0.1 Straumann - Mechanik 2015

0.1.1 Exercise 1.2 - Free vertical fall with friction - NOT DONE YET

Equation of motion

$$m\ddot{y} + mg - \alpha\dot{y}^2 = 0 \tag{1}$$
$$\ddot{y} + g - \beta\dot{y}^2 = 0 \tag{2}$$

Now we can substitude $v = \dot{y}$ and obtain

$$\dot{v} + g - \beta v^2 = 0 \tag{3}$$
$$\int \frac{dv}{v^2 - g/\beta} = \beta(t+c) \tag{4}$$
$$\int \frac{dv}{v - \sqrt{g/\beta}} - \int \frac{dv}{v + \sqrt{g/\beta}} = 2\sqrt{\frac{g}{\beta}}\beta(t+c) \tag{5}$$
$$\log\left(v - \sqrt{g/\beta}\right) - \log\left(v + \sqrt{g/\beta}\right) = 2\sqrt{g\beta}(t+c) \tag{6}$$
$$\frac{1}{2}\log\frac{v + \sqrt{g/\beta}}{v - \sqrt{g/\beta}} = -\sqrt{g\beta}(t+c) \tag{7}$$
$$\frac{1}{2}\log\frac{\sqrt{\beta/g}v + 1}{\sqrt{\beta/g}v - 1} = -\sqrt{g\beta}(t+c) \tag{8}$$
$$\text{arctanh}\sqrt{\frac{\beta}{g}}v + \frac{1}{2}\log(-1) = -\sqrt{g\beta}(t+c) \tag{9}$$

Limit velocity ($\ddot{y} = 0$)

$$v_\infty = \sqrt{\frac{mg}{\alpha}} \tag{10}$$

0.2 Goldstein, Poole, Safko - Classical Mechanics 3rd ed

0.2.1 Exercise 9.1 - Canonical Coordinates

Try the generalized transformation where ($\alpha = 1 = \beta$) is the original trafo

$$Q = \alpha(q + ip) \qquad q = \frac{1}{2\alpha}Q + \frac{1}{2\beta}P \tag{11}$$
$$P = \beta(q - ip) \qquad p = \frac{1}{2i\alpha}Q - \frac{1}{2i\beta}P \tag{12}$$

then

$$\dot{Q} = \frac{\partial Q}{\partial p}\frac{\partial p}{\partial t} + \frac{\partial Q}{\partial q}\frac{\partial q}{\partial t} = -i\alpha\frac{\partial H}{\partial q} + \alpha\frac{\partial H}{\partial p} \tag{13}$$
$$\dot{P} = \frac{\partial P}{\partial p}\frac{\partial p}{\partial t} + \frac{\partial P}{\partial q}\frac{\partial q}{\partial t} = +i\beta\frac{\partial H}{\partial q} + \beta\frac{\partial H}{\partial p} \tag{14}$$

and also

$$\frac{\partial H(q(Q,P),p(q,P))}{\partial Q} = \frac{\partial H}{\partial q}\frac{\partial q}{\partial Q} + \frac{\partial H}{\partial p}\frac{\partial p}{\partial Q} \tag{15}$$

$$= \frac{\partial H}{\partial q}\frac{1}{2\alpha} + \frac{\partial H}{\partial p}\frac{1}{2i\alpha} \tag{16}$$

$$\frac{\partial H(q(Q,P),p(q,P))}{\partial P} = \frac{\partial H}{\partial q}\frac{\partial q}{\partial P} + \frac{\partial H}{\partial p}\frac{\partial p}{\partial P} \tag{17}$$

$$= \frac{\partial H}{\partial q}\frac{1}{2\beta} + \frac{\partial H}{\partial p}\frac{i}{2\beta} \tag{18}$$

which implies

$$\frac{\partial H}{\partial q} = \alpha\frac{\partial H}{\partial Q} + \beta\frac{\partial H}{\partial P} \tag{19}$$

$$\frac{\partial H}{\partial p} = \frac{1}{i}\left(\beta\frac{\partial H}{\partial P} - \alpha\frac{\partial H}{\partial Q}\right) \tag{20}$$

which finally results in

$$\dot{Q} = -i\alpha\left(\alpha\frac{\partial H}{\partial Q} + \beta\frac{\partial H}{\partial P}\right) + \alpha\frac{1}{i}\left(\beta\frac{\partial H}{\partial P} - \alpha\frac{\partial H}{\partial Q}\right) \tag{21}$$

$$= -2i\alpha\beta\frac{\partial H}{\partial P} \tag{22}$$

$$\dot{P} = 2i\alpha\beta\frac{\partial H}{\partial Q} \tag{23}$$

So we see

- for $\alpha = 1\beta$ the equations are not canonical
- for $\alpha = \frac{i}{2}$ and $\beta = 1$ the equations are canonical

0.2.2 Exercise 12.5 - Anharmonic oscillator - NOT DONE YET

$$L = \frac{1}{2}m\dot{x}^2 - 2\cdot\frac{1}{2}k[\sqrt{a^2+x^2} - b]^2 \tag{24}$$