, Z—were each born in a different year, 1971 through 1978. The following is known about their ages.

W is older than V.
S is younger than both Y and V.
T is not younger than Y.
Z is younger than Y, but older than U.

1. Which one of the following is a possible sequence of births from first to last?

   (A) W V T Y Z U X S
   (B) W V T U Y Z X S
   (C) U T Y W V S Z X
   (D) T W Y S Z U X V
   (E) T Y W V S U X Z

2. If S was born in 1975, which one of the following must be false?

   (A) V was born in 1973.
   (B) V was born in 1972.
   (C) Z was born in 1977.
   (D) Y was born in 1974.
   (E) Z was born in 1974.

3. If S was born in 1976 and X was born in 1973, then the year of birth of exactly how many other people can be determined?

   (A) 0
   (B) 1
   (C) 2
   (D) 3
   (E) 4

4. If S was born before X, then which one of the following could be true?

   (A) Z was born before T.
   (B) V was born before W.
   (C) U was born before S.
   (D) W was born after S.
   (E) W was born in 1976.

5. If the condition "S is younger than both Y and V" is dropped, then the year of birth of exactly how many people can be determined?

   (A) 0
   (B) 1
   (C) 2
   (D) 3
   (E) 4

This game does not have any fixed elements (such as T was born in 1972), so the flow chart method is indicated. We'll use an arrow to indicate that one person is older than another. The condition *"W is older than V"* is naturally symbolized as W—>V. The other conditions are symbolized in like manner, giving the following schematic:

1. **W—>V**
2. **Y—>S**
3. **V—>S**
4. **T—>Y**
5. **Y—>Z**
6. **Z—>U**

Now we construct the diagram. Following the guidelines on page 111, look for the element that occurs in the greatest number of conditions. It is Y, so we build the chart around Y. Start with condition 4:

$$T \longrightarrow Y$$

Adding conditions 2 and 5 gives

$$\begin{tikzcd} & & Z \\ T \arrow[r] & Y \arrow[r] \arrow[ur] & S \end{tikzcd}$$

Adding conditions 1 and 3 gives

$$\begin{tikzcd} & & Z \\ T \arrow[r] & Y \arrow[r] \arrow[ur] & S \\ W \arrow[r] & V \arrow[ur] & \end{tikzcd}$$

Finally, adding condition 6 gives

$$\begin{tikzcd} & & Z \arrow[r] & U \\ T \arrow[r] & Y \arrow[r] \arrow[ur] & S & \\ W \arrow[r] & V \arrow[ur] & & \end{tikzcd}$$

There are no conditions on the element X, so it can not be placed in the diagram. However, we note it below the diagram as follows:

$$\begin{tikzcd} & & Z \arrow[r] & U \\ T \arrow[r] & Y \arrow[r] \arrow[ur] & S & \\ W \arrow[r] & V \arrow[ur] & & \end{tikzcd}$$

(X "wild")

Two properties of the diagram should be noted before turning to the questions. First, if two elements are in different rows and no sequence of arrows connects them, then the diagram does not tell us which one is older. For example, since W and Y are in different rows and are not connected by a sequence of arrows, the diagram does not tell us who is older. However, the diagram *does* tell us that T is older than U, because the arrows "flow" from T to Y to Z to U. Second, the diagram tells us that only T, W, or X can be the oldest, and likewise that only U, S, or X can be the youngest.

1. **Which one of the following is a possible sequence of births from first to last?**

   (A) W V T Y Z U X S
   (B) W V T U Y Z X S
   (C) U T Y W V S Z X
   (D) T W Y S Z U X V
   (E) T Y W V S U X Z

This is a straightforward elimination problem. (B) and (E) are not possible sequences because the diagram shows that Z must be older than U. (C) is not a possible sequence because the diagram shows that T must be older than U. The arrows "flow" from T to Y to Z to U. Finally, (D) is not a possible sequence because the diagram shows that V must be older than S. Hence, by process of elimination, the answer is (A).

2. **If S was born in 1975, which one of the following must be false?**

   (A) V was born in 1973.
   (B) V was born in 1972.
   (C) Z was born in 1977.
   (D) Y was born in 1974.
   (E) Z was born in 1974.

The diagram shows that T, Y, W, and V—not necessarily in that order—were all born before S. So they must have been born in the years '71 through '74. This gives the following *possible* diagram. (Because one of the births, S, is fixed, it is now more convenient to use a *line-up* diagram.)

<u>71</u>	<u>72</u>	<u>73</u>	<u>74</u>	<u>75</u>	<u>76</u>	<u>77</u>	<u>78</u>
T	Y	W	V	S			

Clearly, this diagram shows that Z must have been born after '75. Choice (E), therefore, makes the necessarily false statement. The answer is (E).

3. **If S was born in 1976 and X was born in 1973, then the year of birth of exactly how many other people can be determined?**

   (A) 0
   (B) 1
   (C) 2
   (D) 3
   (E) 4

Since two births—S and X—are fixed, we again revert to a *line-up* diagram:

<u>71</u>	<u>72</u>	<u>73</u>	<u>74</u>	<u>75</u>	<u>76</u>	<u>77</u>	<u>78</u>
		X			S		

The original diagram shows that W, V, T, and Y were all born before S, so they must be placed to the left of S on the new diagram. However, we cannot uniquely determine their positions: W and V could have been born in '71 and '72, respectively, or T and Y could have been. So one *possible* diagram is

71	72	73	74	75	76	77	78
W	V	X	T	Y	S		

Clearly this diagram forces Z and U to have been born in the years '77 and '78, respectively. Hence only two other births can be determined. The answer is (C).

4. **If S was born before X, then which one of the following could be true?**

   (A) Z was born before T.
   (B) V was born before W.
   (C) U was born before S.
   (D) W was born after S.
   (E) W was born in 1976.

Add the condition *"S was born before X"* to the diagram:

$$\begin{array}{l} \qquad\qquad\quad\nearrow Z \longrightarrow U \\ T \longrightarrow Y \longrightarrow S \longrightarrow X \\ \qquad\qquad\quad\nearrow \\ W \longrightarrow V \end{array}$$

In this diagram U and S are in different rows and are not connected by a sequence of arrows, so U could have been born before S. The answer, therefore, is (C).

5. **If the condition "S is younger than both Y and V" is dropped, then the year of birth of exactly how many people could be determined?**

   (A) 0
   (B) 1
   (C) 2
   (D) 3
   (E) 4

The condition *"S is younger than both Y and V"* anchored the diagram. Without it we get the following diagram:

T—>Y—>Z—>U
W—>V
(X and S "wild")

Because the two parts of this diagram are independent (they are not connected by a sequence of arrows), W and V could have been born before T or after U. Hence the year of birth cannot be determined for any of the people. The answer is (A).

Points to Remember

1. The three types of sequential games are

   - ❑ Chronological (before, after, etc.)
   - ❑ Quantifiable (size, height, etc.)
   - ❑ Ranking (first, second, etc.)

2. Most sequential games can be solved most efficiently with a flow chart.

3. In general, a game with no fixed elements should be solved using a flow chart.

4. When constructing a flow chart, follow these guidelines.

   1. Look for a condition that starts the "flow".
   2. Build the chart around the element that occurs in the greatest number of conditions.
   3. Keep the chart flexible; it will probably have to evolve with the changing conditions.

There will be no mentor exercise in this section.

EXERCISE

Directions: The following group of questions is based on a set of conditions. Choose the response that most accurately and completely answers each question. Answers and solutions begin on page 119.

Questions 1-5

Ten children—Anna, Bob, Charles, Don, Emily, Frank, Gina, Hank, Irene, and Jane—are comparing their heights.

Jane is taller than Don.
Hank and Irene are the same height and both are shorter than Don.
Irene is taller than Bob.
Both Anna and Emily are taller than Gina.
Both Charles and Frank are shorter than Gina.

1. Which one of the following can be false?

(A) Anna is taller than Frank.
(B) Bob is shorter than Jane.
(C) Jane is shorter than Emily.
(D) Frank is shorter than Anna.
(E) Anna is taller than Charles.

2. What is the minimum number of different heights the ten children can have?

(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

3. If Gina is the same height as Irene, then which one of the following can be false?

(A) Don is taller than Emily.
(B) Jane is taller than Bob.
(C) Emily is taller than Charles.
(D) Gina is the same height as Hank.
(E) Hank is taller than Frank.

4. Which of the following statements supplies information already contained in the original conditions?

I. Frank is shorter than Emily.
II. Jane is taller than Gina.
III. Charles is taller than Frank.

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

5. Which one of the following children could NOT be the third tallest?

(A) Jane
(B) Hank
(C) Irene
(D) Gina
(E) Bob

The following game is taken from a recent LSAT.

Questions 6-7

A law firm has exactly nine partners: Fox, Glassen, Hae, Inman, Jacoby, Kohn, Lopez, Malloy, and Nassar.

Kohn's salary is greater than both Inman's and Lopez's.
Lopez's salary is greater than Nassar's.
Inman's salary is greater than Fox's.
Fox's salary is greater than Malloy's.
Malloy's salary is greater than Glassen's.
Glassen's salary is greater than Jacoby's.
Jacoby's salary is greater than Hae's.

6. If Nassar's salary is the same as that of one other partner of the firm, which one of the following must be false?

(A) Inman's salary is less than Lopez's.
(B) Jacoby's salary is less than Lopez's.
(C) Lopez's salary is less than Fox's.
(D) Lopez's salary is less than Hae's.
(E) Nassar's salary is less than Glassen's.

7. What is the minimum number of different salaries earned by the nine partners of the firm?

(A) 5
(B) 6
(C) 7
(D) 8
(E) 9

Questions 8–11

Six items—H, I, J, K, L, and M—are being packed in a cylindrical carton. The order in which the items are placed in the carton must conform to the following rules:

Both items M and L must be placed in the carton before item H.
Item I must be placed in the carton after items H and K.
Item K cannot be placed in the carton next to item J.
Any red item must be placed in the carton before any non-red item, provided that none of the preceding rules are violated.

8. The items can be placed in the carton in which one of the following sequences?

(A) M, L, H, K, J, I
(B) K, I, M, L, J, H
(C) K, L, M, J, H, I
(D) J, L, K, H, M, I
(E) J, K, M, L, H, I

9. If H and L are the only red items, then the items can be placed in the carton in which one of the following sequences?

(A) L, H, M, K, I, J
(B) L, M, K, H, I, J
(C) L, M, H, K, I, J
(D) L, M, H, J, I, K
(E) L, J, M, H, K, I

10. If M and I are the only red items, which of the following must be true?

(A) J is the last item placed in the carton.
(B) L is the second item placed in the carton.
(C) K is the fourth item placed in the carton.
(D) J is the fourth item placed in the carton.
(E) I is the fourth item placed in the carton.

11. If red items MUST be placed in the carton before non-red items, which of the following cannot be true?

(A) Both K and M are red.
(B) There are two non-red items neither of which is I.
(C) Both H and I are red
(D) There are two non-red items neither of which is H.
(E) Both H and J are red

Answers and Solutions to Exercise

Questions 1-5

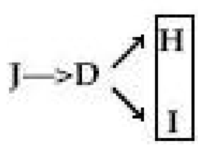
Since this game has no fixed elements (i.e., none of the heights of the children are given), a flow chart is indicated. The condition *"Jane is taller than Don"* can be symbolized as J—>D, where the arrow stands for *is taller than.* The condition *"Hank and Irene are the same height and both are shorter than Don"* can be symbolized as D—>(H=I). The condition *"Both Anna and Emily are taller than Gina"* can be symbolized as A/E—>G. The remaining conditions can be symbolized in like manner, giving the following schematic:

J—>D
D—>(H=I)
I—>B
A/E—>G
G—>C/F

Now we construct a flow chart from these conditions. D occurs in the top two conditions and G occurs in the bottom two conditions; this indicates that there will probably be two charts for this problem. Start with the first condition:

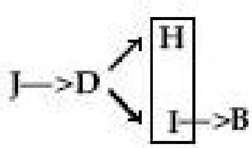
J—>D

Next, add the condition D—>(H=I):

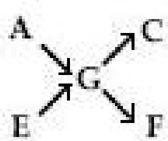


(The rectangle around H and I indicates that they are the same height.)

Then, add the condition I—>B:



The conditions A/E—>G and G—>C/F cannot be added to the diagram, so we build a separate diagram for them:



1. **Which one of the following can be false?**
   (A) Anna is taller than Frank.
   (B) Bob is shorter than Jane.
   (C) Jane is shorter than Emily.
   (D) Frank is shorter than Anna.
   (E) Anna is taller than Charles.

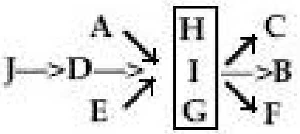
If two elements are not connected by a sequence of arrows or if they are in different diagrams, then their heights are independent of each other. Jane and Emily are in different diagrams, so either could be shorter than the other. The answer is (C).

2. **What is the minimum number of different heights the ten children can have?**
   (A) 3
   (B) 4
   (C) 5
   (D) 6
   (E) 7

The first diagram generates four different heights (Remember H and I are the same height). Since the second diagram is independent of the first, A, E, and J could be the same height, D and G could be the same height, and B, C, and F could be the same height. So the second diagram does not necessarily add any more distinct heights. The answer is (B).

3. **If Gina is the same height as Irene, then which one of the following can be false?**
   (A) Don is taller than Emily.
   (B) Jane is taller than Bob.
   (C) Emily is taller than Charles.
   (D) Gina is the same height as Hank.
   (E) Hank is taller than Frank.

The new condition combines the two original diagrams as follows:



Now since E and D are not connected by a sequence of arrows, either could be taller than the other. The answer is (A).

4. **Which of the following statements supplies information already contained in the original conditions?**
   I. Frank is shorter than Emily.
   II. Jane is taller than Gina.
   III. Charles is taller than Frank.

   (A) I only
   (B) II only
   (C) III only
   (D) I and III only
   (E) II and III only

From the second diagram, we see that Frank must be shorter than Emily (the arrows "flow" from Emily to Gina to Frank). Hence Statement I *is* contained in the original conditions. This eliminates (B), (C), and (E)—they don't contain Statement I. Note that only (A) and (D) remain and neither contains Statement II. Thus we need not check Statement II. As to Statement III, it is false. In the diagram, C and F are not connected by a sequence of arrows, so either could be taller than the other. Hence, by process of elimination, the answer is (A).

5. **Which one of the following children could NOT be the third tallest?**
   (A) Jane
   (B) Hank
   (C) Irene
   (D) Gina
   (E) Bob

From the first diagram, we see that J, D, H, and I are all taller than B. Therefore, B cannot be the third-tallest child. The answer is (E).

Questions 6-7

The condition *"Kohn's salary is greater than both Inman's and Lopez's"* starts the flow:

$$K \begin{array}{l} \nearrow I \\ \searrow L \end{array}$$

Next, add the condition *"Lopez's salary is greater than Nassar's"*:

$$K \begin{array}{l} \nearrow I \\ \searrow L \to N \end{array}$$

Adding the remaining conditions yields the following flow chart:

$$K \begin{array}{l} \nearrow I \to F \to M \to G \to J \to H \\ \searrow L \to N \end{array}$$

6. **If Nassar's salary is the same as that of one other partner of the firm, which one of the following must be false?**

   (A) Inman's salary is less than Lopez's.
   (B) Jacoby's salary is less than Lopez's.
   (C) Lopez's salary is less than Fox's.
   (D) Lopez's salary is less than Hae's.
   (E) Nassar's salary is less than Glassen's.

Suppose Nassar's salary were as low as possible. Then since his salary is the same as that of one other partner, he would have to have the same salary as does Hae. Adding this to the diagram gives

$$K \begin{array}{l} \nearrow I \to F \to M \to G \to J \searrow \\ \searrow \longrightarrow L \longrightarrow \nearrow \end{array} \boxed{\begin{array}{c} H \\ N \end{array}}$$

Even in this diagram, Lopez's salary is greater than Hae's. The answer is (D).

7. **What is the minimum number of different salaries earned by the nine partners of the firm?**

   (A) 5
   (B) 6
   (C) 7
   (D) 8
   (E) 9

The top row of the original chart shows that K, I, F, M, G, J, and H all earn different salaries. Now L could earn the same salary as I, and N could earn the same salary as F (since they're in unconnected rows). This yields the following diagram

$$K \to \boxed{\begin{array}{c} I \\ L \end{array}} \to \boxed{\begin{array}{c} F \\ N \end{array}} \to M \to G \to J \to H$$

This diagram clearly displays seven different salaries. The answer is (C).

Questions 8–11

The conditions of the game are naturally symbolized as follows:

1. M—>H
2. L—>H
3. H—>I
4. K—>I
5. ~(JK)
6. **Red items before non-red items (when possible)**

Combining conditions 1 and 3 yields

M—>H—>I

Adding conditions 2 and 4 to the diagram yields

K ↘
M—>H—>I
L ↗

8. **The items can be placed in the carton in which one of the following sequences?**

   (A) M, L, H, K, J, I
   (B) K, I, M, L, J, H
   (C) K, L, M, J, H, I
   (D) J, L, K, H, M, I
   (E) J, K, M, L, H, I

Choices (A) and (E) violate the condition ~(JK), eliminate. From the diagram, I cannot come before M, which eliminates choice (B). Choice (D) violates the condition M—>H, eliminate. Hence, by process of elimination, the answer is (C).

9. **If H and L are the only red items, then the items can be placed in the carton in which of the following sequences?**

   (A) L, H, M, K, I, J
   (B) L, M, K, H, I, J
   (C) L, M, H, K, I, J
   (D) L, M, H, J, I, K
   (E) L, J, M, H, K, I

Choice (A) violates the condition M—>H, eliminate. Choice (B) has K before H, which violates the condition **Red items before non-red items (when possible)**, eliminate. Choice (D) violates the condition K—>I, eliminate. Choice (E) has J before H, which violates the condition **Red items before non-red items (when possible)**, eliminate. Hence, by process of elimination, the answer is (C).

It is also instructive to solve this problem directly. Since L is red, it must come first; and the original diagram becomes

K ↘
L—>M—>H—>I

Since H is red, K must come after H:

L—>M—>H—>K—>I

Finally, since J is not red and cannot be next to K, J must be last:

L—>M—>H—>K—>I—>J

This uniquely determined order is choice (C).

10. **If M and I are the only red items, which of the following must be true?**

    (A) **J is the last item placed in the carton.**
    (B) **L is the second item placed in the carton.**
    (C) **K is the fourth item placed in the carton.**
    (D) **J is the fourth item placed in the carton.**
    (E) **I is the fourth item placed in the carton.**

Since M is red, the original diagram becomes

$$\begin{array}{l} \qquad\qquad\quad K\searrow \\ M \longrightarrow L \longrightarrow H \longrightarrow I \end{array}$$

Now, since I is red, J must be placed after I:

$$\begin{array}{l} \qquad\qquad\quad K\searrow \\ M \longrightarrow L \longrightarrow H \longrightarrow I \longrightarrow J \end{array}$$

Hence, J is last and the answer is (A). Note, since there is no sequence of arrows connecting K with either L or H, K can come before L, directly after L, or after H. Hence, choices (B) and (C) are not necessarily true.

11. **If red items MUST be placed in the carton before non-red items, which of the following cannot be true?**

    (A) **Both K and M are red.**
    (B) **There are two non-red items neither of which is I.**
    (C) **Both H and I are red**
    (D) **There are two non-red items neither of which is H.**
    (E) **Both H and J are red**

Begin with choice (A). Since both K and M are red, the original diagram becomes

$$\boxed{\begin{matrix} M \\ K \end{matrix}} \longrightarrow L \longrightarrow H \longrightarrow I$$

where the box around M and K indicates that they are first and second, not necessarily in that order. Now, J can be placed last in this diagram without violating any condition. Hence, choice (A) can be true, eliminate.

Turning to choice (B), I must be red since neither of the two non-red items is I. Now, the original diagram shows that M, H, K, and L all precede I and therefore must all be red. Hence, J and only J can be non-red. Therefore, choice (B) cannot be true and therefore is the answer.

Circular Ordering

We have thoroughly studied the ordering of elements in a straight line—the most common type of LSAT game. In the next most common type of ordering game the elements are placed around a circle—typically, people who are evenly spaced around a table. Circular diagrams have a few interesting properties not found in linear diagrams.

First, circular diagrams—unlike linear diagrams—are not fixed. That is, circular diagrams do not have a first, second, . . ., or last position. You can envision a circle as derived from a line by bending the line until the left end point (say, the first) and the right end point (say, the last) meet—forcing the first and last elements to become one and the same. Hence there is no beginning or end on a circle.

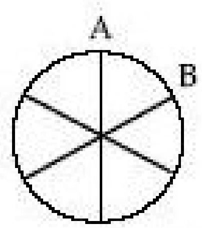
For this reason, you can initially place an element anywhere on the diagram—it can be fixed only in relation to other elements. It is conventional to place the first element at the top of the circle. Then place any additional elements (where applicable) to the left of it, clockwise around the circle.*

Next, although there is no first, second, etc., on a circle, there is left-right orientation (at least locally). So if a condition states that one element is next to another element but does not state whether it's to the left or the right, then two diagrams that are mirror images of each other will be possible.

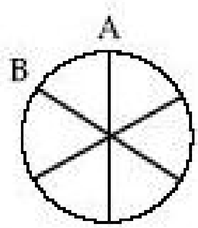
However, if there is no mention of the circle's orientation (left or right), then the mirror image of the diagram need not be considered.

For example, if it is given that A is next to B, and it is not specified whether A is to the left or right of B, then only one of the following two possible diagrams need be considered. They will generate the same answer to any question.

I



II



To see this more clearly, hold [illegible] the right of B. Clearly, during this process, the relationship between A and B (their relative position) did not change—only your perspective did. Thus Figure II is not fundamentally different from Figure I.

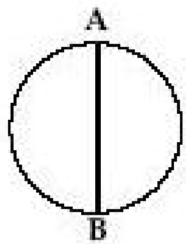
When you draw your circle, insert spokes. Invariably, circular games involve an even number of people (usually 6 or 8) spaced evenly around a circle. Therefore, a particular element will always be directly opposite another element. Drawing spokes inside the circle clarifies and highlights whether two elements are directly opposite each other, which often is a relevant issue.

Now that we have our circle drawn with spokes inserted, we come to **the decision**: which element(s) do we place on the diagram first.

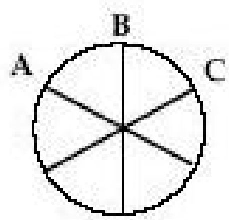
Always place elements whose positions are fixed relative to one another first.

Recall that with linear ordering games we first place any element whose position is fixed (first, second, last, etc.). Then we place any elements whose positions are fixed relative to one another (e.g., B comes after C). Circular diagrams, however, are not fixed. Hence the first step does not apply, and we start with the second step.

The relative position of elements around a circle can be fixed in either of two major ways. First, two elements can be directly opposite each other. This forms a **base axis**, which separates all the remaining elements to either side of it. **Place the base axis on your diagram first.** For example, if A is directly opposite B, then A and B form a base axis as follows:



Second, the elements can be immediately next to each other. This forms a **base group**. For example, if B is immediately between A and C, then we have the following base group:



Place the base group on your circle after you have placed the base axis. Place it first if there is no base axis.

Now let's apply these properties and strategies to a circular game of medium difficulty.

Circular Game

Six people—Alice, Bob, Carol, Dave, Emily, Frank—are seated evenly spaced around a circular table according to the following conditions:

> Alice does not sit next to Carol.
> Bob sits next to Carol or Dave.
> Frank sits next to Dave.
> If Emily sits next to Frank, then she does not sit next to Carol.

1. Of the following, which one is a possible seating arrangement of the six people?

   (A) Alice, Frank, Dave, Carol, Emily, Bob
   (B) Alice, Bob, Carol, Frank, Dave, Emily
   (C) Alice, Bob, Carol, Emily, Frank, Dave
   (D) Alice, Emily, Frank, Dave, Bob, Carol
   (E) Alice, Dave, Bob, Emily, Carol, Frank

2. If Bob is seated next to Frank, then in which one of the following pairs must the people be seated next to each other?

   (A) Alice and Emily
   (B) Bob and Dave
   (C) Bob and Emily
   (D) Carol and Dave
   (E) Carol and Frank

3. If Dave and Carol sit next to each other, then Alice could sit immediately between

   (A) Bob and Carol
   (B) Bob and Frank
   (C) Dave and Emily
   (D) Dave and Frank
   (E) Frank and Emily

4. If Bob sits next to Carol, then which one of the following is a complete and accurate list of the people who could also sit next to Bob?

   (A) Alice
   (B) Alice, Dave
   (C) Dave, Frank
   (D) Alice, Emily, Frank
   (E) Alice, Dave, Emily, Frank

5. Which of the following must be false if Bob sits next to Dave?

   I. Emily sits next to Frank.
   II. Carol sits directly opposite Bob.
   III. Carol sits immediately between Emily and Bob.

   (A) I only
   (B) III only
   (C) I and II only
   (D) II and III only
   (E) I, II, and III

6. If Alice sits next to Emily, then Bob CANNOT sit immediately between

   (A) Alice and Carol
   (B) Alice and Dave
   (C) Carol and Dave
   (D) Carol and Frank
   (E) Dave and Emily

As usual we construct a diagram to aid in answering the questions. First, translate the given conditions into symbols—abbreviating each name with its first letter.

The most concrete condition is *"Frank sits next to Dave"*; it is naturally symbolized as **FD**.

Next, we symbolize the second most concrete condition, *"Bob sits next to Carol or Dave,"* as **BC or BD**. The "or" in this symbol is inclusive. That is, it includes the case in which B sits next to both C and D—in other words, immediately between them. (Unless otherwise stated, the meaning of "or" is inclusive on the LSAT.)

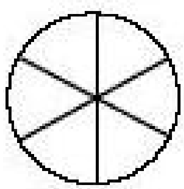
Next, *"Alice does not sit next to Carol"* is symbolized as **~(AC)**.

Finally, we come to the last and most complicated condition, *"If Emily sits next to Frank, then she does not sit next to Carol."* An initial symbol for this sentence might be **(EF)—>~(EC)**, where the arrow stands for "if..., then...." However, we can derive a more descriptive symbol as follows: If Emily were to sit next to both Frank and Carol, then she would be seated immediately between them. This is not allowed. Thus the more concise symbol **~(FEC)** is equivalent to our original symbol **(EF)—>~(EC)**.

We now have the following schematic for our conditions:

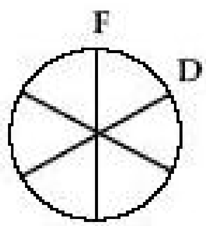
ABCDEF
FD
BC or BD
~(AC)
~(FEC)

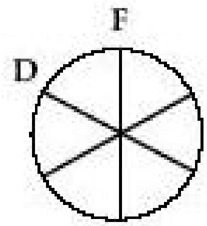
We need, however, a diagram to fill out our scheme. To this end, draw the following circle with spokes inside:



Next, following the strategies developed earlier, we scan the initial conditions for a base axis. There is none. So we look for a base group. The only condition that fixes the relative position of two of the elements is **FD**; it forms our base group.

Since circle diagrams are not fixed, we may initially place **FD** anywhere on the circle. Following convention, however, we put F at the top of the circle, creating the following two possible diagrams:

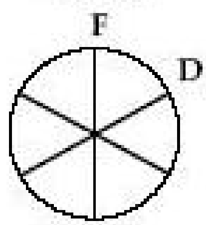




In this problem, there is no mention of the circle's orientation (left or right). We, therefore, need to consider only the first diagram, the other being the mirror image of it.

No other conditions fix the relative positions of the other elements, so our schematic is complete with diagram as follows:

ABCDEF
FD
BC or BD
~(AC)
~(FEC)



This schematic is self-contained. There is no need to refer to the original problem, which should be avoided whenever possible. Now we'll use this schematic to answer the questions.

1. **Of the following, which one is a possible seating arrangement of the six people?**

   (A) Alice, Frank, Dave, Carol, Emily, Bob
   (B) Alice, Bob, Carol, Frank, Dave, Emily
   (C) Alice, Bob, Carol, Emily, Frank, Dave
   (D) Alice, Emily, Frank, Dave, Bob, Carol
   (E) Alice, Dave, Bob, Emily, Carol, Frank

This is a straightforward elimination question. We merely take the initial conditions in succession and test them against each answer-choice, eliminating any answer-choices that do not satisfy the conditions. The last remaining answer-choice will be the answer.

Let's start the elimination process with the condition **FD**. All the answer-choices have F next to D except choice (E). This eliminates (E). Next, we use the condition **BC or BD**. Choices (B), (C), and (D) all satisfy this condition; (A) does not. This eliminates (A). Next, using the condition **~(AC)**, we eliminate choice (D), which has A next to C. Note: Since this is a circular ordering, the list A, E, F, D, B, C does not end at C (recall that there is no first or last on a circle).

Instead, the sequence returns to A and repeats the cycle. This is shown more clearly by the following "flow chart":

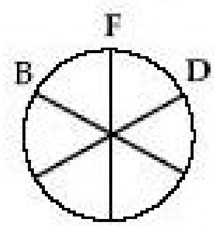
-> A-> E-> F-> D-> B-> C

Finally, choice (C) contradicts the condition ~(FEC). This eliminates (C). Hence, by process of elimination, the answer is (B)—the only answer-choice remaining.

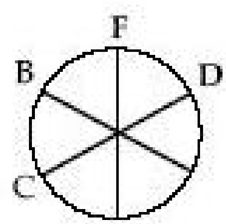
2. **If Bob is seated next to Frank, then in which one of the following pairs must the people be seated next to each other?**

   (A) Alice and Emily
   (B) Bob and Dave
   (C) Bob and Emily
   (D) Carol and Dave
   (E) Carol and Frank

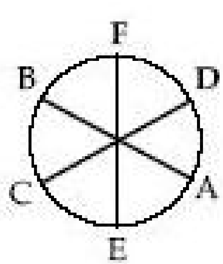
The new condition, *"Bob is seated next to Frank,"* is naturally symbolized as **BF**. Adding this condition to our original diagram gives



Next, from the condition **BC or BD**, we see that B must be next to C, as it is not next to D in the diagram. Our diagram, therefore, is as follows:



Finally, the condition ~(AC) forces A next to D (otherwise it would be next to C), which in turn forces E between A and C. Thus our uniquely determined diagram is



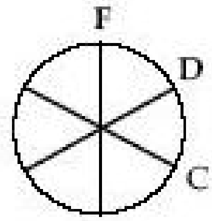
From this diagram, we see that A must sit next to E and therefore the answer is (A).

3. **If Dave and Carol sit next to each other, then Alice could sit immediately between**

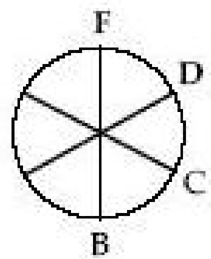
   (A) Bob and Carol
   (B) Bob and Frank
   (C) Dave and Emily
   (D) Dave and Frank
   (E) Frank and Emily

Remember that the questions in a game problem are independent of one another. So the condition **BF**, in Question 2, does not apply to this question.

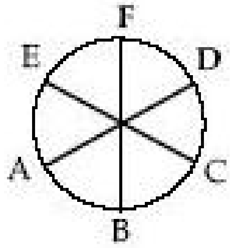
Begin by adding the new condition *"Dave sits next to Carol"*—DC—to the original diagram:

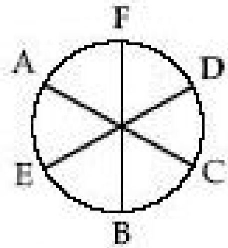


As in Question 2, the second condition, **BC or BD**, forces B next to C, and our diagram becomes



This diagram also satisfies the remaining initial conditions—**~(AC)** and **~(FEC)**. [Why?] Therefore the placement of A and E is arbitrary, and the following two diagrams are possible:





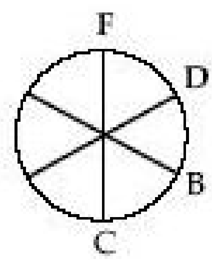
The second diagram satisfies choice (E). The answer, therefore, is (E).

4. **If Bob sits next to Carol, then which one of the following is a complete and accurate list of people who could also sit next to Bob?**

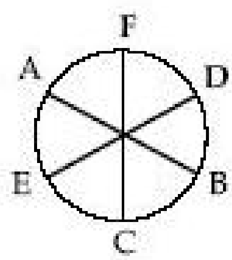
   (A) Alice
   (B) Alice, Dave
   (C) Dave, Frank
   (D) Alice, Emily, Frank
   (E) Alice, Dave, Emily, Frank

This question illustrates that during the test you should not erase previously derived diagrams, for we can use the diagrams derived in solving Questions 2 and 3 to help solve this question. (Note by "a complete and accurate list" the writers of the LSAT mean a list of all <u>possible</u> people, and <u>only</u> those people.)

Referring to the final diagram in Question 2, which has B seated next to C, we see that F *can* be next to B. This eliminates both (A) and (B)—they don't contain F. Next, referring to the final two diagrams in Question 3, we see that both A and E *can* sit next to B. This eliminates (C). Finally, we need to decide between choices (D) and (E). Choice (E) differs from choice (D) only in that it contains D. So we place D next to BC in our original diagram and then check whether this leads to a contradiction of the conditions:



Now if we place A next to F, and E next to C, then all the initial conditions are satisfied by the following diagram:



Hence it <u>is possible</u> for D to be next to B, and ADEF is therefore the complete and accurate list of people who can sit next to B. The answer is (E).

5. **Which of the following must be false if Bob sits next to Dave?**

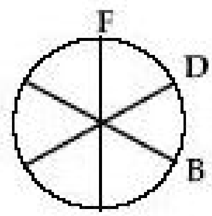
   I. **Emily sits next to Frank.**
   II. **Carol sits directly opposite Bob.**
   III. **Carol sits immediately between Emily and Bob.**

   (A) **I only**
   (B) **III only**
   (C) **I and II only**
   (D) **II and III only**
   (E) **I, II, and III**

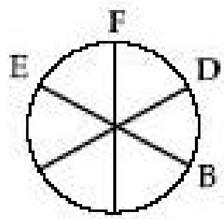
The most efficient way to solve triple-multiple-choice questions is to eliminate answer-choices as you check each sub-statement. Additionally, this method often gives a bonus: you may not need to check the final statement, which typically is the hardest. Even if you're not able to solve the problem, elimination allows you to make an educated guess. (Remember there is no guessing penalty on the LSAT.)

The logic of this question is convoluted because the correct answer will always make a false statement! This question would be much easier if it were worded, "Which of the following is possible?" (See Obfuscation.)

Let's begin our solution by adding the new condition *"Bob sits next to Dave"*—**BD**—to the original diagram:



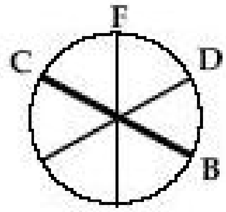
Next, test the first statement "Emily sits next to Frank"—**EF**. To this end, place it on the diagram as follows



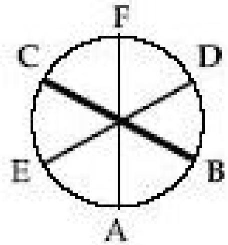
This diagram forces A and C next to each other, which violates the condition ~(AC). Hence Statement I is always false and therefore it is a correct choice.* This eliminates choices (B) and (D); they don't contain I.

Next, test the second statement "Carol sits directly opposite Bob," **C<—>B**, which forms a base axis. Place it on the original diagram as follows:

* Note: I used "correct" instead of "true," because in this context "true" would have been perfectly confusing.



Then placing A next to B—otherwise it would be next to C, which violates the condition ~(AC)—and placing E next to C gives



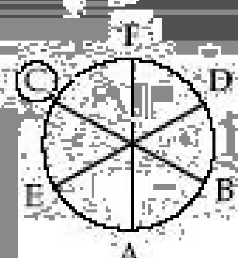
This diagram satisfies all the initial conditions, so it is true. Hence the statement [illegible] nates both (C) and (E), since they both contain II. By elimination, therefore, the answer is (A), and there is no need to check the third statement.

6. If Alice sits next to Emily, then Bob CANNOT sit immediately between

(A) Alice and Carol
(B) Alice and Dave
(C) Carol and Dave
(D) Carol and Frank
(E) Dave and Emily

This question is hard (or at least long) because there are many places where Alice and Emily may sit. However, the answers and diagrams we derived for previous questions will help here. The final diagram in Question 4 has B immediately between C and D. This eliminates choice (C). Furthermore, the final diagrams in Questions 2 and 3 have B immediately between C and F, and A and C, respectively. This eliminates choices (D) and (A). Now the question is not so daunting: we need only to decide between choices (B) and (E).

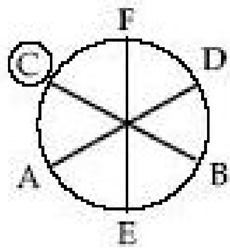
Let's test choice (B), first. If Bob sits immediately between Alice and Dave, i.e., **ABD**, then combining this condition with "Alice sits next to Emily," **AE**, generates the base group **EABD**. Adding this to the original diagram gives



(Note the circle around C indicates that it was forced into that position by the other conditions.)

This diagram satisfies all four of the initial conditions, which eliminates choice (B). Therefore, by the elimination method, we have learned that the answer is (E).

It is, however, instructive to verify that B cannot sit immediately between D and E. To this end, form the symbol **DBEA** and place it on the diagram as follows:



This diagram clearly violates the condition ~(AC).

Points to Remember

1. Circular games are, in general, harder than linear games.
2. Circular diagrams—unlike linear diagrams—are not fixed. That is, circular diagrams do not have a first, second, . . ., or last position. (But there is a left and right.)
3. If there is no mention of the circle's orientation (e.g., A is to the left of B), then the mirror-image diagram need not be considered. However, if a question asks for the number of distinct orderings, then you must count the mirror-image diagram.
4. Draw the circle with spokes inside.
5. Place the base axis on the diagram first. Then place the base group.
6. Left-right is taken from the perspective of the reader looking inward toward the center of the circle.
7. Counting problems are nearly always hard. Consider this when deciding whether to "skip" a question.

WARM-UP DRILLS

(Answers and solutions begin on page 137.)

Drill A

Four people—A, B, C, D—are seated, evenly spaced, around a circular table. For each of the following conditions, count the number of possible orderings of the people around the table.

1. B sits to the immediate left of A.

2. A sits next to B.

3. A sits opposite B.

Drill B

Six people are seated evenly spaced around a circular table. For each of the following pairs of conditions, count the number of possible orderings of the people around the table.

1. A sits directly opposite B.
   C sits to the immediate left of A.

2. A sits opposite B.
   C sits to the immediate left of A.
   If D sits between B and C, then E sits directly opposite D.

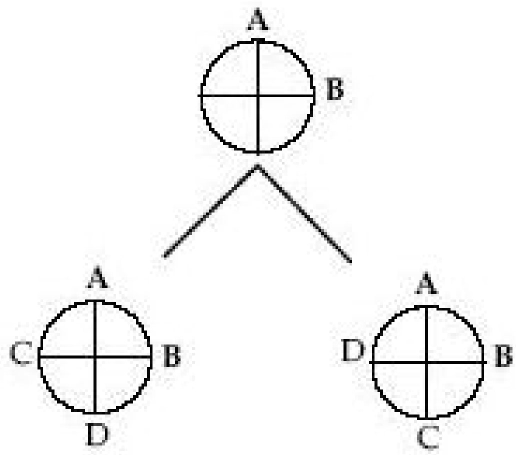
3. A sits opposite B.
   C sits to the immediate left of A.
   If D does not sit between B and C, then D sits next to E.

Answers and Solutions to Warm-Up Drills

Drill A

1. **2 orderings.**

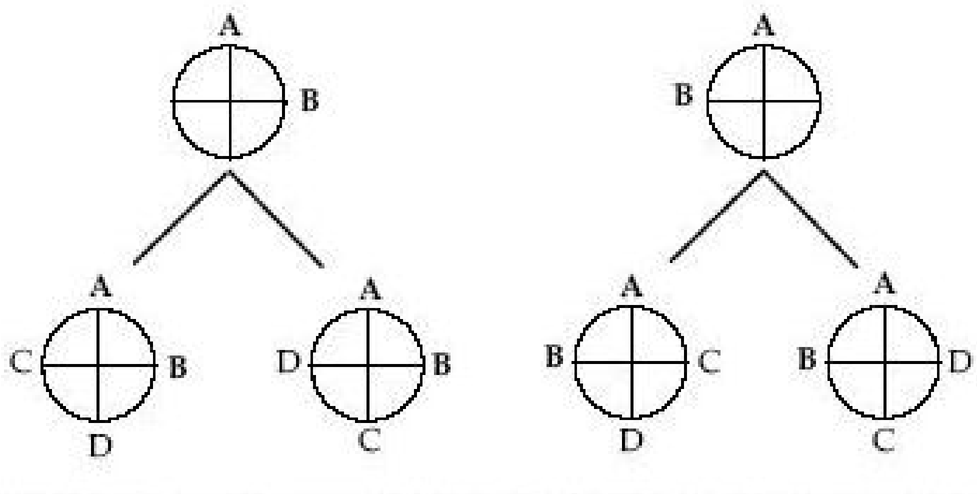
   Following convention, place A at the top of the circle and B to the immediate left of A. The following schematic "tree" shows the possible positions of C and D:



   From the bottom of the diagram, we see that there two possible orderings.

2. **4 orderings.**

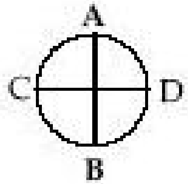
   We must now consider two "trees", one with B to the left of A and one with B to the right of A:

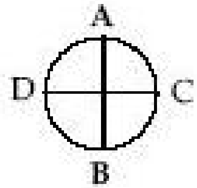


Counting the number of distinct orderings along the bottom of these "trees", we get 4.

3. **2 orderings.**

A and B form a base axis that forces C to one side and D to the other. Two distinct orderings are thereby formed as follows:

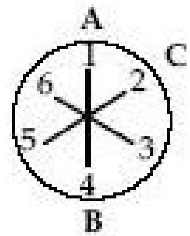




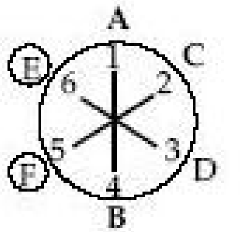
Drill B

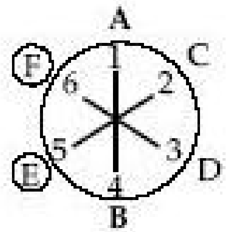
1. **6 orderings.**

The initial conditions generate the following diagram, which is numbered consecutively from 1 through 6 solely to facilitate the explanations that follow.



Next, placing D in position 3 generates the following two possible diagrams:

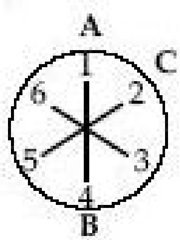




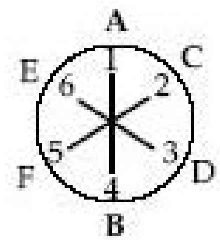
Finally, placing D in positions 5 and 6 will likewise generate two orderings each. So there is a total of 6 possible orderings.

2. **5 orderings.**

From the first two conditions, we get the same diagram as in Problem 1:



Next, if D occupies position 3, then the condition *"if D sits between B and C, then E sits directly opposite D"* forces F between E and B. Thus our diagram is as follows:



This diagram represents one ordering. But if D is in position 5 or 6, then the third condition does not apply. We get, in these cases, the same number of orderings as in Problem 1—four. So five distinct orderings are possible.

MENTOR EXERCISE

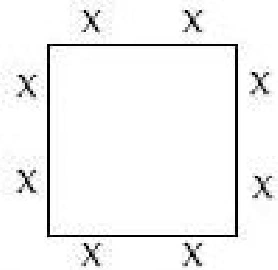
Directions: The following group of questions is based on a set of conditions. Choose the response that most accurately and completely answers each question. Hints, insights, and the answers are provided in the right-hand column.

Questions 1-6

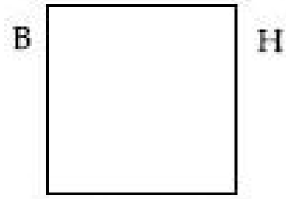
Eight people—Adam, Bob, Carrie, Dan, Elaine, Fred, Greg, Hans—are seated around a square table. There are two people to a side, and each person sits directly opposite another.

> Bob sits directly opposite Hans.
> Adam sits immediately between Greg and Fred.

Although this game involves a square, all the properties derived for circles still hold. The condition *"Bob sits directly opposite Hans"* is naturally symbolized as **B<—>H**, and the condition *"Adam sits immediately between Greg and Fred"* can be symbolized as **GAF** (The flip-flop symbol will not be explicitly written, just remember that G and F can be interchanged.) The diagram will consist of a square:



The condition **B<—>H** forms the base axis. Since the properties of circular diagrams hold for square diagrams, we can place the base axis anywhere on the diagram. Place it at the top as follows:

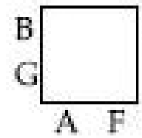


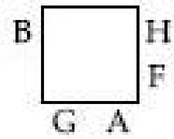
The base group **GAF** cannot be placed on the diagram at this time. Since there is no mention of orientation of the square, we don't have to consider the diagram's mirror image, i.e., the diagram obtained by interchanging B and H (See, however, Questions 4 and 5).

1. **If Elaine does not sit next to Bob, which one of the following must be true?**

   (A) Adam or Greg must sit next to Hans.
   (B) Greg must sit directly opposite Elaine.
   (C) Either Carrie or Dan must sit next to Bob.
   (D) Adam must sit next to Hans.
   (E) Fred must sit next to Bob.

1. There are two possible positions for the condition GAF:





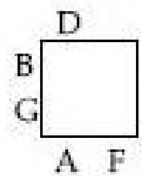
Note the flip-flop of G and F is not needed in this problem. Use these two diagrams to deduce that the answer is (C).

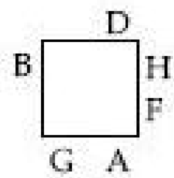
2. **If Dan sits directly across from Adam, and Elaine cannot sit next to Bob, then which of the following people could sit next to Bob?**

   I. Dan
   II. Carrie
   III. Fred

   (A) I only
   (B) II only
   (C) III only
   (D) I and II only
   (E) I and III only

2. There are two possible places for D:





We reject the second diagram since it forces E next to B, violating the supplementary condition ~(EB).

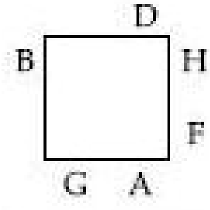
Be careful: The flip-flop of G and F is needed for this question.

The answer is (E).

3. **If Carrie does not sit next to Bob, which one of the following must be false?**

   (A) Hans sits next to Fred.
   (B) Adam sits opposite Carrie.
   (C) Greg sits next to Bob.
   (D) Dan sits to the immediate right of Hans.
   (E) Hans sits between and next to Dan and Fred.

3. If Hans sits between and next to Dan and Fred, then we get the following diagram:



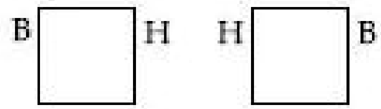
But this diagram forces C next to B, violating the supplementary condition ~(CB).

The answer is (E).

4. **If Greg sits to the immediate left of Adam, then which one of the following is possible?**

   (A) Adam sits next to Bob.
   (B) Hans sits next to Bob.
   (C) Adam sits next to Hans.
   (D) Fred sits next to Bob.
   (E) Bob sits directly opposite Carrie.

4. **Caution:** Since this question applies an orientation to the square, you <u>must</u> consider the mirror image diagram:



The answer is (D).

5. **If Dan sits next to Elaine who sits next to Hans, what is the maximum number of distinct arrangements of the eight people around the square?**

   (A) 2
   (B) 4
   (C) 6
   (D) 8
   (E) 10

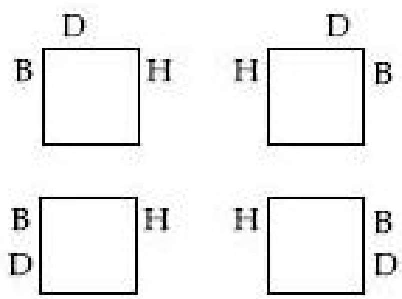
5. **Caution:** Although there is no mention of orientation in this question, you still must consider the mirror image diagram, since it is still a distinct order.

The answer is (D).

6. **If Greg sits directly opposite Dan, which one of the following is complete and accurate list of the people any one of whom could sit next to Bob?**

   (A) Greg
   (B) Greg, Fred, Dan
   (C) Greg, Fred, Elaine, Carrie
   (D) Greg, Fred, Dan, Carrie, Elaine
   (E) Greg, Fred, Dan, Carrie, Adam

6. There are four possible positions for D:



Use these diagrams to deduce that the answer is (D).

EXERCISE

Directions: The following group of questions is based on a set of conditions. Choose the response that most accurately and completely answers each question. Answers and solutions begin on page 145.

Four couples—the Potters, the Regans, the Stewarts, the Wilsons—are seated evenly spaced about a circular table.

The Regans do not sit next to the Stewarts.
The Stewarts sit next to each other.
Mr. Potter sits directly opposite Mr. Wilson.

1. Which of the following are possible?

   I. Mrs. Regan sits next to Mr. Potter.
   II. Mrs. Stewart sits next to Mr. Potter.
   III. Mrs. Potter sits between and next to Mr. Regan and Mrs. Wilson.

   (A) I only
   (B) II only
   (C) III only
   (D) I and II only
   (E) I and III only

2. If Mr. Regan sits midway between Mr. Potter and Mr. Wilson, then which of the following persons could sit directly across the table from Mr. Regan?

   (A) Mrs. Regan
   (B) Mrs. Potter
   (C) Mrs. Stewart
   (D) Mr. Wilson
   (E) Mrs. Wilson

3. If Mrs. Potter sits to the left of Mr. Stewart, then which one of the following is a complete and accurate list of the people any one of whom could sit next to Mr. Potter?

   (A) Mrs. Potter, Mrs. Regan
   (B) Mrs. Potter, Mr. Regan, Mrs. Wilson, Mrs. Stewart
   (C) Mrs. Regan, Mrs. Potter, Mr. Regan, Mrs. Wilson
   (D) Mrs. Regan, Mrs. Potter, Mr. Regan, Mrs. Wilson, Mrs. Stewart
   (E) Mrs. Regan, Mrs. Potter, Mr. Regan, Mrs. Wilson, Mrs. Stewart, Mr. Wilson

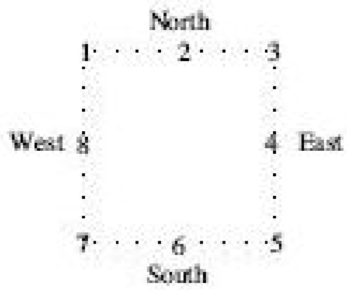
4. If Mr. Potter is between and next to Mr. Stewart and Mrs. Wilson, then how many different seating arrangements of the eight people are possible?

   (A) 1
   (B) 2
   (C) 3
   (D) 4
   (E) 5

The following game appeared on a recent LSAT.

Questions 5–10

A square parking lot has exactly eight lights numbered 1 through 8 situated along its perimeter as diagrammed below.



The lot must always be illuminated in such a way that the following specifications are met:

At least one of any three consecutively numbered lights is off.
Light 8 is on.
Neither light 2 nor light 7 is on when light 1 is on.
At least one of the three lights on each side is on.
If any side has exactly one of its three lights on, then that light is its center light.
Two of the lights on the north side are on.

5. Which one of the following could be a complete and accurate list of lights that are on together?
   (A) 1, 3, 5, 7
   (B) 2, 4, 6, 8
   (C) 2, 3, 5, 6, 8
   (D) 3, 4, 6, 7, 8
   (E) 1, 2, 4, 5, 6, 8

6. Which one of the following lights must be on?
   (A) light 2
   (B) light 3
   (C) light 4
   (D) light 5
   (E) light 6

7. If light 1 is off, which one of the following is a light that must also be off?
   (A) light 3
   (B) light 4
   (C) light 5
   (D) light 6
   (E) light 7

8. Which one of the following statements must be true?
   (A) If light 2 is on, then light 6 is off.
   (B) If light 3 is on, then light 2 is on.
   (C) If light 4 is on, then light 3 is off.
   (D) If light 5 is off, then light 4 is on.
   (E) If light 6 is off, then light 1 is on.

9. If light 5 is on, which one of the following could be true?
   (A) Light 1 is off and light 6 is off.
   (B) Light 1 is on and light 7 is on.
   (C) Light 2 is off and light 4 is on.
   (D) Light 2 is off and light 6 is off.
   (E) Light 6 is on and light 7 is on.

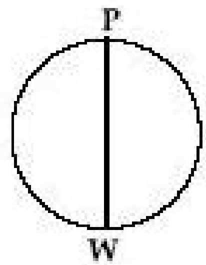
10. If light 4 is on, each of the following statements must be true EXCEPT:
    (A) Light 1 is on.
    (B) Light 2 is on.
    (C) Light 5 is off.
    (D) Light 6 is on.
    (E) Light 7 is off.

ANSWERS AND SOLUTIONS TO EXERCISE

This is a moderately hard game. Let's use the first letter of a name to denote the name, with bold letters denoting men and bold, shadow letters denoting women.

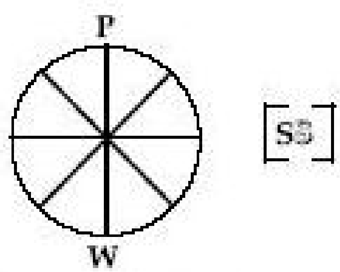
The most concrete condition is *"Mr. Potter sits directly opposite Mr. Wilson"*; it is naturally symbolized as **P<—>W**, where the arrow means "sits directly opposite." This forms a base axis. The next most concrete condition is *"the Stewarts sit next to each other"*; it is naturally symbolized as **S𝕊**. This forms a base group. Finally, the least concrete condition *"the Regans do not sit next to the Stewarts"* can be symbolized as ~(**Rℝ**/**S𝕊**).*

Now we must decide the order in which to place the conditions on the circle. Following the guidelines derived earlier, place the base axis first:



This axis separates the other elements to either side of it.

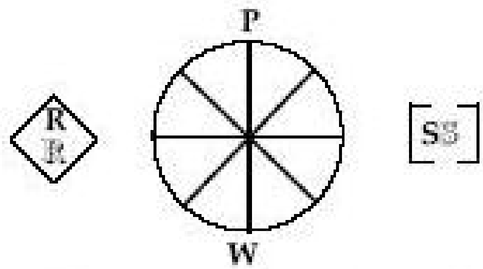
Next, we place the base group **S𝕊**. However, unlike the base axis, **P<—>W**, the position of this base group is not fixed—it may "float" about the circle. Furthermore, since there is no mention of the circle's orientation (that is, left or right), we place the Stewarts on only one side of the axis. Placing them on the other side of the axis will only generate a mirror-image diagram. (See, however, Questions 3 and 4.) Let's use brackets to indicate that the Stewarts sit <u>somewhere</u> on the right side of the circle:



Finally, the fact that the Regans do not sit next to the Stewarts forces the Regans to the other side of the circle, since on the right side of the base axis there are only three spaces between **P** and **W**, two of which are already taken by the Stewarts. The Regans, too, can "float" about their side of the circle. Furthermore, the Regans, unlike the Stewarts, do not necessarily sit next to each other—an im-

* Note: This is the best symbol that I could create, others such as ~(**RS**)/~(**R𝕊**)/~(**ℝS**)/~(**ℝ𝕊**) being too long and unwieldy. You may prefer a different symbol. Whatever symbol you choose is fine so long as it is short and functional.

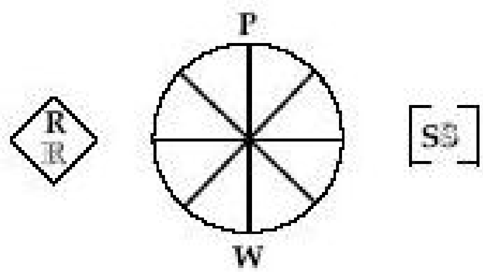
portant distinction for the questions that follow. We denote this in the diagram by writing **R** above ℝ as follows:



Note the elements ℙ and 𝕎 are independent because there are no direct conditions on them. **Remember independent elements can be placed in more positions than dependent elements. Think of independent elements as "wild cards".**

Our schematic with diagram is now compete:

P ℙ **R** ℝ **S** 𝕊 **W** 𝕎 (ℙ and 𝕎 are "wild")
P<—>W
S𝕊
~(**R**ℝ/**S**𝕊)

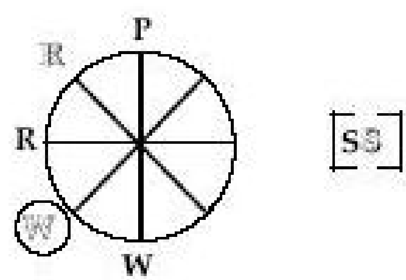


1. Which of the following are possible?

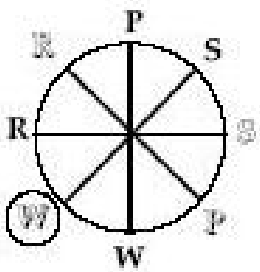
- **I. Mrs. Regan sits next to Mr. Potter.**
- **II. Mrs. Stewart sits next to Mr. Potter.**
- **III. Mrs. Potter sits between and next to Mr. Regan and Mrs. Wilson.**

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I and III only

First check sub-question I. Placing Mrs. Regan, ℝ next to Mr. Potter, **P**, and next to her husband, **R**, gives the following diagram

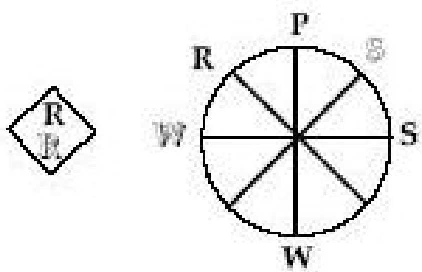


Now, place **S** next to **P** and next to his wife:

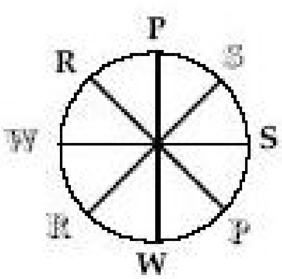


This diagram does not violate any of the initial conditions. Therefore, sub-question I <u>is</u> possible. This eliminates choices (B) and (C) since they do not contain I.

Next, to check sub-question II, place $\mathbb{S}$ next to **P**. Then place **R** between and next to **P** and $\mathbb{W}$. This gives the following diagram:



This diagram forces $\mathbb{R}$ and $\mathbb{P}$ to the left and right of **W**, respectively:

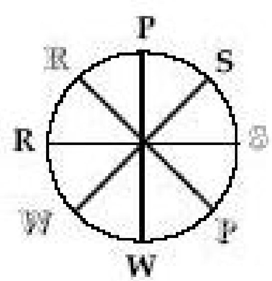


This final diagram does not violate any of the initial conditions. Therefore, sub-question II <u>is</u> possible, which eliminates (A) and (E) since they do not contain II. Hence, by process of elimination, the answer is (D). There is no need to check sub-question III.

2. **If Mr. Regan sits midway between Mr. Potter and Mr. Wilson, then which of the following persons could sit directly across the table from Mr. Regan?**

    (A) Mrs. Regan
    (B) Mrs. Potter
    (C) Mrs. Stewart
    (D) Mr. Wilson
    (E) Mrs. Wilson

The key to the solution of this problem is the condition **S**$\mathbb{S}$. Although the Stewarts may 'float' about their side of the table, they must always be next to each other—forcing one of them to sit midway between **P** and **W**. This means that one of the Stewarts will sit directly across the table from **R**. The following is one of many possible diagrams that do not violate any of the initial conditions:



The answer, therefore, is (C).

3. If Mrs. Potter sits to the left of Mr. Stewart, then which one of the following is a complete and accurate list of the people any one of whom could sit next to Mr. Potter?

(A) Mrs. Potter, Mrs. Regan
(B) Mrs. Potter, Mr. Regan, Mrs. Wilson, Mrs. Stewart
(C) Mrs. Regan, Mrs. Potter, Mr. Regan, Mrs. Wilson
(D) Mrs. Regan, Mrs. Potter, Mr. Regan, Mrs. Wilson, Mrs. Stewart
(E) Mrs. Regan, Mrs. Potter, Mr. Regan, Mrs. Wilson, Mrs. Stewart, Mr. Wilson

The new condition "Mrs. Potter sits to the left of Mr. Stewart" suggests the following diagrams:

Diagram I

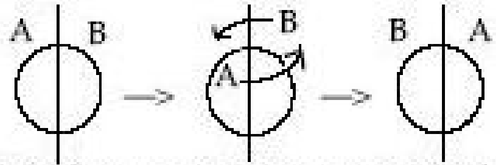
Diagram II

Advanced Concepts

If a diagram is the mirror image of another, then spinning the diagram 180 degrees about a base axis will create the mirror image diagram:

Figure I

Figure II



But if you spin Diagram I 180 degrees about the vertical axis **P**<—>**W**, then P will be in the position of S (not P) in Diagram II. The two diagrams, therefore, are not mirror images of each other.

Now, clearly in Diagram I besides S any one of R, **R**, or W could sit next to **P**. Similarly in Diagram II besides P any one of R, R, or W could sit

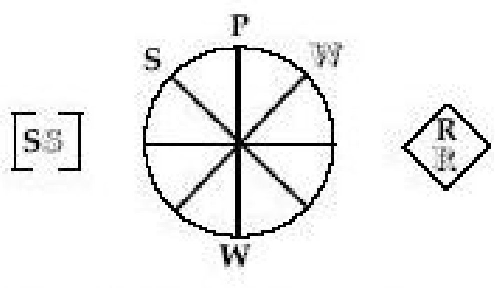
4. If Mr. Potter is between and next to Mr. Stewart and Mrs. Wilson, then how many different seating arrangements of the eight people are possible?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

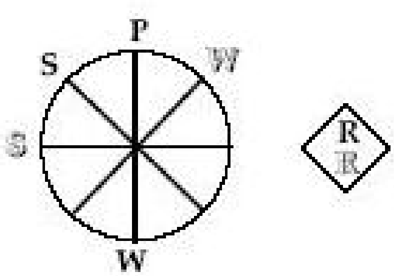
Counting problems, such as this one, are nearly always hard. Counting may have been one of man's first thought processes; nevertheless, counting possibilities is deceptively hard. Keep this in mind when deciding whether to skip a particular question.

This problem has the added subtlety that the mirror image of the diagram must be considered even though there is no mention of the circle's orientation.

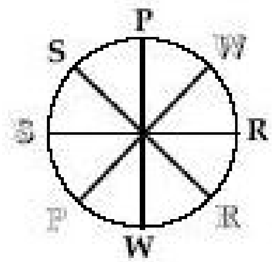
Let's start by adding the new condition SPW to the original diagram as follows:

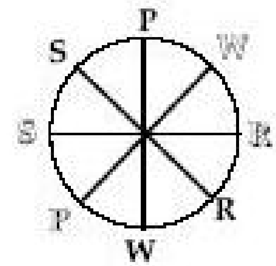


Next, the condition SS gives



Now P must be between S and W because both R and R are on the right. This suggests the following two valid diagrams—one for each of the two possible positions of R.





The mirror images of each of these two diagrams, however, must also be considered distinct orderings of the people around the table. Hence there are four possible seating arrangements of the eight people, and the answer is (D).

Note: Earlier it was stated that if a circular diagram does not have an orientation (left or right), then its mirror image need not be considered. This question, nevertheless, does not violate that rule. Although the mirror-image diagram will generate the same answer to any *relational* question, it is still a distinct ordering and therefore must be counted.

Questions 3 and 4 illustrate some of the subtleties that make circular ordering games, in general, harder than linear ordering games.

Questions 5–10

Adding the condition *"light 8 is on"* to the diagram gives

```
              North
        1 · · · · 2 · · · 3
        ·                 ·
        ·                 ·
West    8 on              4   East
        ·                 ·
        ·                 ·
        7 · · · · 6 · · · 5
              South
```

5. Which one of the following could be a complete and accurate list of lights that are on together?

(A) 1, 3, 5, 7
(B) 2, 4, 6, 8
(C) 2, 3, 5, 6, 8
(D) 3, 4, 6, 7, 8
(E) 1, 2, 4, 5, 6, 8

(A) No. This violates the condition *"Neither light 2 nor light 7 is on when light 1 is on."*
(B) No. This violates the condition *"Two of the lights on the north side are on."*
(C) **Yes.** Placing the information on the diagram yields

```
              North
        1 · · · · 2 · · · 3
        ·        on    on ·
        ·                 ·
West    8 on              4   East
        ·                 ·
        ·                 ·
                 on    on
        7 · · · · 6 · · · 5
              South
```

Since this is to be a complete list of the lights that could be on, the remaining lights must be off:

```
              North
        1 · · · · 2 · · · 3
        · off    on    on ·
        ·                 ·
West    8 on          off 4   East
        ·                 ·
        ·                 ·
          off    on    on
        7 · · · · 6 · · · 5
              South
```

This diagram does not violate any of the conditions: There are not three consecutively numbered lights on. Light 8 is on. Light 1 is off and therefore the condition *"Neither light 2 nor light 7 is on when light 1 is on"* does not apply. Each side has a light on. The west side has exactly one light on and it is the center light. Two lights on the north side, 2 and 3, are on.
(D) No. Just as in choice (B), this violates the condition *"Two of the lights on the north side are on."*
(E) No. This violates the condition *"At least one of any three consecutively numbered lights is off."*

6. Which one of the following lights must be on?

(A) light 2
(B) light 3
(C) light 4
(D) light 5
(E) light 6

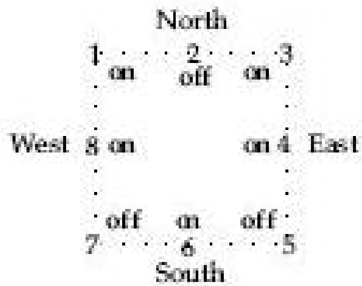
(A) No. The following diagram has light 2 off and does not violate any of the conditions:

```
              North
        1 · · · · 2 · · · 3
        · on     off   on ·
        ·                 ·
West    8 on          off 4   East
        ·                 ·
        ·                 ·
        · off    on    on ·
        7 · · · · 6 · · · 5
              South
```

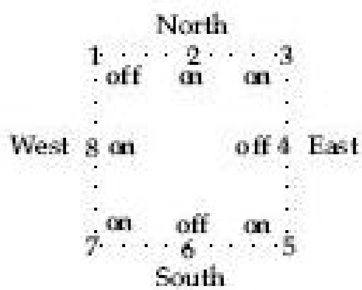
(B) Yes. Suppose light 3 is off. Then from the condition *"Two of the lights on the north side are on,"* we know that lights 1 and 2 must be on. However, this contradicts the condition *"Neither light 2 nor light 7 is on when light 1 is on."* Hence, light 3 must be on.

(C) No. The diagram for choice (A) also shows that light 4 need not be on.

(D) No. The following diagram has light 5 off and does not violate any of the conditions:



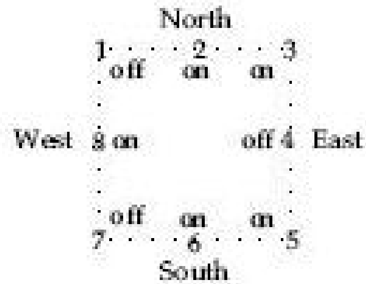
(E) No. The following diagram has light 6 off and does not violate any of the conditions:



7. If light 1 is off, which one of the following is a light that must also be off?

(A) light 3
(B) light 4
(C) light 5
(D) light 6
(E) light 7

(A) No. The following diagram has light 3 on and does not violate any of the conditions:

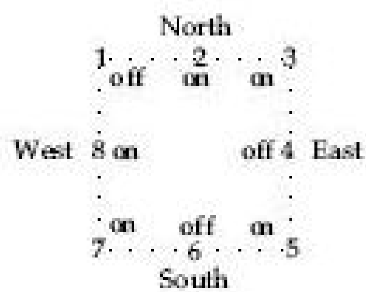


(B) Yes. Suppose light 4 is on. If light 1 is off, then from the condition *"Two of the lights on the north side are on"* we know that lights 2 and 3 must be on. This, however, has three consecutively numbered lights on—2, 3, and 4—contradicting the condition *"At least one of any three consecutively numbered lights is off."* Hence, light 4 must be off.

(C) No. The diagram for choice (A) has light 5 on.

(D) No. The diagram for choice (A) has light 6 on.

(E) No. The following diagram has light 7 on and does not violate any of the conditions:



8. Which one of the following statements must be true?

(A) If light 2 is on, then light 6 is off.
(B) If light 3 is on, then light 2 is on.
(C) If light 4 is on, then light 3 is off.
(D) If light 5 is off, then light 4 is on.
(E) If light 6 is off, then light 1 is on.

(A) No. The following diagram has lights 2 and 6 on and does not violate any of the conditions:

```
            North
      1 · · · 2 · · · 3
      · off   on   on ·
      ·               ·
West  8 on        off 4  East
      ·               ·
      · off   on   on ·
      7 · · · 6 · · · 5
            South
```

(B) No. The following diagram has light 3 on and light 2 off, and does not violate any of the conditions:

```
            North
      1 · · · 2 · · · 3
      · on   off   on ·
      ·               ·
West  8 on         on 4  East
      ·               ·
      · off   on  off ·
      7 · · · 6 · · · 5
            South
```

(C) No. The diagram for choice (B) has lights 4 and 3 on.

(D) Yes. Suppose light 5 is off. If light 4 is also off, then light 3 must be on since *"At least one of the three lights on each side is on."* However, light 3 is not the middle light, which contradicts the condition *"If any side has exactly one of its three lights on, then that light is its center light."*

(E) No. The following diagram has lights 6 and 1 off and does not violate any of the conditions:

```
            North
      1 · · · 2 · · · 3
      · off   on   on ·
      ·               ·
West  8 on        off 4  East
      ·               ·
      · on   off   on ·
      7 · · · 6 · · · 5
            South
```

9. If light 5 is on, which one of the following could be true?

(A) Light 1 is off and light 6 is off.
(B) Light 1 is on and light 7 is on.
(C) Light 2 is off and light 4 is on.
(D) Light 2 is off and light 6 is off.
(E) Light 6 is on and light 7 is on.

(A) Yes. Suppose lights 1 and 6 are off:

```
            North
      1 · · · 2 · · · 3
      · off           ·
      ·               ·
West  8 on            4  East
      ·               ·
      ·      off   on ·
      7 · · · 6 · · · 5
            South
```

Since two lights on the north side must be on, lights 2 and 3 must be on:

```
            North
      1 · · · 2 · · · 3
      · off   on   on ·
      ·               ·
West  8 on            4  East
      ·               ·
      ·      off   on ·
      7 · · · 6 · · · 5
            South
```

Since three consecutively numbered lights cannot be on, light 4 must be off:

```
           North
      1· · · · 2 · · · ·3
      : off    on    on :
      :                 :
West  8 on          off 4  East
      :                 :
      :      off    on  :
      7· · · · 6 · · · ·5
           South
```

Finally, light 7 must be on—otherwise on the south side only light 5 would be on, which would violate the condition *"If any side has exactly one of its three lights on, then that light is its center light."* This yields the following unique diagram, which does not violate any of the conditions:

```
           North
      1· · · · 2 · · · ·3
      : off    on    on :
      :                 :
West  8 on          off 4  East
      :                 :
      : on   off    on  :
      7· · · · 6 · · · ·5
           South
```

Thus, lights 1 and 6 can both be off.

(B) No. This violates the condition *"Neither light 2 nor light 7 is on when light 1 is on."*

(C) No. If light 2 is off, then from the condition *"Two of the lights on the north side are on"* lights 1 and 3 must be on. However, this scenario has three consecutively numbered lights on—3, 4, and 5—violating the condition *"At least one of any three consecutively numbered lights is off."*

(D) No. Since light 2 is off, the condition *"Two of the lights on the north side are on"* forces lights 1 and 3 to be on:

```
           North
      1· · · · 2 · · · ·3
      : on    off    on :
      :                 :
West  8 on              4  East
      :                 :
      :      off    on  :
      7· · · · 6 · · · ·5
           South
```

This in turn forces light 4 to be off—otherwise three consecutively numbered lights would be on: 3, 4, and 5.

```
           North
      1· · · · 2 · · · ·3
      : on    off    on :
      :                 :
West  8 on          off 4  East
      :                 :
      :      off    on  :
      7· · · · 6 · · · ·5
           South
```

Also, light 7 must be off since light 1 and light 7 cannot both be on :

```
           North
      1· · · · 2 · · · ·3
      : on    off    on :
      :                 :
West  8 on          off 4  East
      :                 :
      : off  off    on  :
      7────────6────────5
           South
```

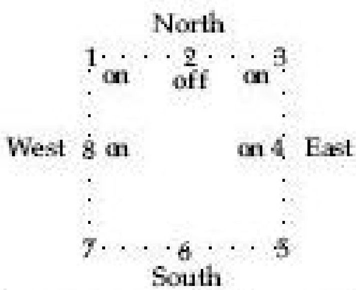
However, the south side of this diagram violates the condition *"If any side has exactly one of its three lights on, then that light is its center light."*

(E) No. This scenario has three consecutively numbered lights on—5, 6, and 7.

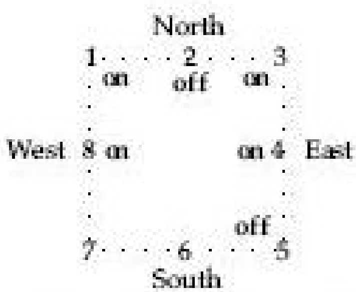
10. **If light 4 is on, each of the following statements must be true EXCEPT:**
    (A) Light 1 is on.
    (B) Light 2 is on.
    (C) Light 5 is off.
    (D) Light 6 is on.
    (E) Light 7 is off.

(A) No. Suppose light 1 is off. Then from the condition *"Two of the lights on the north side are on,"* lights 2 and 3 must be on. However, this scenario has three consecutively numbered lights on—2, 3, and 4.

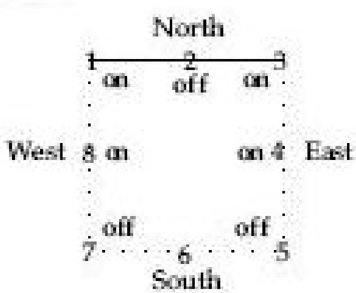
(B) Yes. Suppose light 2 is off. Then from the condition *"Two of the lights on the north side are on,"* lights 1 and 3 must be on:



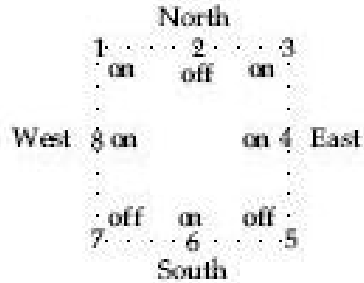
Since three consecutively numbered lights cannot be on, light 5 must be off:



Further, since light 1 is on, light 7 must be off:

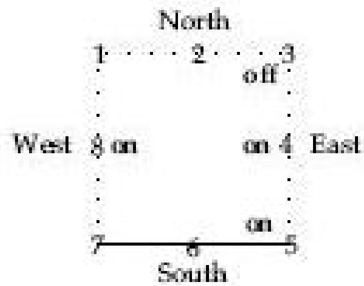


Finally, since *"At least one of the three lights on each side is on,"* light 6 is on:



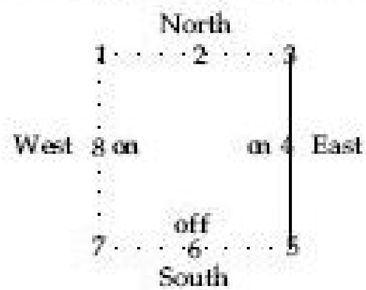
This diagram satisfies all the conditions. Hence, light 2 need not be on.

(C) No. Suppose light 5 is on. Since three consecutively numbered lights cannot be on, light 3 must be off:



Now, since two lights on the north side must be on, lights 1 and 2 must be on. However, this violates the condition *"Neither light 2 nor light 7 is on when light 1 is on."*

(D) No. Suppose light 6 is off:



CASE I: If light 1 is off, then from the condition *"Two of the lights on the north side are on"* lights 2 and 3 must be on. However, this scenario has three consecutively numbered lights on—2, 3, and 4.

CASE II: If light 1 is on, then light 7 must be off since *"neither light 2 nor light 7 is on when light 1 is on."*

```
            North
        1-----2-----3
        . on        .
        .           .
West    8 on     on 4   East
        .           .
        . off  off  .
        7 . . 6 . . 5
            South
```

Since *"At least one of the three lights on each side is on,"* light 5 must be on. However, this scenario has light 5 as the only light on the south side on, which violates the condition *"If any side has exactly one of its three lights on, then that light is its center light."*

(E) No. Suppose light 7 is on:

```
            North
        1 . . 2 . . 3
        .           .
        .           .
West    8 on     on 4   East
        .           .
        . on        .
        7 . . 6 . . 5
            South
```

Since *"neither light 2 nor light 7 is on when light 1 is on,"* light 1 must be off:

```
            North
        1 . . 2 . . 3
        . off       .
        .           .
West    8 on     on 4   East
        .           .
        . on        .
        7 . . 6 . . 5
            South
```

This diagram in turn forces lights 2 and 3 to be on since *"Two of the lights on the north side are on"*:

```
            North
        1 . . 2 . . 3
        . off  on  on .
        .           .
West    8 on     on 4   East
        .           .
        . on        .
        7 . . 6 . . 5
            South
```

However, this diagram has three consecutively numbered lights on—2, 3, and 4.

Generating Formulas

The previous linear ordering games we studied were static and finite. We were given a fixed number of elements and were asked questions about their possible orderings. Generating formulas, however, tend to be dynamic, in the sense that a basic sequence is given that is used to "generate" other sequences by repeated applications of the formulas. Because the formulas can be applied indefinitely, the sequences often have no end—though typically we are interested in only the beginning of the sequence.

Example:

A particular computer code uses only the letters A, B, C, and D. A "word" is formed in the code according to the following rules:

ABC is the basic word from which all other words are constructed.
D must appear in a word more than once, if at all.
Interchanging the first and last letters in a word creates a new word.
Adding a pair of Ds to the end of a word creates another word.

Notice that the third and fourth conditions are permissive. That is, they *could be* applied but don't have to be.

With permissive conditions, the contrapositive rule of logic does not apply.

The second condition, on the other hand, is mandatory: if D occurs in a word, it *must* occur at least once more.

With mandatory conditions, the contrapositive does apply.

There are only two basic types of questions to these games:

I. **Those that ask you to derive a new sequence from a basic sentence.** In the game above, for example, you may be given the word ABC and then asked to derive a new word by applying the fourth and third rules, in that order.

2. **Those that ask you to "discover" from where a sequence was derived.** In the game above, for example, you may be asked "From which word was the word DBCDA derived?"

The latter type of question tends to be more difficult since there are many paths you can retrace, only one of which will lead to the correct answer.

Because working backwards is often difficult, look for opportunities to reverse the direction by using the contrapositive. But apply the contrapositive only to *mandatory* conditions.

Generating-formula games are one of the few types of games for which it is not advisable to draw a diagram. In fact, typically you cannot draw a diagram. Nevertheless, you may want to symbolize the "rules" for easy reference.

Generating Formulas

In a secret code that uses only the letters A, B, C, and D, a word is formed by applying the following rules:

Rule 1: A B C D is the base word.
Rule 2: If C immediately follows B, then C can be moved to the front of the word.
Rule 3: One letter of the same type can be added immediately after an A, a B, or a C.

1. Which one of the following is not a code word?

(A) A B C D

4. If a fourth rule is added to the other three rules which states that whenever B or D ends a word the sequence obtained by dropping either B or D is still

1. Which one of the following is not a code word?

(A) A B C D
(B) D A B C
(C) C A B D
(D) A A B B C C D
(E) C C C A B D

Let's use elimination on this question. A B C D is the base word. So (A) is a word—eliminate (A). Applying Rule 2 to the base word gives C A B D. So (C) is a word—eliminate. Applying Rule 3 to the base word *three* times gives A A B B C C D. So (D) is a word—eliminate. Finally, applying Rule 2 to the base word gives C A B D; then applying Rule 3 to C *twice* gives C C C A B D. So (E) is a word—eliminate. Hence, by process of elimination, the answer is (B).

2. Which of the following letters can start a word?

I. A
II. B
III. C

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I and III only

A can start a word since it starts the base word. This eliminates choices (B) and (C) since they don't contain I. C can start a word since C A B D is formed from the base word by using Rule 2. This eliminates (A) and (D) since they don't contain III. Hence the answer is (E), and there is no need to check Statement II.

3. The word C A A B C C D can be formed from the base word by applying the rules in which one of the following orders?

(A) 22333
(B) 23232
(C) 32233
(D) 3223
(E) 3233

This question is hard, because we don't know to which letter(s) in the base word to apply the rules. Furthermore, there is more than one way to generate the word—but, of course, only one of those ways is listed as an answer-choice. We can, however, narrow the number of answer-choices by analyzing the word C A A B C C D. Notice that C occurs three times and A two times. So Rule 3 must have been applied three times, twice to C and once to A. This eliminates choices (B) and (D) since neither has three 3's. Next, turning to choice (A), we apply Rule 2 to the base word giving C A B D. Now Rule 2 cannot be applied to this word again, since C is not immediately after B. This eliminates (A). Next, choice (C) seems at first glance to be plausible. It begins the same way as does choice (E). But notice in choice (C) that Rule 2 is applied twice in a row. A little fiddling shows that if this is done, then two C's in a row would come at the beginning of the word. So eliminate (C). Hence, by process of elimination, the answer is (E). As a matter of test taking strategy this would be sufficient analysis of the question. However, it is instructive to verify that the answer is (E). To that end, apply Rule 3 to C in the base word A B C D which gives A B C C D. Next, apply Rule 2 which gives C A B C D. Finally, apply Rule 3 to A and then to C which gives C A A B C C D.

4. **If a fourth rule is added to the other three rules which states that whenever B or D ends a word the sequence obtained by dropping either B or D is still a word, then which of the following would be true?**

   I. Some words could end with A.
   II. Some words could start with C and end with C.
   III. A C D would be a word.

   (A) I only
   (B) II only
   (C) I and II only
   (D) I and III only
   (E) I, II, and III

Start with the base word A B C D. Applying the new rule gives A B C. Then applying Rule 2 gives C A B. Finally, applying the new rule again gives CA. Hence some words could end with A. So I is true. This eliminates choice (B). Next, starting again with the base word A B C D, apply Rule 3 to C which gives A B C C D. Then apply Rule 2 which gives C A B C D. Finally, apply the new rule which gives C A B C. So some words could start with C and end with C. This eliminates choices (A) and (D). Unfortunately, we have to check the third gives C A B C D. Finally applying the new rule gives C A B C D D C B A C. So eliminate (C). Finally, choice (D) is a word because it can be derived by applying Rule 2 to the base word which gives C A B D. Then applying the new rule gives C A B D D B A C. So eliminate (D). Thus, by process of elimination, the answer is (E).

Turning to the other method, we now show that (E) violates one of the rules. The only way that A can end a word is if the reversed sequence of a word is added to the word itself.* But D C B A is not the reversed sequence of C B C D, which isn't even a word.

Although the latter method was faster than deriving the four words, it can be deceptively hard to spot the choice that violates one or more of the rules.

Points to Remember

1. With permissive conditions, the contrapositive rule of logic does *not* apply.

2. With mandatory conditions, the contrapositive *does* apply.

3. There are only two basic types of questions to these games:

   I. Those that ask you to derive a new sequence from a basic sentence.

   II. Those that ask you to "discover" from where a sequence was derived.

4. Generating-formula games are one of the few types of games for which it is not advisable to draw a diagram. Nevertheless, you may want to symbolize the "rules" for easy reference.

* Recall that supplementary conditions apply only to the questions in which they are introduced. Hence the rule introduced in Question 4 no longer applies.

There will be no mentor exercise for this section.

Exercise

Directions: The following group of questions is based on a set of conditions. Choose the response that most accurately and completely answers each question. Answers and solutions begin on page 163.

Questions 1-4

In the game of Sense, a word is formed by combining the letters G, H, I, J, K:

G cannot be the first or last letter in a word.
I and J cannot be next to each other.
If J occurs in a word, then it occurs an odd number of times.
H cannot begin a word unless K ends the word.

1. Which one of the following is a word in the game of Sense?

(A) H J I K
(B) G
(C) I G J K H
(D) J H K J K
(E) H I K I

2. In the word □ I K J, which of the following could be placed in the box to make a word?

I. H
II. I
III. J

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) II and III only

3. Which one of the following is NOT a word but could become a word by adding one or more letters to its right end?

(A) G
(B) I J
(C) J
(D) H J J G
(E) H J G K

4. Of the following strings of letters, which can be made into words by adding a J and then reordering the letters?

I. G H J I J
II. I J
III. H K G

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) I, II, and III

Answers and Solutions to Exercise

We will not use a diagram to solve this game; however, we will symbolize the conditions for easy reference and to better understand them. The condition *"G cannot be the first or last letter in a word"* can be symbolized as G ≠ First/Last. The condition *"I and J cannot be next to each other"* is naturally symbolized as ~(IJ). The condition *"If J occurs in a word, then it occurs an odd number of times"* can be symbolized as J—>(J=odd#). Finally, the condition *"H cannot begin a word unless K ends the word"* can be symbolized as (H=First)—>(K=Last). Note that all the conditions in this game are mandatory, so we may safely apply the contrapositive to any of them. Summarizing the conditions yields the following schematic:

G ≠ First/Last
~(IJ)
J—>(J=odd#)
(H=First)—>(K=Last)

1. **Which one of the following is a word in the game of Sense?**

   (A) H J I K
   (B) G
   (C) I G J K H
   (D) J H K J K
   (E) H I K I

(A) is not a word because it violates the condition ~(IJ). (B) is not a word because it violates the condition G ≠ First/Last. If there is only one element in a word, then it both begins and ends the word. (C) is a word: G does not begin or end it; I is not next to J; J appears only once, which is an odd number of times; and the condition (H=First)—>(K=last) does not apply since H is not first. The answer is (C).

2. **In the word ☐ I K J, which of the following could be placed in the box to make a word?**

   I. H
   II. I
   III. J

   (A) I only
   (B) II only
   (C) III only
   (D) I and II only
   (E) II and III only

Start with H. Now, H I K J is not a word because H is first and K is not last. This eliminates (A) and (D). Next, place I in the box: I I K J. This sequences of letters satisfies all the conditions, so it is a word. This eliminates (C). Unfortunately, we have to check Statement III. Placing J in the box gives J I K J. This, however, is not a word since J appears an even number of times, which violates the condition J—>(J=odd#). The answer is (B).

3. **Which one of the following is NOT a word but could become a word by adding one or more letters to its right end?**

   (A) G
   (B) I J
   (C) J
   (D) H J J G
   (E) H J G K

G cannot be made into a word by adding letters to its right end because G can never be the first letter of a word. This eliminates (A). Next I J cannot be made into a word by adding letters to its right end because I can never be next to J in a word. Next, H J J G is not a word because H is first but K is not last; additionally, J appears twice. But if we add J and K, in that order, then there will be an odd number of Js and K will be the last letter, which makes H J J G J K a word. The answer is (D).

Don't make the mistake of choosing (C) or (E); both are already words.* Remember we are looking for a string of letters that is NOT a word but can be made into one by adding one or more letters to its right end.

4. **Of the following strings of letters, which can be made into words by adding a J and then reordering the letters?**

   I. G H J I J
   II. I J
   III. H K G

   (A) I only
   (B) II only
   (C) I and II only
   (D) I and III only
   (E) I, II, and III

G H J I J can be made into a word by first adding another J, which yields three Js (an odd number). Then moving I to the left of G. Next, I J cannot be made into a word by adding another J because that would result in two Js (an even number). Finally, H K G can be made into a word: First, add J to the right of G. Then flip-flop H and K, which yields K H G J. The answer is (D).

* J is a word because it appears an odd number of times (once) and none of the other conditions are violated.

Paths and Flow Charts

Although flow charts and paths are not, strictly speaking, ordering games, they have many of the properties found in ordering games.* Thus it is natural to analyze them here.

Flow charts and paths tend to be highly determinative. Once the chart has been constructed, the questions typically can be answered with little additional thought—often all the answers can be discerned by merely reading the chart.

The catch is that the chart may not be easy to derive. Because this type of game typically has many conditions, the chart can easily get out of control. Charting is an art. However, there are some guidelines that help:

1. **Look for a condition that starts the "flow" or that contains a lot of information.**
2. **Look for an element that occurs in many conditions.**
3. **Keep the chart flexible; it will probably have to evolve with the changing conditions.**

Before we start, we need to address some of the hazards and symbols common to these games. Because flow charts and paths involve a "flowing" of information, the *if-then* symbol, —>, is the workhorse for these games. Because the information can often "flow" in both directions, the symbol "<—>" also comes into play. A slash through a symbol indicates that information cannot flow in that direction. For example: A-/->B means information cannot flow from A to B.

As you work through these games be alert to any opportunity to apply the contrapositive rule of logic. Often negative conditions can be expressed more clearly by rewording them in the contrapositive. For example, the statement

"if it is not sunny, then Biff is not going to the beach"

can be reworded more directly as

"if Biff is going to the beach, then it is sunny."

It is not necessary that both parts of the *if-then* statement be negative for this technique to be effective. For example, the statement *"if Linda is hired, then Roland is not"* can be recast as *"if Roland is hired, then Linda is not."* Although in this case the contrapositive statement is no simpler than the original, it may, and often does, open up connections to other conditions.

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* In fact, as we saw with sequential ordering, many games can be solved more easily and more efficiently using flow charts.

We need to review two common fallacies associated with the contrapositive. From the statement "if A, then B" we can conclude, using the contrapositive, "if not B, then not A." It would be fallacious, however, to conclude either *"if not A, then not B"* or *"if B, then A."* Also note that *some* means "at least one and perhaps all."

Until this point, our discussion has been out of chronological order: we have discussed how to solve flow and path games without discussing how to identify them. Path games are easy to identify; typically they involve the actual movement of an element or of information. Some examples are

- ❑ Four cities are connected by six roads.
- ❑ A memo can be passed from Sara to Helen, but not from Sara to John.
- ❑ If a litigant filed his case in federal court and lost, then he may appeal to the 4th District Court and from there to the Supreme Court.

Flow charts are harder to identify than paths. In fact, they can be quite cryptic. However, a game with many *if-then* conditions is often a tip-off to a flow-chart game. Unfortunately, the *if-then* thought is often embedded in other equivalent structures. For example, the sentence "All A's are B's" can be reworded as "If x is an A, then x is a B." For a more subtle example take the sentence "Linda and Sara are not both hired"; it can be recast as "if Linda is hired, then Sara is not" (or "if Sara is hired, then Linda is not").

The following drill will help you identify embedded *if-then* statements.

If-then Drill

Directions: Translate each of the following conditions into an equivalent *if-then* statement. Answers and explanations are on page 172.

Condition	*If-then* form
1. No A is a B.	
2. Alice will go to the party only if Bobby goes.	
3. Anyone who is not an A cannot be a B.	
4. Only A's are B's.	
5. Of two light switches A and B, A and B cannot both be on.	
6. Of two light switches A and B: A is off, when B is off; A is on, when B is on.	

As you analyze a flow chart, look for "loops" that connect groups of elements. An example will illustrate:

Flow Chart

Six debutantes—Alison, Bridgette, Courtney, Dominique, Emily, Francine—meet at a party. During the time they have been at the party some girls have come to like certain other girls.

Amiable Alison likes every girl at the party.
Aloof, yet popular Bridgette likes no one at the party, but everyone likes her.
Courtney likes only two girls, one of whom is Dominique.
Dominique likes three girls, none of whom are Courtney or Francine.
Emily and Francine each like only one girl.

1. Of the following girls, who likes Emily?

    I. Alison
    II. Dominique
    III. Francine

    (A) I only
    (B) II only
    (C) III only
    (D) I and II only
    (E) I and III only

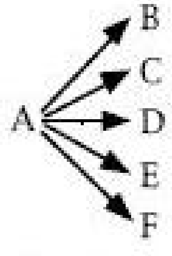
2. A "click" is a group of two or more girls who like one another. How many clicks are formed amongst the six girls?

    (A) 0
    (B) 1
    (C) 2
    (D) 3
    (E) 4

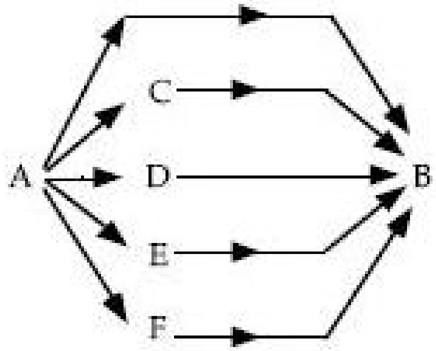
3. How many girls at the party like at least one girl whose feelings are not reciprocal?

    (A) 2
    (B) 3
    (C) 4
    (D) 5
    (E) 6

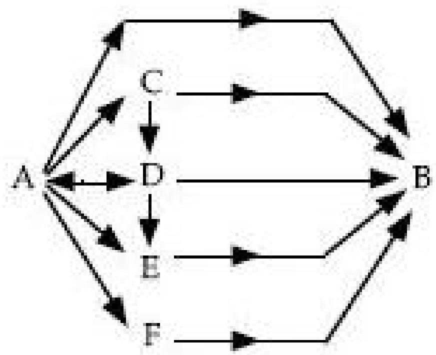
Alison likes every girl, so we start the "flow" with her:



Next, every girl likes Bridgette, but she does not like any of them. So we end the "flow" with Bridgette. (Note how the diagram evolves.)



Next, since Dominique likes three girls, two of whom are neither Courtney nor Francine, she must like both Alison and Emily, in addition to Bridgette. Adding this result plus the third condition, *"Courtney likes Dominique,"* to the diagram gives



Finally, since Emily and Francine each like only one girl and everyone likes Bridgette, Emily and Francine each must like Bridgette only. So there is nothing else to add to the diagram.

Note A,C,D forms a "loop", because from A the arrows can be followed all the way around the "loop" back to A. But A,D,E does not form a loop, because from A you cannot get back to A, whether you go first to D, or first to E.

1. **Of the following girls, who likes Emily?**

   I. Alison
   II. Dominique
   III. Francine

   (A) I only
   (B) II only
   (C) III only
   (D) I and II only

2. A "click" is a group of two or more girls who like one another. How many clicks are formed amongst the six girls?

   (A) 0
   (B) 1
   (C) 2
   (D) 3
   (E) 4

There is only one. In the chart, a two-way arrow connects A and D, so they form a click. The loop A,C,D does not form a click because it's not two-way: A likes C, but that feeling is not reciprocal. The answer is (B).

3. How many girls at the party like at least one girl whose feelings are not reciprocal?

   (A) 2
   (B) 3
   (C) 4
   (D) 5
   (E) 6

In the chart, there are 5 arrows pointing to B, so 5 girls like B. There are no arrows emanating from B, so none of those feelings are reciprocal. The answer is (D).

Flow-chart games can bring to the fore some subtle issues, as the following difficult game illustrates.

Circuit

Six lights—J, K, L, M, N, O—are connected in a circuit. Each light can be either on or off.

If K is on, L is off.
J and N cannot both be on.
M is off if and only if either J or N is on.
If O is on, N is on; if O is off, N is off.

1. How many lights in the circuit must be on?

   (A) 0
   (B) 1
   (C) 2
   (D) 3
   (E) 4

2. If J is on, which one of the following could be a complete and accurate list of the other lights that are on as well?

   (A) K and O
   (B) L and N
   (C) L and O
   (D) M, N, and K
   (E) L

To keep the notation simple, let the letter itself stand for *"the light is on,"* and place a tilde before the letter to indicate that the light is off. The condition *"If K is on, L is off"* is naturally symbolized as K—>~L. The condition *"J and N cannot* *both be on"* means that if one is on the other must be off: N—>~J.[*] The condition *"M is off if and only if either J or N is on"* is naturally symbolized as ~M<—>(J or N). Finally, the condition *"If O is on, N is on; if O is off, N is off"* means that O is on if and only if N is on: O<—>N.

Now we come to the crucial decision—with which condition should we start our chart? Following the guidelines on page 165, look for the element that occurs in the greatest number of conditions; it is N. Of the three conditions that contain N, the condition O<—>N is the most restrictive, so we start the flow with it:

O<—>N

Next, adding the condition N—>~J gives

O<—>N—>~J

Then, adding the condition ~M<—>(J or N) gives

O<—>N—>~J
↓
~M<—>(J or N)

Finally, the condition K—>~L is independent of the other conditions, so the flow chart consists of two distinct parts:

O<—>N—>~J
↓
~M<—>(J or N)

K—>~L

We will use only this chart to answer the following questions.

1. **How many lights in the circuit must be on?**

   (A) 0
   (B) 1
   (C) 2
   (D) 3
   (E) 4

Since this is a counting problem, we anticipate that it will be hard. It won't disappoint us.

From the bottom chart, we see that both K and L can be off. (The condition K—>~L states only that if K is on then L is off. It says nothing about the case when K is off, so both K and L *could be* off.) Furthermore, since the bottom chart is independent of the top chart, the status of K and L (whether on or off) does not affect the other lights. Next, from the top chart if both O and N are on, then both J and M must be off. Thus it is possible for only two lights, O and N, to be on, which eliminates (D) and (E). Next, we check the alternative circumstance where

---

[*] J—>~N would also suffice, but we don't need both.

both O and N are off. From the top chart, we see that if M is also off, then J must be on. While if M is on, J can not be on. Combining these cases (M *on* or *off*), we see that one and only one of M, J, and N must be on. Hence it is possible for only one element to be on, which eliminates (C). Finally, we check whether all the lights could be off. Again, the top chart shows that this is not possible: if M is off, then either N or J must be on, which eliminates (A)—at least one light must be on. The answer, therefore, is (B).

2. **If J is on, which one of the following could be a complete and accurate list of the other lights that are on as well?**

   (A) K and O
   (B) L and N
   (C) L and O
   (D) M, N, and K
   (E) L

Notice that (A), (B), (C), and (D) all contain either N or O, but not both. However, the condition O<—>N states that they are always on at the same time. This eliminates (A) through (D).

Points to Remember

1. When constructing a flow chart, use the following guidelines:

   (a) Look for a condition that starts the flow.
   (b) Look for an element that occurs in many conditions.
   (c) Keep the chart flexible.

2. Be aware of the fallacies associated with the contrapositive:

   From A—>B you can conclude

   ~B—>~A;

   but you <u>cannot</u> conclude either

   ~A—>~B, or B—>A.

3. *Some* means "at least one and perhaps <u>all</u>."

4. The following statements contain embedded *if-then* statements.

<u>Statement</u>	<u>*If-then* form</u>
All A's are B's.	If x is an A, then x is a B.
A and B are not both C's.	If A is a C, then B is not.
No A is a B.	If A, then not B.
Only A's are B's.	If B, then A.

Solutions to *If-then* Drill

Condition	*If-then form*
1. No A is a B.	If A, then not B.
2. Alice will go to the party only if Bobby goes.	If Alice goes, then Bobby goes.

Explanation: This common structure causes students much confusion. It states only that if Alice is at the party, then Bobby must also be at the party. (Note, this condition is not reciprocal; the statement "if Bobby is at the party, then Alice is also" is not necessarily true.)

3. Anyone who is not an A cannot be a B.	If not A, then not B.

Explanation: The contrapositive further simplifies this to "if B, then A."

4. Only As are Bs.	If B, then A.
5. Of two light switches A and B, A and B cannot both be on.	If A is on, then B is not.

Explanation: "If B is on, then A is not" will also suffice, but it is not necessary to state both—one is the contrapositive of the other.

6. Of two light switches A and B: A is off, when B is off; A is on, when B is on.	A is off if and only if B is off.

Explanation: "A is on if and only if B is on" will also suffice, but again it is not necessary to state both.

MENTOR EXERCISE

Directions: The following group of questions is based on a set of conditions. Choose the response that most accurately and completely answers each question. Hints, insights, partial solutions, and the answers are provided in the right-hand column.

Questions 1–6

Six people—Albert, Ben, Carrie, Darlene, Emily, and Fred—are competing in a gymnastics event. Two of them compete on the horse, two compete in the vault, and two compete on the parallel bars.

> Ben competes on the horse if and only if Carrie competes in the vault.
>
> If Darlene does not compete on the parallel bars, then Fred competes in the vault.
>
> If Emily competes in the vault, then Fred does not.

This is a rather hard game. Its underlying structure is actually simple, but there's lots of information to wade through. We start by symbolizing the conditions. We'll use an equal sign to indicate that a person competes in a particular event. The first condition, *"Ben competes on the horse if and only if Carrie competes in the vault,"* can be symbolized as (B=H)<—>(C=V). The second condition, *"If Darlene does not compete on the parallel bars, then Fred competes in the vault,"* can be symbolized as (D≠P)—>(F=V). This in turn can be recast, using the contrapositive, as (F≠V)—>(D=P). Finally, the condition *"If Emily competes in the vault, then Fred does not"* can be symbolized as (E=V)—>(F≠V). This gives the following schematic:

(B=H)<—>(C=V)
(F≠V)—>(D=P)
(E=V)—>(F≠V)

To start the flow chart, look for the element that occurs in the greatest number of conditions; it is F. So build the chart around it. Start with the third condition:

(E=V)—>(F≠V)

Next, add the second condition:

(E=V)—>(F≠V)—>(D=P)

Finally, the condition (B=H)<—>(C=V) cannot be added to the chart, so it forms an independent flow chart:

(E=V)—>(F≠V)—>(D=P)

(B=H)<—>(C=V)

Note that A is "wild" since it is not contained in the diagram.

1. If Ben competes on the horse, then which one of the following can be true?

   (A) Both Emily and Albert compete in the vault.
   (B) Emily competes on the horse and Darlene competes in the vault.
   (C) Darlene does not compete on the parallel bars and Albert competes in the vault.
   (D) Albert competes on the parallel bars and Carrie competes in the vault.
   (E) Albert competes on the horse and Darlene does not compete on the parallel bars.

1. Since Ben competes on the horse, we know from the bottom half of the chart that Carrie competes in the vault. Furthermore, if Darlene does not compete on the parallel bars, then applying the contrapositive to the top part of the chart, we see that Fred also competes in the vault. This fills both slots for the vault, so no one else can compete in that event. These restrictions are sufficient to eliminate choices (A), (B), (C), and (E).

The answer is (D).

2. If Darlene does not compete on the parallel bars, then which one of the following cannot be true?

   (A) Ben competes on the horse.
   (B) Fred competes in the vault.
   (C) Albert competes on the parallel bars.
   (D) Emily competes in the vault.
   (E) Both Ben and Fred compete in the vault.

2. Hint: Apply the contrapositive to the top half of the diagram.

The answer is (D).

3. If Ben and Carrie compete in the same event, then which one of the following can be true?

   I. Albert competes on the horse.
   II. Emily competes in the vault.
   III. Darlene does not compete on the parallel bars.

   (A) I only
   (B) II only
   (C) III only
   (D) I and III only
   (E) I, II, and III

3. From the condition (B=H)<—>(C=V), we know that Ben and Carrie must both compete on the parallel bars. (Why?) As to I, since Albert is an independent element, we intuitively expect that he could compete on the horse, but you should verify this. As to II, if Emily competes in the vault, then from the diagram Darlene must compete on the parallel bars. This, however, puts three people in the parallel bar event, contradicting the condition that there are two people in each event. As to III, if Darlene does not compete on the parallel bars, then applying the contrapositive to the top diagram shows that Fred must compete in the vault, and Emily cannot compete in the vault. Now it's easy to work out a schedule with these restrictions.

The answer is (D).

4. Suppose the condition "If Carrie does not compete in the vault, then Emily does" is added to the given conditions. Which one of the following cannot be true if Emily and Darlene do not compete in the same event?

   (A) Ben does not compete on the horse and Darlene does.
   (B) Fred competes on the horse.
   (C) Ben does not compete on the horse and Darlene competes on the parallel bars.
   (D) Albert competes on the parallel bars.
   (E) Emily competes in the vault.

4. This question is difficult because there are six different ways to assign different events to Carrie and Emily. Additionally, the string of inferences needed to answer the question is quite long.

To begin, add the new condition (C≠V)—>(E=V) to the diagram:

(C≠V)—>(E=V)—>(F≠V)—>(D=P)

(B=H)<—>(C=V)

Now assume that Ben does not compete on the horse and Darlene does, choice (A). Then use the above diagram to derive a contradiction—namely that Darlene also competes on the parallel bars.

The answer is (A).

5. If Darlene competes in the vault, then how many different people could possibly compete on the horse?

   (A) 2
   (B) 3
   (C) 4
   (D) 5
   (E) 6

5. Applying the contrapositive, along with the new condition *"Darlene competes in the vault"* to the original diagram, shows that Fred competes in the vault and Emily does not. Now Ben cannot compete on the horse. (Why?) From these conditions you should be able to work out three valid schedules.

The answer is (B).

6. Suppose the condition "if Fred does not compete in the vault, then Emily does" is added to the original conditions. Of the following, which one cannot be true?

   (A) Ben competes on the horse and Albert competes in the vault.
   (B) Ben competes on the horse and Emily competes in the vault.
   (C) Darlene competes on the parallel bars.
   (D) Albert competes on the parallel bars.
   (E) Fred competes on the parallel bars and Albert competes on the horse.

6. This question is hard, or at least long, because it actually contains five questions. The new condition changes the diagram only slightly:

(E=V)<—>(F≠V)—>(D=P)

(B=H)<—>(C=V)

Start with (A). If Ben competes on the horse, then from the bottom half of the new diagram Carrie must compete on the vault along with Albert. Turning to the top diagram, clearly Emily cannot compete in the vault, since that would put three people—Emily, Albert, and Carrie—in one event. But if Emily does not compete on the vault, then again from the top diagram and the contrapositive Fred must compete in the vault, which leads to the same contradiction.

The answer is (A).

EXERCISE

Directions: The following group of questions is based on a set of conditions. Choose the response that most accurately and completely answers each question. Answers and solutions begin on page 180.

Questions 1–5

Seven small towns—H, I, J, K, L, M, N—are serviced by three roads—Routes 1, 2, and 3.

Route 1 ends at N and L, and passes through M only.
Route 2 starts at H. Then passes through L and I, and ends back at H.
Route 3 ends at N and K, and passes through J only.

Two towns are directly connected if a person can drive from one of the towns to the other without passing through any other town.

1. Of the following towns, which one has the greatest number of direct connections?

   (A) H
   (B) N
   (C) J
   (D) K
   (E) L

2. Which one of the following towns has the fewest number of connections, direct or otherwise, with the other towns?

   (A) H
   (B) K
   (C) N
   (D) L
   (E) I

3. If a new road were built directly connecting I and J, and if all the direct connections between the towns were of equal distance, then in which one of the following pairs of towns is neither town on the shortest route connecting L and K?

   (A) J, H
   (B) H, I
   (C) I, J
   (D) N, I
   (E) M, N

4. If a new road were built directly connecting H and K, then the maximum number of paths connecting L to J, which do not pass through any town more than once, would be

   (A) 1
   (B) 2
   (C) 3
   (D) 4
   (E) 5

5. If two new roads were built directly connecting L to J and K to H, then which one of the following is NOT a complete and accurate list of the towns through which a person could pass on a single trip from L to H?

   (A) I
   (B) J, K
   (C) M, N, J, K
   (D) No town
   (E) M, N, K

Questions 6–7

Six lights—J, K, L, M, N, and O—are connected in a circuit. Each light can be either on or off.

If K is on, N is on.
J and N cannot both be on.
If O is on, N is on; if O is off, N is off.

6. If K is on, which one of the following must be true?

   (A) L is on.
   (B) L is off.
   (C) J is off.
   (D) M is on.
   (E) N is off.

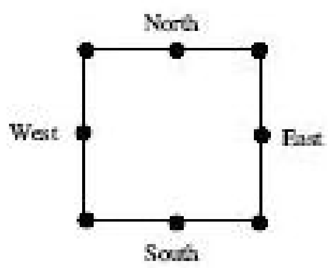
7. If J is on, what is the maximum number of lights that can be off?

   (A) 1
   (B) 2
   (C) 3
   (D) 4
   (E) 5

The next three games appeared on recent LSATs.

Questions 8–9

Eight benches—J, K, L, T, U, X, Y, and Z—are arranged along the perimeter of a park as shown below:



The following is true:

J, K, and L are green; T and U are red; X, Y, and Z are pink.
The green benches stand next to one another along the park's perimeter.
The pink benches stand next to one another along the park's perimeter.
No green bench stands next to a pink bench.
The bench on the southeast corner is T.
J stands at the center of the park's north side.
If T stands next to X, then T does not also stand next to L.

8. For which one of the following benches are there two and no more than two locations either one of which could be the location the bench occupies?

   (A) K
   (B) T
   (C) X
   (D) Y
   (E) Z

9. If Z is directly north of Y, which one of the following statements must be true?

   (A) J is directly west of K.
   (B) K is directly east of U.
   (C) U is directly north of X.
   (D) X is directly south of J.
   (E) Z is directly south of J.

Questions 10–11

Six people—Julio, Kevin, May, Norma, Olivia, and Tamio—participate in a track meet. Two of them enter the marathon, two enter the relay, and two enter the sprint.

Each participant enters only one event.
If Kevin enters the marathon, then both Julio and May enter the relay, and if both Julio and May enter the relay, then Kevin enters the marathon.
If Norma does not enter the sprint, then Tamio enters the relay.
If Olivia enters the relay, then Julio does not enter the relay.
Olivia and Tamio do not both enter the relay.

10. If Kevin enters the marathon, then which one of the following cannot be true?

    (A) Julio enters the relay and Norma enters the sprint.
    (B) Olivia enters the marathon and Norma enters the sprint.
    (C) Tamio enters the marathon and Olivia enters the sprint.
    (D) Tamio enters the relay and Olivia enters the sprint.
    (E) Tamio enters the marathon and May enters the relay.

11. If both Kevin and Olivia enter the relay, then which one of the following must be true?

    (A) Julio and Tamio enter the marathon.
    (B) May enters the marathon.
    (C) May enters the sprint.
    (D) Tamio enters the sprint.
    (E) Norma enters the sprint.

Questions 12–13

A lake contains exactly five islands—J, K, L, M, O—which are unconnected by bridges. Contractors will build a network of bridges that satisfies the following specifications:

Each bridge directly connects exactly two islands with each other, and no two bridges intersect.
No more than one bridge directly connects any two islands.
No island has more than three bridges that directly connect it with other islands.
J, K, and L are each directly connected by bridge with one or both of M and O.
J is directly connected by bridge with exactly two islands.
K is directly connected by bridge with exactly one island.
A bridge directly connects J with O, and a bridge directly connects M with O.

12. If a bridge directly connects K with O, then which one of the following could be true?

    (A) No bridge directly connects L with M.
    (B) A bridge directly connects J with L.
    (C) A bridge directly connects L with O.
    (D) There are exactly three bridges directly connecting L with other islands.
    (E) There are exactly two bridges directly connecting O with other islands.

13. If no island that is directly connected by bridge with M is also directly connected by bridge with O, then there must be a bridge directly connecting

    (A) J with L
    (B) J with M
    (C) K with O
    (D) L with M
    (E) L with O

Questions 14–17

There are five employees—G, H, I, J, and K—in an office. Rumors spread through the office according to the following rules.

Rumors can pass from G to H, but not vice versa.
Rumors can pass from G to I, but not vice versa.
Rumors can pass from G to J, but not vice versa.
Rumors can pass in either direction between H and I.
Rumors can pass from H to J, but not vice versa.
Rumors can pass from J to I, but not vice versa.
Rumors can pass in either direction between J and K.

A direct one-way path from one person to another is called a segment.

14. A rumor begun by I that reaches K will be known by all the following employees EXCEPT:
    (A) I
    (B) G
    (C) K
    (D) H
    (E) J

15. If all segments have the same length and if rumors always follow the shortest path, then the longest path any rumor follows in the system is the path from
    (A) H to J
    (B) J to K
    (C) H to G
    (D) I to J
    (E) I to K

16. Which of the following is a complete and accurate list of the people to whom a rumor can be spread along exactly one segment from H?
    (A) I
    (B) J
    (C) I, J
    (D) I, J, K
    (E) I, J, G

17. If a two-way segment from K to G is added to the rumor mill, which of the following segments would have to be added to the system so that each person could spread a rumor directly to at least two other people and receive a rumor directly from at least two other people?
    (A) I to J
    (B) H to G
    (C) K to H
    (D) I to G
    (E) J to H

Questions 18–19

A Hollywood production company is hiring six people—John, Kent, Mary, Nora, Olivia, and Tom. Two of them are hired as editors, two as gaffers, and two as actors.

Each person is hired for only one job.
If Kent is hired as an editor, then both John and Mary are hired as gaffers, and if both John and Mary hired as gaffers, then Kent is hired as an editor.
If Nora is not hired as an actor, then Tom is hired as a gaffer.

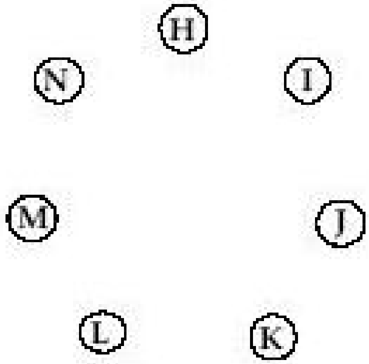
18. If Kent is hired as an editor, then which one of the following cannot be true?
    (A) Nora and Tom are hired as actors.
    (B) Olivia is hired as an actor, and Tom as an editor.
    (C) Olivia and Nora are hired as actors.
    (D) Tom is hired as a gaffer and Olivia is hired as an actor.
    (E) Neither John nor Mary are hired as editors.

19. If both Kent and Olivia are hired as gaffers, then which one of the following must be true?
    (A) Nora is hired as an editor.
    (B) Tom is hired as an actor.
    (C) John is hired as an actor.
    (D) John and Tom are hired as editors.
    (E) Nora is hired as an actor.

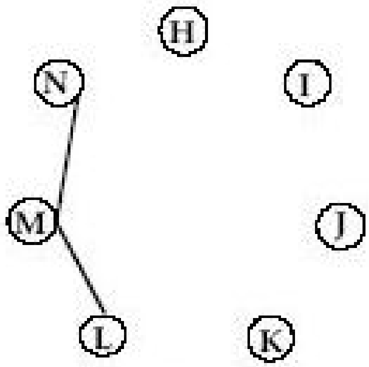
Answers and Solutions to Exercise

Questions 1–5

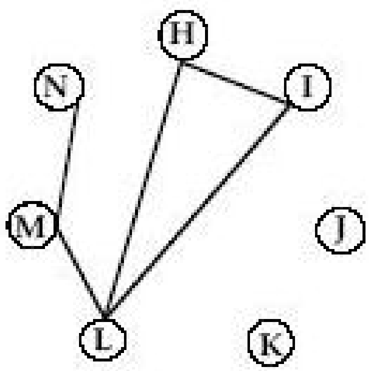
Diagramming makes this a fairly easy game. Start by placing the seven towns in a circle:*



Next, add Route 1 to the diagram:

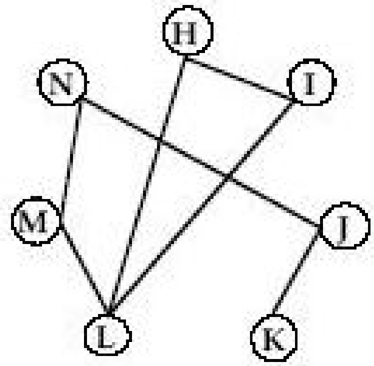


Then, add Route 2:



Finally, add Route 3:

* This is not the most efficient way to arrange the towns, but it is the most natural.



1. **Of the following towns, which one has the greatest number of direct connections?**

   (A) H
   (B) N
   (C) J
   (D) K
   (E) L

From the chart, there are three roads going to L, and no other town has more than two roads going to it. Hence the answer is (E).

2. **Which one of the following towns has the fewest number of connections, direct or otherwise, with the other towns?**

   (A) H
   (B) K
   (C) N
   (D) L
   (E) I

Again from the chart, there is only one road going to K, and every other town has at least two roads going to it. So K has the fewest number of connections. The answer is (B).

3. If a new road were built directly connecting I and J, and if all the direct connections between the towns were of equal distance, then in which one of the following pairs of towns is neither town on the shortest route connecting L and K?

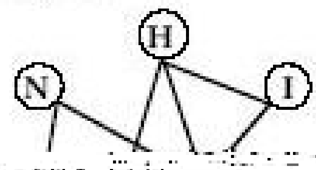
(A) J, H
(B) H, I
(C) I, J
(D) N, I
(E) M, N

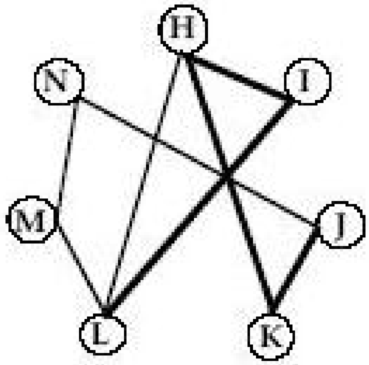
Adding the new road to the chart gives

4. If a new road were built directly connecting H and K, then the maximum number of paths connecting L to J, which do not pass through any town more than once, would be

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Adding the new road to the original diagram yields

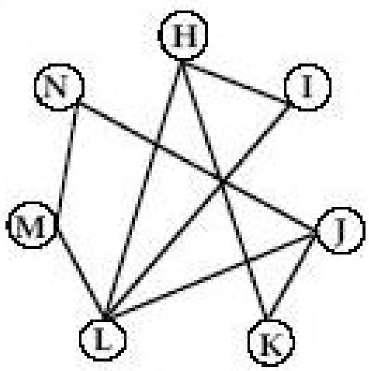




Hence the answer is (C).

5. **If two new roads were built directly connecting L to J and K to H, then which one of the following is NOT a complete and accurate list of the towns through which a person could pass on a single trip from L to H?**
   - (A) I
   - (B) J, K
   - (C) M, N, J, K
   - (D) No town
   - (E) M, N, K

Adding the new roads to the chart gives



The route in choice (E) directly connects either M or N to K, but in the chart there is no road directly connecting M or N to K. Hence this is impossible, and the answer is (E).

<u>Questions 6–7</u>

Let the letter itself stand for *"the light is on,"* and place a tilde before the letter to indicate that the light is off. We begin the chart with the condition *"If O is on, N is on; if O is off, N is off,"* which means that O is on if and only if N is on:

O<—>N

Next, the condition *"J and N cannot both be on"* means that if one is on the other must be off: N—>~J. Adding this to the diagram yields

O<—>N—>~J

Finally, adding the condition *"If K is on, N is on,"* K—>N, to the chart gives

$$\begin{array}{c} O \longleftrightarrow N \longrightarrow \sim J \\ \uparrow \\ K \end{array}$$

6. **If K is on, which one of the following must be true?**
   - (A) L is on.
   - (B) L is off.
   - (C) J is off.
   - (D) M is on.
   - (E) N is off.

In the diagram, a sequence of arrows "flows" from K to N to ~J. Hence, J must be off. The answer is (C).

7. **If J is on, what is the maximum number of lights that can be off?**
   - (A) 1
   - (B) 2
   - (C) 3
   - (D) 4
   - (E) 5

The fact that J is on prompts us to apply the contrapositive to the diagram:

~O<—>~N<—J
↓
~K

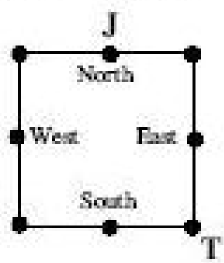
This shows that at least 3 lights must be off. Now, no conditions apply to either L or M. So their status—whether on or off—is independent of the other lights. Hence, both could be off. This gives a maximum of 5 lights off. The answer is (E).

<u>Questions 8–9</u>

The condition *"If T stands next to X, then T does not also stand next to L"* can be symbolized as TX—>~(TL). Summarizing the remaining conditions yields the following diagram:

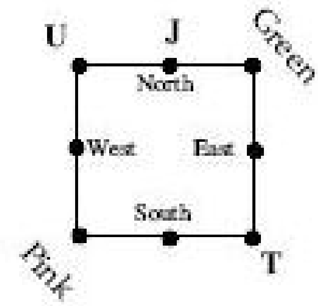
Green (J K L) Red T U Pink (X Y Z)

~(Green/Pink)
TX—>~(TL)



The parentheses around the green and the pink benches remind us that these benches form groups.

Since the green benches cannot be next to the pink benches, the red benches—T and U—must separate them. Now, the pink benches form a group of three, so they must be located on the west and south borders where there is sufficient room. This in turn forces U into the northwest corner of the park, separating the pink benches from the green benches.



8. For which one of the following benches are there two and no more than two locations either one of which could be the location the bench occupies?

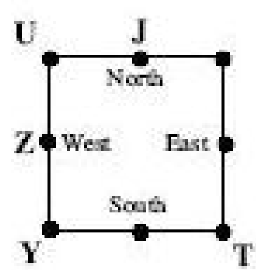
(A) K
(B) T
(C) X
(D) Y
(E) Z

Since K is green, we see from the diagram it must be either the middle bench on the east side or the bench on the northeast corner. The answer is (A).

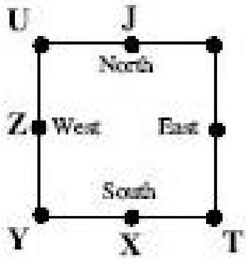
9. If Z is directly north of Y, which one of the following statements must be true?

(A) J is directly west of K.
(B) K is directly east of U.
(C) U is directly north of X.
(D) X is directly south of J.
(E) Z is directly south of J.

Placing the condition *"Z is directly north of Y"* on the diagram yields



This forces X between Y and T:



This diagram clearly shows X is directly south of J. The answer is (D).

Questions 10–11

The condition *"If Kevin enters the marathon, then both Julio and May enter the relay, and if both Julio and May enter the relay, then Kevin enters the marathon"* means that Kevin enters the marathon if and only if both Julio and May enter the relay. This can be symbolized as

(K=Ma)<—>(J & M=R)

The condition *"Olivia and Tamio do not both enter the relay"* means that if Olivia enters the relay, then Tamio does not. This in turn can be symbolized as

(O=R)—>(T≠R)*

The remaining conditions can be symbolized in like manner, which yields the following schematic:

(K=Ma)<—>(J & M=R)
(O=R)—>(T≠R)
(N≠S)—>(T=R)
(O=R)—>(J≠R)

* The condition (T=R)—>(O≠R) would also work, but we don't need both.

10. **If Kevin enters the marathon, then which one of the following cannot be true?**

    (A) Julio enters the relay and Norma enters the sprint.
    (B) Olivia enters the marathon and Norma enters the sprint.
    (C) Tamio enters the marathon and Olivia enters the sprint.
    (D) Tamio enters the relay and Olivia enters the sprint.
    (E) Tamio enters the marathon and May enters the relay.

If Kevin enters the marathon, then from (K=Ma)<—>(J & M=R) we know that both Julio and May enter the relay. Look at choice (D); it has Tamio in the relay. This puts three people—Julio, May, and Tamio—in the relay, violating the fact that only two people enter each event. The answer is (D).

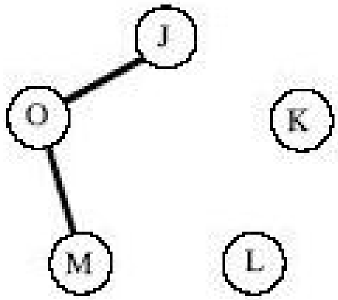
11. **If both Kevin and Olivia enter the relay, then which one of the following must be true?**

    (A) Julio and Tamio enter the marathon.
    (B) May enters the marathon.
    (C) May enters the sprint.
    (D) Tamio enters the sprint.
    (E) Norma enters the sprint.

Since Kevin and Olivia fill the two relay entries, no one else enters the relay. Consider the condition (N≠S)—>(T=R) and take the contrapositive to obtain (T≠R)—>(N=S). Now, since Tamio cannot enter the relay, Norma enters the sprint. The answer is (E).

Questions 12–13

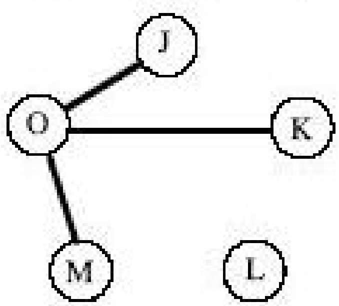
At first glance, this problem appears daunting. However, a diagram will greatly simplify it. Place the condition *"A bridge directly connects J with O, and a bridge directly connects M with O"* on a diagram:



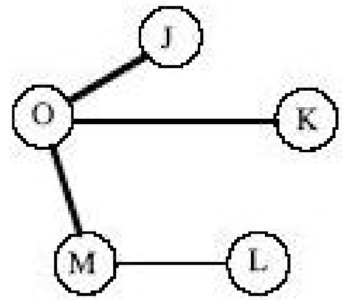
12. If a bridge directly connects K with O, then which one of the following could be true?

    (A) No bridge directly connects L with M.
    (B) A bridge directly connects J with L.
    (C) A bridge directly connects L with O.
    (D) There are exactly three bridges directly connecting L with other islands.
    (E) There are exactly two bridges directly connecting O with other islands.

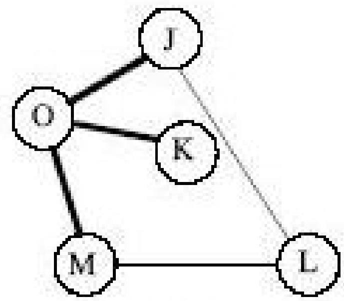
Add the condition *"a bridge directly connects K with O"* to the diagram:



The condition *"J, K, and L are each directly connected by bridge with one or both of M and O"* tells us that L must be directly connected to either M or O. However, L cannot be directly connected to O since O is already directly connected to three other islands. Hence, L must be directly connected to M:



This diagram eliminates choice (A). Turning to choice (B), suppose a bridge directly connects J with L:



This diagram directly connects J, K, and L with M or O; J with exactly two islands; K with exactly one island; J with O; and M with O. Hence, J *could be* directly connected with L. The answer is (B).

13. If no island that is directly connected by bridge with M is also directly connected by bridge with O, then there must be a bridge directly connecting

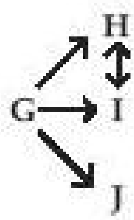
    (A) J with L
    (B) J with M
    (C) K with O
    (D) L with M
    (E) L with O

J cannot be directly connected to M since J is directly connected to O (see original diagram)—otherwise it would violate the premise of this question. Further, J cannot be directly connected to K since K is directly connected to exactly one other island, M or O. Therefore, as J must be directly connected to exactly two other islands, it must be directly connected to L. The answer is (A).

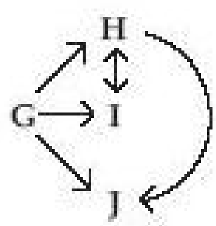
Questions 14–17

The first three conditions yield the following diagram:

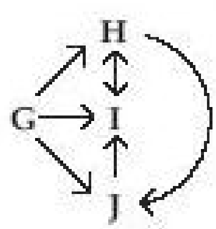
Adding the fourth condition yields



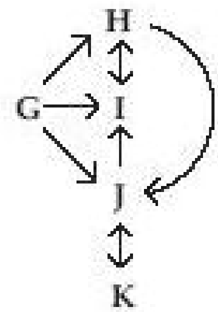
Adding the fifth condition yields



Adding the sixth condition yields



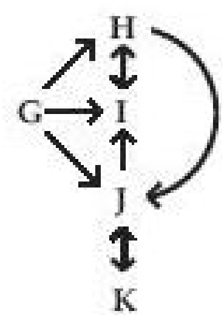
Finally, adding the last condition yields



14. A rumor begun by I that reaches K will be known by all the following employees EXCEPT:

(A) I
(B) G
(C) K
(D) H
(E) J

The rumor can pass from I to H, and from H to J, and from J to K. This scenario has the rumor passing from I to K without passing through G. The answer is choice (B). The following diagram illustrates the path from I to K:



15. If all segments have the same length and if rumors always follow the shortest path, then the longest path any rumor follows in the system is the path from

(A) H to J
(B) J to K
(C) H to G
(D) I to J
(E) I to K

Begin with choice (A). The original diagram shows that a rumor can spread directly from H to J, so this is unlikely to be the longest path. Turning to choice (B), the diagram shows that a rumor can also spread directly from J to K. Since there cannot be two answers, this eliminates both (A) and (B). Turning to choice (C), the diagram clearly shows that no rumor can spread from H to G, eliminate. Turning to choice (D), the diagram shows that a rumor originating at I can spread directly only to H and then from H to J. This rumor consists of 2 segments. Finally, the path taken in choice (E) includes the path in choice (D) plus one additional segment from J to K. Hence, the answer is (E).

16. Which of the following is a complete and accurate list of the people to whom a rumor can be spread along exactly one segment from H?

(A) I
(B) J
(C) I, J
(D) I, J, K
(E) I, J, G

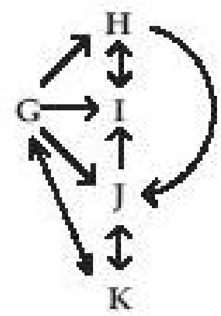
The original diagram shows arrows leading directly from H to I and to J, and only to these two. Hence, the answer is (C).

As to the other choices, both (A) and (B) are incomplete since we just showed that there are direct paths leading from H to both I and J. Choice (D) is inaccurate since a rumor cannot pass from H to K without passing through J. Hence, a rumor needs at least two segments to reach K from H. Finally, choice (E) is inaccurate: No arrow points from H (nor from any other person) to G. Hence, no rumors can pass from H (nor from any other person) to G.

17. If a two-way segment from K to G is added to the rumor mill, which of the following segments would have to be added to the system so that each person could spread a rumor directly to at least two other people and receive a rumor directly from at least two other people?

(A) I to J
(B) H to G
(C) K to H
(D) I to G
(E) J to H

Adding the new condition to the diagram yields



Now, look at choice (D). If there were a segment from I to G, then I could spread rumors to both H and G and could receive rumors from both H and G. Further, H could spread rumors to both I and J and could receive rumors from both G and I. J could spread rumors to both I and K and could receive rumors from both G and K. K could spread rumors to both G and J and could receive rumors from both G and J. Finally, G could spread rumors to both I and K and could receive rumors from both I and K. The answer is choice (D).

Questions 18–19

The condition *"If Kent is hired as an editor, then both John and Mary are hired as gaffers, and if both John and Mary hired as gaffers, then Kent is hired as an editor"* means that Kent is hired as a editor if and only if both John and Mary are hired as gaffers. This can be symbolized as

(K=E)<—>(J & M=G)

The condition *"If Nora is not hired as an actor, then Tom is hired as a gaffer."* can be symbolized as

(N≠A)—>(T=G)

18. If Kent is hired as an editor, then which one of the following cannot be true?

(A) Nora and Tom are hired as actors.
(B) Olivia is hired as an actor, and Tom as an editor.
(C) Olivia and Nora are hired as actors.
(D) Tom is hired as a gaffer and Olivia is hired as an actor.
(E) Neither John nor Mary are hired as editors.

If Kent is hired as an editor, then from (K=E)<—>(J & M=G) we know that both John and Mary are hired as gaffers. Look at choice (D); it has Tom hired as a gaffer. This forces three people—John, Mary, and Tom—to be gaffers, violating the fact that only two people are hired for each position. The answer is (D).

19. If both Kent and Olivia are hired as gaffers, then which one of the following must be true?

(A) Nora is hired as an editor.
(B) Tom is hired as an actor.
(C) John is hired as an actor.
(D) John and Tom are hired as editors.
(E) Nora is hired as an actor.

Since Kent and Olivia fill the two gaffer positions, no one else is hired as a gaffer. Consider the condition (N≠A)—>(T=G) and take the contrapositive to obtain

(T≠G)—>(N=A)

Now, since Tom cannot be hired as a gaffer (those two positions are already filled by Kent and Olivia), Nora is hired as an actor. The answer is (E).

Grouping Games

We have thoroughly studied various ways to order elements. In this chapter, we'll study various ways to group elements. We got a taste of the tasks involved in grouping elements when we studied hybrid games, which both order and group items.

Because grouping games partition elements into sets, the number of elements is often an issue. As mentioned before, counting can be challenging. This tends to make grouping games more difficult than ordering games.

Pay close attention to the maximum or minimum number of elements in a group; this is often the heart of the game.

Grouping games can be classified as those that partition the elements into 2 groups, and those that partition the elements into 3 or more groups. The former are sometimes called selection games because they "select" elements from a pool, dividing the pool into two groups: those selected and those not selected. The example on the next page will illustrate.

Selection Game

The starting line-up for the Olympic basketball "Dream Team" is chosen from the following two groups:

Group A	Group B
Johnson, Drexler, Bird, Ewing	Laettner, Robinson, Jordan, Malone, Pippen

The following requirements must be meet:

Two players are chosen from Group A, and three from Group B.
Jordan starts only if Bird starts.
Drexler and Bird do not both start.
If Jordan starts, then Malone does not.
Exactly 3 of the four fast-break specialists—Johnson, Bird, Jordan, Pippen—must be chosen.

1. If Jordan starts, which of the following must also start?

   (A) Malone or Johnson
   (B) Drexler or Laettner
   (C) Drexler or Johnson
   (D) Johnson or Pippen
   (E) Malone or Robinson

2. All of the following pairs of players can start together EXCEPT

   (A) Ewing and Drexler
   (B) Jordan and Johnson
   (C) Robinson and Johnson
   (D) Johnson and Bird
   (E) Pippen and Malone

3. If the condition "Bird starts only if Pippen doesn't" is added to the other conditions, then which of the following must be false?

   (A) Johnson starts with Bird
   (B) Laettner starts with Malone
   (C) Laettner starts with Bird
   (D) Jordan starts with Robinson
   (E) Jordan starts with Bird

4. If Malone starts, which one of the following is a complete and accurate list of the players from Group A any one of whom could also start?

   (A) Johnson
   (B) Johnson, Drexler
   (C) Johnson, Bird
   (D) Johnson, Drexler, Bird
   (E) Johnson, Ewing, Bird

5. Which one of the following players must start?

   (A) Pippen
   (B) Johnson
   (C) Jordan
   (D) Malone
   (E) Bird

This problem is rather convoluted because not only are there direct conditions on the players, such as *"Drexler and Bird do not both start,"* but there are also constraining numerical conditions, such as *"exactly three fast-break specialists must be chosen."*

It is best to solve this problem without a diagram; however, we will still symbolize the conditions for clarity and easy reference. The condition *"Jordan starts only if Bird starts"* implies only that if Jordan is starting then Bird must be starting as well. So we symbolize it as **Jordan—>Bird**. The condition *"Drexler and Bird do not both start"* means that if one starts then the other does not. So we symbolize it as **Drexler—>~Bird.*** Students often misinterpret this condition to mean that neither of them starts. To state that neither starts, put *both* at the beginning of the sentence: *Both* Drexler and Bird do not start.

The condition *"if Jordan starts, then Malone does not"* is naturally symbolized as **Jordan—>~Malone**. It tells us that if J starts then M does not, but tells us nothing when M does not start. Such a condition, where the two parts of an *if-then* statement do not similarly affect each other, is called a nonreciprocal condition. On the other hand, a condition such as Jordan<—>~Malone affects J and M equally. In this case, we are told that if J starts then M does not as before, but we are told additionally that if M does not start then J does. It is important to keep the distinction between reciprocal and nonreciprocal relations clear; a common mistake is to interpret a nonreciprocal relation as reciprocal (see Unwarranted Assumptions, page 31). The remaining conditions cannot be easily written in symbol form, but we will paraphrase them in the schematic:

Jordan—>Bird
Drexler—>~Bird
Jordan—>~Malone
2 from Group A 3 from Group B
fast-break specialists: Johnson, Bird, Jordan, Pippen
3 fast-break specialists
Ewing, Laettner, Robinson are "wild"

Note: Ewing, Laettner, and Robinson are independent because there are no conditions that refer directly to them. We now turn to the questions.

1. **If Jordan starts, which of the following must also start?**

   (A) Malone or Johnson
   (B) Drexler or Laettner
   (C) Drexler or Johnson
   (D) Johnson or Pippen
   (E) Malone or Robinson

From the condition **Jordan—>Bird**, we know that if Jordan starts, then Bird must start as well. Now both Jordan and Bird are fast-break specialists, and three of the four fast-break specialists must start. So at least one of the remaining fast-break specialists—Johnson or Pippen—must also start. The answer is (D).

* Bird—>~Drexler would also suffice, but we don't need both.

2. **All of the following pairs of players can start together EXCEPT:**

   (A) Ewing and Drexler
   (B) Jordan and Johnson
   (C) Robinson and Johnson
   (D) Johnson and Bird
   (E) Pippen and Malone

We shall use the method of indirect proof to solve this problem: That is, assume that a particular answer-choice is true. Then check whether it leads to a contradiction or an impossible situation. If so, it is the answer; if not, then select another answer-choice and repeat the process until a contradiction is found.

Begin with choice (A). Both Ewing and Drexler are from Group A, so the remaining 3 starters must be chosen from Group B. Additionally, they must all be fast-break specialists since neither E nor D is—there are exactly 3 fast-break specialists. But Jordan and Pippen are the only fast-break specialists in Group B. So the third fast-break specialist cannot be chosen. The answer therefore is (A). This type of question can be time consuming because you may have to check all the answer-choices—save these questions for last.

3. **If the condition "Bird starts only if Pippen doesn't" is added to the other conditions, then which one of the following must be false?**

   (A) Johnson starts with Bird
   (B) Laettner starts with Malone
   (C) Laettner starts with Bird
   (D) Jordan starts with Robinson
   (E) Jordan starts with Bird

This problem is both long and hard. Again, we use an indirect proof. Start with (A). Both Johnson and Bird are from Group A, and both are fast-break specialists. So the remaining 3 starters must be chosen from Group B, one of which must be a fast-break specialist. Now if Jordan, Robinson, and Laettner are chosen, there will be three fast-break specialists and none of the initial conditions will be violated. So (A) is not necessarily false; eliminate it. Next, we check (B). Both Laettner and Malone are from Group B, and neither is a fast-break specialist. So the three remaining starters must all be fast-break specialists, and two of them must be from Group A—Johnson and Bird. This leaves only Jordan and Pippen to choose from. Jordan cannot be chosen because Malone has already been chosen (**Jordan—>~Malone**), and from the new condition Pippen cannot be chosen because Bird has already been chosen. Hence the answer is (B).

4. **If Malone starts, which one of the following is a complete and accurate list of the players from Group A any one of whom could also start?**

   (A) Johnson
   (B) Johnson, Drexler
   (C) Johnson, Bird
   (D) Johnson, Drexler, Bird
   (E) Johnson, Ewing, Bird

Jordan *cannot* start with Malone according to the condition **Jordan—>~Malone**. To play three fast-break specialists, therefore, Johnson, Bird, and Pippen are all *required* to start. Since both Johnson and Bird are from Group A and exactly two players from that group start, these two players comprise the complete list of starters from Group A when Malone also starts. The answer is (C).

5. **Which one of the following players must start?**

   (A) Pippen
   (B) Johnson
   (C) Jordan
   (D) Malone
   (E) Bird

Suppose Bird does not start. Then the 3 fast-break specialists must be Johnson, Jordan, and Pippen. But if Jordan starts, then from the initial conditions Bird must also start. Hence Bird must always start. The answer is (E).

Grouping by Threes

Three committees are formed from eight people—F, G, H, I, J, K, L, M. Two of the committees have three members, and one of the committees has only two members.

G serves with M.
L serves with only one other person.
F does not serve with M.

1. Which one of the following is a committee?

   (A) M, L, I
   (B) G, F, M
   (C) G, L
   (D) G, H, I
   (E) K, G, M

2. If F cannot serve with K, and K cannot serve with M, which one of the following must be false?

   (A) F serves with L.
   (B) F serves with J.
   (C) L serves with H.
   (D) H serves with I.
   (E) I serves with M.

3. If H serves with K, which one of the following cannot be true?

   (A) F serves with K.
   (B) J serves with F.
   (C) I serves with M.
   (D) F serves with L.
   (E) J serves with L.

4. If K, J, and I serve on different committees, which one of the following must be true?

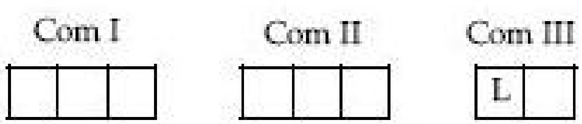
   (A) K serves with G.
   (B) I serves on a committee of two.
   (C) J serves on a committee of two.
   (D) H serves with F.
   (E) J serves with F.

5. Which one of the following conditions is inconsistent with the given conditions?

   (A) K serves on a committee of three.
   (B) M serves with H.
   (C) M, H, and I serve together.
   (D) F does not serve with G.
   (E) H serves with L.

We start by symbolizing the conditions. The condition *"G serves with M"* is naturally symbolized as $G = M$. The condition *"F does not serve with M"* is symbolized as $F \neq M$. The condition *"L serves with only one other person"* means that L is on the committee of two; we symbolize it as $L = 2$. The diagram will consist of three compartmentalized boxes. This gives the following schematic:

F G H I J K L M (H, I, J, K "wild")
$G = M$
$F \neq M$
$L = 2$



Before turning to the questions, two readily derived conditions should be noted. First, since G serves with M, and F does not serve with M, F cannot serve with G. Second, since L serves on the two-person committee, L cannot serve with G or M (otherwise L would be on a three-person committee).

1. **Which one of the following is a committee?**

   (A) M, L, I
   (B) G, F, M
   (C) G, L
   (D) G, H, I
   (E) K, G, M

(A) is not a committee since L must serve on a committee of two. (B) is not a committee since F cannot serve with G. Neither (C) nor (D) is a committee since G and M must serve together. Hence, by process of elimination, the answer is (E).

2. **If F cannot serve with K, and K cannot serve with M, which one of the following must be false?**

   (A) F serves with L.
   (B) F serves with J.
   (C) L serves with H.
   (D) H serves with I.
   (E) I serves with M.

We shall use an indirect proof. Start with (A). If F serves with L, then G and M could serve on Committee I, K on Committee II, and the remaining people could serve at random without violating any initial condition. So F could serve with L. This eliminates (A). Next, test (B). If F serves with J on Committee I, then G and M would have to serve on Committee II. And the remaining people could be placed as follows:

Com I			Com II			Com III	
F	J	H	G	M	I	L	K

This diagram does not violate any initial condition, so F could serve with J. This eliminates (B). Next, test (C). There are two possible places for the pair G and M, Committee I and Committee II. If G and M serve on Committee I, then F would have to serve on Committee II:

Com I			Com II			Com III	
G	M		F			L	H

Clearly this diagram leaves no room for K since K cannot serve with either M or F. The case with the pair G and M serving on Committee II leads to a similar result. Hence L cannot serve with H. The answer is (C).

3. If H serves with K, which one of the following cannot be true?

(A) F serves with K.
(B) J serves with F.
(C) I serves with M.
(D) F serves with L.
(E) J serves with L.

If H serves with K on Committee I, then G and M must serve on Committee II. (Why?) This gives the following diagram:

Com I			Com II			Com III	
H	K		G	M		L	

(The diagram with H and K on Committee II is not presented because it generates the same results.) Again we apply an indirect proof. Start with (A). If F serves with K, then from the above diagram F must serve on Committee I. And we can place I and J on Committees II and III, respectively:

Com I			Com II			Com III	
H	K	F	G	M	I	L	J

This diagram does not violate any initial condition, so F could serve with K. This eliminates (A). Next, test (B). J and F cannot serve on Committee I, since from the above diagram H and K are already there. Likewise, J and F cannot serve on Committees II and III. Hence the answer is (B).

4. If K, J, and I serve on different committees, which one of the following must be true?

(A) K serves with G.
(B) I serves on a committee of two.
(C) J serves on a committee of two
(D) H serves with F.
(E) J serves with F.

We shall construct counter-examples for four of the answer-choices; the one for which we cannot construct a counter-example will be the answer. Start with (A). Suppose K serves on Committee I and G serves on Committee II. Then from the condition G = M, we know that M must also serve on Committee II. And the remaining people can be placed without violating any initial condition as follows:

Com I			Com II			Com III	
K	H	F	J	G	M	L	I

This diagram is a counter-example not only to (A) but to (C) and (E) as well. This eliminates (A), (C), and (E). Next, test choice (B). Suppose that I serves on Committee I, with G and M. Then the remaining people can be grouped as follows:

Com I	Com II	Com III
I G M	F H J	L K

This diagram does not violate any initial condition, so it is a counter-example to (B). Hence, by process of elimination, the answer is (D).

5. **Which one of the following conditions is inconsistent with the given conditions?**

   (A) K serves on a committee of three.
   (B) M serves with H.
   (C) M, H, and I serve together.
   (D) F does not serve with G.
   (E) H serves with L.

The first counter-example in Question 4 shows that K can serve on a three-person committee. This eliminates (A). Next, turning to choice (B), suppose M serves with H on Committee I. This forces G to also serve on Committee I. Now place F on Committee III and the remaining people as follows:

Com I	Com II	Com III
M H G	J K I	L F

This diagram does not violate any initial condition, so "M serves with H" is consistent with the initial conditions. This eliminates (B). Next, turning to choice (C), suppose M, H, and I serve together on Committee I. But since M must serve with G, there would then be four people on Committee I. The same result occurs when M, H, and G are on Committee II. Hence (C) is inconsistent with the initial conditions, and the answer is (C).

Points to Remember

1. Pay close attention to the maximum or minimum number of elements in a group, for this is often the heart of the game.

2. Grouping games are classified as those that divide the elements into two sets—Selection games—and those that divide the elements into three or more sets.

3. A reciprocal condition affects both elements equally.

4. Don't interpret a nonreciprocal condition as reciprocal.

5. The method of indirect proof is used often with grouping games.

WARM-UP DRILL

Answers and solutions are on page 199.

1. How many groups can be formed from A, B, C?

2. In how many ways can two elements be selected from A, B, C, and D?

3. In how many ways can a group of four be chosen from the sets {A, B, C} and {D, E, F}, given that two elements must be selected from each set, and A can be selected only if D is selected?

4. In how many ways can 3 elements be selected from the sets {A, B} and {C, D, E} if some elements must be selected from {A, B}, and C and D cannot both be selected?

5. How many groups of 3 can be selected from U, V, W, X, Y, Z given that

   V is not selected unless Z is selected.
   Y and W cannot both be selected.
   U is selected only if V is not.
   Either V or Y, but not both, is selected.

Answers and Solutions to Warm-up Drill

1. **7 groups.**
   A; B; C
   AB; AC; BC
   ABC

2. **6 ways.**
   AB; AC; AD
   BC; BD
   CD

3. **7 ways.**

   AB: DE, DF, ~~EF~~

   AC: DE, DF, ~~EF~~

   BC: DE, DF, EF

4. **7 ways.**

   If A is chosen, then either CE or DE may be added: ACE, ADE

   If B is chosen, then either CE or DE may be added: BCE, BDE

   If both A and B are chosen, then C, D, or E may be added: ABC, ABD, ABE

5. **5 groups.**
   The condition *"V is not selected unless Z is selected"* is equivalent to *"if V is selected, then Z is selected."* The condition *"Y and W cannot both be selected"* is equivalent to *"if Y is selected, then W is not."* The condition *"U is selected only if V is not"* is equivalent to *"if U is selected, then V is not."*

   Now suppose V is selected. Then Z must be selected and neither U nor Y can be selected. Combining the pair VZ with the remaining elements—W and X—gives the following two groups: VZW and VZX.

   Next, suppose Y is selected. Then V cannot be selected, nor can W. Combining Y with the remaining elements—U, X, and Z—gives the following three groups: YUX, YUZ, and YXZ. Hence a total of five groups is possible.

There will be no more mentor exercises for the games.

EXERCISE

The first three games are taken from recent LSATs.

Questions 1–6

Seven buildings are located in an urban development.

Three of the buildings are residential; the other four are commercial.
Each of the residential buildings is made of either brick or wood.
Each of the commercial buildings is made of either wood, concrete, brick, or steel.
All of the residential buildings and all of the brick buildings have fire escapes, but no other buildings do.
Exactly four of the buildings have fire escapes.

1. Which of the following must be true?

    I. At least one of the buildings is made of wood.
    II. At least one of the buildings is made of steel.
    III. At least one of the buildings is made of brick.

    (A) I only
    (B) II only
    (C) III only
    (D) I and II only
    (E) I and III only

2. If exactly two of the commercial buildings are made of concrete, then which one of the following must be true?

    (A) At least one of the commercial buildings is made of wood.
    (B) Exactly two buildings are made of wood.
    (C) Exactly one of the commercial buildings is made of steel.
    (D) No commercial building is made of wood.
    (E) At most one building is made of steel.

3. If there is at least one wooden building, one concrete building, one brick building, and one steel building, then which one of the following must be false?

    (A) Exactly four buildings are made of brick.
    (B) Exactly three buildings are made of wood.
    (C) Exactly two buildings are made of wood and exactly two buildings are made of steel.
    (D) Exactly two buildings are made of steel and exactly two buildings are made of concrete.
    (E) Exactly two buildings are made of wood and exactly two buildings are made of brick.

4. If there are exactly three brick buildings and one steel building, then any of the following can be true EXCEPT:

    (A) there is exactly one wooden building
    (B) there are no wooden buildings
    (C) there are exactly three wooden buildings
    (D) there are no concrete buildings
    (E) there are exactly two concrete buildings

5. If exactly half of the buildings with fire escapes are wooden, then which one of the following must be false?

    (A) There are more wooden buildings than brick buildings.
    (B) There are more steel buildings than wooden buildings.
    (C) There are exactly three wooden buildings.
    (D) There are exactly three brick buildings.
    (E) The number of steel buildings is equal to the number of concrete buildings.

6. If as many as possible of the buildings with fire escapes are wooden, then which of the following must be true?

    I. There are exactly three wooden buildings.
    II. There is exactly one brick building.
    III. There are fewer concrete buildings than wooden buildings.

    (A) I only
    (B) II only
    (C) I and III only
    (D) II and III only
    (E) I, II, and III

Questions 7–8

A hobbyist is stocking her aquarium with exactly three fish of different types and with exactly two species of plants. The only fish under consideration are a G, an H, a J, a K, and an L, and the only kinds of plants under consideration are of the species W, X, Y, and Z. She will observe the following conditions:

If she selects the G, she can select neither the H nor a Y.
She cannot select the H unless she selects the K.
She cannot select the J unless she selects a W.
If she selects the K, she must select an X.

7. If the hobbyist selects the H, which one of the following must also be true?

   (A) She selects at least one W.
   (B) She selects at least one X.
   (C) She selects the J, but no Y's.
   (D) She selects the K, but no X's.
   (E) She selects at least one X, but no Y's.

8. If the hobbyist selects a Y, which one of the following must be the group of fish she selects?

   (A) G, H, K
   (B) H, J, K
   (C) H, J, L
   (D) H, K, L
   (E) J, K, L

Questions 9–10

Petworld has exactly fourteen animals (three gerbils, three hamsters, three lizards, five snakes) that are kept in four separate cages (W, X, Y, Z) according to the following conditions:

Each cage contains exactly two, four, or six animals.
Any cage containing a gerbil also contains at least one hamster; any cage containing a hamster also contains at least one gerbil.
Any cage containing a lizard also contains at least one snake; any cage containing a snake also contains at least one lizard.
Neither cage Y nor cage Z contains a gerbil.
Neither cage W nor cage X contains a lizard.

9. At most, how many snakes can occupy cage Y at any one time?

   (A) one
   (B) two
   (C) three
   (D) four
   (E) five

10. If there are exactly two hamsters in cage W and the number of gerbils in cage X is equal to the number of snakes in cage Y, then the number of snakes in cage Z must be exactly

   (A) one
   (B) two
   (C) three
   (D) four
   (E) five

Questions 11–14

A group of four items is selected from seven items—G, H, I, J, K, L, and M—according to the following rules:

Either G or I must be selected.
Either H or K must be selected.
Neither K nor I can be selected with H.
Neither L nor G can be selected unless the other is also selected.

11. Which of the following groups is an acceptable selection of the items?
    (A) G, I, L, M
    (B) I, K, M, H
    (C) G, K, I, M
    (D) G, L, J, M
    (E) I, G, K, L

12. Which of the following groups of items cannot be among the items selected?
    (A) H, J
    (B) H, J, M
    (C) L, K, I
    (D) G, H, M
    (E) L, H, J

13. If I and M are selected, which of the following items must also be selected?
    (A) G, L
    (B) J, H
    (C) H
    (D) K, J
    (E) L

14. There would be only one possible way to select the four items if which of the following restrictions were added to the original set of conditions?
    (A) If I is selected, then G is selected.
    (B) Both I and G are selected.
    (C) If J is selected, then M is selected.
    (D) Either L or M is selected.
    (E) If I is selected, then K is selected.

Questions 15–18

Six items—U, V, W, X, Y, Z—are being separated into 3 groups—Group 1, Group 2, Group 3—according to the following conditions:

The number of items in Group 1 is less than or equal to the number of items in Group 2.
The number of items in Group 2 is less than or equal to the number of items in Group 3.
V and W cannot be in the same group.
X can be in Group 3 only if Y is in Group 3.

15. Which one of the following is an acceptable grouping of the six items?

	Group 1	Group 2	Group 3
(A)	UV	WXY	Z
(B)	X	Y	VZUW
(C)	V	YW	XUZ
(D)	V	Z	XYUW
(E)	UW	YZ	XV

16. If Group 1 contains only the item Y, which of the following must be true?
    (A) Group 3 contains four items.
    (B) Group 2 contains the same number of items as Group 3.
    (C) V is in Group 3.
    (D) Group 2 contains three items.
    (E) X is in Group 2.

17. If W and Y are in the same group and V is in Group 3, then which of the following must be false?
    (A) W and Y are in Group 2.
    (B) U is the only item in Group 1.
    (C) X is the only item in Group 1.
    (D) U is in Group 3.
    (E) Group 3 contains 2 items.

18. If Group 2 contains only one item which is neither W nor V, which of the following must be true?
    (A) Group 1 contains only V or only W.
    (B) Group 3 contains W.
    (C) Group 1 contains both U and V.
    (D) Group 2 contains Z.
    (E) Group 1 contains only W.

Answers and Solutions to Exercise

Questions 1–6

This is a moderately hard game. To solve it, we'll use the following schematic:

3	4	4
Res	(Res, B) = Fx	Com
B or W		B, W, C, or S

[Where Fx stands for fire escape, and the condition **(Res, B) = Fx** means that all residential and all brick buildings have fire escapes.] The key to this problem is the condition **(Res, B) = Fx**.

1. **Which of the following must be true?**

   I. **At least one of the buildings is made of wood.**
   II. **At least one of the buildings is made of steel.**
   III. **At least one of the buildings is made of brick.**

   (A) **I only**
   (B) **II only**
   (C) **III only**
   (D) **I and II only**
   (E) **I and III only**

Statement I is false. All three residential buildings could be brick and one commercial building could also be brick to give four buildings with fire escapes. This eliminates (A), (D), and (E). Statement II is false. With the same scenario as in Statement I, the three remaining commercial buildings could all be concrete. This eliminates (B). Hence, we have learned that the answer is (C), without having to check Statement III. But let's verify that it's true.

The greatest number of wooden buildings with fire escapes is obtained when all three residential buildings are wooden. Now in order to have a fourth building with a fire escape, one of the commercial buildings must be brick.

2. **If exactly two of the commercial buildings are made of concrete, then which one of the following must be true?**

   (A) **At least one of the commercial buildings is made of wood.**
   (B) **Exactly two buildings are made of wood.**
   (C) **Exactly one of the commercial buildings is made of steel.**
   (D) **No commercial building is made of wood.**
   (E) **At most one building is made of steel.**

Now we use a diagram consisting of two boxes, one for the residential buildings and one for the commercial buildings. Place the two concrete buildings as follows:

3	4
Res	Com
	C C

Next, attempt to construct a counterexample for each answer-choice. The one for which this cannot be done will be the answer. Start with (A). Suppose all three residential buildings are wood:

3	4
Res	Com
W W W	C C

Now in order to have four buildings with fire escapes, one of the commercial buildings must be brick and one more could be steel:

3	4
Res	Com
W W W	B C C S

This diagram is a counterexample for (A) and (B). Furthermore, replacing S with a W gives a counterexample to (C) and (D). Hence, by process of elimination, the answer is (E). But again it is instructive to verify this. To that end, assume two of the buildings are steel:

3	4
Res	Com
	C C S S

Now only the three remaining residential buildings have fire escapes, but there must be four buildings with fire escapes. Hence, it is impossible to have more than one steel building.

3. **If there is at least one wooden building, one concrete building, one brick building, and one steel building, then which one of the following must be false?**

   (A) Exactly four buildings are made of brick.

   (B) Exactly three buildings are made of wood.

   (C) Exactly two buildings are made of wood and exactly two buildings are made of steel.

   (D) Exactly two buildings are made of steel and exactly two buildings are made of concrete.

   (E) Exactly two buildings are made of wood and exactly two buildings are made of brick.

Our work at the end of Question 2 shows that the answer is (D).

Notice that we did not use the

4. **If there are exactly three brick buildings and one steel building, then any of the following can be true EXCEPT**

   (A) there is exactly one wooden building

   (B) there are no wooden buildings

   (C) there are exactly three wooden buildings

   (D) there are no concrete buildings

   (E) there are exactly two concrete buildings

If there are no wooden buildings, then the three residential buildings must all be brick. Now we need a fourth building that has a fire escape. But all the remaining commercial buildings are either steel or concrete, none of which can have a fire escape. Hence the answer is (B). Notice that we needed only the first part of the supplementary condition.

5. **If exactly half of the buildings with fire escapes are wooden, then which one of the following must be false?**

   (A) There are more wooden buildings than brick buildings.

   (B) There are more steel buildings than wooden buildings.

   (C) There are exactly three wooden buildings.

   (D) There are exactly three brick buildings.

   (E) The number of steel buildings is equal to the number of concrete buildings.

All brick buildings have fire escapes. Now if three of the buildings are brick, choice (D), then there must be three wooden buildings with fire escapes—since half of the buildings with fire escapes are wooden. This

6. **If as many as possible of the buildings with fire escapes are wooden, then which of the following must be true?**

   I. **There are exactly three wooden buildings.**
   II. **There is exactly one brick building.**
   III. **There are fewer concrete buildings than wooden buildings.**

   (A) **I only**
   (B) **II only**
   (C) **I and III only**
   (D) **II and III only**
   (E) **I, II, and III**

In order to have as many of the wooden buildings with fire escapes as possible, all three residential buildings must be wooden.

3 Res W W W	4 Com

Now in order to have four buildings with fire escapes, exactly one of the commercial buildings must be brick:

3 Res W W W	4 Com B

This shows that Statement II is true, which eliminates (A) and (C). Next, three concrete commercial buildings can be added to this diagram without violating any given condition:

3 Res W W W	4 Com B C C C

This diagram has the same number of wooden as concrete buildings. Hence Statement III is false. This eliminates (D) and (E). So without having to check Statement I, we have learned the answer is (B).

<u>Questions 7–8</u>

The first and last conditions are naturally symbolized as follows:

1. G—>(~H & ~Y)
2. K—>X

The second and third conditions are rather complicated structures. Remember, the statement A unless B translates as ~B—>A. So the condition *"She cannot select the H unless she selects the K"* translates as ~K—>~H. Likewise, the third condition translates as ~W—>~J. The contrapositive simplifies these conditions to*

3. H—>K
4. J—>W

The transitive property combines the second and third conditions into

5. H—>K—>X

Summarizing what we have developed gives

Fish (3)	**Plants (2)**
G, H, J, K, L	W, X, Y, Z

G—>(~H & ~Y)
H—>K—>X
J—>W

* Note: Two negatives make a positive, so symbol statements such as ~~H were simplified to H.

7. **If the hobbyist selects the H, which one of the following must also be true?**

   (A) She selects at least one W.
   (B) She selects at least one X.
   (C) She selects the J, but no Y's.
   (D) She selects the K, but no X's.
   (E) She selects at least one X, but no Y's.

If H is selected, then the condition H—>K—>X shows that X is also selected. The answer is (B).

8. **If the hobbyist selects a Y, which one of the following must be the group of fish she selects?**

   (A) G, H, K
   (B) H, J, K
   (C) H, J, L
   (D) H, K, L
   (E) J, K, L

Begin with choice (A). If she selects G, then the condition G—>(~H & ~Y) indicates she did not select Y. This, however, contradicts the premise that she selected Y—eliminate choice (A). As to choice (B), since she selects an H, the condition H—>K—>X shows she also selects an X. Further, since she selects a J, the condition J—>W shows she also selects a W. This has her selecting three plants—Y, X, and W, which violates the condition that she selects exactly two plants—eliminate choice (B). A similar analysis eliminates choices (C) and (E). Thus, by process of elimination, the answer is (D).

<u>Questions 9–10</u>

The condition *"Any cage containing a gerbil also contains at least one hamster"* can be symbolized as G—>H. The condition *"any cage containing a hamster also contains at least one gerbil"* can be symbolized as H—>G. Combining these two conditions gives G<—>H. In other words, a cage contains a gerbil if and only if it contains a hamster. The condition *"Neither cage Y nor cage Z contains a gerbil"* can be symbolized as G ≠ Y, Z. Symbolizing the other conditions in like manner yields

Gerbils	Hamsters	Lizards	Snakes
3	3	3	5

G<—>H
L<—>S
G ≠ Y, Z
L ≠ W, X

2, 4, or 6 per cage

A few properties should be noted before turning to the questions: Since lizards and snakes are always together and lizards cannot be in cages W and X, snakes also cannot be in cages W and X. Similarly, hamsters cannot be in cages Y and Z.

9. **At most, how many snakes can occupy cage Y at any one time?**

   (A) one
   (B) two
   (C) three
   (D) four
   (E) five

For questions that ask you to maximize a situation, begin with the highest number. Then the next highest, and so on. The first choice for which a valid diagram can be constructed will be the answer. So we begin with five snakes in cage Y:

W	X	Y	Z
		SSSSS	

The condition **L<—>S** forces an L into cage Y, which fills up cage Y and forces the other L's into cage Z—as they cannot be in cages W or X:

W	X	Y	Z
		SSSSSL	LL

Now, every cage containing a lizard must also contain a snake. So cage Z must contain a snake. This is impossible, however, since all five snakes have already been placed in cage Y. This eliminates choice (E).

Turning to choice (D), place four snakes in cage Y, along with two lizards:

W	X	Y	Z
		SSSSLL	

This forces the other snake and lizard into cage Z:

W	X	Y	Z
		SSSSLL	SL

The remaining animals can be placed as follows:

W	X	Y	Z
GGHH	GH	SSSSLL	SL

This diagram satisfies every condition. Hence, four snakes can occupy cage Y. The answer is (D).

10. **If there are exactly two hamsters in cage W and the number of gerbils in cage X is equal to the number of snakes in cage Y, then the number of snakes in cage Z must be exactly**

    (A) one
    (B) two
    (C) three
    (D) four
    (E) five

Begin with two hamsters in cage W:

W	X	Y	Z
HH			

The condition G<—>H forces a gerbil into cage W. Further, the condition **2, 4, or 6 per cage** forces another gerbil into cage W:

W	X	Y	Z
HHGG			

Since gerbils cannot be in cages Y or Z, the remaining hamster and gerbil must be in cage X:

W	X	Y	Z
HHGG	HG		

If *"the number of gerbils in cage X is equal to the number of snakes in cage Y,"* then there is one snake in cage Y. Hence, the 4 remaining snakes must be in cage Z:

W	X	Y	Z
HHGG	HG	S	SSSS

The answer is (D)

Questions 11–14

The condition *"Neither K nor I can be selected with H"* can be symbolized as

H—>~K
H—>~I

The condition *"Neither L nor G can be selected unless the other is also selected"* simply means that if either L or G is selected then both must be selected:

L<—>G

Symbolizing the remaining conditions yields the following schematic:

G or I
H or K
H—>~K
H—>~I
L<—>G

11. Which of the following groups is an acceptable selection of the items?

(A) G, I, L, M
(B) I, K, M, H
(C) G, K, I, M
(D) G, L, J, M
(E) I, G, K, L

Choices (A) and (D) violate the condition **H or K**. Choice (B) violates the condition H—>~K. Choice (C) violates the condition L<—>G. Hence, by process of elimination, the answer is (E).

12. Which of the following groups of items cannot be among the items selected?

(A) H, J
(B) H, J, M
(C) L, K, I
(D) G, H, M
(E) L, H, J

Begin with (A). Selecting both H and J will satisfy all the conditions, eliminate (A). Turning to choice (B), since H is selected, I cannot be selected (H—>~I). Hence, from the condition **G or I**, we must select G. Now, from the condition L<—>G, we must select L. This scenario has five items being selected, violating the fact that only four items are selected. The answer is (B).

13. If I and M are selected, which of the following items must also be selected?

(A) G, L
(B) J, H
(C) H
(D) K, J
(E) L

Since I is selected, the condition H—>~I† prevents H from being selected. Hence, the condition **H or K** forces K to be selected. Now, neither G nor L can be selected since they must be selected together, which would yield a group of five. This leaves only J to be selected. The answer is (D).

14. There would be only one possible way to select the four items if which of the following restrictions were added to the original set of conditions?

(A) If I is selected, then G is selected.
(B) Both I and G are selected.
(C) If J is selected, then M is selected.
(D) Either L or M is selected.
(E) If I is selected, then K is selected.

Begin with choice (A). If I is actually selected, then the four items selected would be fully determined. But choice (A) does not require that I be

† Taking the contrapositive of H—>~I yields I—>~H.

selected. Suppose G is selected.* Then L must be selected since G and L must be selected together. Now, we can satisfy all the conditions by selecting either H and M or H and J. Hence, the items selected are not fully determined, eliminate choice (A). Turning to choice (B), from the condition L<—>G, we know L must be selected. Now, since I has been selected, H cannot be selected (H—>~I). Hence, from the condition H or K, we know K must be selected. Thus, the four items are uniquely determined—I, G, L, K. The answer is choice (B).

Questions 15–18

15. Which one of the following is an acceptable grouping of the six items?

	Group 1	Group 2	Group 3
(A)	UV	WXY	Z
(B)	X	Y	VZUW
(C)	V	YW	XUZ
(D)	V	Z	XYUW
(E)	UW	YZ	XV

(A) No. This violates the condition "The number of items in Group 2 is less than or equal to the number of items in Group 3."
(B) No. This violates the condition "V and W cannot be in the same group."
(C) No. This violates the condition "X can be in Group 3 only if Y is in Group 3."
(D) Yes. The four conditions are satisfied:

(1) Group 1 has the same number of items as Group 2.
(2) Group 2 has fewer items than Group 3.
(3) V and W are in different groups.
(4) Y is with X in Group 3.

* Remember either G or I must be selected.

(E) No. This violates the condition "X can be in Group 3 only if Y is in Group 3."

16. If Group 1 contains only the item Y, which of the following must be true?

(A) Group 3 contains four items.
(B) Group 2 contains the same number of items as Group 3.
(C) V is in Group 3.
(D) Group 2 contains three items.
(E) X is in Group 2.

X cannot be in Group 1 since "Group 1 contains only the item Y." Suppose X is in Group 3. Then Y must also be in Group 3 since "X can be in Group 3 only if Y is in Group 3." But this violates the fact that Y is in Group 1. Since we have shown that X cannot be in either Group 1 or Group 3, it must be in Group 2. The answer is (E).

17. If W and Y are in the same group and V is in Group 3, then which of the following must be false?

(A) W and Y are in Group 2.
(B) U is the only item in Group 1.
(C) X is the only item in Group 1.
(D) U is in Group 3.
(E) Group 3 contains 2 items.

(A) No. The following grouping satisfies all the conditions and has W and Y in Group 2:

Group 1	Group 2	Group 3
X	WY	UVZ

(B) Yes. Place U and V on a diagram:

Group 1	Group 2	Group 3
U		V

Since "V and W cannot be in the same group," W must be in Group 2. Further, since "W and Y are in the same group," Y must also be in Group 2:

Group 1	Group 2	Group 3
U	WY	V

Now, since "the number of items in Group 2 is less than or equal to the number of items in Group 3," the remaining items must be in Group 3:

Group 1	Group 2	Group 3
U	WY	VXZ

However, this diagram violates the condition "X can be in Group 3 only if Y is in Group 3."

(C) No. The following grouping satisfies all the conditions and has X as the only item in Group 1:

Group 1	Group 2	Group 3
X	WY	UVZ

(D) No. The following grouping satisfies all the conditions and has U in Group 3:

Group 1	Group 2	Group 3
WY	XZ	VU

(E) No. The following grouping satisfies all the conditions and has 2 items in Group 3:

Group 1	Group 2	Group 3
WY	XZ	VU

18. If Group 2 contains only one item which is neither W nor V, which of the following must be true?

(A) Group 1 contains only V or only W.
(B) Group 3 contains W.
(C) Group 1 contains both U and V.
(D) Group 2 contains Z.
(E) Group 1 contains only W.

Since Group 2 contains only one item, Group 1 must contain only one item: "The number of items in Group 1 is less than or equal to the number of items in Group 2." Now, Group 3 cannot contain both V and W since "V and W cannot be in the same group." Hence, either V or W must be in Group 1 as neither can be in Group 2. The answer is (A).

Assignment Games

We have discussed various ways to order elements—**ordering games**, and various ways to group elements—**grouping games**. Now we will discuss various ways to assign characteristics to elements—these are **assignment games**. Assignment games will wind up our discussion of the three major types of games. They tend to be the hardest games, so it's wise to save them for last.

Assignment games match a characteristic with an element of the game. For example, you may be asked to assign a schedule: Bob works only Monday, Tuesday, or Friday. Or you may be told that a person is either a Democrat or a Republican.

Because the characteristics are typically assigned to groups of elements, assignment games can look very similar to grouping games. Additionally, in assigning

Example: *(Assignment game)*

There are eight players on a particular basketball team—A, B, C, D, F, G, H, I. Three are guards, three are forwards, and two are centers. Each player is either a free agent or not a free agent.

All of the guards are free agents.
A and C are forwards; F and H are not forwards.
Only one forward is a free agent.

In this game the team has already been selected; now the goal is to assign a position (characteristic) to each player and decide whether he is a free agent (characteristic). Notice that conditions, such as *"All of the guards are free agents,"* apply to groups of individuals; this makes the game at first glance appear to be a grouping game.

Many assignment games can be solved very efficiently by using a elimination grid. An example will illustrate this method of diagramming.

Elimination Grid

Dean Peterson, Head of the Math Department at Peabody Polytech, is making the fall teaching schedule. Besides himself there are four other professors—Warren, Novak, Dornan, and Emerson. Their availability is subject to the following constraints.

Warren cannot teach on Monday or Thursday.
Dornan cannot teach on Wednesday.
Emerson cannot teach on Monday or Friday.
Associate Professor Novak can teach at any time.
Dean Peterson cannot teach evening classes.
Warren can teach only evening classes.
Dean Peterson cannot teach on Wednesday if Novak teaches on Thursday, and Novak teaches on Thursday if Dean Peterson cannot teach on Wednesday.
At any given time there are always three classes being taught.

1. At which one of the following times can Warren, Dornan, and Emerson all be teaching?
   (A) Monday morning
   (B) Friday evening
   (C) Tuesday evening
   (D) Friday morning
   (E) Wednesday morning

2. For which day will the dean have to hire a part-time teacher?
   (A) Monday
   (B) Tuesday
   (C) Wednesday
   (D) Thursday
   (E) Friday

3. Which one of the following must be false?
   (A) Dornan does not work on Tuesday.
   (B) Emerson does not work on Tuesday morning.
   (C) Peterson works every day of the week except Wednesday.
   (D) Novak works every day of the week except Wednesday.
   (E) Dornan works every day of the week except Wednesday.

4. If Novak does not work on Thursday, then which one of the following must be true?
   (A) Peterson works Tuesday morning.
   (B) Dornan works Tuesday morning.
   (C) Emerson works Tuesday.
   (D) Peterson works on Wednesday.
   (E) Warren works Tuesday morning.

We indicate that a teacher does not work at a particular time by placing an X on the elimination grid. Placing the two conditions *"Warren cannot teach on Monday or Thursday"* and *"Warren can teach only evening classes"* on the grid gives

	M	T	W	TH	F	
Warren	X	X	X	X	X	a.m.
	X			X		p.m.
Dorman						a.m.
						p.m.
Novak						a.m.
						p.m.
Emerson						a.m.
						p.m.
Peterson						a.m.
						p.m.

Placing the remaining conditions in like manner gives

	M	T	W	TH	F	
Warren	X	X	X	X	X	a.m.
	X			X		p.m.
Dorman			X			a.m.
			X			p.m.
Novak						a.m.
						p.m.
Emerson	X				X	a.m.
	X				X	p.m.
Peterson						a.m.
	X	X	X	X	X	p.m.

To answer the following questions, we will refer only to the grid, not the original problem.

1. **At which one of the following times can Warren, Dornan, and Emerson all be teaching?**

   (A) **Monday morning**
   (B) **Friday evening**
   (C) **Tuesday evening**
   (D) **Friday morning**
   (E) **Wednesday morning**

The grid clearly shows that all three can work on Tuesday night. The answer is (C).

2. **For which day will the dean have to hire a part-time teacher?**

   (A) **Monday**
   (B) **Tuesday**
   (C) **Wednesday**
   (D) **Thursday**
   (E) **Friday**

Dornan and Novak are the only people who can work Monday evenings, and three classes are always in session, so extra help will be needed for Monday evenings. The answer is (A).

3. **Which one of the following must be false?**

   (A) **Dornan does not work on Tuesday.**
   (B) **Emerson does not work on Tuesday morning.**
   (C) **Peterson works every day of the week except Wednesday.**
   (D) **Novak works every day of the week except Wednesday.**
   (E) **Dornan works every day of the week except Wednesday.**

The condition *"Dean Peterson cannot teach on Wednesday if Novak teaches on Thursday, and Novak teaches on Thursday if Dean Peterson cannot teach on Wednesday"* can be symbolized as **(P≠W)<—>(N=TH)**. Now, if Novak works every day of the week, except Wednesday, then in particular he works Thursday. So from the condition **(P≠W)<—>(N=TH)**, we know that Dean Peterson cannot work on Wednesday. But from the grid this leaves only Novak and Emerson to teach the three Wednesday morning classes. Hence the answer is (D).

4. **If Novak does not work on Thursday, then which one of the following must be true?**

   (A) **Peterson works Tuesday morning.**
   (B) **Dornan works Tuesday morning.**
   (C) **Emerson works on Tuesday.**
   (D) **Peterson works on Wednesday.**
   (E) **Warren works on Tuesday morning.**

If you remember to think of an *if-and-only-if* statement as an equality, then this will be an easy problem. Negating both sides of the condition

(P≠W)<—>(N=TH)

gives

Multiple-Choice Game

There are four partners in a particular law firm. Each partner is an expert in at least one of three fields: criminal law, worker's compensation, and patent law. These are the only areas of law that the partners of the firm practice.

D and F both practice in at least one of the same fields.
D practices in worker's compensation and patent law.
F practices in only two fields.
D and E do not practice in the same field.
F and H do not practice in the same field.

1. Which one of the following must be false?

   (A) F practices in exactly two fields.
   (B) H practices in exactly one field.
   (C) E practices in more than one field.
   (D) E practices in only one field.
   (E) D practices in exactly two fields.

2. The people in which one of the following pairs could practice in exactly the same fields?

   (A) D and H
   (B) E and F
   (C) D and E
   (D) E and H
   (E) H and F

3. If the combination of fields in which F practices is different from any of the combinations in which her colleagues practice, then which one of the following must be true?

   (A) H does not practice patent law.
   (B) F does not practice patent law.
   (C) H does not practice worker's compensation.
   (D) F practices criminal law.
   (E) F and H practice in the same fields.

4. If a new partner who practices in exactly two fields joins the firm, then he cannot practice in all of the fields that the combination

   (A) D and F do
   (B) E and H do
   (C) E and F do
   (D) D and H do
   (E) F and H do

In this game, the goal is to assign one or more characteristics (fields of practice) to each element (partner). Hence it is a multiple-choice game, and therefore an elimination grid is unwarranted.

The diagram for this game will consist of four compartments, one for each of the partners D, E, F, and H:

D	E	F	H

Let the letters C, W, and P stand for "practices in criminal law," "practices in worker's comp.," and "practices in patent law," in that order. Placing the condition *"D practices in worker's comp. and patent law"* on the diagram gives

D	E	F	H
W&P			

Next, the condition *"D and E do not practice in the same field"* means that E practices only criminal law, and D practices only worker's comp. and patent law—otherwise they would practice in some of the same fields. Adding this to the diagram yields

D	E	F	H
W&P	C		
2	1		

Next, the condition *"D and F both practice in at least one of the same fields"* means that F must practice in either worker's comp. or patent law, or both. Adding this to the diagram along with the condition *"F practices in only two fields"* gives

D	E	F	H
W&P	C	W or P	
2	1	2	

Finally, the condition *"H and F do not practice in the same field"* means that if F practices worker's comp., then H does not; and if F practices patent law, then H does not. In other words, ~W or ~P. In the diagram, we use an arrow to indicate this conditional relationship between F and H as follows:*

D	E	F	H
W&P	C	W or P →	~Wor~P
2	1	2	1

This diagram is a bit more restrictive than the situation warrants: F could practice criminal law. A more precise diagram would be

D	E	F	H
W&P	C	WorPorC →	~Wor~Por~C
2	1	2	1

However, the previous diagram is sufficient for answering the questions that follow.

* Note because F practices in two fields, and H and F do not practice in the same fields, H can practice in only one field.

1. **Which one of the following must be false?**

   (A) F practices in exactly two fields.
   (B) H practices in exactly one field.
   (C) E practices in more than one field.
   (D) E practices in only one field.
   (E) D practices in exactly two fields.

The diagram clearly shows that (A), (B), (D), and (E) are true and that (C) is false. Thus the answer is (C).

2. **The people in which one of the following pairs could practice in exactly the same fields?**

   (A) D and H
   (B) E and F
   (C) D and E
   (D) E and H
   (E) H and F

From the diagram, we see that D and H cannot practice in exactly the same fields because D practices in two fields, whereas H practices in only one. This dismisses (A). A similar analysis dismisses choices (B), (C), and (E). As a matter of test taking strategy this would be sufficient to mark the answer (D), but it is instructive to work out a possible assignment. You should verify that the following diagram is consistent with all the initial conditions:

D	E	F	H
W&P	C	W&P	C
2	1	2	1

3. **If the combination of fields in which F practices is different from any of the combinations in which her colleagues practice, then which one of the following must be true?**

   (A) H does not practice patent law.
   (B) F does not practice patent law.
   (C) H does not practice worker's compensation.
   (D) F practices criminal law.
   (E) F and H practice in the same fields.

From the diagram, we know that F must practice either worker's comp. or patent law; but because of the new condition, she cannot practice both—otherwise she would practice in the same fields as D. So F must practice criminal law, and the answer is (D).

4. If a new partner who practices in exactly two fields joins the firm, then he cannot practice in all of the fields that the combination

   (A) D and F do
   (B) E and H do
   (C) E and F do
   (D) D and H do
   (E) F and H do

Again from the diagram, we see that F and H practice in mutually exclusive fields. Furthermore, F practices in two fields and H practices in one field, so between them they practice in all three fields. But we are told that the new partner practices in only two fields. Hence he cannot practice in as many fields as do F and H combined. The answer is (E).

You probably have noticed that once the diagram has been constructed, assignment games are somewhat manageable. However, the diagram may not be easy to construct, and it may require considerable inspiration to figure out what kind of diagram to use. As you work the exercises in this section, you will develop more intuition in this regard.

Points to Remember

1. Assignment games tend to be the hardest, so save them for last.

2. Assignment games <u>pair</u> each element with one or more characteristics, whereas grouping games <u>partition</u> the elements into two or more groups.

3. It is important that you identify whether you are dealing with an assignment game or a grouping game because different methods are used to solve each type of game.

4. Elimination grids are very effective when they can be applied, which is about one-third of the time.

5. Exact criteria cannot be given for when to use an elimination grid. But if only two options (characteristics) are available to the elements—yes/no, on/off, etc.—then an elimination grid is probably indicated.

EXERCISE

Directions: The following group of questions is based on a set of conditions. Choose the response that most accurately and completely answers each question. Answers and solutions begin on page 222.

Questions 1–6

The Mom & Pop liquor store employs five cashiers—Adams, Bates, Cox, Drake, and Edwards—each of whom works alone on exactly one day, Monday through Friday.

Adams works only Mondays or Wednesdays.
Bates will not work Wednesdays or Fridays.
Drake and Edwards work on consecutive days.

1. Which one of the following is a possible work schedule?
   (A) Edwards, Bates, Adams, Drake, Cox
   (B) Bates, Adams, Cox, Edwards, Drake
   (C) Edwards, Drake, Adams, Cox, Bates
   (D) Adams, Bates, Edwards, Cox, Drake
   (E) Drake, Edwards, Adams, Bates, Cox

2. If Cox works on Tuesday, then all of the following statements must be true EXCEPT:
   (A) Bates works on Monday.
   (B) Adams works on Wednesday.
   (C) Drake could work on Thursday.
   (D) Edwards could work on Friday.
   (E) Drake could work on Wednesday.

3. Which one of the following CANNOT be true?
   (A) Cox works on Thursday.
   (B) Edwards works on Monday.
   (C) Adams and Bates work on consecutive days.
   (D) Drake and Edwards work on consecutive days.
   (E) Cox works on Monday.

4. If Bates works Thursday, which one of the following must be true?
   (A) Adams works Wednesday.
   (B) Drake works Tuesday.
   (C) Cox works Friday.
   (D) Edwards works Wednesday.
   (E) Adams works Monday.

5. If Adams and Bates CANNOT work on consecutive days, then which one of the following must be false?
   (A) Cox works Tuesday.
   (B) Edwards works Monday.
   (C) Drake works Tuesday.
   (D) Edwards works Wednesday.
   (E) Adams works Monday.

6. If Bates CANNOT work either immediately before or after Edwards, then which one of the following must be false?
   (A) Edwards works on Monday.
   (B) Edwards works on Tuesday.
   (C) Edwards works on Wednesday.
   (D) Edwards works on Thursday.
   (E) Edwards works on Friday.

The following games are taken from recent LSATs.

Questions 7–10

A street cleaning crew works only Monday to Friday, and only during the day. It takes the crew an entire morning or an entire afternoon to clean a street. During one week the crew cleaned exactly eight streets—First, Second, Third, Fourth, Fifth, Sixth, Seventh, and Eighth streets. The following is known about the crew's schedule for the week:

The crew cleaned no street on Friday morning.
The crew cleaned no street on Wednesday afternoon.
It cleaned Fourth Street on Tuesday morning.
It cleaned Seventh Street on Thursday morning.
It cleaned Fourth Street before Sixth Street and after Eighth Street.
It cleaned Second, Fifth, and Eighth streets on afternoons.

7. If the crew cleaned Second Street earlier in the week than Seventh Street, then it must have cleaned which one of the following streets on Tuesday afternoon?
   (A) First Street
   (B) Second Street
   (C) Third Street
   (D) Fifth Street
   (E) Eighth Street

8. If the crew cleaned Sixth Street on a morning and cleaned Second Street before Seventh Street, then what is the maximum number of streets whose cleaning times cannot be determined?
   (A) 1
   (B) 2
   (C) 3
   (D) 4
   (E) 5

9. What is the maximum possible number of streets any one of which could be the one the crew cleaned on Friday afternoon?
   (A) 1
   (B) 2
   (C) 3
   (D) 4
   (E) 5

10. If the crew cleaned First Street earlier in the week than Third Street, then which one of the following statements must be false?
   (A) The crew cleaned First Street on Tuesday afternoon.
   (B) The crew cleaned Second Street on Thursday afternoon.
   (C) The crew cleaned Third Street on Wednesday morning.
   (D) The crew cleaned Fifth Street on Thursday afternoon.
   (E) The crew cleaned Sixth Street on Friday afternoon.

Questions 11–12

A store sells shirts only in small, medium, and large sizes, and only in red, yellow, and blue colors. Casey buys exactly three shirts from the store.

A shirt type consists of both a size and a color.
Casey does not buy two shirts of the same type.
Casey does not buy both a small shirt and a large shirt.
No small red shirts are available.
No large blue shirts are available.

11. If Casey buys a small blue shirt, which one of the following must be false?
   (A) Casey buys two blue shirts.
   (B) Casey buys two red shirts.
   (C) Casey buys two yellow shirts.
   (D) Casey buys two small shirts.
   (E) Casey buys two medium shirts.

12. If Casey buys exactly one medium shirt and does not buy two shirts of the same color, then she cannot buy which one of the following?
   (A) a medium red shirt
   (B) a medium yellow shirt
   (C) a medium blue shirt
   (D) a large red shirt
   (E) a large yellow shirt

Answers and Solutions to Exercise

Questions 1–6

This is a scheduling game of medium difficulty. We shall use an elimination grid to solve it. To indicate that a person does not work on a particular day, place an X on the grid. To indicate that a person does work on a particular day, place a W on the grid. Placing the conditions on the grid yields

	M	T	W	TH	F
Adams		X		X	X
Bates			X		X
Cox					
Drake					
Edwards					

1. **Which one of the following is a possible work schedule?**

   (A) Edwards, Bates, Adams, Drake, Cox
   (B) Bates, Adams, Cox, Edwards, Drake
   (C) Edwards, Drake, Adams, Cox, Bates
   (D) Adams, Bates, Edwards, Cox, Drake
   (E) Drake, Edwards, Adams, Bates, Cox

(A) and (D) are not possible work schedules since Drake and Edwards must work on consecutive days. (B) is not a possible work schedule since Adams must work on Monday or Wednesday. (C) is not a possible work schedule since Bates will not work on Friday. Hence, by process of elimination, the answer is (E).

2. **If Cox works on Tuesday, then all of the following statements must be true EXCEPT:**

   (A) Bates works on Monday.
   (B) Adams works on Wednesday.
   (C) Drake could work on Thursday.
   (D) Edwards could work on Friday.
   (E) Drake could work on Wednesday.

Adding the new condition to the grid (and recalling only one person works each day) yields

	M	T	W	TH	F
Adams		X		X	X
Bates		X	X		X
Cox	X	W	X	X	X
Drake		X			
Edwards		X			

Clearly, from the grid, either Drake or Edwards must work Friday. Further, since Drake and Edwards work on consecutive days, they must work Thursday and Friday. So Drake cannot work Wednesday. The answer is (E).

3. **Which one of the following CANNOT be true?**

   (A) Cox works on Thursday.
   (B) Edwards works on Monday.
   (C) Adams and Bates work on consecutive days.
   (D) Drake and Edwards work on consecutive days.
   (E) Cox works on Monday.

Begin with (A). Add *"Cox works on Thursday"* to the grid:

	M	T	W	TH	F
Adams		X		X	X
Bates			X	X	X
Cox	X	X	X	W	X
Drake				X	
Edwards				X	

Now, the condition *"Drake and Edwards work on consecutive days"* generates two grids—one with Drake and Edwards working on Monday and Tuesday (not necessarily in that order) and one with Drake and Edwards working on Tuesday and Wednesday (not necessarily in that order):

Diagram 1

	M	T	W	TH	F
Adams	X	X		X	X
Bates	X	X	X	X	X
Cox	X	X	X	W	X
Drake	W	X	X	X	X
Edwards	X	W	X	X	X

Diagram II

	M	T	W	TH	F
Adams		X	X	X	X
Bates		X	X	X	X
Cox	X	X	X	W	X
Drake	X	W	X	X	X
Edwards	X	X	W	X	X

Clearly, Diagram I leaves no day for Bates to work. And Diagram II forces Adams and Bates to work together on Monday, violating the condition that only one employee works at a time. Hence, Cox cannot work on Thursday. The answer is (A).

4. **If Bates works Thursday, which one of the following must be true?**

   (A) Adams works Wednesday.
   (B) Drake works Tuesday.
   (C) Cox works Friday.
   (D) Edwards works Wednesday.
   (E) Adams works Monday.

Adding the condition *"Bates works Thursday"* yields

	M	T	W	TH	F
Adams		X		X	X
Bates	X	X	X	W	X
Cox				X	
Drake				X	
Edwards				X	

Since Drake and Edwards must work on consecutive days, neither can work on Friday. This leaves only Cox to work on Friday. The answer is (C).

5. If Adams and Bates CANNOT work on consecutive days, then which one of the following must be false?

(A) Cox works Tuesday.
(B) Edwards works Monday.
(C) Drake works Tuesday.
(D) Edwards works Wednesday.
(E) Adams works Monday.

Begin with (A). Place Cox on the grid:

	M	T	W	TH	F
Adams		X		X	X
Bates		X	X		X
Cox	X	W	X	X	X
Drake		X			
Edwards		X			

Now, Drake and Edwards could work on Thursday and Friday:

	M	T	W	TH	F
Adams		X		X	X
Bates		X	X	X	X
Cox	X	W	X	X	X
Drake	X	X	X	W	X
Edwards	X	X	X	X	W

Clearly, this diagram forces Bates to work on Monday and Adams to work on Wednesday:

	M	T	W	TH	F
Adams	X	X	**W**	X	X
Bates	**W**	X	X	X	X
Cox	X	**W**	X	X	X
Drake	X	X	X	**W**	X
Edwards	X	X	X	X	**W**

This diagram satisfies all the initial conditions and Adams and Bates are not working on consecutive days. This eliminates (A).

Turning to (B), place Edwards on the grid:

	M	T	W	TH	F
Adams	X	X		X	X
Bates	X		X		X
Cox	X				
Drake	X				
Edwards	**W**	X	X	X	X

Since Drake and Edwards must work on consecutive days, we know that Drake must work Tuesday:

	M	T	W	TH	F
Adams	X	X		X	X
Bates	X	X	X		X
Cox	X	X			
Drake	X	W	X	X	X
Edwards	W	X	X	X	X

Clearly, this diagram forces Adams to work on Wednesday and Bates to work on Thursday. However, this violates the supplemental condition *"Adams and Bates CANNOT work on consecutive days."* The answer is (B).

6. If Bates CANNOT work either immediately before or after Edwards, then which one of the following must be false?

(A) Edwards works on Monday.
(B) Edwards works on Tuesday.
(C) Edwards works on Wednesday.
(D) Edwards works on Thursday.
(E) Edwards works on Friday.

Before starting, you should scan the answer-choices for one that eliminates many positions for Bates. Now, if Edwards works Wednesday, then Bates cannot work either Tuesday or Thursday. So we begin with Edwards working Wednesday:

	M	T	W	TH	F
Adams		X	X	X	X
Bates			X		X
Cox			X		
Drake			X		
Edwards	X	X	W	X	X

Again, this prevents Bates from working either Tuesday or Thursday:

	M	T	W	TH	F
Adams		X	X	X	X
Bates		X	X	X	X
Cox			X		
Drake			X		
Edwards	X	X	W	X	X

Clearly, this diagram does not leave room for both Adams and Bates since they cannot work the same day. The answer is (C).

Questions 7–10

This is a hard assignment (schedule) game, mainly because it is quite long and contains a labyrinth of information. However, a well-chosen diagram will greatly simplify it.

We symbolize only the last two conditions. The condition *"It cleaned Fourth Street before Sixth Street and after Eighth Street"* can be symbolized as **8th—>4th—>6th**. The condition *"It cleaned Second, Fifth, and Eighth streets on afternoons"* can be symbolized as **2d, 5th, 8th = afternoon**. Place an X on the diagram to indicate that a street is not cleaned at a particular time. The first two conditions state that no street is cleaned on either Wednesday afternoon or Friday morning:

	M	T	W	TH	F
a.m.					X
p.m.			X		

Next, use a street's number (1st, 2nd, etc.) to indicate it is cleaned at a particular time. (Note: although the problem does not state it, you are to assume that each street is cleaned only once a week.) The third and fourth conditions say that Fourth Street and Seventh Street were cleaned on Tuesday and Thursday mornings, respectively:

	M	T	W	TH	F
a.m.		4		7	X
p.m.			X		

Now, from the condition **8th—>4th—>6th** and the fact that Fourth Street was cleaned on Tuesday, we see that Eighth Street was cleaned on Monday. Combining this with the condition **2d, 5th, 8th = afternoon** shows that Eighth Street was cleaned on Monday afternoon:

	M	T	W	TH	F
a.m.		4		7	X
p.m.	8		X		

7. **If the crew cleaned Second Street earlier in the week than Seventh Street, then it must have cleaned which one of the following streets on Tuesday afternoon?**

   (A) First Street
   (B) Second Street
   (C) Third Street
   (D) Fifth Street
   (E) Eighth Street

From the diagram and the condition **2d, 5th, 8th = afternoon**, we see that the crew must have cleaned Second Street on Tuesday afternoon:

	M	T	W	TH	F
a.m.		4		7	X
p.m.	8	2	X		

The answer is (B).

8. **If the crew cleaned Sixth Street on a morning and cleaned Second Street before Seventh Street, then what is the maximum number of streets whose cleaning times cannot be determined?**

   (A) 1
   (B) 2
   (C) 3
   (D) 4
   (E) 5

The new condition, *"the crew cleaned Sixth Street on a morning,"* and the condition **8th—>4th—>6th** force Sixth Street to have been cleaned on Wednesday morning:

	M	T	W	TH	F
a.m.		4	6	7	X
p.m.	8		X		

Next, the new condition, *"[the crew] cleaned Second Street before Seventh Street"* and the condition **2d, 5th, 8th = afternoon** force Second Street to have been cleaned on Tuesday afternoon:

	M	T	W	TH	F
a.m.		4	6	7	X
p.m.	8	2	X		

This chart leaves undetermined the cleaning times of only three streets—First, Third, and Fifth. The answer is (C).

9. What is the maximum possible number of streets any one of which could be the one the crew cleaned on Friday afternoon?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

The original diagram

	M	T	W	TH	F
a.m.		4		7	X
p.m.	8		X		

shows that the cleaning times of only 4th, 7th, and 8th streets are determined. Combining this with the condition **2d, 5th, 8th = afternoon** shows that all the remaining streets could have been cleaned on Friday afternoon. The answer is (E).

10. If the crew cleaned First Street earlier in the week than Third Street, then which one of the following statements must be false?

(A) The crew cleaned First Street on Tuesday afternoon.
(B) The crew cleaned Second Street on Thursday afternoon.
(C) The crew cleaned Third Street on Wednesday morning.
(D) The crew cleaned Fifth Street on Thursday afternoon.
(E) The crew cleaned Sixth Street on Friday afternoon.

Suppose the crew cleaned First Street on Tuesday afternoon, choice (A). Then since Second and Fifth streets must be cleaned in the afternoon (**2d, 5th, 8th = afternoon**), we see from the original diagram that they must be cleaned on Thursday and Friday afternoons:

	M	T	W	TH	F
a.m.		4		7	X
p.m.	8	1	X		

2nd / 5th

The new condition, *"the crew cleaned First Street earlier in the week than Third Street,"* forces Third Street to be cleaned on Wednesday morning. Further, the condition **8th—>4th—>6th** forces Sixth Street to also be cleaned on Wednesday morning. But only one street at a time can be cleaned. The answer is (A).

Questions 11–12

Be careful not to interpret the condition *"Casey does not buy both a small shirt and a large shirt"* to mean that she buys neither a small shirt nor a large shirt.* We symbolize the condition as ~(S&L). The condition *"No small red shirts are available"* can be symbolized as **~(SR)**. The remaining conditions are symbolized in like manner:

~(same type)
~(S&L)
~(SR)
~(LB)

11. If Casey buys a small blue shirt, which one of the following must be false?

(A) Casey buys two blue shirts.
(B) Casey buys two red shirts.
(C) Casey buys two yellow shirts.
(D) Casey buys two small shirts.
(E) Casey buys two medium shirts.

We shall attempt to construct a diagram for each answer-choice. The one for which this cannot be done will be the answer. Begin with (A). Suppose Casey buys two blue shirts, one of which is small:

S B	B	

If the other blue shirt is a medium and the third shirt is a medium yellow, the diagram becomes

S B	M B	M Y

* To give the sentence that meaning rewrite it as "Casey does not buy a small shirt nor a large shirt."

This diagram does not have two shirts of the same type, does not have both a small and a large shirt, does not have a small red shirt, and does not have a large blue shirt. Hence, it satisfies every condition. This eliminates (A). Turning to choice (B), if Casey buys two red shirts, the diagram becomes

S		
B	R	R

Now, neither red shirt can be small, ~**(SR)**. Further, neither red shirt can be large since Casey does not buy both small and large shirts, ~**(S&L)**. Hence, both red shirts must be mediums. However, this violates the fact that she does not buy two shirts of the same type. Therefore, she cannot buy two red shirts. The answer is (B).

12. If Casey buys exactly one medium shirt and does not buy two shirts of the same color, then she cannot buy which one of the following?

(A) a medium red shirt
(B) a medium yellow shirt
(C) a medium blue shirt
(D) a large red shirt
(E) a large yellow shirt

The condition *"[she] does not buy two shirts of the same color"* means she buys one shirt of each color. Begin with (A). Suppose Casey buys a medium red shirt:

M		
R	B	Y

Then the remaining shirts could be a small blue and a small yellow:

M	S	S
R	B	Y

This diagram satisfies every condition. Hence, she can buy a medium red shirt—eliminate (A).

As to (B), if she buys a medium yellow shirt, the diagram becomes

M		
Y	R	B

Since she does not buy both small and large shirts, the two remaining shirts must be both small or both large:

Diagram I

M	S	S
Y	R	B

Diagram II

M	L	L
Y	R	B

Diagram I violates the condition ~**(SR)**, and Diagram II violates the condition ~**(LB)**. Hence, she cannot buy a medium yellow shirt. The answer is (B).

LSAT GAME SECTION

Give yourself 35 minutes to complete this LSAT section. It is important that you time yourself so that you can find your optimum working pace, and so that you will know what to expect when you take the test.

Note, you will not be allowed any scratch paper during the actual LSAT; all your work must be done on the test booklet. To accustom yourself to writing in a confined space, you should write all your scratch work in the book.

Answers and solutions begin on page 238.

LSAT SECTION

Time—35 minutes
24 Questions

Directions: Each group of questions in this section is based on a set of conditions. In answering some of the questions, it may be useful to draw a rough diagram. Choose the response that most accurately and completely answers each question and blacken the corresponding space on your answer sheet.

Questions 1–5

The Mammoth Corporation has just completed hiring nine new workers: Brandt, Calva, Duvall, Eberle, Fu, Garcia, Haga, Irving, and Jessup.

> Fu and Irving were hired on the same day as each other, and no one else was hired that day.
> Calva and Garcia were hired on the same day as each other, and no one else was hired that day.
> On each of the other days of hiring, exactly one worker was hired.
> Eberle was hired before Brandt.
> Haga was hired before Duvall.
> Duvall was hired after Irving but before Eberle.
> Garcia was hired after both Jessup and Brandt.
> Brandt was hired before Jessup.

1. Who were the last two workers to be hired?
   (A) Eberle and Jessup
   (B) Brandt and Garcia
   (C) Brandt and Calva
   (D) Garcia and Calva
   (E) Jessup and Brandt

2. Who was hired on the fourth day of hiring?
   (A) Eberle
   (B) Brandt
   (C) Irving
   (D) Garcia
   (E) Jessup

3. Exactly how many workers were hired before Jessup?
   (A) 6
   (B) 5
   (C) 4
   (D) 3
   (E) 2

4. Which one of the following must be true?
   (A) Duvall was the first worker to be hired.
   (B) Haga was the first worker to be hired.
   (C) Fu and Irving were the first two workers to be hired.
   (D) Haga and Fu were the first two workers to be hired.
   (E) Either Haga was the first worker to be hired or Fu and Irving were the first two workers to be hired.

5. If Eberle was hired on a Monday, what is the earliest day on which Garcia could have been hired?
   (A) Monday
   (B) Tuesday
   (C) Wednesday
   (D) Thursday
   (E) Friday

GO ON TO THE NEXT PAGE.

Questions 6–12

An apartment building has five floors. Each floor has either one or two apartments. There are exactly eight apartments in the building. The residents of the building are J, K, L, M, N, O, P, and Q, who each live in a different apartment.

J lives on a floor with two apartments.
K lives on the floor directly above P.
The second floor is made up of only one apartment.
M and N live on the same floor.
O does not live on the same floor as Q.
L lives in the only apartment on her floor.
Q does not live on the first or second floor.

6. Which one of the following must be true?
   (A) Q lives on the third floor.
   (B) Q lives on the fifth floor.
   (C) L does not live on the fourth floor.
   (D) N does not live on the second floor.
   (E) J lives on the first floor.

7. Which one of the following CANNOT be true?
   (A) K lives on the second floor.
   (B) M lives on the first floor.
   (C) N lives on the fourth floor.
   (D) O lives on the third floor.
   (E) P lives on the fifth floor.

8. If J lives on the fourth floor and K lives on the fifth floor, which one of the following can be true?
   (A) O lives on the first floor.
   (B) Q lives on the fourth floor.
   (C) N lives on the fifth floor.
   (D) L lives on the fourth floor.
   (E) P lives on the third floor.

9. If O lives on the second floor, which one of the following CANNOT be true?
   (A) K lives on the fourth floor.
   (B) K lives on the fifth floor.
   (C) L lives on the first floor.
   (D) L lives on the third floor.
   (E) L lives on the fourth floor.

10. If M lives on the fourth floor, which one of the following must be false?
   (A) O lives on the fifth floor.
   (B) J lives on the first floor.
   (C) L lives on the second floor.
   (D) Q lives on the third floor.
   (E) P lives on the first floor.

11. Which one of the following must be true?
   (A) If J lives on the fourth floor, then Q does not live on the fifth floor.
   (B) If O lives on the second floor, then L does not live on the fourth floor.
   (C) If N lives on the fourth floor, then K does not live on the second floor.
   (D) If K lives on the third floor, then O does not live on the fifth floor.
   (E) If P lives on the fourth floor, then M does not live on the third floor.

12. If O lives on the fourth floor and P lives on the second floor, which one of the following must be true?
   (A) L lives on the first floor.
   (B) M lives on the third floor.
   (C) Q lives on the third floor.
   (D) N lives on the fifth floor.
   (E) Q lives on the fifth floor.

GO ON TO THE NEXT PAGE.

Questions 13–17

Hannah spends 14 days, exclusive of travel time, in a total of six cities.

Each city she visits is in one of three countries—X, Y, or Z.
Each of the three countries has many cities.
Hannah visits at least one city in each of the three countries.
She spends at least two days in each city she visits.
She spends only whole days in any city.

13. If Hannah spends exactly eight days in the cities of country X, then which one of the following CANNOT be true?
    - (A) She visits exactly two cities in country X.
    - (B) She visits exactly two cities in country Y.
    - (C) She visits exactly two cities in country Z.
    - (D) She visits more cities in country Y than in country Z.
    - (E) She visits more cities in country Z than in country Y.

14. If Hannah visits an equal number of cities in each of the countries, what is the greatest total number of days she can spend visiting cities in country X?
    - (A) 3
    - (B) 4
    - (C) 5
    - (D) 6
    - (E) 7

15. If Hannah spends three days in the cities of country Y and seven days in the cities of country Z, then which one of the following must be false?
    - (A) She visits more cities in country X than in country Y.
    - (B) She visits exactly two cities in country X.
    - (C) She visits more cities in country Z than in country X.
    - (D) She visits exactly two cities in country Z.
    - (E) She visits exactly three cities in country Z.

16. If the city of Nomo is in country X, and if Hannah spends as many days as possible in Nomo and as few days as possible in each of the other cities that she visits, then which one of the following must be true?
    - (A) Hannah cannot visit any other cities in country X.
    - (B) Hannah can visit four cities in country Y.
    - (C) Hannah can spend six days in Nomo.
    - (D) Hannah cannot spend more than four days in country Z.
    - (E) Hannah can visit, at most, a total of four cities in countries Y and Z.

17. If Hannah visits a combined total of four cities in countries X and Y, what is the greatest total number of days she can spend visiting cities in country Y?
    - (A) 6
    - (B) 7
    - (C) 8
    - (D) 9
    - (E) 10

GO ON TO THE NEXT PAGE.

Questions 18–24

Exactly six dogs—P, Q, R, S, T, and U—are entered in a dog show. The judge of the show awards exactly four ribbons, one for each of first, second, third, and fourth places, to four of the dogs. The information that follows is all that is available about the six dogs:

Each dog is either a greyhound or a labrador, but not both.
Two of the six dogs are female and four are male.
The judge awards ribbons to both female dogs, exactly one of which is a labrador.
Exactly one labrador wins a ribbon.
Dogs P and R place ahead of dog S, and dog S places ahead of dogs Q and T.
Dogs P and R are greyhounds.
Dogs S and U are labradors.

18. Which one of the following is a complete and accurate list of the dogs that can be greyhounds?

(A) P, Q
(B) P, R
(C) P, Q, R
(D) P, R, T
(E) P, Q, R, T

19. Which one of the following statements CANNOT be true?

(A) A female greyhound wins the second place ribbon.
(B) A female labrador wins the second place ribbon.
(C) A female labrador wins the third place ribbon.
(D) A male greyhound wins the fourth place ribbon.
(E) A female greyhound wins the fourth place ribbon.

20. Which one of the following dogs must be male?

(A) dog P
(B) dog R
(C) dog S
(D) dog T
(E) dog U

21. Which one of the following statements can be false?

(A) Dog P places ahead of dog R.
(B) Dog P places ahead of dog T.
(C) Dog R places ahead of dog U.
(D) Dog R places ahead of dog T.
(E) Dog S places ahead of dog U.

22. If dog Q is female, which one of the following statements can be false?

(A) Dog P is male.
(B) Dog R is male.
(C) Dog Q wins the fourth place ribbon.
(D) Dog Q is a greyhound.
(E) Dog T is a greyhound.

23. If dog T wins the fourth place ribbon, then which one of the following statements must be true?

(A) Dog P is male.
(B) Dog Q is male.
(C) Dog T is male.
(D) Dog Q is a labrador.
(E) Dog T is a labrador.

24. Which one of the following statements could be true?

(A) Dog P does not win a ribbon.
(B) Dog R does not win a ribbon.
(C) Dog S does not win a ribbon.
(D) Dog T wins a ribbon.
(E) Dog U wins a ribbon.

S T O P

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS SECTION ONLY.
DO NOT WORK ON ANY OTHER SECTION IN THE TEST.

ANSWERS AND SOLUTIONS TO LSAT SECTION

Answers to Questions

1.	D	9.	E	17.	C
2.	A	10.	C	18.	E
3.	A	11.	B	19.	B
4.	E	12.	C	20.	E
5.	D	13.	A	21.	A
6.	D	14.	D	22.	E
7.	E	15.	D	23.	B
8.	A	16.	B	24.	D

Questions 1–5

This is a rather easy linear ordering game. Following the strategies developed earlier, we let the first letter of each name stand for the name and then symbolize the conditions. *"Fu and Irving were hired on the same day as each other, and no one else was hired that day"* is naturally symbolized as F = I. We'll use an arrow to indicate that a person was hired before another. The condition *"Duvall was hired after Irving but before Eberle"* can be symbolized as I—>D—>E. The remaining conditions can be symbolized in like manner, giving the following schematic:

1. F = I
2. C = G
3. *
4. E—>B
5. H—>D
6. I—>D—>E
7. J—>G
8. B—>G
9. B—>J

(Note: The * reminds us of the condition *"On each of the other days of hiring, exactly one worker was hired,"* which could not be readily symbolized.) Since the game has no fixed elements (such as "E was hired first"), we'll use a flow chart to solve it. Condition 6 contains the most information, so build the chart around it:

I—>D—>E

Adding Conditions 1 and 5 gives

H ↘
$\boxed{\begin{matrix} I \\ F \end{matrix}}$ —>D —>E

(Note: The rectangle around I and F indicates that they were hired on the same day.)

Adding Conditions 4 and 9 gives

```
 H
   ↘
[I] —>D—>E—>B—>J
[F]
```

Finally, adding Conditions 2 and 7 gives (Note: Condition 8 is superfluous.)

```
 H
   ↘
[I] —>D—>E—>B—>J—>[G]
[F]               [C]
```

This diagram is quite determinative: from it we'll be able to read off the answers to all the questions.

1. **Who were the last two workers to be hired?**

   (A) Eberle and Jessup
   (B) Brandt and Garcia
   (C) Brandt and Calva
   (D) Garcia and Calva
   (E) Jessup and Brandt

Clearly from the diagram, both Garcia and Calva were hired on the last day. The answer is (D).

2. **Who was hired on the fourth day of hiring?**

   (A) Eberle
   (B) Brandt
   (C) Irving
   (D) Garcia
   (E) Jessup

Again from the diagram, we see that Irving and Fu could have been hired on the first day, Haga could have been hired on the second day (or vice versa, since they are not connected by a sequence of arrows), and Duvall must have been hired on the third day. Hence Eberle was hired on the fourth day. The answer is (A).

3. **Exactly how many workers were hired before Jessup?**

   (A) 6
   (B) 5
   (C) 4
   (D) 3
   (E) 2

Again, we merely read off the answer from the diagram. Checking the diagram, we see that six people are to the left of Jessup. Therefore, six people were hired before Jessup. The answer is (A).

4. Which one of the following must be true?

    (A) Duvall was the first worker to be hired.
    (B) Haga was the first worker to be hired.
    (C) Fu and Irving were the first two workers to be hired.
    (D) Haga and Fu were the first two workers to be hired.
    (E) Either Haga was the first worker to be hired or Fu and Irving were the first two workers to be hired.

In the diagram, Haga is in a different row than Fu and Irving, so we cannot determine whether Haga was hired first or Fu and Irving were hired first. The answer is (E).

5. If Eberle was hired on a Monday, what is the earliest day on which Garcia could have been hired?

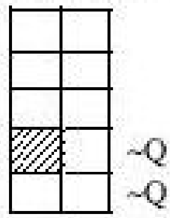
    (A) Monday
    (B) Tuesday
    (C) Wednesday
    (D) Thursday
    (E) Friday

If Eberle was hired on Monday, then from the chart we see that Brandt was hired at the earliest on Tuesday, Jessup on Wednesday, and both Garcia and Calva on Thursday. The answer is (D).

Questions 6–12

This is a hybrid (ordering / grouping) game of medium difficulty. As before, we construct a diagram to help answer the questions. The condition *"J lives on a floor with two apartments"* can be symbolized as **J = 2 apts**. The condition *"K lives on the floor directly above P"* is naturally symbolized as **K/P**. The condition *"The second floor is made up of only one apartment"* can be symbolized as **2d = alone**. The condition *"M and N live on the same floor"* is naturally symbolized as **M = N**. The condition *"O does not live on the same floor as Q"* is naturally symbolized as **O ≠ Q**. The condition *"L lives in the only apartment on her floor"* can be symbolized as **L = alone**. Finally, the condition *"Q does not live on the first or second floor"* is naturally symbolized as **Q ≠ 1st, 2d**. This gives the following schematic:

J,K,L,M,N,O,P,Q
J = 2 apts
K/P
2d = alone
M = N
O ≠ Q
L = alone
Q ≠ 1st, 2d



6. **Which one of the following must be true?**
   - (A) Q lives on the third floor.
   - (B) Q lives on the fifth floor.
   - (C) L does not live on the fourth floor.
   - (D) N does not live on the second floor.
   - (E) J lives on the first floor.

From the condition **M = N**, we know that M and N live on the same floor. But there is only one apartment on the second floor, **2d = alone**. Therefore, N cannot live on the second floor. The answer is (D).

7. Which one of the following CANNOT be true?

(A) K lives on the second floor.
(B) M lives on the first floor.
(C) N lives on the fourth floor.
(D) O lives on the third floor.
(E) P lives on the fifth floor.

From the condition K/P, we know that P must live directly below K and thus cannot possibly live on the top floor. The answer is (E).

8. If J lives on the fourth floor and K lives on the fifth floor, which one of the following can be true?

(A) O lives on the first floor.
(B) Q lives on the fourth floor.
(C) N lives on the fifth floor.
(D) L lives on the fourth floor.
(E) P lives on the third floor.

Adding the supplementary conditions to the diagram gives

K		
J		
		~Q
		~Q

From the condition K/P, we know that P must live on the fourth floor:

K		
J	P	
		~Q
		~Q

Now we try to construct a valid diagram for each of the answer-choices, starting with (A). Placing O on the bottom floor gives

K		
J	P	
		~Q
O		~Q

From the condition M = N, we see that both M and N must live on the 3d floor:

K		
J	P	
M	N	
		~Q
O		~Q

Now from the condition **L = alone**, we know that L must live on the second floor:

K		
J	P	
M	N	
	L	
O		~Q

Finally, the condition **O ≠ Q** forces Q to the top floor:

K	Q
J	P
M	N
	L
O	

This diagram satisfies all the original and supplementary conditions, so O *can* live on the first floor. The answer is (A).

9. If O lives on the second floor, which one of the following CANNOT be true?

(A) K lives on the fourth floor.
(B) K lives on the fifth floor.
(C) L lives on the first floor.
(D) L lives on the third floor.
(E) L lives on the fourth floor.

Add the new condition *"O lives on the second floor"* to the diagram:

	O	
		~Q

Now we attack the answer-choices, attempting to construct a diagram for each one. The answer-choice for which a valid diagram cannot be constructed will be the answer. Start with choice (E). Place L on the 4th floor:

Next, place J on the top floor and the condition **K/P** on the 2d and 3d floors:

O	J	
M	N	
	K	
	P	
		~Q

Finally, place Q on the 3d floor, and place L by itself on the bottom floor:

O	J
M	N
Q	K
	P
L	

This diagram does not violate any initial condition. Hence O *can* live on the top floor, which eliminates choice (A). Turning to (B), place J on the first floor:

M	N	
		~Q
J		~Q

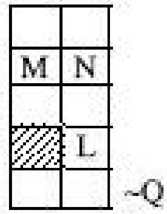
Next, place the condition **K/P** on the 2d and 3d floors, which forces L to the top floor (**L = alone**):

	L	
M	N	
	K	
	P	
J		~Q

Finally, place O and Q on the 1st and 3d floors, respectively:

	L
M	N
Q	K
	P
J	O

This diagram does not violate any initial condition. Hence J *can* live on the 1st floor, which eliminates choice (B). Turning to (C), place L on the 2d floor:



Clearly, this diagram does not allow for the placement of the condition K/P. Hence L cannot live on the second floor, and the answer is (C).

11. **Which one of the following must be true?**

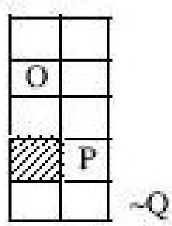
    (A) If J lives on the fourth floor, then Q does not live on the fifth floor.
    (B) If O lives on the second floor, then L does not live on the fourth floor.
    (C) If N lives on the fourth floor, then K does not live on the second floor.
    (D) If K lives on the third floor, then O does not live on the fifth floor.
    (E) If P lives on the fourth floor, then M does not live on the third floor.

This question is long because it actually contains five distinct questions. During the test you should save such a question for last. However, if you were alert, you may have noticed that there is a shortcut to this particular question: Notice that answer-choice (B) is merely a rewording of Question 9 and its answer. In Question 9, we learned that if O lives on the second floor, then L cannot live on the fourth floor. This is exactly what choice (B) says. Hence the answer is (B). **Remember, it is not uncommon for the LSAT writers to repeat a question in a different form.**

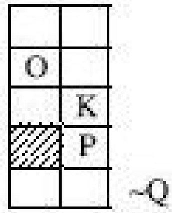
12. **If O lives on the fourth floor and P lives on the second floor, which one of the following must be true?**

    (A) L lives on the first floor.
    (B) M lives on the third floor.
    (C) Q lives on the third floor.
    (D) N lives on the fifth floor.
    (E) Q lives on the fifth floor.

Add the new conditions to the diagram:



Next, add the condition K/P:



Now the condition **M = N** can be placed on either the bottom or the top floor. We construct separate diagrams for each case:

Diagram 1

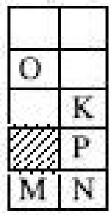
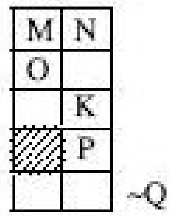


Diagram II



Next, since L must be alone, she must be on top floor in Diagram 1, and on the bottom floor in Diagram 2:

Diagram 1

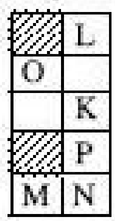
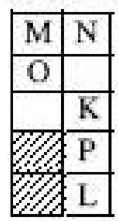


Diagram II

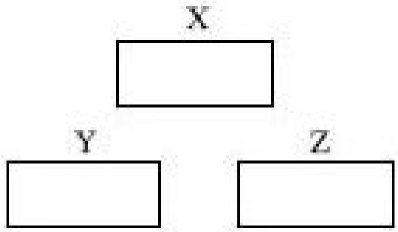


Clearly in both diagrams, the condition **O ≠ Q** forces Q onto the 3d floor. Hence the answer is (C).

Questions 13–17

Although a diagram is not needed to solve this game efficiently, we will draw one to help keep track of the information. It will consist of three rectangles, representing the three countries:

14 days
6 cities
Visits at least 1 city in each country
Spends at least 2 days in each city



13. **If Hannah spends exactly eight days in the cities of country X, then which one of the following CANNOT be true?**

    (A) She visits exactly two cities in country X.
    (B) She visits exactly two cities in country Y.
    (C) She visits exactly two cities in country Z.
    (D) She visits more cities in country Y than in country Z.
    (E) She visits more cities in country Z than in country Y.

Since Hannah spends 8 days out of a total of 14 visiting the cities of country X, she must spend the remaining 6 days in the cities of countries Y and Z. Further, since she must spend at least 2 days in each city, she can visit at most a total of 3 cities in countries Y and Z. Now if she visits only 2 cities in country X, then she will have visited a total of only 5 cities, which contradicts the fact that she visits 6 cities. Hence the answer is (A).

14. **If Hannah visits an equal number of cities in each of the countries, what is the greatest total number of days she can spend visiting cities in country X?**

    (A) 3
    (B) 4
    (C) 5
    (D) 6
    (E) 7

To visit an equal number of cities in each country, she must visit 2 cities in each country:

X

2 cities

Y | Z
2 cities | 2 cities

To spend the *greatest* number of days in country X, she must spend the *least* number of days possible in countries Y and Z. Since she must spend at least 2 days in each city, she must spend 4 days in country Y and 4 days in country Z:

X

2 cities

Y | Z
2 cities 4 days | 2 cities 4 days

This puts her in country X for 14 – 8 = 6 days. Hence the answer is (D).

15. If Hannah spends three days in the cities of country Y and seven days in the cities of country Z, then which one of the following must be false?

(A) She visits more cities in country X than in country Y.
(B) She visits exactly two cities in country X.
(C) She visits more cities in country Z than in country X.
(D) She visits exactly two cities in country Z.
(E) She visits exactly three cities in country Z.

First, since Hannah spends a total of 10 days visiting cities in countries Y and Z, we know she must spend 4 days in country X (14 – 10 = 4) and can visit a maximum of two cities. Additionally, since she spends 3 days in country Y and must spend at least 2 days in each city, we know that she visits only one city in country Y.

X

2 cities max
4 days

Y | Z
1 city 3 days | 7 days

So if she visits 2 cities in country Z, she will have visited at most 5 cities (1 + 2 + 2 = 5). But this contradicts the fact that she visits a total of 6 cities. Hence, the answer is (D).

16. **If the city of Nomo is in country X, and if Hannah spends as many days as possible in Nomo and as few days as possible in each of the other cities that she visits, then which one of the following must be true?**

    (A) **Hannah cannot visit any other cities in country X.**
    (B) **Hannah can visit four cities in country Y.**
    (C) **Hannah can spend six days in Nomo.**
    (D) **Hannah cannot spend more than four days in country Z.**
    (E) **Hannah can visit, at most, a total of four cities in countries Y and Z.**

To maximize her time in Nomo, Hannah must spend a minimum of two days in each of the other 5 cities she visits, for a total of 10 days outside Nomo. Hence, the most time she can spend in Nomo is four days. This eliminates (C). Now, the other 5 cities could all be in countries Y and Z, which eliminates (E). On the other hand, up to 3 cities could be in country X (and the remaining cities in countries Y and Z), which eliminates (A). If it happens that three of the five cities besides Nomo are located in country Z, Hannah will spend more than four days in that country, eliminating choice (D). This leaves choice (B) as the answer.

Let's check that Hannah can indeed visit four cities in country Y: She visits Nomo in country X for four days. Then if she visits 4 cities in country Y for eight days and 1 city in country Z for two days, she fulfills all the conditions for her trip. So indeed statement (B) must be true.

Note the unusual and subtle wording of this question: it asks which one of 5 *possibilities* must be true. To say that a *possibility* must be true is to claim that under some circumstances (but not necessarily all) the object of that possibility can be true. For example, to say "it must be true that John can run a four minute mile" is to say that "John ran this particular mile in four minutes can be true." The key point is that the statement may be true but doesn't have to be. The possibility must exist, but it need not be realized in every instance. So, in choice (B), the statement "Hannah can visit four cities in Y" must be true because there is a valid scenario in which she does visit four such cities—even though there are other scenarios as well.

17. **If Hannah visits a combined total of four cities in countries X and Y, what is the greatest total number of days she can spend visiting cities in country Y?**

    (A) 6
    (B) 7
    (C) 8
    (D) 9
    (E) 10

To spend the greatest number of days in the cities of country Y, Hannah must visit only one city in country X and for only 2 days. So she visits three cities in country Y. Now since she visits a total of six cities, she must visit two cities in country Z for a total of at least 4 days. Hence she spends a total of 6 days (2 + 4) outside country Y. Thus she can spend at most 8 days (14 – 6) in the cities of country Y. The answer is (C).

Questions 18–24

This game is difficult because it contains reams of information and the conditions are rather subtle. The game requires you to order, group, and assign elements, though it is mainly an assignment game. You are asked to assign characteristics (male, female; labrador, greyhound) to the elements (dogs).

The condition *"Dogs P and R place ahead of dog S, and dog S places ahead of dogs Q and T"* can be symbolized as $\boxed{\begin{matrix}P\\R\end{matrix}} \longrightarrow S \longrightarrow \boxed{\begin{matrix}Q\\T\end{matrix}}$. This diagram will form the core of the schematic. Although symbolizing the remaining conditions will clutter up the schematic, we will do so for consistency and for easy reference. During the test, it would probably be best to work just with the above diagram and make a mental note of the other conditions. Symbolizing the other conditions gives the following schematic:

P Q R S T U

P,R=Grey S,U=Lab 1 Lab=Rib 4 Males

2 Females, both won ribbons and one is a lab

$$\boxed{\begin{matrix}P\\R\end{matrix}} \longrightarrow S \longrightarrow \boxed{\begin{matrix}Q\\T\end{matrix}}$$

P Q R S T U

2 Females, both won ribbons and one is a lab

P,R=Grey　　S,U=Lab　　1 Lab=Rib　　4 Males

S → Rib　　U → No Rib → 5th or 6th

Ribs $\boxed{\begin{matrix}P\\R\end{matrix}} \rightarrow S \rightarrow \boxed{\begin{matrix}Q\\T\end{matrix}}$

Although this diagram took a bit of work to construct, it will greatly simplify the following problems.

18. **Which one of the following is a complete and accurate list of the dogs that can be greyhounds?**

    (A) P, Q
    (B) P, R
    (C) P, Q, R
    (D) P, R, T
    (E) P, Q, R, T

Since U finished either 5th or 6th, we see from the condition Ribs $\boxed{\begin{matrix}P\\R\end{matrix}} \rightarrow S \rightarrow \boxed{\begin{matrix}Q\\T\end{matrix}}$ that Q or T (but not both) placed fourth. Now suppose Q placed fourth. Then since S is the only labrador to win a ribbon, Q must be a greyhound. Now if we assume that T placed fourth, the same analysis shows that it must be a greyhound. Hence both Q and T can be greyhounds and of course P and R are greyhounds (we were given that). The answer is (E).

19. **Which one of the following statements CANNOT be true?**

    (A) A female greyhound wins the second place ribbon.
    (B) A female labrador wins the second place ribbon.
    (C) A female labrador wins the third place ribbon.
    (D) A male greyhound wins the fourth place ribbon.
    (E) A female greyhound wins the fourth place ribbon.

Earlier, we derived that S is the only labrador awarded a ribbon. Now from the condition Ribs $\boxed{\begin{matrix}P\\R\end{matrix}} \rightarrow S \rightarrow \boxed{\begin{matrix}Q\\T\end{matrix}}$, we see that S did not place second. Hence no labrador placed second. The answer is (B). (Note the fact that the labrador is female was not used.)

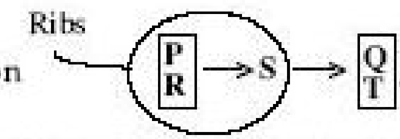
20. Which one of the following dogs must be male?

- **(A)** dog P
- **(B)** dog R
- **(C)** dog S
- **(D)** dog T
- **(E)** dog U

Since both females received ribbons and U did not, U must be male. The answer is (E).

21. Which one of the following statements can be false?

- **(A)** Dog P places ahead of dog R.
- **(B)** Dog P places ahead of dog T.
- **(C)** Dog R places ahead of dog U.
- **(D)** Dog R places ahead of dog T.
- **(E)** Dog S places ahead of dog U.

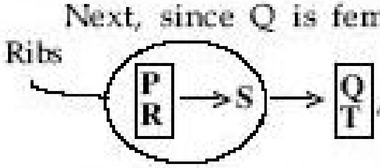
We'll use elimination. From the condition 

, we see that both P and R placed ahead of T. Hence both choices (B) and (D) are necessarily true—eliminate. Next, since U placed either 5th or 6th, both R and S placed ahead of U. Hence both choices (C) and (E) are necessarily true—eliminate. Therefore, by process of elimination, the answer is (A).

22. If dog Q is female, which one of the following statements can be false?

- **(A)** Dog P is male.
- **(B)** Dog R is male.
- **(C)** Dog Q wins the fourth place ribbon.
- **(D)** Dog Q is a greyhound.
- **(E)** Dog T is a greyhound.

Again, we use elimination. First, we show that S is female: S is a labrador and S wins a ribbon. Only one labrador wins a ribbon, and we are given that a female labrador wins a ribbon. Therefore, S is female. Now if Q is female, then all other dogs <u>must</u> be male. (Remember: 4 males and 2 females) This eliminates both (A) and (B).

Next, since Q is female, she won a ribbon. Now from the condition 

, we see that she placed 4th. This eliminates (C).

Finally, because we know Q wins a ribbon and only one lab, S, wins a ribbon, Q must be a greyhound. This eliminates (D). Thus, by process of elimination, the answer is (E).

23. If dog T wins the fourth place ribbon, then which one of the following statements must be true?

(A) Dog P is male.
(B) Dog Q is male.
(C) Dog T is male.
(D) Dog Q is a labrador.
(E) Dog T is a labrador.

Since dog T placed fourth, the condition Ribs $\boxed{\begin{matrix}P\\R\end{matrix}} \to S \to \boxed{\begin{matrix}Q\\T\end{matrix}}$ becomes

Ribs $\boxed{\begin{matrix}P\\R\end{matrix}} \to S \to T \to Q$. This diagram shows that Q placed fifth and therefore did not win a ribbon. But both females won ribbons, so Q must be male. The answer is (B).

24. Which one of the following statements could be true?

(A) Dog P does not win a ribbon.
(B) Dog R does not win a ribbon.
(C) Dog S does not win a ribbon.
(D) Dog T wins a ribbon.
(E) Dog U wins a ribbon.

This question is a freebie. Question 23 has T in fourth place and therefore winning a ribbon. Hence the answer is (D).