

```
In [30]: # HW_4.Y_Chang_Tiange
import numpy as np
import matplotlib.pyplot as plt
import scipy.stats as sp
from scipy.stats import expon
```

```
In [31]: M = 1000
mylambda = 2
sca = 1/mylambda
random_x = sp.norm.rvs(sca,sca,M)
print(random_x)
```

```
[ 3.77778444e-01  5.26332484e-01  7.09917346e-01  6.97388027e-01
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6.07151689e-01	3.70458966e-02	8.68619103e-01	-4.53452490e-01
2.98013159e-01	6.63852313e-01	7.01475304e-01	4.54882328e-01
7.98637671e-01	2.53236371e-01	-2.51151555e-01	1.94867647e-01
9.14285473e-01	1.77467294e+00	2.15046974e+00	1.21662447e+00
-1.51706888e-02	5.31342123e-01	2.55420394e-01	1.68893327e+00
1.50895960e+00	4.90710789e-01	5.87328405e-01	-1.76356825e-01
1.09515364e-01	7.88841268e-01	6.61635873e-01	7.68117872e-01
2.10922591e-01	4.43505420e-01	-2.82428817e-01	4.44900704e-02
2.21070459e-01	1.46961946e+00	1.18773884e+00	7.13632750e-01
2.37496369e-01	-5.55472072e-02	7.09928681e-01	4.78589508e-01
1.73221136e-01	8.47439384e-01	-3.55574060e-01	1.28802083e+00
6.96598559e-01	3.14295673e-01	6.63267015e-01	3.35088933e-01
1.31955922e-01	9.02958076e-01	8.45248022e-01	3.96348482e-01
1.14125985e+00	1.36378968e-01	-3.60719540e-01	4.76508284e-01
4.98335397e-01	2.37071301e-01	6.09949248e-01	8.90997518e-01
-2.12607704e-01	3.10885428e-01	-3.97977350e-02	3.82022171e-01
1.14356635e+00	3.03749604e-01	3.47230849e-01	6.17669567e-01
7.96918495e-01	6.64508111e-01	9.14334306e-01	5.52826284e-01
4.32559357e-01	1.16404130e+00	3.86964499e-01	-7.80522906e-02
5.57812724e-01	7.83985268e-01	4.67787638e-01	4.38832225e-02
3.58091389e-01	1.03806941e+00	8.75264053e-02	-3.17931969e-01
1.54149026e-01	7.38950491e-01	2.59050846e-01	7.13558470e-02
3.16032423e-01	6.30121851e-01	4.79077497e-01	7.28799534e-01
1.58289326e-01	-8.17811333e-02	-9.08617049e-02	5.58357376e-01
7.53293543e-01	5.96838713e-01	7.90924401e-02	7.58042846e-02
2.73000413e-02	4.95563229e-01	1.25415375e+00	3.41632382e-01
1.16791931e+00	-2.09250925e-01	5.65427782e-01	-5.99784958e-02
-3.72499357e-01	3.38926652e-01	6.86551968e-02	-7.11667923e-02
-5.72764059e-01	1.82746302e-01	5.16365058e-01	7.15912750e-01
1.01432024e+00	2.49675848e-01	8.22265759e-01	8.13018911e-01
7.80313341e-01	2.85043154e-01	1.25215028e+00	7.68986882e-01
6.74265236e-01	8.31288787e-01	3.95985869e-01	3.87485924e-01

```

3.77051781e-01  4.19662906e-01  1.10975266e+00  9.34554689e-01
-5.69927087e-02 -2.99594664e-01  4.11081502e-01  1.09679654e+00
5.35248411e-01  4.17356213e-01  9.60987956e-01  1.01564757e+00
2.74649548e-01  5.12594761e-01  1.96371660e-01 -3.99806205e-01
3.75866934e-01 -5.28950664e-01  6.69579814e-01 -3.87952153e-01
7.44135753e-01  3.31623448e-01  2.41935279e-01  8.98576323e-01
3.88314151e-01  5.06379551e-01  6.09820923e-01  4.89255034e-01
-3.59044991e-01  6.25015531e-01  4.09565092e-01  2.09759343e-01
5.35310040e-01  1.02531774e+00  9.52849904e-01  4.88698204e-01
4.24621382e-01  1.81487216e-01  3.47425234e-01  1.11633433e+00
-8.08475124e-02  4.07218672e-01 -3.50743947e-01  1.37296504e+00
4.94689301e-01 -6.38166276e-01  9.71570444e-01  1.15532774e+00
-1.48003710e-01 -3.00528631e-01  1.12896160e+00 -2.56787050e-01
3.50900014e-01  8.31761594e-01  3.38452923e-01  1.94020543e+00
6.97473883e-02  1.16914425e-01  5.58531827e-01  5.45023178e-01]

```

```

In [32]: sca_x = np.mean(random_x)
print('mean exact = ',sca)
print('mean sample = ',sca_x)

```

```

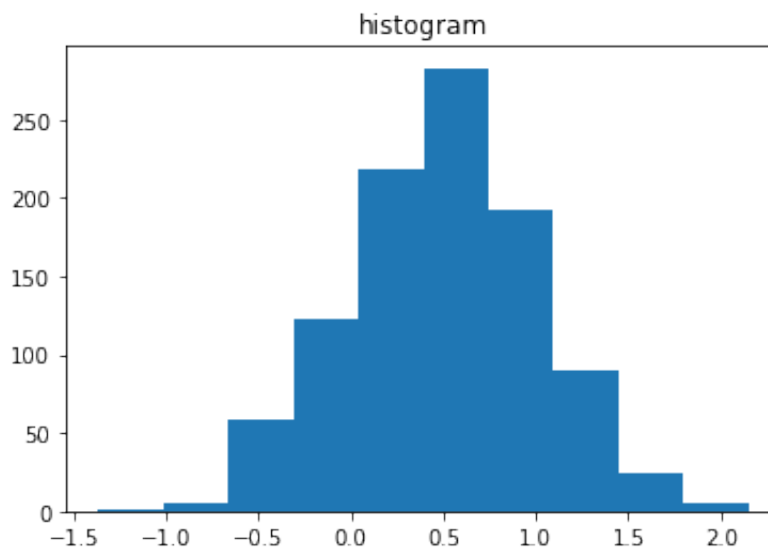
mean exact = 0.5
mean sample = 0.49515061292336837

```

```

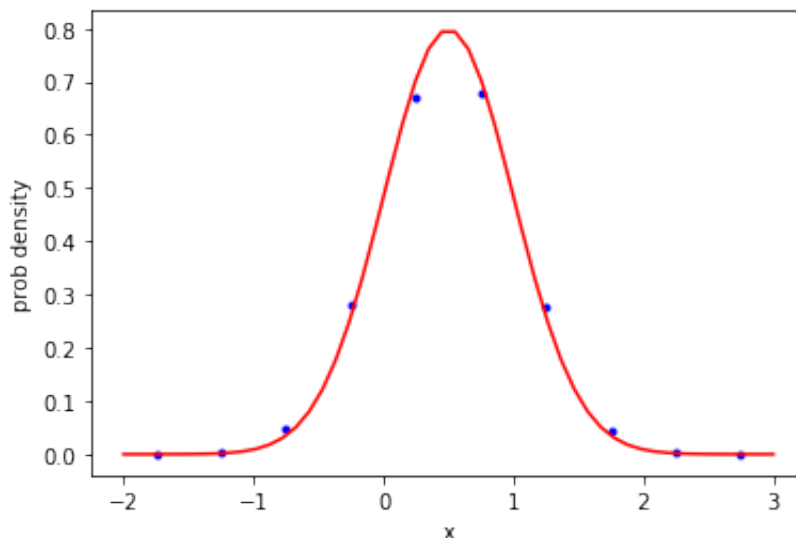
In [33]: plt.hist(random_x, bins = 10)
plt.title("histogram")
plt.show()

```

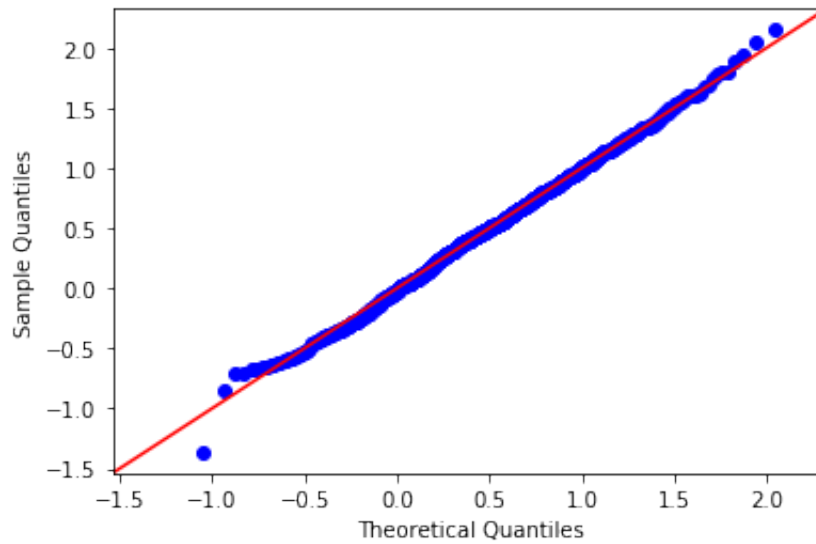


```
In [36]: ### kernel density estimate
nbins = 10 # number of bins
b = 3      # right end (from looking at histogram)
a = -2     # left end (from looking at histogram)
dx = (b-a)/nbins # bin width
bin_edges = np.linspace(a,b,nbins+1) # nbins+1 points for nbins bins
print(bin_edges)
# counts in each bin
counts, bin_edges = np.histogram(random_x,bins=bin_edges)
print(counts)
# find bin centers and kernel density
bin_centers = (bin_edges[1:] + bin_edges[:-1])/2
print(bin_centers)
kernel_density = counts/(M*dx)
print(kernel_density)
# plot of kernel density estimate and exact prob density
plt.plot(bin_centers,kernel_density,'b.')
xfine = np.linspace(a,b) # fine array in x for plotting exact curve
plt.plot(xfine,sp.norm.pdf(xfine,sca,sca),'r-')
#plt.ylim((0,1))
plt.xlabel('x')
plt.ylabel('prob density')
plt.show()
```

```
[-2.  -1.5 -1.  -0.5  0.   0.5  1.   1.5  2.   2.5  3. ]
[ 0  1 24 140 335 339 138 21  2  0]
[-1.75 -1.25 -0.75 -0.25  0.25  0.75  1.25  1.75  2.25  2.75]
[0.      0.002 0.048 0.28  0.67  0.678 0.276 0.042 0.004 0.   ]
```



```
In [37]: # quantile-quantile plot (qqplot)
import statsmodels.api as sm # statsmodels needed for qq plot
#sm.qqplot(random_x, dist=sp.norm(1,2), line = '45') <- OLD VERSION
sm.qqplot(random_x, dist=sp.norm, loc=sca, scale=sca, line = '45')
plt.show()
```



```
In [ ]:
```