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IFAC PapersOnLine 51-13 (2018) 390–395

Portfolio modeling for an algorithmic trading based on control theory

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Abstract: In the present paper, a mathematical model for a portfolio is proposed. This model is valid for operations of buying and selling shares of an asset in constant periods of time, additionally, it has a state space form which can be used to design a control law using control theory. The designed control law can be interpreted as a trading signal to reach a portfolio value desired. The mathematical model and control law proposed are validated by means simulations using real daily prices of Mexican stock exchange.

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Keywords: Control of Nonlinear Systems, Trading Algorithm, Portfolio Modeling

1. INTRODUCTION

The stock market is constantly evolving with the help of new technologies and new products that can be traded. Particularly, technology has made it possible for trading operations to be carried out faster and take advantage of market opportunities. An algorithm is an specific set of clearly defined instructions aimed to carry out a task or process. Algorithmic trading is the process which use computers programmed to define a set of instructions for placing a trade in order to generate profits at a speed and frequency that is impossible for a human trader. The defined rules set are based on timing, price quantity or mathematical model Seth (2017); Kirchner (2015); Yadav (2016).

Currently, algorithmic trading has a relevant importance in the large securities markets, in such a way that regulations for this type of operations in the stock market have been proposed Yadav (2016); BUSCH (2016). Different algorithm strategies have emerged to make profits in terms of improve earnings or cost reduction. The common trading strategies used in algo-trading are the Trend Following Strategies, Arbitrage Opportunities, Index Fund Rebalancing, Trading Range (Mean Reversion), Volume Weighted Average Price (VWAP), Time Weighted Average Price (TWAP), Percentage of Volume (POV), Implementation Shortfall and Mathematical Model Based Strategies Seth (2017)TRELEAVEN et al. (2013).

In the mathematical model based strategies, the machine learning and artificial intelligence have served as the basis for trading algorithm design since 15 years ago. In R.J. Kuo and Hwang (2001), a genetic algorithm based on a fuzzy neural network is proposed as knowledge base to measure the qualitative effect on the stock market. The qualitative effect estimated is used by the neural network to take the trading decision. The complexity in the implementation of these algorithms makes it necessary a high computing capacity to perform online trading. In Bendtsen and Pena (2016), a gated Bayesian network model is used to learn the buy and sell decisions in a trading systems; and the trading algorithm performance is compared to the benchmark investment strategy buy-and-hold.

In Gunter Schmidt and Kersch (2010), the empirical performance comparison between a threat-based, reservation price average price and buy-and-hold algorithms is presented. In that, the performance of the threat-based algorithm is better than all investigated algorithms. In Peter DeMarzo and Mansour (2006), an algorithm to price the current value of an option assuming that there are no arbitrage opportunities, is proposed. The valuation considers the obtaining of the maximum and minimum limits for the price. In all these papers, the objective is to propose a model to estimate the behavior of the instrument price and, based on that estimation make the trading decision.

There are published contributions on the application of control theory for optimal design of portfolios. In Sethi and Thompson (1970), a cash balance problem is formulated as a control theory problem, where the financtial interpretation of the Hamiltonian function and, the adjoint function, is stated. In Fleming and Sheu (2000); Fleming and Pang (2004), considering the hyperbolic absolute risk aversion (HARA) as the utility function, a portfolio optimization is reformulated as an infinite time horizon control problem. There, the solutions of the dynamic programming equation are used to derive the optimal investment policies. These papers consider the portfolio optimization in continuous time, which can cause problems in a real-time implementation, because the stock prices generally change in discrete time.

In this paper, a dynamic mathematical model is proposed for a portfolio composed of shares in the stock exchange. The proposed model has the form of state space in discrete time, which can be used to propose a trading signal based on control theory. The proposed control law can

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10.1016/j.ifacol.2018.07.310

be adapted to different investor profiles by means of the correct selection of the control gain. The performance of the control law is validated through simulations using real prices of the Mexican stock exchange. This paper is organized as follows: in section 2, the procedure to obtain the portfolio model is detailed. In section 3, the trading signal is obtained using a nonlinear control law. The simulation results of the proposed algorithm are shown in section 4. Finally, section 5 presents the conclusions of the paper.

2. PORTFOLIO MODELING

A portfolio is a grouping of financial assets such as stocks, bonds and cash equivalents, as well as their funds counterparts, including mutual, exchange-traded and closed funds. Portfolios are held directly by investors and/or managed by financial professionals INVESTOPEDIA (2017). Particularly for a shares portfolio, the portfolio value can be obtained as a sum based on the share value and, the shares number available for each asset.

$$v_p = n_{a1}p_1 + n_{a2}p_2 + \cdots + n_{an}p_n, \tag{1}$$

where v_p is the portfolio value, n_{an} is the shares number of the nth asset and p_n is the stock market price of the corresponding nth asset. When there is available capital to make another investment, the portfolio value may include this capital as follows:

$$v_p = n_{a1}p_1 + n_{a2}p_2 + \cdots + n_{an}p_n + m_{cap}, \tag{2}$$

where m_{cap} is the capital available.

In general, the value of a portfolio can be expressed as a matrix operation:

$$v_p = P_a^T N + m_{cap}, \tag{3}$$

where $N = [\, n_{a1}\ n_{a2}\ \cdots\ n_{an}\,]^T$ and $P_a^T = [\, p_1\ p_2\ \cdots\ p_n\,]$.

In order to facilitate the obtention of a dynamic model for a portfolio, a single asset portfolio will be considered; after that, the mathematical model is generalized.

2.1 Buy and Sell

In a financial market, it is normal to carry out stock purchase and sale operations, driven by the interest of increasing the value of the portfolio. A **purchase** transaction means to exchange money for contracts that hold us as owners of the shares, and correspondingly a **sale** operation means to change the shares for their value in money. Unfortunately, to operate in a financial market, there are intermediaries who charge commissions for the use of platforms, information and, investment advice. That commission usually varies depending on the amount or type of stocks or assets being traded. In this paper, r_{com} is used as the percentage of the intermediary charges for the operations that are carry out.

In order to simplify the development of the model proposed in this paper, it is considered that there is an amount m to be invested in a single type of asset through the purchase of n_a shares. Then, if is adquired a number n_a of shares at a price p, the amount that has to be paid ($real\_cost$) is defined as:

$$real\_cost = (1 + r_{com})p \times n_a. \tag{4}$$

At the same way, when selling n_a shares at a price p, the $real\_value$ received for those shares is defined as:

$$real\_value = (1 - r_{com})p \times n_a. \tag{5}$$

Then, assuming an initial amount $m_{(0)}$ for investment, after a buy operation execution of $n_{am(0)}$ shares at the price $p_{(0)}$, the remaining amount $m_{(1)}$ can be calculated as $m_{(0)} - real\_cost$, then:

$$m_{(1)} = m_{(0)} - (1 + r_{com})p_{(0)}n_{am(0)}. \tag{6}$$

Proposing the variable k to indicate the time step, which is considered as time interval constant, the amount available after a buy operation could be calculated as:

$$m_{(k+1)} = m_{(k)} - (1 + r_{com})p_{(k)}n_{am(k)}. \tag{7}$$

On the other hand, if the same reasoning for a sale operation is applied, the new available amount is calculated as:

$$m_{(k+1)} = m_{(k)} + (1 - r_{com})p_{(k)}n_{am(k)}, \tag{8}$$

in both equations (7) and (8), $n_{am(k)}$ denotes the shares number in the operation at the time step k.

The shares number resulting $n_{a(k+1)}$ is a little easier to calculate because it can be obtained as a sum or subtraction operation. For the buy case:

$$n_{a(k+1)} = n_{a(k)} + n_{am(k)}, \tag{9}$$

and, for a sale case:

$$n_{a(k+1)} = n_{a(k)} - n_{am(k)}. \tag{10}$$

Then, the calculation of the amount and the number of shares available can be expressed by the following equations systems:

Buy:

$$\begin{aligned} m_{(k+1)} &= m_{(k)} - (1 + r_{com})p_{(k)}n_{am(k)}, \\ n_{a(k+1)} &= n_{a(k)} + n_{am(k)}, \end{aligned} \tag{11}$$

Sell:

$$\begin{aligned} m_{(k+1)} &= m_{(k)} + (1 - r_{com})p_{(k)}n_{am(k)}, \\ n_{a(k+1)} &= n_{a(k)} - n_{am(k)}. \end{aligned} \tag{12}$$

Assuming that it is not permitted to ask for a loan to invest and neither short sales, the number of shares available to buy can be bounded as follows:

$$0 \le n_{am(k)} \le floor\left(\frac{m_{(k)}}{(1 + r_{com})p_{(k)}}\right), \tag{13}$$

and the number of shares available to sale is bounded by:

$$0 \le n_{am(k)} \le n_{a(k)}. \tag{14}$$

2.2 Generalizing the model

From (11) and (12), it can be seen that the commission (r_{com}) appears in a buy or sale operation. So, this commission can be considered as an uncontrollable known perturbation in the system. Then, the buy and sell operation can be rewritten as follows:

Buy:

$$\begin{aligned} m_{(k+1)} &= m_{(k)} - p_{(k)}n_{am(k)} - r_{com}p_{(k)}n_{am(k)}, \\ n_{a(k+1)} &= n_{a(k)} + n_{am(k)}, \end{aligned} \tag{15}$$

Sell:

$$\begin{aligned} m_{(k+1)} &= m_{(k)} + p_{(k)} n_{am(k)} - r_{com} p_{(k)} n_{am(k)}, \\ n_{a(k+1)} &= n_{a(k)} - n_{am(k)}. \end{aligned} \tag{16}$$

So far, two operation conditions have been considered separately. A unique model can be constructed by adding a variable that indicates whether the stock is a buy or sell, α_{cv}, which is defined as:

$$\alpha_{cv} = \begin{cases} 1 & , Buy \\ -1 & , Sell \end{cases} . \tag{17}$$

Then, the equations (15) and (16) can be expresed as an unique model for both operations.

$$\begin{aligned} m_{(k+1)} &= m_{(k)} - p_{(k)} \alpha_{cv} n_{am(k)} - r_{com} p_{(k)} n_{am(k)}, \\ n_{a(k+1)} &= n_{a(k)} + \alpha_{cv} n_{am(k)}. \end{aligned} \tag{18}$$

It can be seen that the proposed dynamic model is directly influenced by buying and selling operations (α_{cv} value) and the number of shares that are required to operate ($n_{am(k)}$ value). Then, if $u_{(k)} = \alpha_{cv} n_{am(k)}$ is defined, the system can explicitly have a control signal that alters their behavior. This control signal can be seen as a trading signal where the sign and magnitude of $u_{(k)}$ corresponds to a decision of purchase or sale and, the number of shares respectively.

$$\begin{aligned} m_{(k+1)} &= m_{(k)} - p_{(k)} u_{(k)} - r_{com} p_{(k)} \|u_{(k)}\|, \\ n_{a(k+1)} &= n_{a(k)} + u_{(k)}. \end{aligned} \tag{19}$$

Including the portfolio value v_p, the complete model proposed is defined as:

$$\begin{aligned} v_{p(k)} &= p_{(k)} n_{a(k)} + m_{(k)}, \\ m_{(k+1)} &= m_{(k)} - p_{(k)} u_{(k)} - r_{com} p_{(k)} \|u_{(k)}\|, \\ n_{a(k+1)} &= n_{a(k)} + u_{(k)}. \end{aligned} \tag{20}$$

The model (20) has the form of a time variant state space model as follows:

$$\begin{aligned} y_{(k)} &= C_{(k)} x_{(k)}, \\ x_{(k+1)} &= A x_{(k)} + B_{(k)} u_{(k)} - g(u_{(k)}), \end{aligned}$$

where $y_{(k)} = v_{p(k)}$, $x_{(k)} = \begin{bmatrix} m_{(k)} & n_{a(k)} \end{bmatrix}^T$, $C_{(k)} = \begin{bmatrix} 1 & p_{(k)} \end{bmatrix}$, $A = I_{2\times 2}$, $B_{(k)} = \begin{bmatrix} -p_{(k)} & 1 \end{bmatrix}^T$, and $g(u_{(k)}) = r_{com} p_{(k)} \|u_{(k)}\|$.

In general the model (20) can be rewritten for a portfolio that considers more than one asset to invest as follows:

$$\begin{aligned} V_{p(k)} &= P_{(k)} N_{a(k)} + M_{(k)}, \\ M_{(k+1)} &= M_{(k)} - P_{(k)} U_{(k)} - r_{com} P_{(k)} \|U_{(k)}\|, \\ N_{a(k+1)} &= N_{a(k)} + U_{(k)}, \end{aligned} \tag{21}$$

where q is the assets number where the algorithm can operate, $V_{p(k)} = \begin{bmatrix} v_{p1(k)} & v_{p2(k)} & \dots & v_{pq(k)} \end{bmatrix}^T$ is a vector that includes the portfolio value for each asset, $P_{(k)} = diag\left(\begin{bmatrix} p_{1(k)} & p_{2(k)} & \dots & p_{q(k)} \end{bmatrix}\right)$ is a diagonal matrix with the price at time k for each asset, $N_{a(k)} = \begin{bmatrix} n_{a1(k)} & n_{a2(k)} & \dots & n_{aq(k)} \end{bmatrix}^T$ is a shares number vector, $M_{(k)} = \begin{bmatrix} m_{1(k)} & m_{2(k)} & \dots & m_{q(k)} \end{bmatrix}^T$ is a vector with the amount to invest in each asset, $U_{(k)} = \begin{bmatrix} u_{1(k)} & u_{2(k)} & \dots & u_{q(k)} \end{bmatrix}$ is the control signal vector, or trading signal vector, for each asset.

3. TRADING ALGORITHM BASED ON CONTROL THEORY

In the section 2, the procedure to obtain a generalized model for a portfolio composed of an amount to invest $m_{(k)}$ in a $n_{a(k)}$ shares is presented. Based on the model proposed, a trading algorithm is designed aplying control theory.

3.1 Trading algorithm design

From (20), the v_p at time $k+1$ can be calculated as:

$$v_{p(k+1)} = p_{(k+1)} n_{a(k+1)} + m_{(k+1)}, \tag{22}$$

then using $n_{a(k+1)}$ and $m_{(k+1)}$ from (20), the $v_{p(k+1)}$ is obtained as

$$\begin{aligned} v_{p(k+1)} &= p_{(k+1)}(n_{a(k)} + u_{(k)}) + m_{(k)} - p_{(k)} u_{(k)} \\ &\quad - r_{com}(p_{(k)} \|u_{(k)}\|), \\ v_{p(k+1)} &= p_{(k+1)} n_{a(k)} + m_{(k)} + (p_{(k+1)} - p_{(k)}) u_{(k)} \\ &\quad - r_{com}(p_{(k)} \|u_{(k)}\|). \end{aligned} \tag{23}$$

Considering that the future price $p_{(k+1)}$ can be obtained from the actual price $p_{(k)}$ adding an increment $\Delta p_{(k)}$,

$$p_{(k+1)} = p_{(k)} + \Delta p_{(k)}, \tag{24}$$

then,

$$\begin{aligned} v_{p(k+1)} &= (p_{(k)} + \Delta p_{(k)}) n_{a(k)} + m_{(k)} + \Delta p_{(k)} u_{(k)} \\ &\quad - r_{com} p_{(k)} \|u_{(k)}\|, \\ v_{p(k+1)} &= p_{(k)} n_{a(k)} + \Delta p_{(k)} n_{a(k)} + m_{(k)} + \Delta p_{(k)} u_{(k)} \\ &\quad - r_{com} p_{(k)} \|u_{(k)}\|, \\ v_{p(k+1)} &= v_{p(k)} + \Delta p_{(k)} n_{a(k)} + \Delta p_{(k)} u_{(k)} \\ &\quad - r_{com} p_{(k)} \|u_{(k)}\|. \end{aligned} \tag{25}$$

In (25), it can be seen that the future portfolio value $v_{p(k+1)}$ depends directly on the actual portfolio value $v_{p(k)}$, the share price $p_{(k)}$, the price increment $\Delta p_{(k)}$, the current shares number $n_{a(k)}$ and $u_{(k)}$ value.

Usually, the objective of an investment is to have a portfolio that reaches a target value (v_p^{ref}) or desired profit, then we define an error variable (e_p) to make decisions. Then, the error variable is defined as

$$e_{p(k)} = v_p^{ref} - v_{p(k)}, \tag{26}$$

where v_p^{ref} is a constant desired portfolio value constant. Evaluating the $e_{p(k)}$ at the time $k+1$, the error dynamics can be obtained as:

$$
\begin{aligned}
e_{p(k+1)} &= v_p^{ref} - v_{p(k+1)}, \\
e_{p(k+1)} &= v_p^{ref} - v_{p(k)} - \Delta p_{(k)} n_{a(k)} - \Delta p_{(k)} u_{(k)} \\
&\quad + r_{com} p_{(k)} \|u_{(k)}\|, \\
e_{p(k+1)} &= e_{p(k)} - \Delta p_{(k)} n_{a(k)} - \Delta p_{(k)} u_{(k)} \\
&\quad + r_{com} p_{(k)} \|u_{(k)}\|. \qquad (27)
\end{aligned}
$$

Then, the $u_{(k)}$ signal is designed to manipulate the behavior of the error variable $e_{p(k)}$. The desired dynamic behavior to reach the expected value v_p^{ref} is $e_{p(k+1)} = K_e e_{p(k)}$, then the control signal $u_{(k)}$ is defined as:

$$u_{(k)} = \left(\Delta p_{(k)}\right)^{-1} \left(e_{p(k)} - \Delta p_{(k)} n_{a(k)} - K_e e_{p(k)}\right), \quad (28)$$

where, $\Delta p_{(k)}$ is known and measurable, $\Delta p_{(k)} \neq 0$ and $\|K_e\| < 1$ to ensure convergence. If $\Delta p_{(k)} = 0$, means that there has been no change in the price of the stock. Due that it is not necessary to apply some operation then, $u_{(k)} = 0$.

Substituting (28) in (27), the error dynamics in closed-loop is obtained as follows

$$e_{p(k+1)} = K_e e_{p(k)} + r_{com} p_{(k)} \|u_{(k)}\|. \quad (29)$$

It can be seen that closed-loop dynamics is not asymptotically stable, because the perturbation term $r_{com} p_{(k)} \|u_{(k)}\|$ depends on the magnitude of the control law. The perturbation is bounded due to r_{com} is a small percentage of operation cost $p_{(k)} \|u_{(k)}\|$. Considering (13) and (14), the control signal $u_{(k)}$ is bounded as follows:

$$-n_{a(k)} \leq u_{(k)} \leq floor\left(\frac{m_{(k)}}{(1 + r_{com}) p_{(k)}}\right). \quad (30)$$

3.2 *Stability analysis*

In order to simplify the stability analysis, lets define the control signal limits as $u_{max} = floor\left(\frac{m_{(k)}}{(1+r_{com})p_{(k)}}\right)$ and $u_{min} = -n_{a(k)}$, then control signal limits (30) can be rewrritten as

$$u_{min} \leq u_{(k)} \leq u_{max}. \quad (31)$$

Assumming, $\|K_e\| < 1$, $r_{com} << 1$, $\Delta p_{(k)} \neq 0$, and there is $\alpha_1 > 0$ such that $max(\|u_{min}\|, \|u_{max}\|) < \alpha_1$, then

$$
\begin{aligned}
e_{p(k+1)} &= K_e e_{p(k)} + r_{com} p_{(k)} \|u_{(k)}\|, \\
e_{p(k+1)} &\leq K_e e_{p(k)} + r_{com} p_{(k)} \alpha_1. \qquad (32)
\end{aligned}
$$

Let define a candidate Lyapunov function $V_{(k)} = e_{(k)}^2$, and $\Delta V_{(k)} = V_{(k+1)} - V_{(k)}$. If $0 \leq \|u_k\| \leq \alpha_1$ then

$$
\begin{aligned}
\Delta V_{(k)} &= V_{(k+1)} - V_{(k)}, \\
\Delta V_{(k)} &= e_{(k+1)}^2 - e_{(k)}^2, \\
\Delta V_{(k)} &\leq (K_e e_{p(k)} + r_{com} p_{(k)} \alpha_1)^2 - e_{(k)}^2, \\
\Delta V_{(k)} &\leq K_e^2 e_{p(k)}^2 + 2 K_e e_{p(k)} r_{com} p_{(k)} \alpha_1 \\
&\quad + r_{com}^2 p_{(k)}^2 \alpha_1^2 - e_{(k)}^2, \\
\Delta V_{(k)} &\leq (K_e^2 - 1) e_{p(k)}^2 + 2 K_e e_{p(k)} r_{com} p_{(k)} \alpha_1 \\
&\quad + r_{com}^2 p_{(k)}^2 \alpha_1^2, \\
\Delta V_{(k)} &\leq (K_e^2 - 1) e_{p(k)}^2 + \gamma_1, \qquad (33)
\end{aligned}
$$

where it can be seen that $V_{(k)}$ decreases because the term $(K_e^2 - 1)e_{p(k)}^2$ is always negative, and $\Delta V_{(k)}$ is bounded by $\gamma_1 = 2K_e e_{p(k)} r_{com} p_{(k)} \alpha_1 + r_{com}^2 p_{(k)}^2 \alpha_1^2$. Then, when $e_{p(k)} \to 0$, $\gamma_1 \to r_{com}^2 p_{(k)}^2 \alpha_1^2$.

4. SIMULATION RESULTS

In order to test the behavior of the proposed trading algorithm, simulations with real stock shares were implemented. Software used for simulation was MATLAB [1] and, the stock prices were obtained from Yahoo Finance [2]. Simulation conditions for all tests scenarios are the following:

- Amount to invest: $m_{(0)} = 50000$ MXN (Mexican pesos)
- Operation commision: $r_{com} = 0.29\%$
- Control gain: $K_e = 0.8$
- Simulation period: 249 days (05/23/2016-05/23/2017)
- Portfolio value desired: $v_p^{ref} = 1.1 m_{(0)} = 55000$ MXN
- Shares to test: GRUMAB, AXTELCPO, AMXL and BOLSAA from mexican stock exchange, which are chosen randomly in order to test the algorithm.
- For the simulations it is considered that the buying and selling operations are executed instantly.

Fig. 1-4 will show: 1) stock value variations during the given period, 2) portfolio value calculation based on the control algorithm, stock price value and commissions payed for each operation and, 3) control signal behavior. Additionally, figures will show the number of days it took each scenario to reach the desired portfolio value.

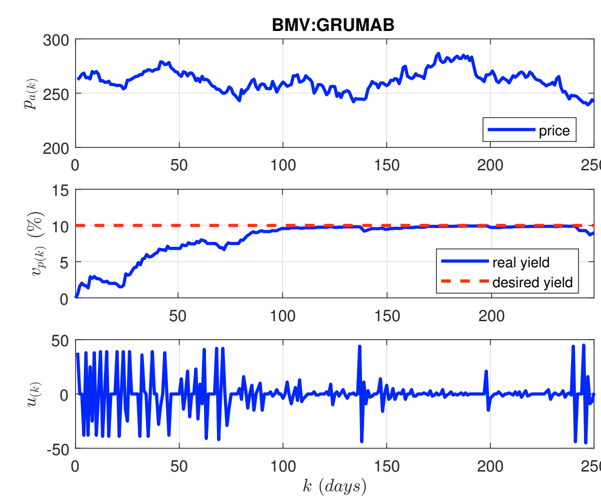

Fig. 1. GRUMAB simulation results

In Fig 1, simulation results using GRUMAB stock prices are shown. In top graph, the GRUMAB prices behavior in the given period is displayed. The portfolio value behavior and the control signal are presented in middle and bottom graphs respectively. It can be seen that, the desired portfolio value (v_p^{ref}) is reached at the day 100, even thought the stock prices had a lateral tendency during the chosen period.

[1] Trademark of The MathWorks, Inc.
[2] https://finance.yahoo.com/

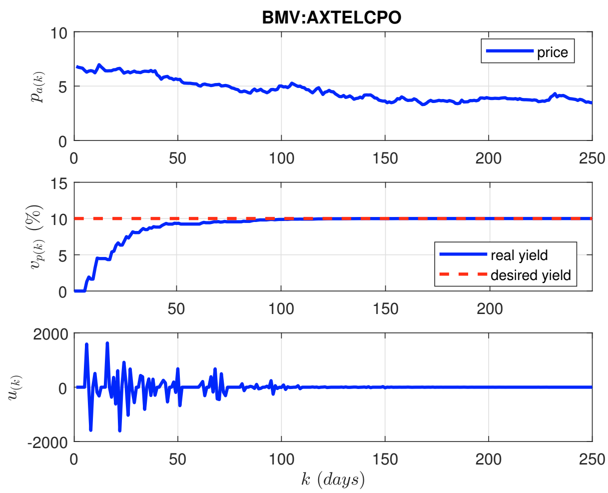

Fig. 2. AXTELCPO simulation results

In Fig. 2, results with the AXTELCPO stock prices are presented. The target (v_p^{ref}) is reached before 100 days of simulation, even when AXTELCPO prices had a downward tendency. Additionally, once the target value v_p^{ref} was reached, the algorithm did not perform more operations.

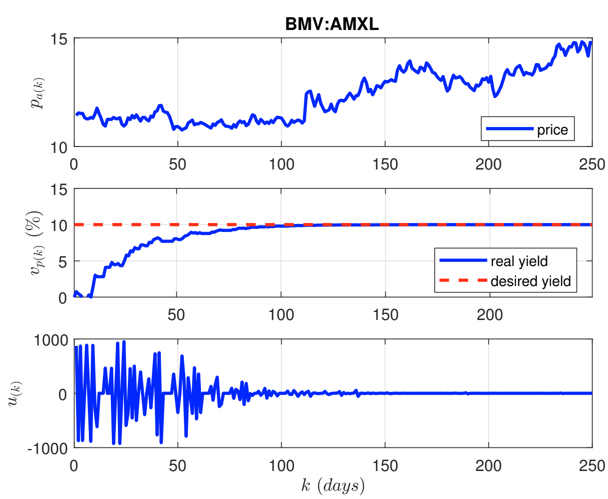

Fig. 3. AMXL simulation results

The simulation results using AMXL stock prices are displayed in Fig. 3. Similarly as previous results, the v_p^{ref} value is reached before the 100 days, and the operations number falls as $v_{p(k)}$ approaches the target value.

The simulation results obtained using the BOLSAA stock prices (Fig. 4) validate the good performance of the trading algorithm proposed. It must be considered that the trading algorithm decreaces the operations number and the amount per operation when the $v_{p(k)}$ approaches the target value v_p^{ref}. This is because the control signal $u_{(k)}$ is calculated as a function of the error variable $e_{(k)}$, then as the error $e_{(k)}$ decreases, the control signal also does. The convergence time depend heavily on the control gain K_e. The proposed value for K_e in the previous simulations is selected considering only the condition $\|K_e\| < 1$, but this control gain can be considered as a parameter that is related to the investor profile. From (29), it can be seen

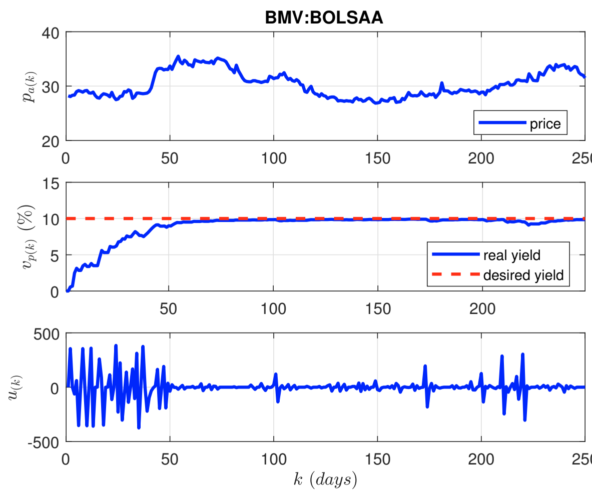

Fig. 4. BOLSAA simulation results

that the $e_{p(k+1)}$ is approximately a proportional fraction of $e_{p(k)}$ determined by the K_e value.

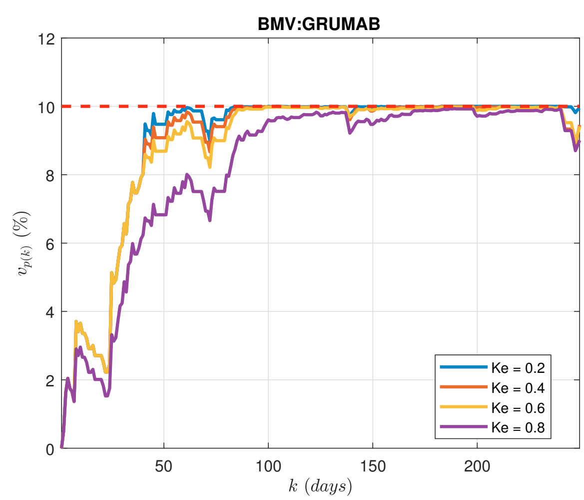

Fig. 5. GRUMAB simulation results with K_e variations

In Fig. 5, the GRUMAB investment behavior with different K_e values is shown. It is easy to see that, for K_e values near to 0, the desired portfolio value is reached in a shorter time. Due to control law ($u_{(k)}$) is limited by (30), in the first days of simulation the performance of the investment is similar for K_e values between 0.2 and 0.6, which implies that in each operation a larger shares amount is traded.

When the target portfolio value (v_p^{ref}) is reached, the trading operations is automatically reduced to maintain the desired level. The v_p^{ref} value can be updated by means of rules proposed by the investor so that the algorithm takes advantage of new market opportunities. In Fig. 6, the simulations results for the trading algorithm proposed with v_p^{ref} update are shown. In this simulation, the v_p^{ref} is updated when the $e_{p(k)}$ is less than 0.5%.

It can be seen that when a reference update occurs, the trades number increases to reach the new target and again it will decrease when this new target is close to being reached.

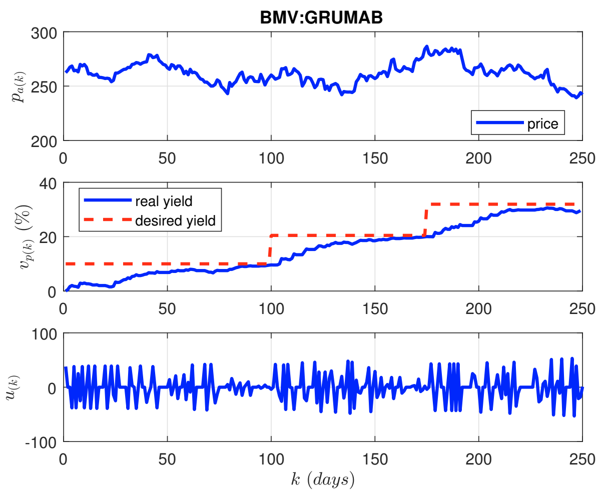

Fig. 6. GRUMAB simulation results with reference update and $K_e = 0.8$

5. CONCLUSIONS

In the present paper, based on the buying and selling operations, a mathematical model is proposed to describe the behavior of the shares number and amount to be invested in an asset, which includes the operation commissions by the platform owner. Based on the portfolio model a control law is designed which can be interpreted as trading signal.

From the simulations results, it can be seen that the trading algorithm achieves the portfolio desired value on every test case. It is important to mention that the trading signal proposed is theoretically realizable because it is necessary to know the price variation ($\Delta p_{(k)}$) in each time interval (which is difficult to satisfy). The main advantage of the trading algorithm proposed is that it is simple and it can be adjusted according to the investor profile by means of gain control K_e, where K_e values near to 0 means a agresive investor profile and K_e values near to 1 corresponds to a conservative profile. The proportional controller proposed in this work is sufficient to achieve the control objective because $\Delta p_{(k)}$ is considered known, otherwise, it is necessary to provide a robust controller and a price estimator.

Additionally, once the target value is reached, the algorithm can continue to take advantage of market opportunities if v_p^{ref} is updated with rules proposed by the investor. Another advantage of the proposed portfolio model is that it can be used to design new controllers based on different control strategies which will be explore in future work.

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