0.1 Goldstein, Poole, Safko - Classical Mechanics 3rd ed

0.1.1 Exercise 9.1 - Canonical Coordinates

Try the generalized transformation where ($\alpha = 1 = \beta$) is the original trafo

$$Q = \alpha(q + ip) \qquad q = \frac{1}{2\alpha}Q + \frac{1}{2\beta}P \tag{1}$$

$$P = \beta(q - ip) \qquad p = \frac{1}{2i\alpha}Q - \frac{1}{2i\beta}P \tag{2}$$

then

$$\dot{Q} = \frac{\partial Q}{\partial p}\frac{\partial p}{\partial t} + \frac{\partial Q}{\partial q}\frac{\partial q}{\partial t} = -i\alpha\frac{\partial H}{\partial q} + \alpha\frac{\partial H}{\partial p} \tag{3}$$

$$\dot{P} = \frac{\partial P}{\partial p}\frac{\partial p}{\partial t} + \frac{\partial P}{\partial q}\frac{\partial q}{\partial t} = +i\beta\frac{\partial H}{\partial q} + \beta\frac{\partial H}{\partial p} \tag{4}$$

and also

$$\frac{\partial H(q(Q,P),p(q,P))}{\partial Q} = \frac{\partial H}{\partial q}\frac{\partial q}{\partial Q} + \frac{\partial H}{\partial p}\frac{\partial p}{\partial Q} \tag{5}$$

$$= \frac{\partial H}{\partial q}\frac{1}{2\alpha} + \frac{\partial H}{\partial p}\frac{1}{2i\alpha} \tag{6}$$

$$\frac{\partial H(q(Q,P),p(q,P))}{\partial P} = \frac{\partial H}{\partial q}\frac{\partial q}{\partial P} + \frac{\partial H}{\partial p}\frac{\partial p}{\partial P} \tag{7}$$

$$= \frac{\partial H}{\partial q}\frac{1}{2\beta} + \frac{\partial H}{\partial p}\frac{i}{2\beta} \tag{8}$$

which implies

$$\frac{\partial H}{\partial q} = \alpha\frac{\partial H}{\partial Q} + \beta\frac{\partial H}{\partial P} \tag{9}$$

$$\frac{\partial H}{\partial p} = \frac{1}{i}\left(\beta\frac{\partial H}{\partial P} - \alpha\frac{\partial H}{\partial Q}\right) \tag{10}$$

which finally results in

$$\dot{Q} = -i\alpha\left(\alpha\frac{\partial H}{\partial Q} + \beta\frac{\partial H}{\partial P}\right) + \alpha\frac{1}{i}\left(\beta\frac{\partial H}{\partial P} - \alpha\frac{\partial H}{\partial Q}\right) \tag{11}$$

$$= -2i\alpha\beta\frac{\partial H}{\partial P} \tag{12}$$

$$\dot{P} = 2i\alpha\beta\frac{\partial H}{\partial Q} \tag{13}$$

So we see

- for $\alpha = 1\beta$ the equations are not canonical
- for $\alpha = \frac{i}{2}$ and $\beta = 1$ the equations are canonical

0.1.2 Exercise 12.5 - Anharmonic oscillator - NOT DONE YET

$$L = \frac{1}{2}m\dot{x}^2 - 2 \cdot \frac{1}{2}k[\sqrt{a^2 + x^2} - b]^2 \tag{14}$$