Relative Value Trading Using the Merton Model

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1 Introduction

In the complex landscape of fixed-income trading, where comprehensive and timely data on bond prices and credit spreads can be elusive, this project focuses on employing the Merton Model as a key tool for Relative Value Trading. The fixed income market's opacity, particularly in the case of less liquid or privately held debt, underscores the need for innovative approaches to credit risk assessment. The Merton Model, with its unique reliance on equity data, offers a practical solution to bridge the informational gap, especially for companies with publicly traded equities, allowing for a more informed evaluation of credit risk.

Investor segmentation between equity and credit poses a challenge, creating a discrepancy in what should be a consistent and disciplined valuation of these two markets. Despite the inherent connection between debt and equity in shaping a company's financial structure, the capital market in practice deals with differing perceptions and preferences among investors. This segmentation actively inhibits a unified approach to valuing debt and equity, preventing consistent enterprise valuation that accurately reflects their combined worth. Consequently, companies face issues in determining their true enterprise value in segmented markets, highlighting the need for innovative methodologies to bridge the valuation gap between debt and equity.

The core strategy we employ in this paper involves Relative Value Trading, utilizing signals from one market to anticipate movements in another. Employing the Merton Model, the project explores trades such as shorting an Equity Put Option versus a Credit Default Swap (CDS) Contract, leveraging the model's implied default probability. This strategy aims to capitalize on valuation differences between equity and credit markets, offering investors a nuanced approach to navigate the challenges posed by the fixed-income market's segmentation data limitations.

2 Exploratory Data Analysis

This project aims to use the Merton Model to predict the default probability of our selected companies which in turn is used to price the options and credit default swaps of the various companies. The Merton Model requires financial data of the selected companies. Therefore, we collected quarterly data from Bloomberg for a 10-year period starting December 2013 and ending September 2023.

The companies we selected are all listed on the US stock exchange and cover 5 different industries (financial, industrial, health care, real estate, technology). The selected companies are Pfizer Inc., Baxter International Inc., Xerox Holdings Corp, Boston Properties Inc., Vornado Realty Trust, APA Corp (US), Ovintiv Inc., Ally Financial Inc., and MBIA Inc. The data collected on the companies consists of the assets, liabilities, and the realized volatilities of the companies at the money options. Further, we collected the companies' realized default probabilities as these probabilities will be compared to the implied default probabilities that we calculate through our Merton Model.

3 The Merton Model

The Merton Model, developed by Robert C. Merton, is a financial model that extends the Black-Scholes-Merton framework to account for the possibility of default by a firm. It introduces the concept of a risky debt, where the firm's assets (A_T) are modeled as a geometric Brownian motion, following the lognormal/Black-Scholes dynamics:

$$dA_t = rdt + \sigma_A A_t dW_t$$

Here, r is the risk-free rate, σ_A is the volatility of the firm's assets, t is time, and W_t is a Wiener process.

The Merton Model involves calculating the probability of default (E_T) at a future time T. The equation for E_T is given by:

$$E_T = A_T N(d_1) - L_T e^{-rt} N(d_2)$$

where:

$$d_1 = \frac{\ln(A_T/L_T) + \left(r + \frac{\sigma_A^2}{2}\right) T}{\sigma_A \sqrt{T}}$$

$$d_2 = d_1 - \sigma_A \sqrt{T}$$

In these equations, $N(\cdot)$ is the cumulative distribution function of the standard normal distribution, L_T is the face value of the firm's debt at time T, and A_T is the firm's asset value at time T. The term e^{-rt} represent the present value of the debt.

These equations provide a framework for assessing the probability of default based on the dynamics of the firm's assets and the Black-Scholes-Merton assumptions, offering insights into the risk associated with corporate debt.

4 Methodology

The Methodology used consists of three steps. The first step employs the Merton Model which addresses challenges in fixed-income trading, particularly the limited availability of comprehensive bond data. The primary goal is to assess credit risk and facilitate Relative Value Trading by leveraging Equity data when fixed-income information is constrained.

Merton's main idea when creating the model was that a company will be incentivized to default if a company's equity is negative, which essentially means that the company is earning negative profits. Therefore, the equity value at time T is equivalent to a vanilla call option which can be expressed in the following payoff structure:

$$E_T = (A_T - L_T)^+$$

Merton assumed that the liabilities of a company are fixed and then modeled the assets as a log-normal stochastic process which in terms gives us a variation of the Black-Scholes pricing formula for the equity portion of a company.

When using this formula, we have the data of A_T which is each company's assets, L_T which is the company's liabilities, and r which is the interest rate. However, we need to find the volatility of the assets which is not easily estimated. Known however is the volatility of the equity which gives the following expression of the asset's volatility in terms of equity:

$$\sigma_A = \sigma_E N(d_1) \frac{E}{A}$$

Solving for the asset's volatility is done simultaneously to solving the equity value using a root-finding algorithm in the Python library. Once we know the equity values and the asset volatilities, we start the second step where we easily calculate the companies' distance to default and their equivalent probability of default through the following formula:

$$DD = \frac{\ln\left(\frac{A_T}{L_T}\right) + \left(r - \frac{\sigma^2}{2} T\right)}{\sigma_A \sqrt{T}}$$

$$Q = 1 - N(DD)$$

Here, Q demonstrates a company's default probability. Thereafter, the last step of the project is to compare the calculated default probabilities to the realized default probabilities to use for our trading strategy.

5 Trading Strategies

The trading strategy we implement is employed to create a directional-neutral position and capitalize on discrepancies between equity and credit markets. The strategy looks at the implied default probability we've calculated using the Merton Model versus the market realized default probability over time for each company. The strategy suggests that if the credit markets are pricing in a lower/higher default probability than the equity markets, it could indicate an opportunity.

Directional neutrality is achieved by balancing the directional exposure of the equity put option with the opposite exposure through the CDS. If there is a default, in the case of the first strategy, the increase in value of the Puts helps to cover our obligations from selling the CDS and vice versa in the second strategy. Additionally, if there is no default, we benefit from the difference between the CDS spread and the option premium. This premium can be viewed as compensation for taking on the risk of potential equity/CDS losses.

Position Type	Instrument
Short Put	Equity Option
Long Protection	Matching CDS

Table 1: Trading Strategy when Credit is Cheaper than Equity

Position Type	Instrument
Long Put	Equity Option
Sell Protection	Matching CDS

Table 2: Trading Strategy when Equity is Cheaper than Credit

The figure below highlights the trading signals that we've created using this methodology. When the realized default probability is higher than the implied default probability, the trading signal tells us to long the CDS spread and short the put option (shown with green triangles), and when the realized default probability is lower than the implied default probability, the trading signal tells us to short the CDS spread and long the put option (shown with red triangles). These triangles as signals that tell us if a company might be in a risky situation (above the threshold) or a safer one (below the threshold). The horizontal dashed lines in each graph represent the market-realized default probability. By looking at this figure, we can quickly spot moments when a company's financial health might be a concern or when things seem more stable and identify our trading strategy.

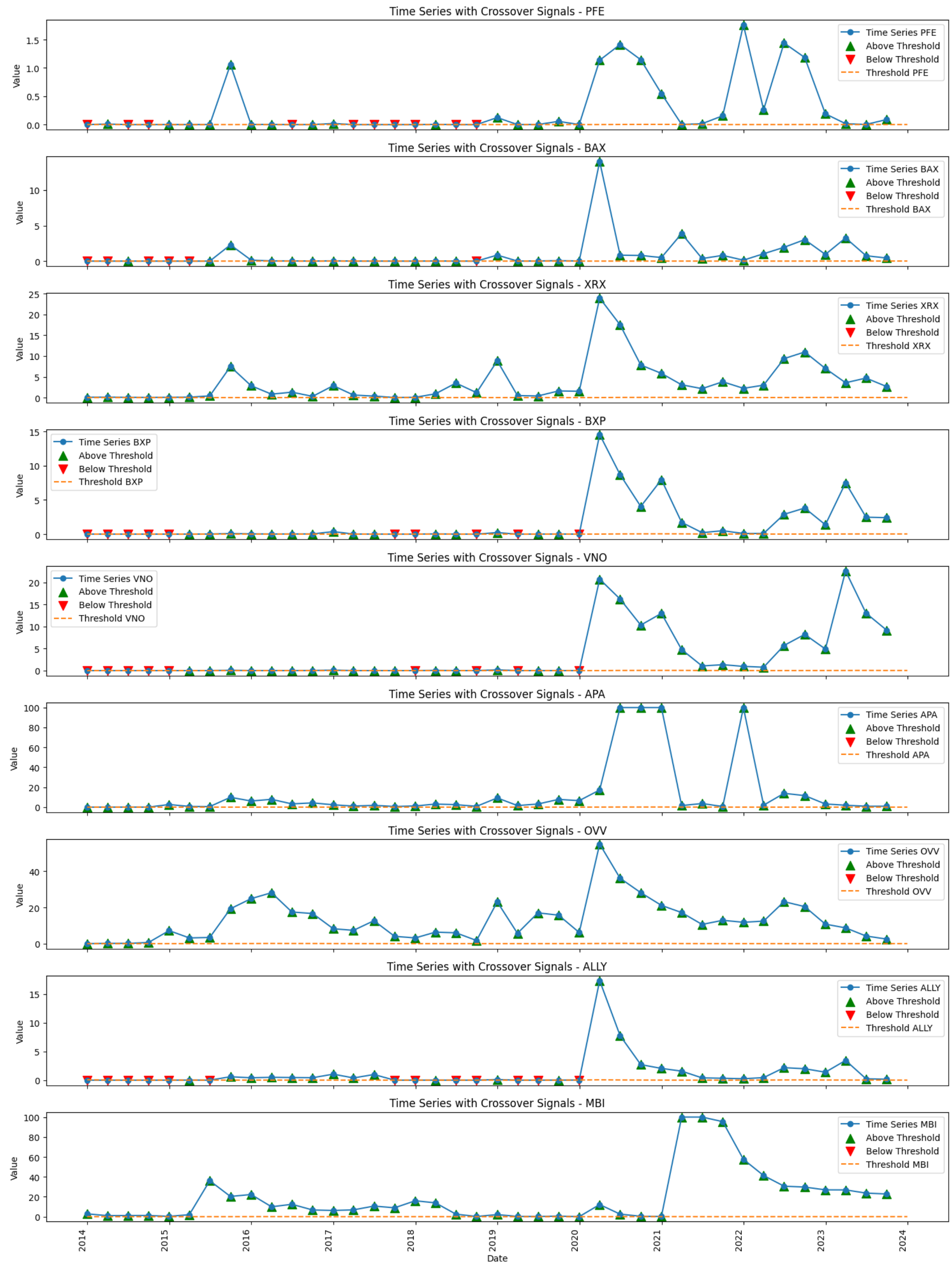

Figure 1: Trading Signals for All Companies

6 Results

As explained in the section above, the foundation of a Relative Value Trade between credit and equity markets using CDS and Options is to harvest the difference in premiums between the CDS Spread and the Options.

In this case, we have considered a notional of \$1,000,000 for both the CDS and Put options contract. For eg, consider Pfizer (PFE), trading at the time of writing for \$28.79, with an at-the-money option trading at \$1.856941. The premium collected for shorting a \$1,000,000 face value of put options would be:

$$Premium(Put) = \frac{(1000000 * 1.8569)}{28.79 * 100} = \$644.98$$

Similarly, a \$1,000,000 notional CDS contract with a spread of 35.227385 (yearly) will result in a quarterly premium of:

$$Premium(CDS) = \frac{(1000000 * 35.227385)}{4 * 10000} = \$880.675$$

Therefore, if we were going long the strategy with these values our returns under a no default scenario would be the difference between the 2 premiums:

$$PnL = (880.675 - 644.98) = \$235.695$$

The plot below shows the per-quarter PnL either by going long the strategy (short option, buy cds) or short the strategy (buy options, sell protection) depending on the Trading Signal generated.

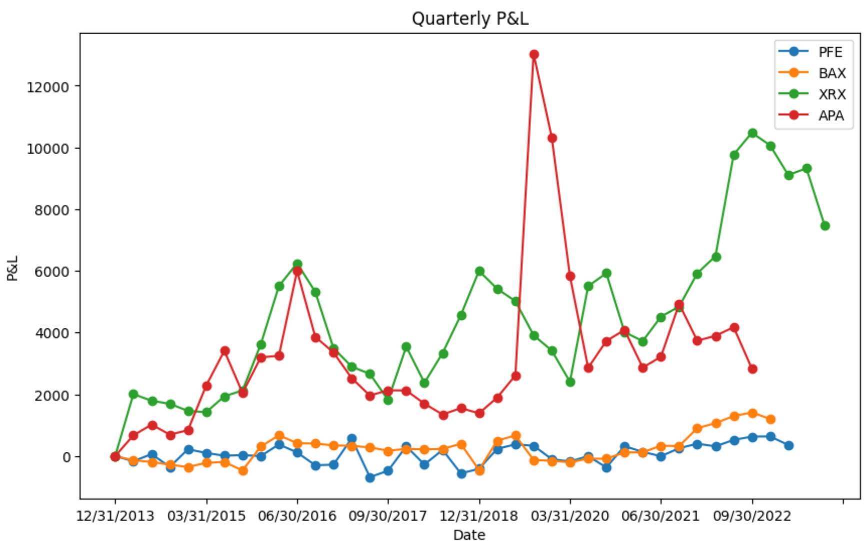

Figure 2: Quarterly profit and Loss

This table below shows a summary of the PnL of each of the 4 companies during the entire time period. Due to a lack of available CDS data for most of the companies in our portfolio, the Trading Strategy could only be conducted on the below 4 companies.

Metrics	**PFE**	**BAX**	**XRX**	**APA**
Max	634.10	1409.50	10471.36	13036.48
Min	-638.00	-448.01	0.00	0.00
Mean	69.80	283.48	4527.87	3204.13
Std. Dev.	336.48	453.62	2610.17	2520.28
Mean as %	0.01%	0.03%	0.45%	0.32%

Table 3: Summary of P&L

7 Conclusion

In the realm of collecting financial data, the identification of missing credit default swap (CDS) spreads and option prices presented a substantive challenge. In response to this, we employed the Black-Scholes Model as a methodological approach for estimating missing option prices. However, The resultant calculated prices, derived from the Black-Scholes model, exhibited a notable deviation from the observed market prices, thereby revealing a limitation in its application within the context of real-world financial data.

The Black-Scholes model, a foundation element in financial mathematics, is instrumental in determining the theoretical valuation of European-style options. Despite its widespread application, the model rests on certain assumptions that may not seamlessly align with intricacies inherent in actual market dynamics. Factors such as transaction costs, market frictions, and fluctuation in volatility contribute to disparities between the model's predictions and empirically observed prices.

The Merton model makes a lot of assumptions that may not be feasible in the practical work. The model uses the market value of debt as a key input. In cases where the debt is not actively traded or when market conditions are illiquid, determining the market value of debt can be challenging and may lead to inaccurate risk assessments. The model also assumes that the returns on the firm's assets are normally distributed. In practice, financial asset returns often exhibit non-normal distributions, with the possibility of extreme events (fat tails) that the model may not capture effectively. This model is a static model that provides a snapshot of credit risk at a specific point in time. It does not capture the dynamic nature of financial markets and may not adequately reflect changes in a firm's risk profile over time.

When using the Merton Model, it is important to note that this model makes a lot of assumptions that are not necessarily true for most companies in today's world. For example, the Merton Model assumes that all debt (liabilities) is the same throughout the company, which we know for a fact is most often not true as companies often have various short-term debts and long-term debts. Additionally, when calculating the volatility of a company's assets, the Merton Model makes a similar assumption on the company's assets namely that all assets are the same. However, a company typically has a smaller portion of its assets being short-term assets in the form of cash and receivables, etc. while it has a bigger portion of long-term assets which are usually not as volatile. Therefore, an adjustment to the model could be to calculate separate equity and asset volatility values for short-term and long-term assets and liabilities and then use an average for the asset volatility value when calculating the distance to default and the probability of default.

In light of the constraints identified in conventional models, the KMV model has emerged as a noteworthy advancement over the Merton model, garnering increased recognition in financial practice. Its dominance lies in its capacity to address some of the limitations inherent in the Merton framework, making it a preferred choice for risk assessment in contemporary financial landscapes.

The KMV model distinguishes itself by augmenting the Merton model's methodology. Notably, it goes beyond the Merton Model's focus on leverage alone. Instead, the KMV model incorporates both long and short-term debts, providing a more nuanced representation of a company's financial obligations. This inclusion is particularly crucial in capturing the diverse and dynamic nature of financial structures in today's markets.

As we conclude, it is evident that the pursuit of more sophisticated models and methodologies, such as the KMV model, is important for refining financial analyses and risk assessments. The continuous evolution of our approaches is essential to meet the demands of an ever-changing financial environment, ensuring accuracy, reliability, and adaptability in our quest for a deeper understanding of credit risk and relative value trading

8 References

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