Estimation and Decision Systems 2023/24- Activity sheet 2

1 - Consider the system

$$y(t) = b_0 u(t) + b_1 u(t-1) + \cdots + b_n u(t-n) + r(t).$$

where $r(t)$ is measurement noise. In system identification, it is common to define the backward shift operator q^{-1} as follows[1]:

$$q^{-1} x(t) = x(t-1)$$

and represent the system as

$$y(t) = B\left(q^{-1}\right) u(t) + r(t)$$

where

$$B\left(q^{-1}\right) = b_0 + b_1 q^{-1} + \cdots + b_n q^{-n}.$$

If we define the vectorss

$$\begin{aligned} \theta &= \begin{bmatrix} b_0 & b_1 & \cdots & b_n \end{bmatrix}^T \in \mathbb{R}^{n+1} \\ \varphi(t) &= \begin{bmatrix} u(t) & u(t-1) & \cdots & u(t-n) \end{bmatrix}^T \in \mathbb{R}^{n+1}, \end{aligned}$$

we can rewrite the system equation as

$$y(t) = \varphi^T(t)\theta + r(t).$$

Suppose you collect observations of the system at time instants $t = 1, \ldots, N$, with $N > n$. You can then describe the observations from $t = n+1, \ldots, N$ using the equation:

$$\begin{bmatrix} y(n+1) \\ y(n+2) \\ \vdots \\ y(N) \end{bmatrix} = \begin{bmatrix} x^T(n+1) \\ x^T(n+2) \\ \vdots \\ x^T(N) \end{bmatrix} \theta + \begin{bmatrix} r(n+1) \\ r(n+2) \\ \vdots \\ r(N) \end{bmatrix}.$$

[1]Some authors refer to this operator as z^{-1}

Defining

$$
\begin{aligned}
Y &= \begin{bmatrix} y(n+1) \\ y(n+2) \\ \vdots \\ y(N) \end{bmatrix} && (1)\\
\Phi &= \begin{bmatrix} \varphi^T(n+1) \\ \varphi^T(n+2) \\ \vdots \\ \varphi^T(N) \end{bmatrix} = \begin{bmatrix} u(n+1) & u(n) & \cdots & u(1) \\ u(n+2) & u(n+1) & \cdots & u(2) \\ \vdots & \vdots & \vdots & \vdots \\ u(N) & u(N-1) & \cdots & u(N-n) \end{bmatrix} && (2)\\
\mathcal{R} &= \begin{bmatrix} r(n+1) \\ r(n+2) \\ \vdots \\ r(N) \end{bmatrix} && (3)
\end{aligned}
$$

the equation of observations can be rewritten in the form:

$$Y = \Phi\theta + \mathcal{R}$$

a) Write a function to estimate the parameters of the model (the coefficients of the polynomial $B\left(q^{-1}\right)$ which are also the elements of the parameter vector θ) using the least squares estimator. The least squares estimator of θ minimizes the criterion:

$$V = \|Y - \Phi\theta\|_2^2 = (Y - \Phi\theta)^T (Y - \Phi\theta)\,.$$

- The function should be named *IdFIR* and should be in the file *IdFIR.m*.

- It has following inputs:
  - **data** - An *IESTiddata* object containing the system input/ouput data.
  - **nb** - The system order.
- And has the following outputs:
  - **sys** - *idpoly* object
  - **Y** - Output measurements (vector defined in equation (1))
  - **Phi** - Regressors' matrix (defined in equation (2)).

b) Using the function you wrote in the previous item and the signals $y_i(t)$ (output) and $u_i(t)$ recorded in *actSheet2Data.mat* estimate the parameters for five models with:

- $n = n_0 = 0$.
- $n = n_1 = 1$.
- $n = n_2 = 2$.

- $n = n_3 = 3$.
- $n = n_4 = 4$.
- $n = n_5 = 5$.

Use the folowing variables: **n0**, **n1**, **n2**,**n3**, **n4** e **n5** for the model orders, **Yi** for the outpu measurements, **Phi0**, **Phi1**, **Phi2**,**Phi3**, **Phi4** e **Phi5** for the regressor matrices, and **sys0**, **sys1**, **sys2**, **sys3**, **sys4** e **sys5** for the identified models.

d) Write the function *CompareModels* Build the *CompareModels* function to compare input-output data with simulations generated by an idpoly model. The function simulates the system using the input signal of the input/ouput data and plots the following curves:

- Measured output (y, the output signal of the input/ouput data);
- Simulated output (ys);
- Simulation error (es=y-ys)

The function has the following inputs:

- **valData** - *iddata* object with the input/output data.
- **sys** - idpoly object with the model.

The ouputs are

- **BFIT** -Best Fit Index defined as

$$
\begin{aligned}
BFIT &= \left\{ 1 - \sqrt{\frac{\sum_{t=1}^{N} [y(t) - y_s(t)]^2}{\sum_{t=1}^{N} [y(t) - \bar{y}]^2}} \right\} \times 100\% \\
&= \left(1 - \frac{\|\mathbf{y} - \mathbf{ys}\|_2}{\|\mathbf{y} - \mathrm{mean}(\mathbf{y})\|_2} \right) \times 100\% \\
\bar{y} &= \frac{1}{N} \sum_{t=1}^{N} y(t) = \mathrm{mean}(\mathbf{y})
\end{aligned}
$$

- **es** - simulation errors

$$
e_s(t) = y(t) - y_s(s)
$$

This function must be in the file *CompareModels.m*.

e) The data file*actSheet2Data.mat* also contains the variables **yv** and **uv**. These variables store the input-output values (**yv** for output and **uv** for input) collected in another experiment with the same system.

e1 - Use the *CompareModels.m* function to compare the six models estimated in item b). Use the variables **BFIT0**, **BFIT1**,

BFIT2, **BFIT3**, **BFIT4** and **BFIT5** afor the Best Fit Indexes and **es0**, **es1**, **es2**, **es3**, **es4** and **es5** for the simulation errors.

e2 - Perform a white noise test on the errors **es0**, **es1**, **es2**, **es3**, **es4**, and **es5**.

REMARK: If you didn't develop a function for this test in the Activity 1, you can use the Matlab System Identification toolbox command *resid* (type *help resid* in the Matlab Command window to know how it works). This command takes as inputs the input-output data in an iddata object and the model whose residuals you want to test. In addition to the function for normalized autocorrelation of residuals, it also displays the cross-correlation between the model's output and input. It always uses a 99% confidence interval ($\alpha = 0.01$).

f) Which of the estimated models do you believe is the most suitable for describing the system, and what is your reasoning behind this choice? Are there benefits tousing data from from a different experiment for of model validation and selection? If so, what are those advantages?

Variable Lists

Questão 1:

- **n0**=0, **n1**=1, **n2**=2, **n3**=3, **n4**=4, **n5**=5 Model orders of the identied systems.
- **Yi** - Output identification data.
- **Phi0**, **Phi01**, **Phi02**, **Phi03**, **Phi04**, **Phi05** - Regressor matrices of the different model orders
- **sys0**, **sys1**, **sys2**, **sys3**, **sys4**,**sys5** -Identified modesl.
- **BFIT0**, **BFIT1**, **BFIT2**, **BFIT3**, **BFIT4**, **BFIT5** - Best fit indexes.
- **es0**, **es1**, **es2**, **es3**, **es4**, **es5** - Simulation errors.

File list:

Question 1:

- *IdFIR.m* - Contains function *IdFIR*
- *CompareModels.m* - Containfunction *CompareModels*
- *ModelSys0.fig*,