Estimation and Decision Systems 2023/24
Activity sheet 3

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1 - Constant Signal:

a) Create the *idpoly* object **sys0** describing the *model*

$$y(t) = \left(2 - 2q^{-1} + q^{-2} + 3q^{-3} - q^{-4}\right) u(t)$$

b) Create the signal $u1(t) = 1$ for all t in the range $1 \leq t \leq 100$ and save it in the variable **u0**.

c) Simulate the system and record the output in the variable **y0**.

d) Estimate two FIR models with orders $n_b = 0$ and $n_b = 1$. Use the following variables:

- **io0i** - *iddata* object with the input-output data.
- **syse00** - *idpoly* object with the estimated model of order 0 ($n_b = 0$).
- **X00** - regressor matrix for the order 0 model ($n_b = 0$).
- **syse01**- *idpoly* object with the estimated model of order 1 ($n_b = 1$).
- **X01** - regressor matrix for the order 1 model ($n_b = 1$).

Remark: Use the *IdFIR* function developed in the previous lab activity.

e) Can you explain the result of attempting to calculate the model with $n_b = 1$? (hint: Calculate the rank of the matrix **X01**). What does this observation imply? Does it apply consistently to all values of $n_b \geq 1$?

f) Observe how the output of the zero order model compares to the estimation data. Save the BFIT index in **BFIT00i** and the simulation errors in the **es00i**.
Remark: Use the *CompareModels* function.

g) Simulate both the true sytem **sys0** and the zero-order estimated model **(syse00)** using as input a 100-sample binary white noise input with a zero mean and an amplitude of 1. Compare the output signals. Use the following variables:

- **uv0** - input signal (binary white noise).

- **yv0** - Output of **sys0**
- **io0v** - *iddata* object with the input-outpu data **uv0**, **yv0**
- **BFIT00v** - Best fit index
- **es00v** - Simulation errors (difference between the outputs of **sys0** and **sye00**).

Remark: Use the *CompareModels* function again.

h) Comment on the results of items f) and g). Based on the results of these items, explain how model validation should be carried out: using the estimation data or alternative data? If you opt for alternative data, how should this data be selected? If you were to compare simulations of sys0 and syse00 with a constant input, U, what results would you expect? Why?

2 - Sine wave

a) Create the signal $u_{s1}(t) = \text{sen}\left(\frac{2\pi}{5}t\right)$ para $t = 1, \ldots, 100$ save it in **us1**.

b) Simulate **sys0** and record the output in **ys1**.

c) Estimate FIR models with orders $n_b = 0, \ldots, 4$. Use the following variables:

- **io1** - *iddata* with the input-output data.
- **syse10**...**syse14** - idpoly objects with the estimated models of orders $n_b = 0, \ldots, 4$, respectively. **X10** ...**X14** - regressor's matrices for $n_b = 0, \ldots, 4$, respectively.

d) Compute the ranks of **X10**, ..., **X14**. Considering the obtained values, what could have been anticipated in the estimation of the orders 2, 3 and 4 ($n_b = 2$, $n_b = 3$, and $n_b = 4$) models. Try to provide an explanation for why the expected results did not materialize.

e) Generate the signal $u_{vs1}(t) = 2\cos\left(\frac{2\pi}{5}t\right)$, $t = 1, \ldots, 100$. Simulate **sys0** using this signal as input. ompare this response to that of the **syse11** model. Also compare the Bode diagrams of **sys0** e **syse11** (use the Matlab command *bode* to plot the Bode diagrams).Comment on the results (examine the differences between the diagrams, identify the frequencies or frequency ranges where these differences are not significant, and relate them to the input signal frequency). Use the following variables:

- **uvs1** - $u_{vs1}(t)$;
- **yvs1** - output signal of **sys0**;
- **io1vs1** - *iddata* object with the input-ouput data;
- **BFIT11vs1** - Best fit index;
- **es11vs1** - Simulation errorr.

f) Simulate **sys0** once again, using a 100-sample realization of binary white noise with zero mean and amplitude 1 as the input. Once more, compare the output of this system to that of the **syse11** model. Use the following variables

- **uv1** - input signal;
- **yv1** - output signal of **sys0**;
- **io1v1** - *iddata* with the input-ouput data;
- **BFIT11v1** - best fit index;
- **es11v1** - simulation errors;

Compare the results obtained in this item with those from the previous item and explain any differences, if applicable.

g) Write the *MCIdFIR* function to perform Monte Carlo simulations of identification experiments. This function has to:

- Have the following inputs:
  - **sys0** - *idpoly* object with the true system;
  - **ui** - input signal;
  - **nb** - Order of the identified system;
  - **sigma** - Standard deviation of the noise;
  - **Nmc** - number of Monte Carlo simulations.
- Perform the following operations:

  1. Simulate the system.

  2. Throughout Nmc iterations:

  2.1 Generate Gaussian zero-mean white noise with the prescribed standard deviation.

  2.2 Disturb **sys0** output signal with noise generated in the previous step

  2.3 Identify the system.
- To have the following output:
  - **Theta** - Matrix with dimensions $Nmc \times (n_b + 1)$ whose rows contain the estimated parameters values of the different iterations (row k represents the parameter vector $\begin{bmatrix}\hat{b}_0 & \hat{b}1 & \cdots & \hat{b}n_b\end{bmatrix}$ estimated in iteration k).

h) Perform a Monte Carlo simulation with 100 identification experiments for a first order model ($n_b = 1$), using input signal **us1** and output noise with a standard deviation of 0.5. Save the results in **Theta11**. Calculate the mean and standard deviation of each column in **Theta11** and store the results in **Theta11Med** and **Theta11DP**, respectively

i) Repeat the Monte Carlo Simulation of the previous item but for a fourth order model ($n_b = 4$). Use the variables **Theta14**, **Theta14Med** and **Theta14DP** for the estimated parameters, their mean an standar deviation, respectively.

j) Compare the standard deviations obtained in the two previous itens and try to explain the differences or similarities.
Remark: "In the case of measurement error being white noise, the covariance of the least squares estimates is given by $(X^T X)^{-1}\sigma^2$, where σ^2 is the variance of the white noise.

3 - Bisinusoidal signal

a) Compute:

$$\begin{aligned} \phi_1 &= 2\pi * \mathbf{rand} - \pi \\ \phi_2 &= 2\pi * \mathbf{rand} - \pi. \end{aligned}$$

Store ϕ_1 and ϕ_2 in **Phi1** and **Phi2**, respectively. Generate the signal $u_{s2}(t) = \sin\left(\frac{2\pi}{5}t + \phi_1\right) + \sin\left(\frac{2\pi}{10}t + \phi_2\right)$ for $t = 1, ; \ldots, ; 100$ and save it in **us2**.
Remark: *rand* refers to a random number with a uniform distribution ranging from 0 to 1. It can be generated by the *rand* Matlab command. Hence, to calculate ϕ_1 use the following Matlab commands

```
>> Phi1=2*pi*rand-pi;
```

To calculate ϕ_2, an identical command can be used. Both ϕ_1 and ϕ_2 are equally distributed random phase shifts. On average, the mean peak values of the sum of sinusoids with different frequencies are minimized when they are subjected to random phase shifts. This is a very important issue in system identification because actuators limit the amplitude of the input signals.

b) Simulate **sys0** and store the output in **ys2**.

c) Identify FIR models with orders ranging from 0 to 4 ($n_b = 0, ; \ldots, ; 4$). Use the following variables:"

- **io2** - *iddata* with the input-output data;
- **syse20**. . . **syse24** - idpoly objects with the estimated models of orders $n_b = 0, \ldots, 4$, respectively.
- **X20** . . . **X24** - regressor's matrices for $n_b = 0, \ldots, 4$, respectively.

d) Calculate the ranks of **X20**, **X21**, . . . , **X24**. Given the values obtained, what would be expected in the estimation of the fourth order mode ($n_b = 4$)? Try to provide an explanation for why the expected outcome did not occur.

e) Calculate:

$$\begin{aligned}\phi_3 &= 2\pi\mathbf{rand} - \pi \\ \phi_4 &= 2\pi\mathbf{rand} - \pi.\end{aligned}$$

Generate $u_{vs2}(t) = 3\cos\left(\frac{2\pi}{5}t + \phi_3\right) + 2\cos\left(\frac{2\pi}{10}t + \phi_4\right),\ t = 1, \ldots,\ 100.$ Simulate **sys0** using this signal as input. Compare this response to that of the **syse23** model. Compare the Bode diagrams of**sys0** e **syse23**. Comment on the results (examine the differences between the diagrams, identify the frequencies or frequency ranges where these differences are not significant, and relate them to the input signal frequency) Use the following variables:

- **Phi3** - ϕ_3;
- **Phi4** - ϕ_4;
- **uvs2** - $u_{vs2}(t)$;
- **yvs2** - output signal of **sys0**;
- **io2vs2** - *iddata* with the input-ouput data;
- **BFIT23vs2** - Best fit index;
- **es23vs2** - simulation errors.

f) Simulate **sys0** once again, using a 100-sample realization of binary white noise with zero mean and amplitude 1 as the input. Compare the output of this system with that of the **syse23** model. Use the following variables:

- **uv2** - input signal;
- **yv2** - output of **sys0**;
- **io2v2** - *iddata* with th input-output data;
- **BFIT23v2** - Best fit index;
- **es23v2** - simulation errors.

Compare the results of this item with those of the previousitem and explain any differences, if applicable

g) Perform a Monte Carlo simulation of 100 identification experiments for a third order model ($n_b = 3$), using the input **us2** and an output noise with a standard deviation of 0.5. Store the parameters in **Theta23**. Calculate the means and standard deviations of the parameters and store the results in **Theta23Med** and **Theta23DP**, respectively.

h) Conduct a Monte Carlo simulation with 100 identification experiments of a fourth order model ($n_b = 4$), using the input **us2** and an output noise with a standard deviation of 0.5. Sotre the estimated parameters in **Theta24**. Calculate the mean and standard deviation the parameters and store them in **Theta24Med** and **Theta24DP**, respectively.

i) Compare the standard deviations obtained in the two previous items and attempt to explain the differences or similarities.

4 - Tri-sinusoidal signa

a) Calculate:

$$\begin{aligned}\phi_5 &= 2\pi\textbf{rand} - \pi \\ \phi_6 &= 2\pi\textbf{rand} - \pi \\ \phi_7 &= 2\pi\textbf{rand} - \pi.\end{aligned}$$

Sotre ϕ_5, ϕ_6 e ϕ_7 in **Phi5 Phi6** e **Phi7**, respectively, create the signal $u_{s3}(t) = \text{sen}\left(\frac{2\pi}{5}t + \phi_5\right) + \text{sen}\left(\frac{2\pi}{10}t + \phi_6\right) + \text{sen}\left(\frac{2\pi}{3}t + \phi_7\right)$ for $t = 1, \ldots, 100$ and store it in **us3**.

b) Simulate **sys0** with $u_{s3}(t)$ as input and store the output in **ys3**.

c) Identify FIR models with orders ranging from 0 to 4 ($n_b = 0, \ldots, 4$). Use the following variables:

- **io3** -*iddata* with the input-output data;
- **syse30**...**syse34** - *idpoly* objects with the estimated models of orders $n_b = 0, \ldots, 4$, respectively.
- **X30**,...,**X34** - regressor's matrices for $n_b = 0, \ldots, 4$, respectively.

d) Compute the ranks of **X30**, ..., **X34**.

e) Simulate **sys0** using a 100-sample realization of binary white noise with zero mean and amplitude 1 as input. Compare the output of this system with that of the **syse34** model Use the following variables

- **uv3** - input signal;
- **yv3** - output of **sys0**;
- **io3v3** - *iddata* object with the input-ouput data;
- **BFIT34v3** - Best fit index;
- **es34v3** - simulation errors.

f) Perform a Monte Carlo simulation with 100 identification experiments of a fourth model ($n_b = 4$), using the input **us3** and output noise with a standard deviation of 0.5. Save the estimated parameters in **Theta34**. Calculate the parameters' mean and standard deviation and store the results in **Theta34Med** and **Theta34DP**, respectively.

g) Based on the results obtained in this Lab Activity, determine the maximum number of parameters that can be identified using a multisinusoidal input signal with m frequencies.