

Programming in Perl

Week Ten

Algorithms and data structuring

Why Are Algorithms Important?

- Algorithms are language independent representations of the steps needed to accomplish a task
- If you have a knowledge of how to accomplish a task before you start working on a program, you can structure your data to make it easier and/or faster to process the data
- We will look at some sorting functions to see how this is accomplished

Searching

- Sorting is one way to structure your data to make it easier to process
- Sorted data is faster to search
- Example:
 - ◆ Suppose you we have an array of student ID's and we need to know if the ID 42 has been used
 - ◆ We want to add the ID if it is not in the array
 - ◆ (No fair using a hash, let's pretend that Perl doesn't have hashes to see how we can do it!)

Searching

- Using grep:

```
# assume @student_id is already defined
my $id = 42;
unless(grep {$_== $id} @student_id) {
    push @student_id, $id;
}
```

- Problem: It looks at every ID, even if it found it early in the search
- This algorithm always takes N tests for an array of length N

Searching

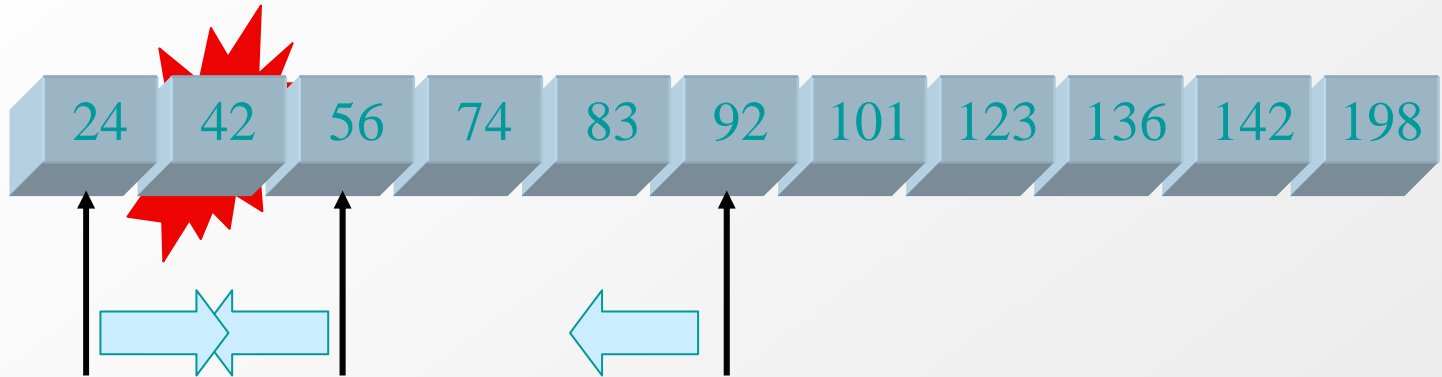
- Refine it a little!

```
my $id = 42;
my $found = 0;
foreach $item (@student_id) {
    if ($item == $id) {
        $found = 1;
        last;
    }
}
push @student_id unless $found;
```

- On the average, this routine takes $N/2$ test to find if the ID is in the array
- Both of the routines are linear searches

Searching

- If the data were sorted into ascending order, we can speed up the tests



This is called a “Binary Sort”

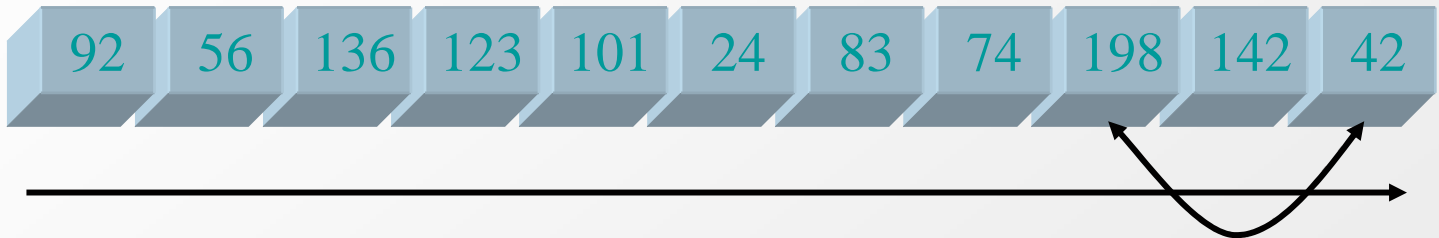
It only works on sorted data!

Sorting

- There are sorting routines from simple to complex
 - ◆ What you use depends on the task: each routine has a set of properties that you must match to the task
 - ◆ Some are fast, but you must resort every time you add data
 - ◆ Others maintain the sort as you insert data, but are slow
 - ◆ Still others are a compromise between the extremes
- The type of data you have can also effect the algorithm you pick
 - ◆ Some work better on lots of data but poorly on small
 - ◆ Some don't preserve the order of identical keys: The “Stability”
 - ◆ Some may not work well with nearly sorted data: The “Sensitivity”

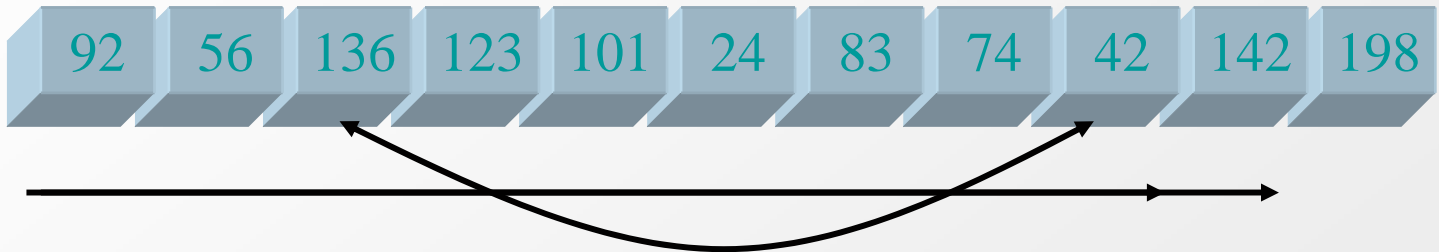
Selection Sort

- One simple sort is the “selection sort”
 - ◆ Find the largest (or smallest) value and put it into the proper place. Lather. Rinse. Repeat.



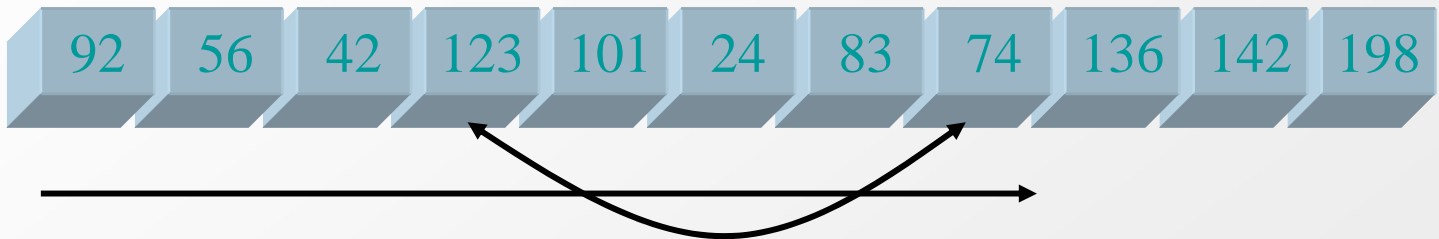
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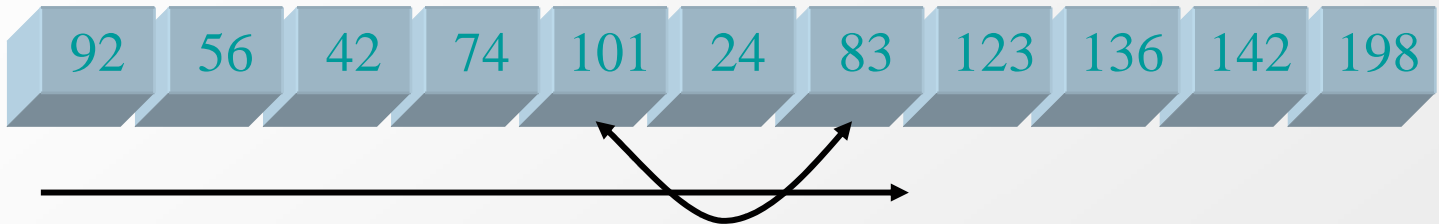
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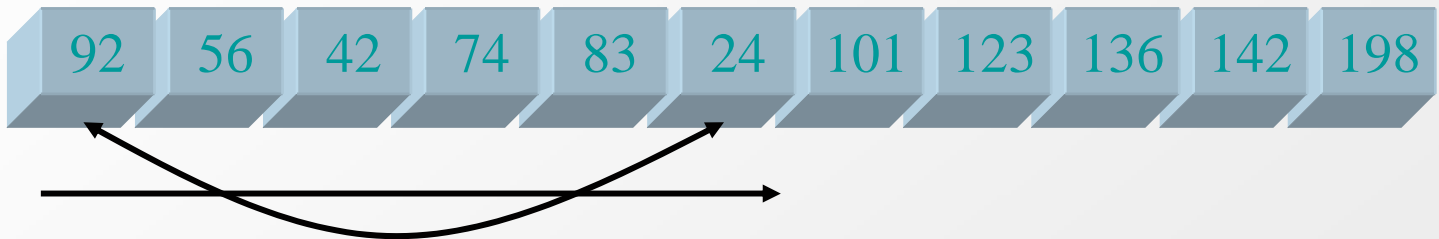
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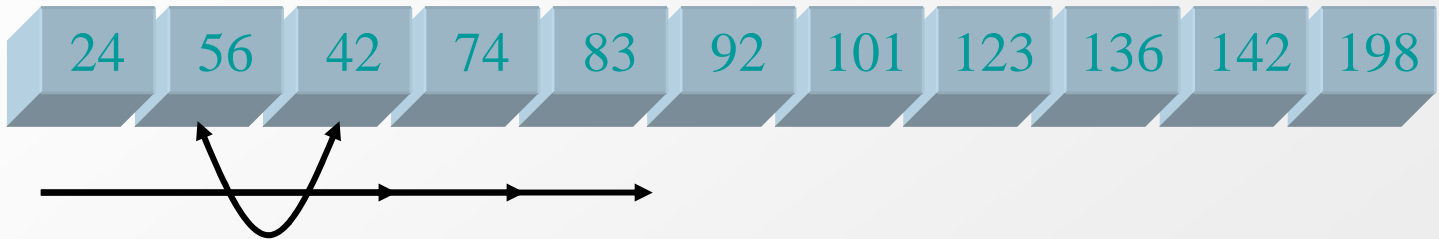
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Selection sort is slow! (N^2)

But is stable and insensitive

Selection Sort

```
sub selection_sort {  
    return unless @_;  
    my $in      = \@_;      # take reference to the input array  
    my $upper = $#_;        # length of input array  
    for (my $i = $upper; $i; $i--) {  
        my $max = $i;  
        for (my $j = 0; $j < $i; $j++) {  
            $max = $j if $in->[$j] > $in->[$max];  
        }  
        ($in->[$i], $in->[$max]) = ($in->[$max], $in->[$i]);  
    }  
}
```

Insertion Sort

- The insertion sort scans through the array, inserting the smallest value in the proper place



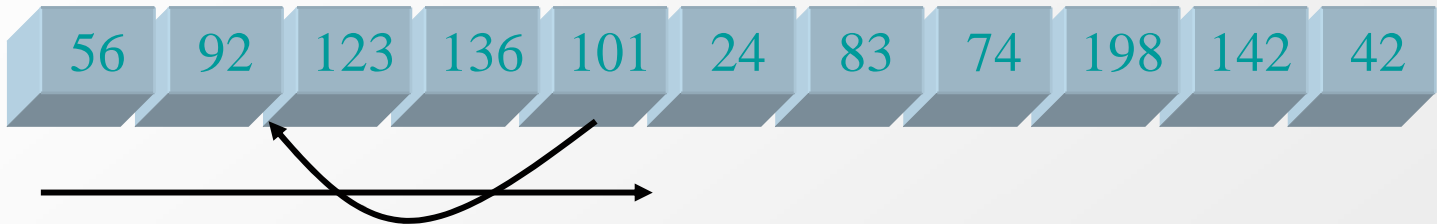
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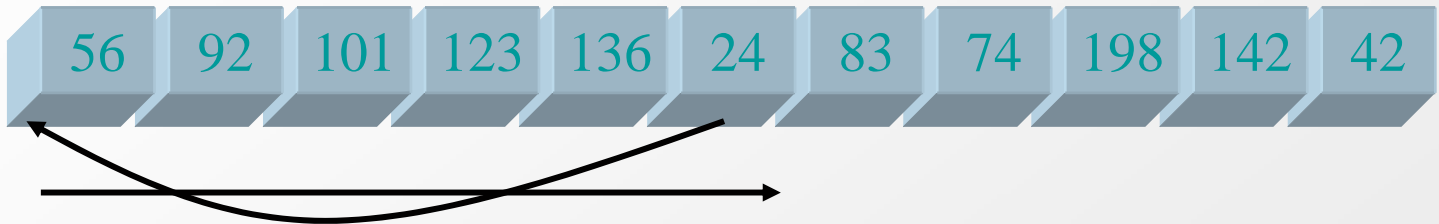
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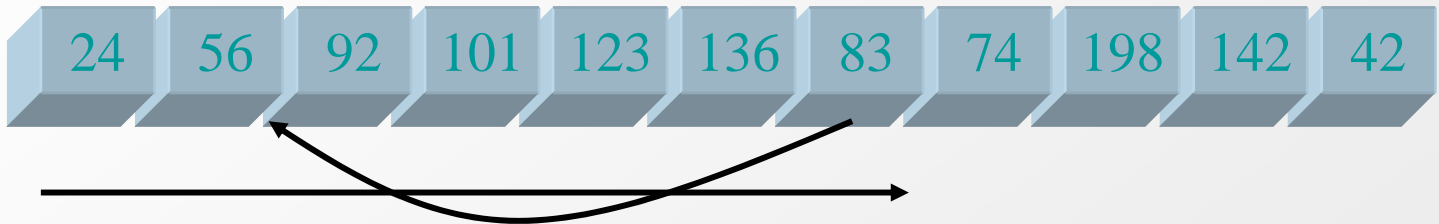
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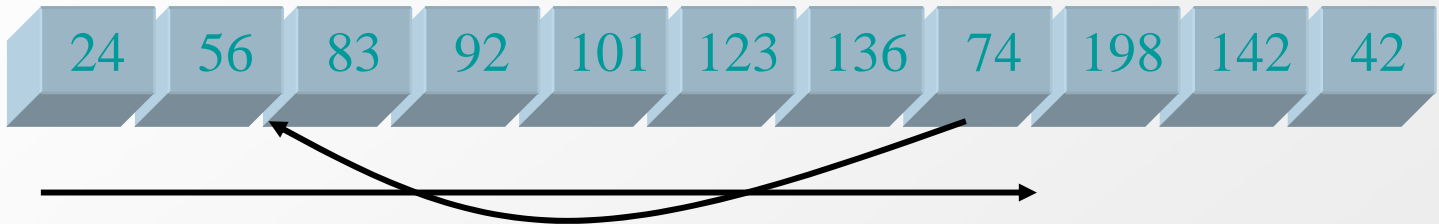
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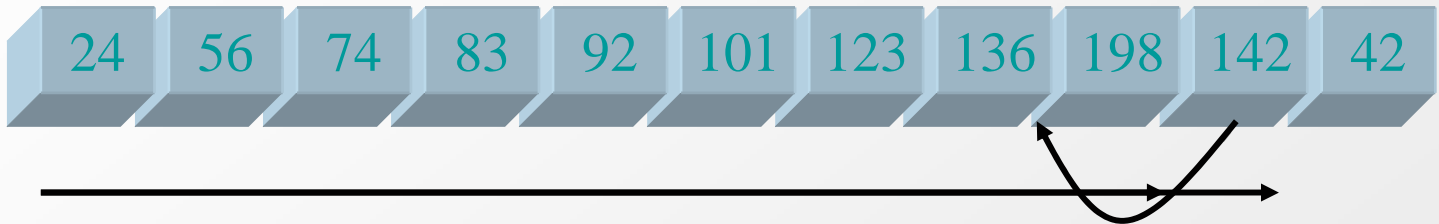
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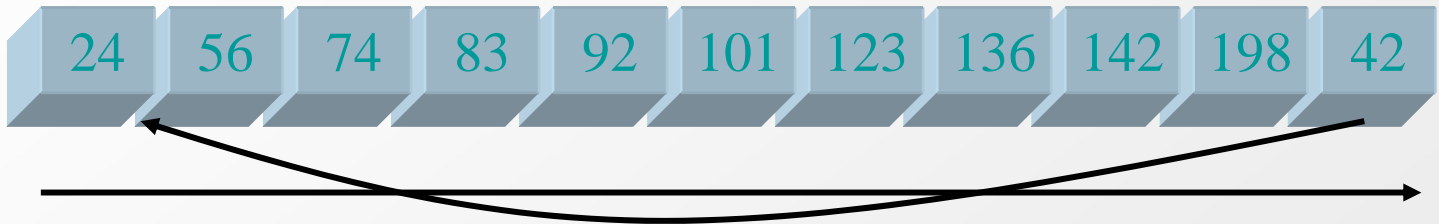
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The insertion sort also works in N^2 worst case but works rather well if the data is almost sorted

Insertion Sort

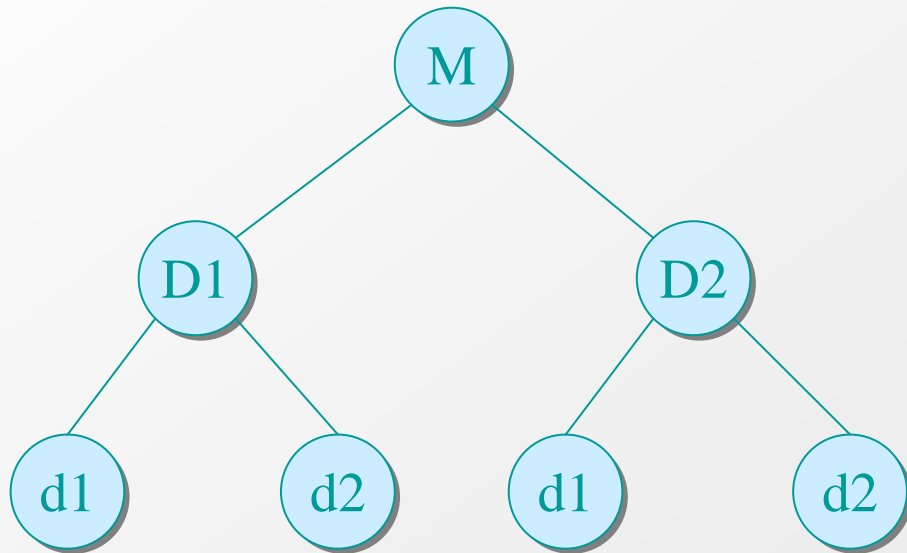
```
sub insertion_sort {  
    return unless @_;  
    my $in = \@_;  
    for (my $j = 1; $j < @$in; $j++) {  
        my $val = $in->[$j];  
        my $i = ($j - 1);  
        while ($i >= 0 and $in->[$i] > $val) {  
            $in->[$i+1] = $in->[$i];  
            $i = $i - 1;  
        }  
        $in->[$i+1] = $val;  
    }  
}
```

Heap Sort

- The previous sorts both had the problem of taking N^2 time to sort
 - ◆ The each added element in the array, the time to sort goes up exponentially
 - ◆ Each also required you to make extensive changes to the array, moving data around in the array, sometimes several times
- By changing the way the data is stored, i.e. in an array, we can speed up the sorting

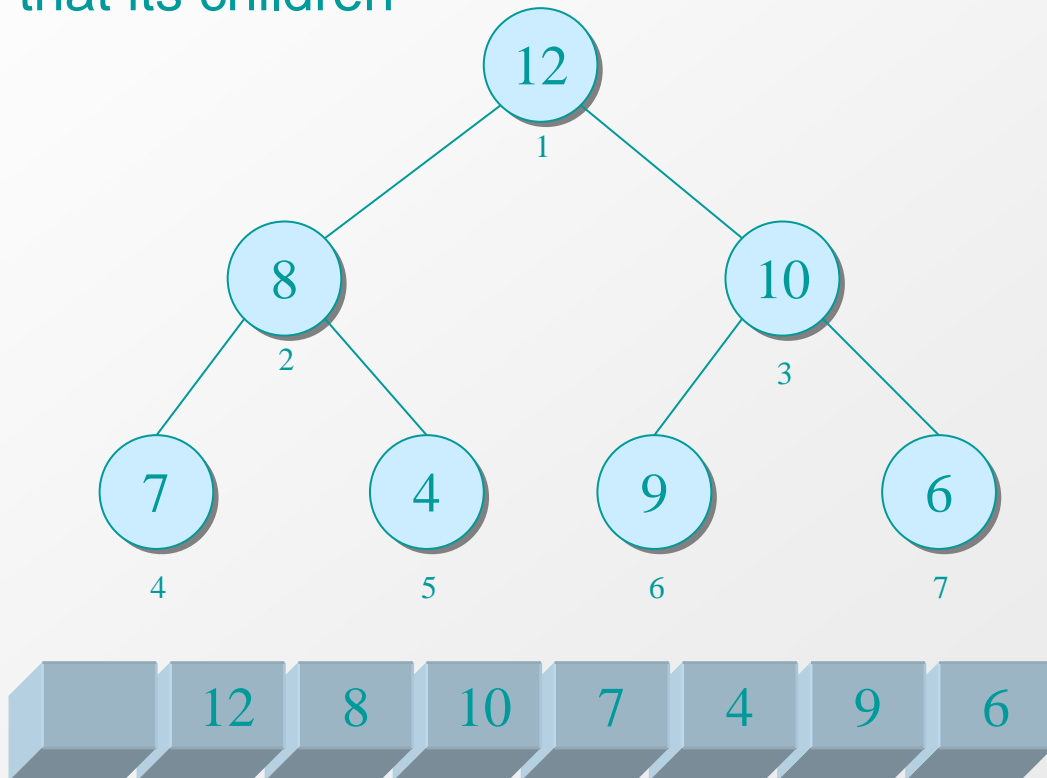
Heap Sort

- A heap is a binary tree structure



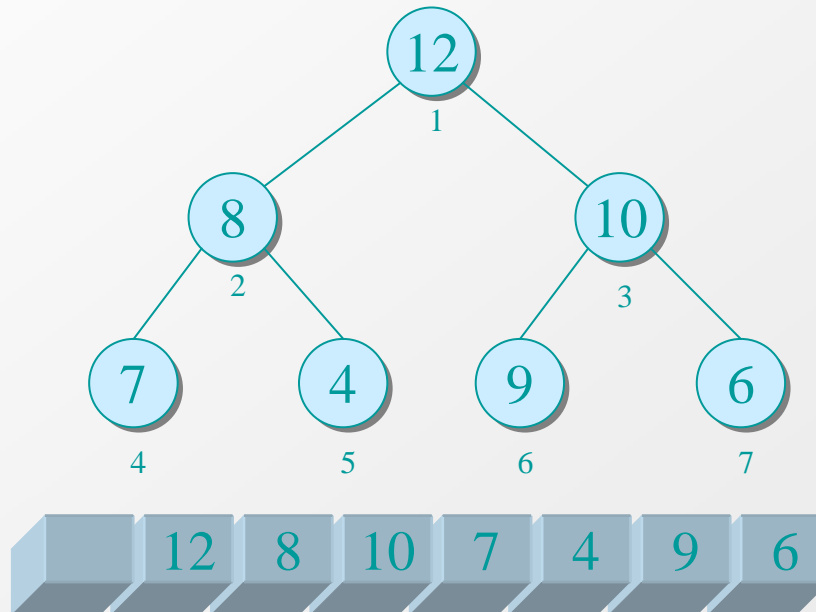
Heap Sort

- A heap has the property that any parent node must be greater than its children



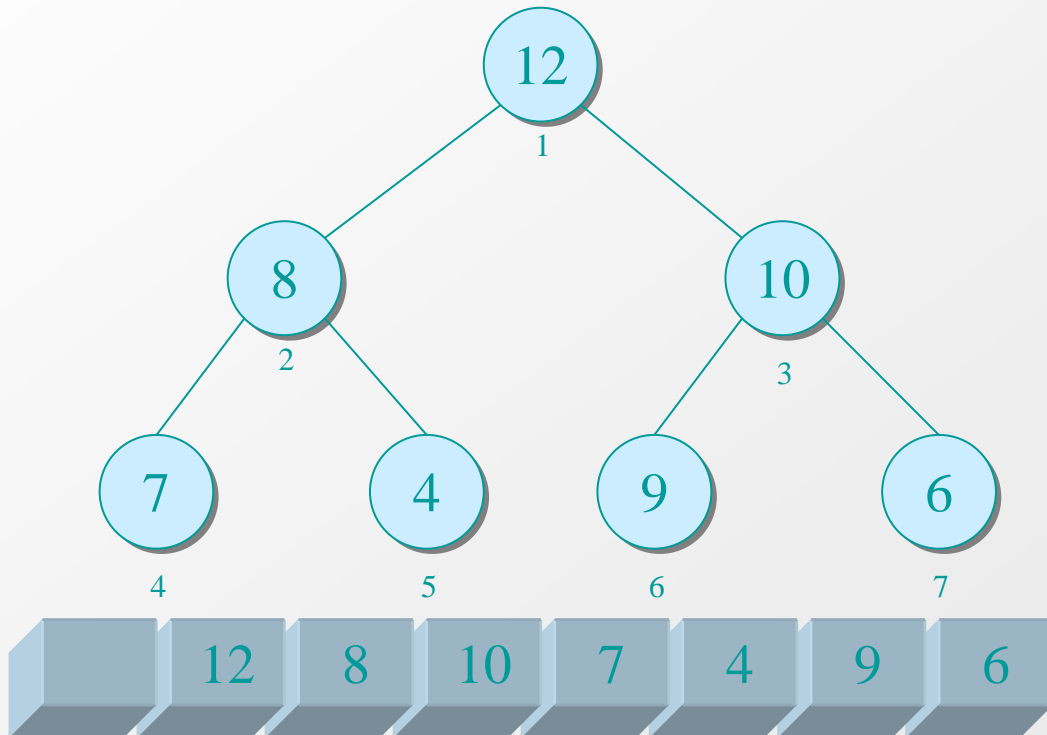
Heap Sort

- We can move through this array by using the properties
 - ◆ The Parent of $\text{node}(i) = \text{node}(\text{int}(i / 2))$
 - ◆ Left Child of $\text{node}(i) = \text{node}(i * 2)$
 - ◆ Right Child of $\text{node}(i) = \text{node}(i * 2 + 1)$



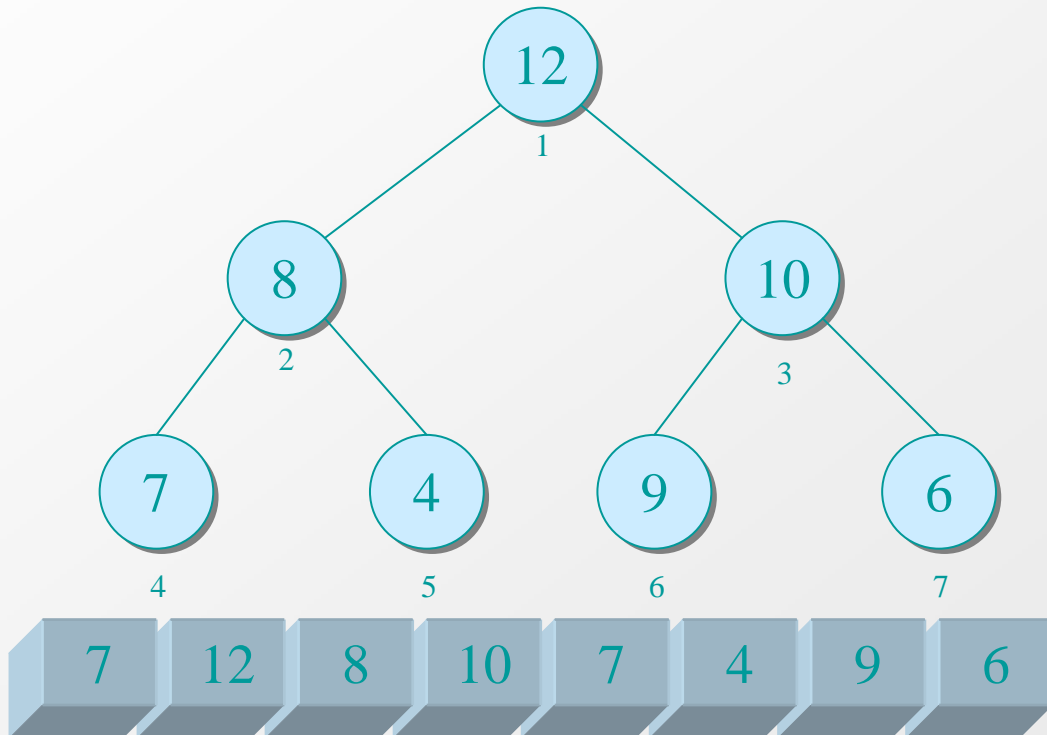
Heap Sort

- To sort a heap
 - ◆ Put the heap size into array[0]



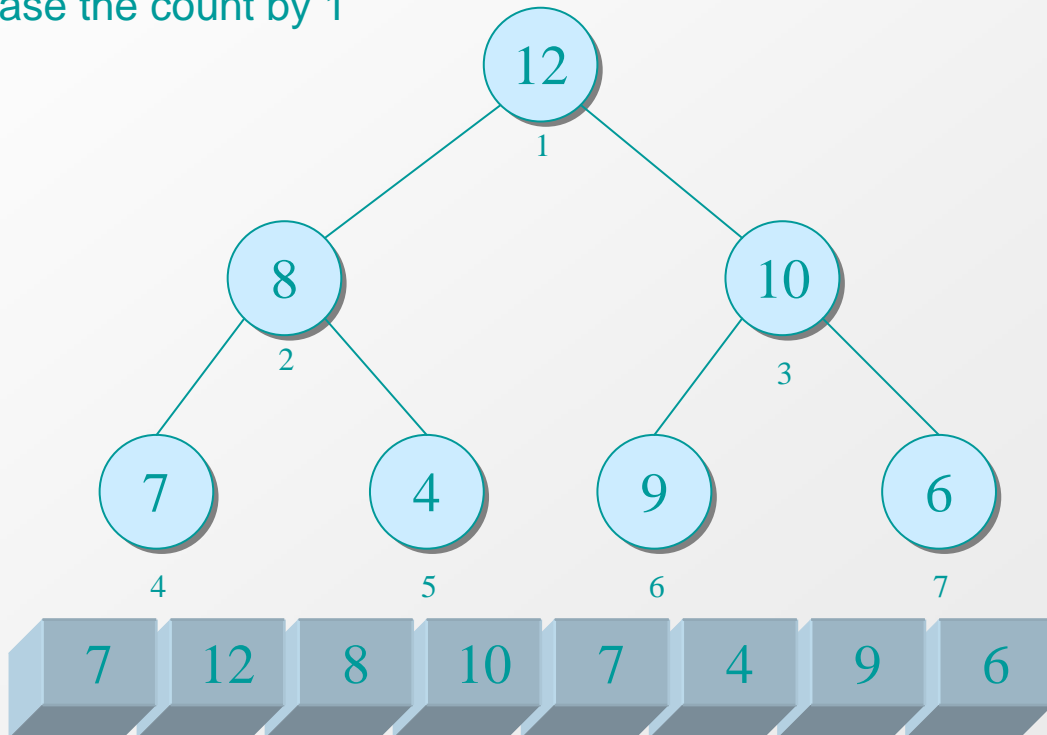
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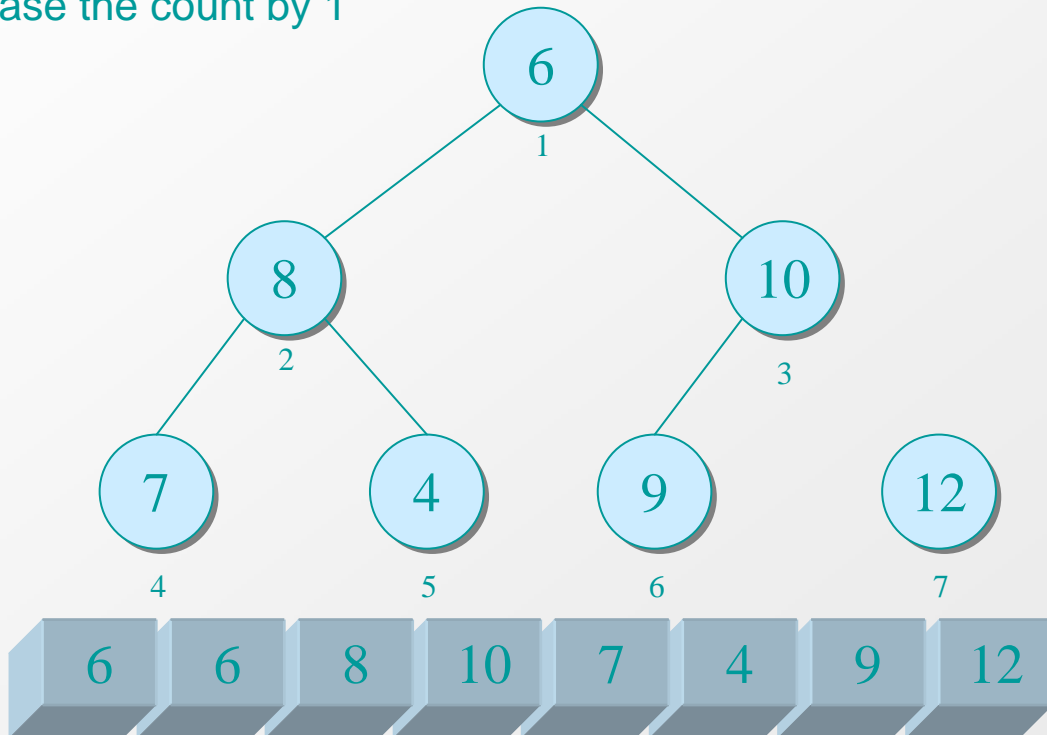
Heap Sort

- To sort a heap
 - ◆ Swap the largest number with the last element of the heap
 - ◆ Decrease the count by 1



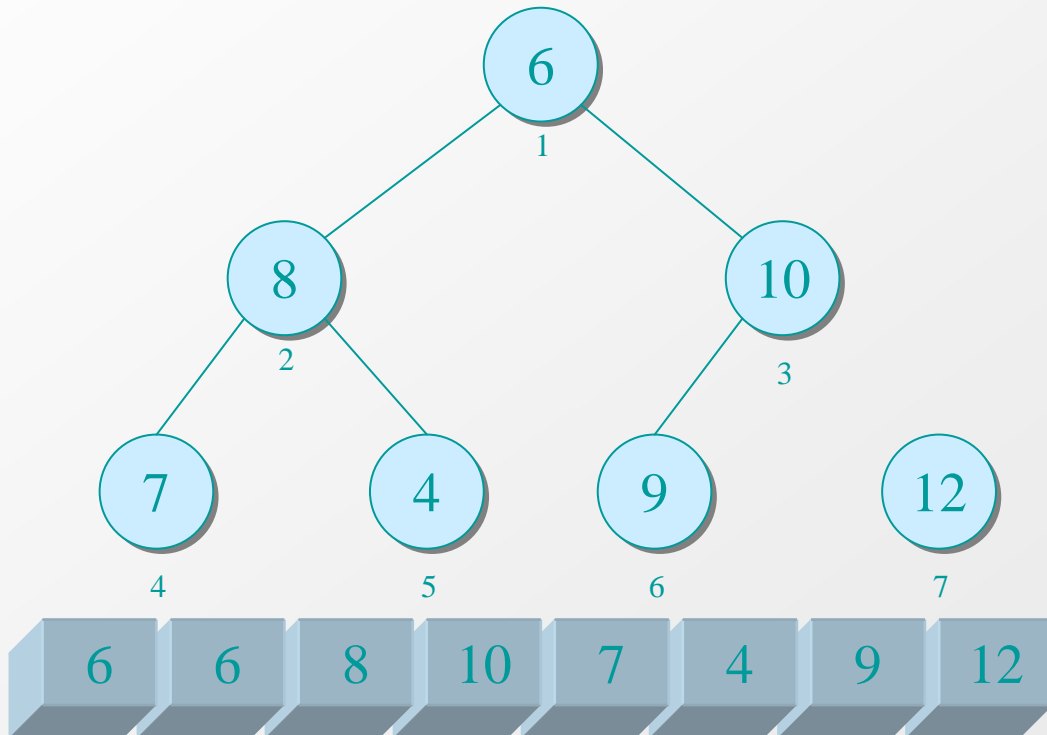
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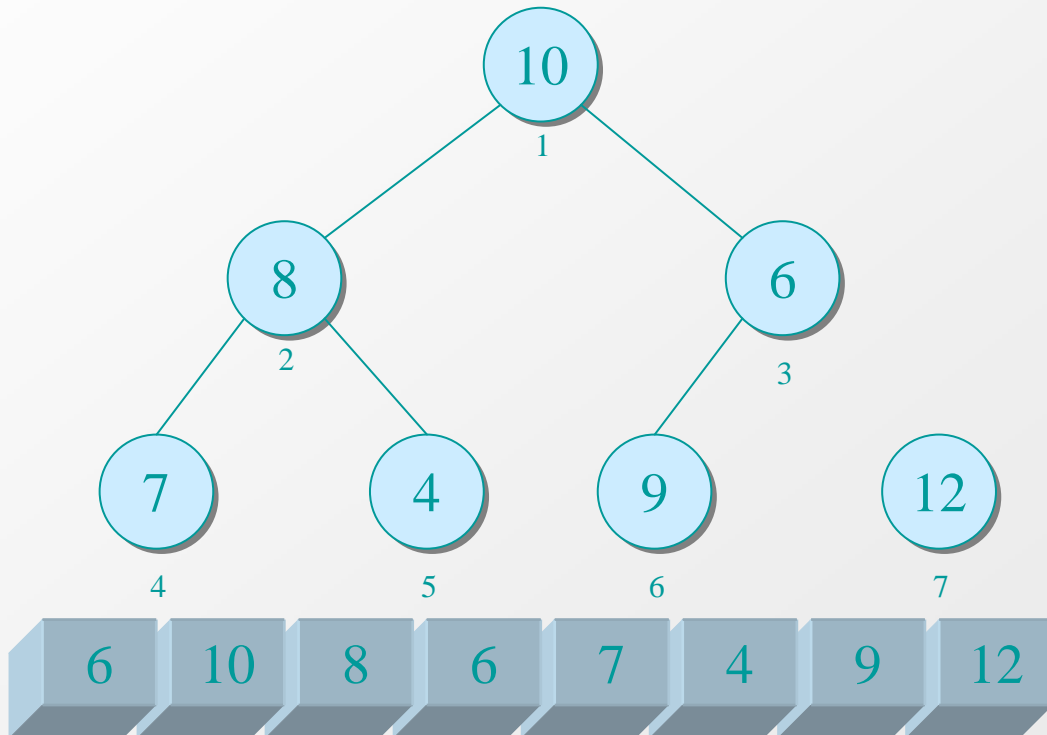
Heap Sort

- To sort a heap
 - ◆ The tree is no longer a heap. Push the value down until it is



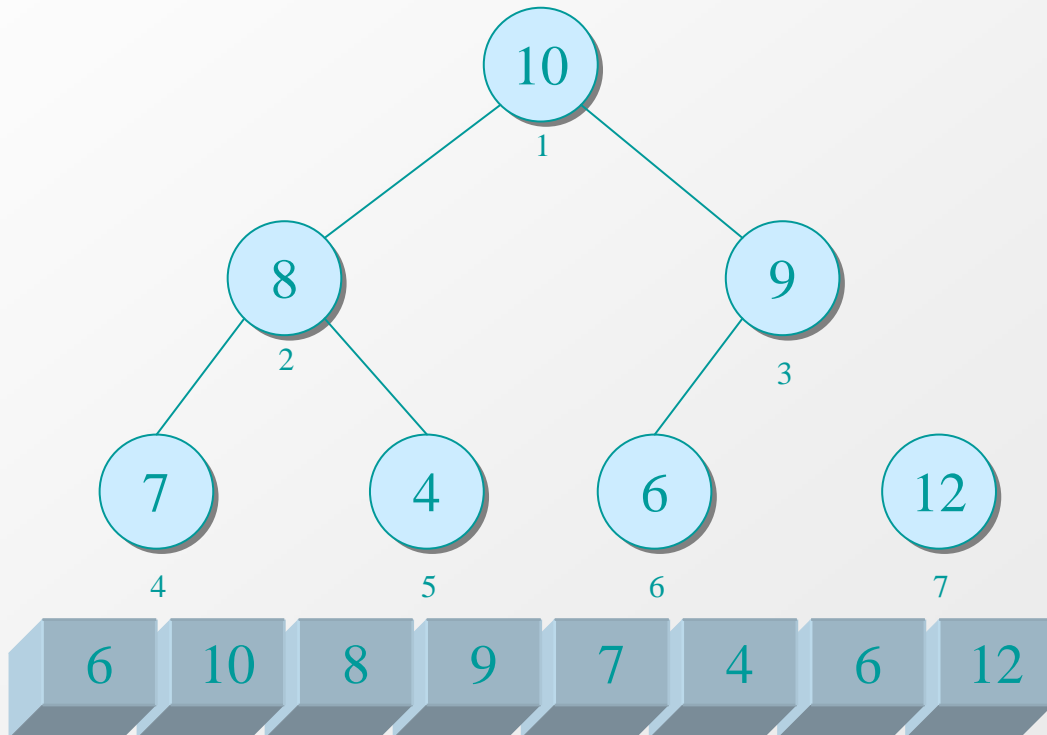
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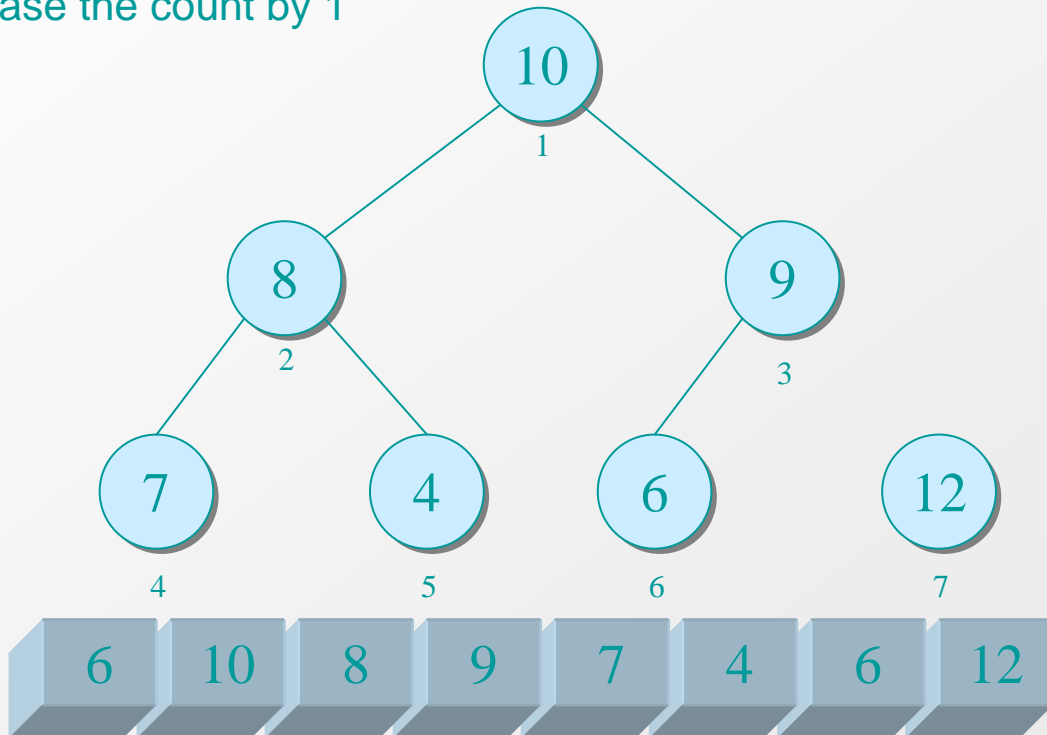
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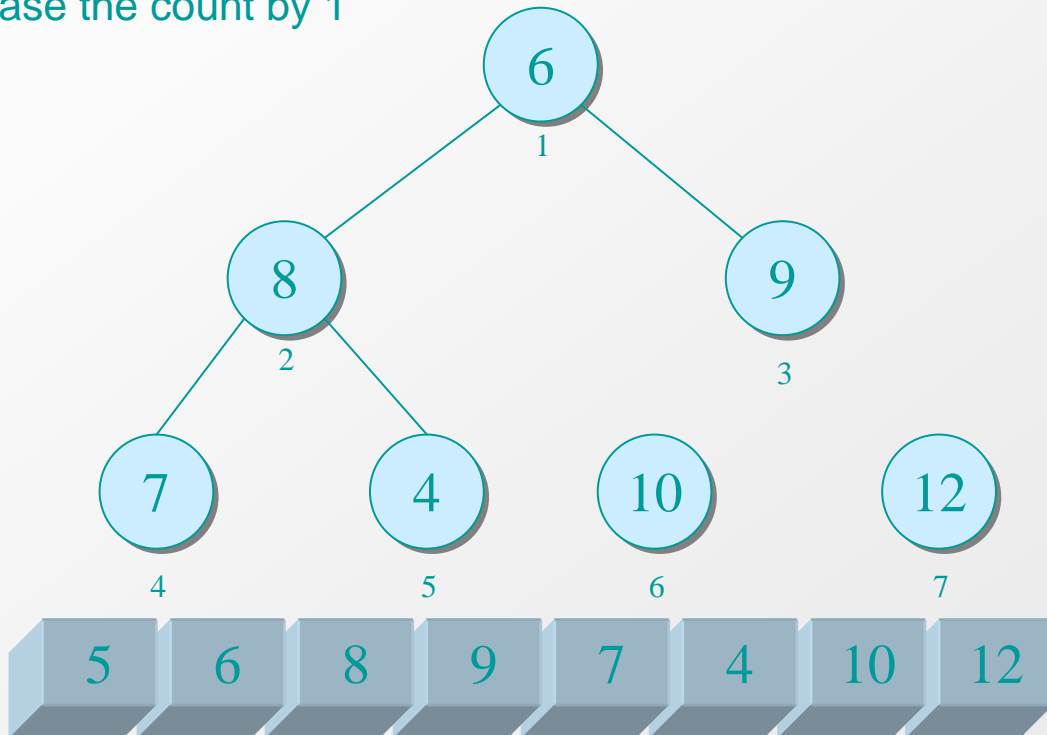
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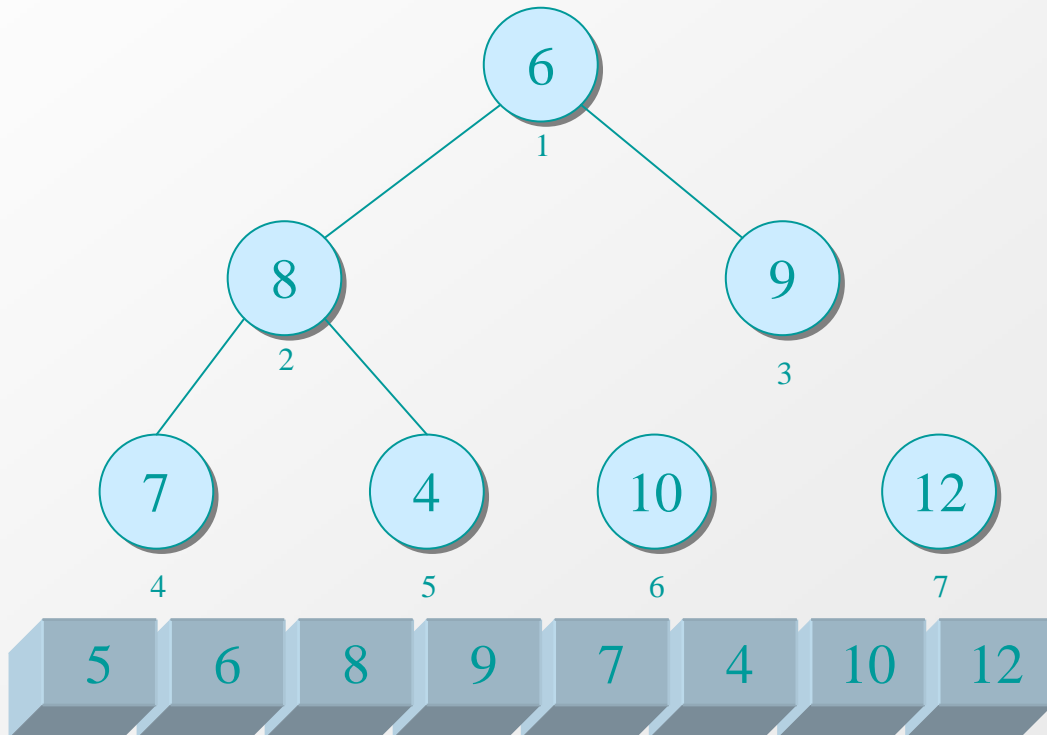
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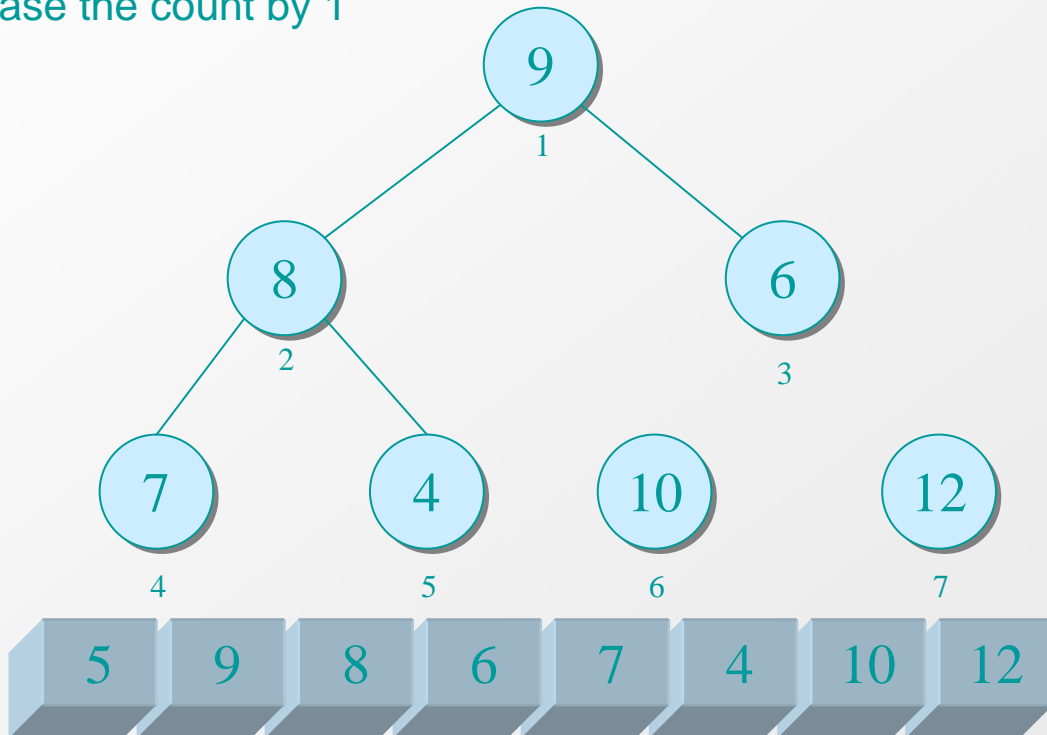
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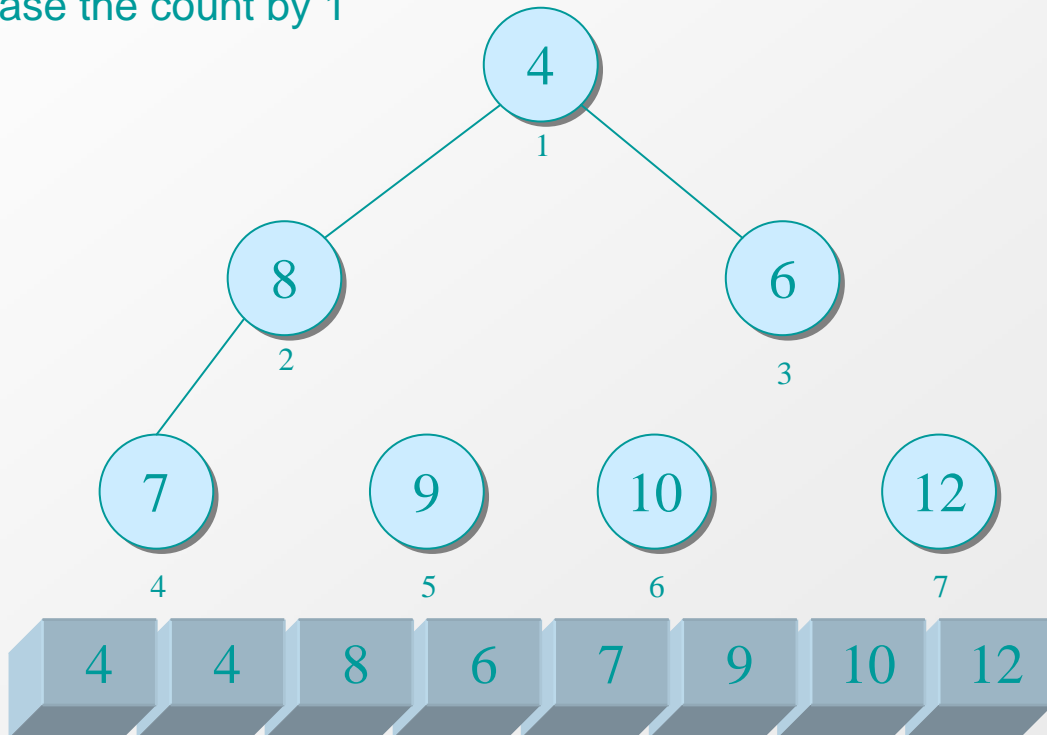
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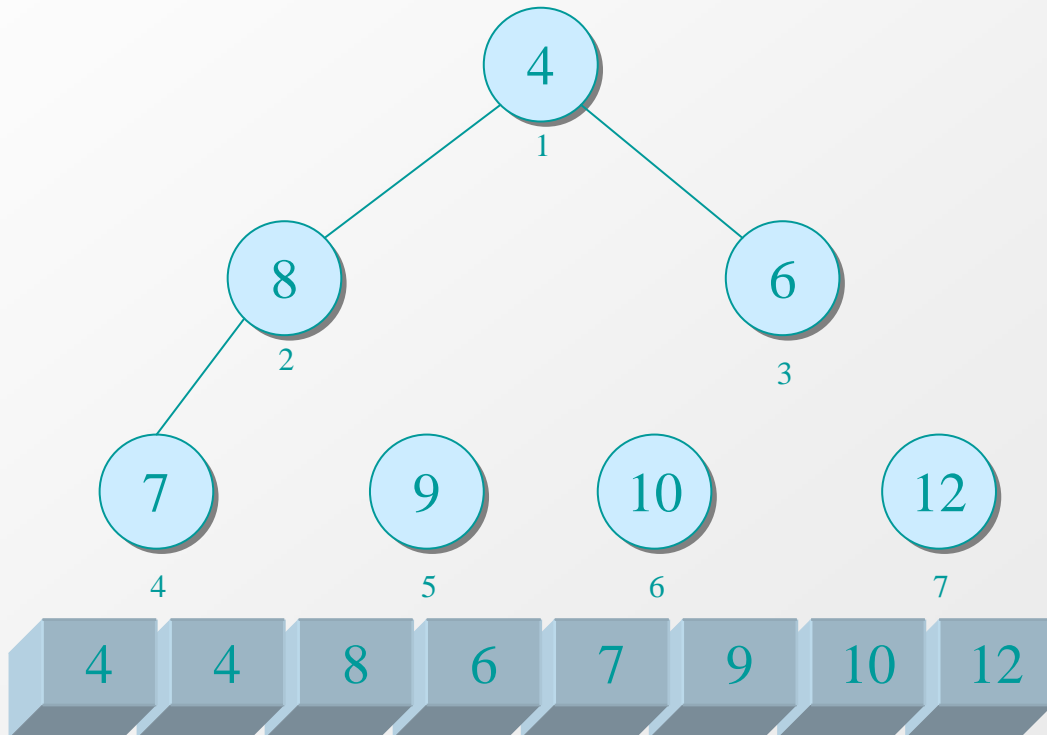
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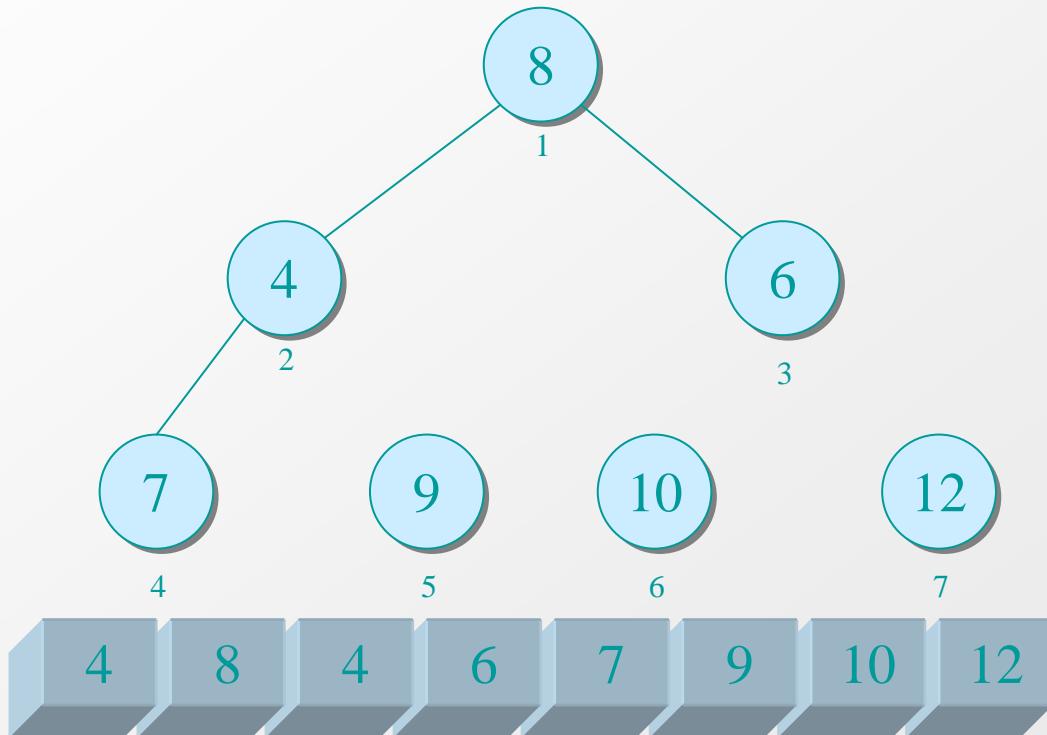
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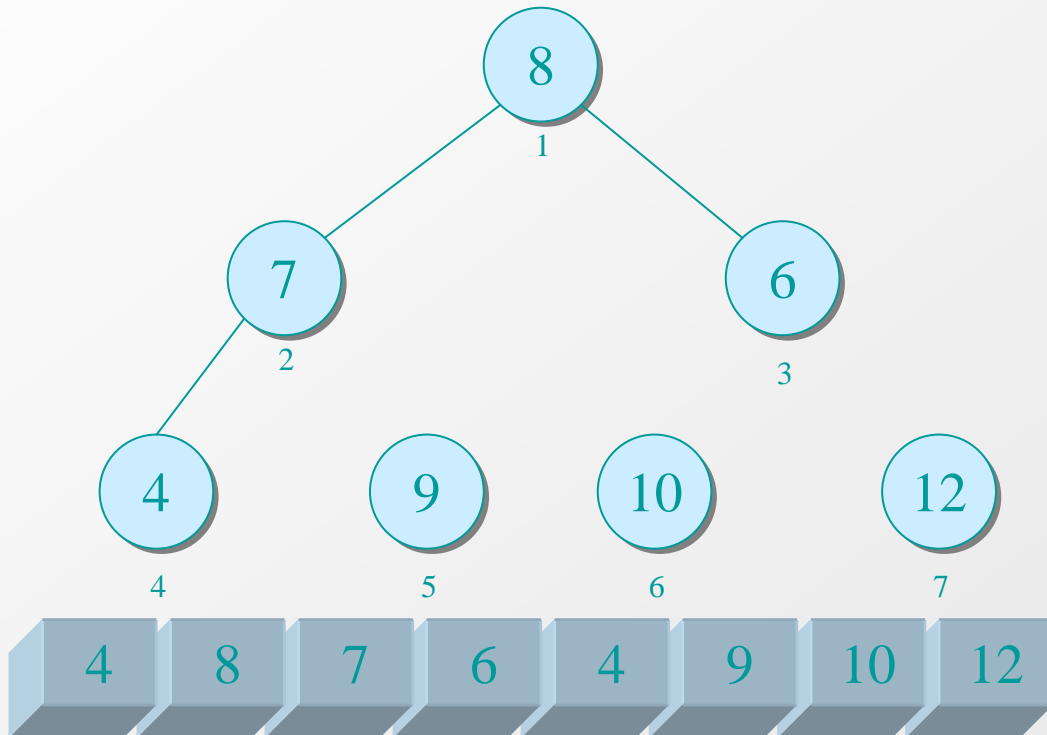
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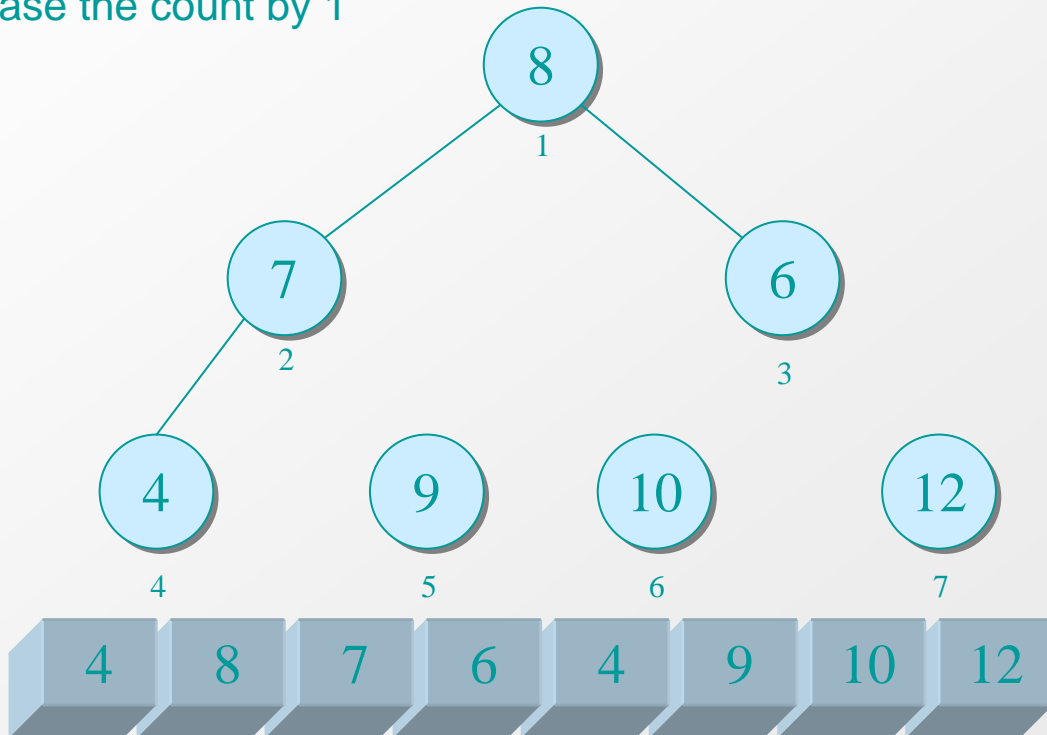
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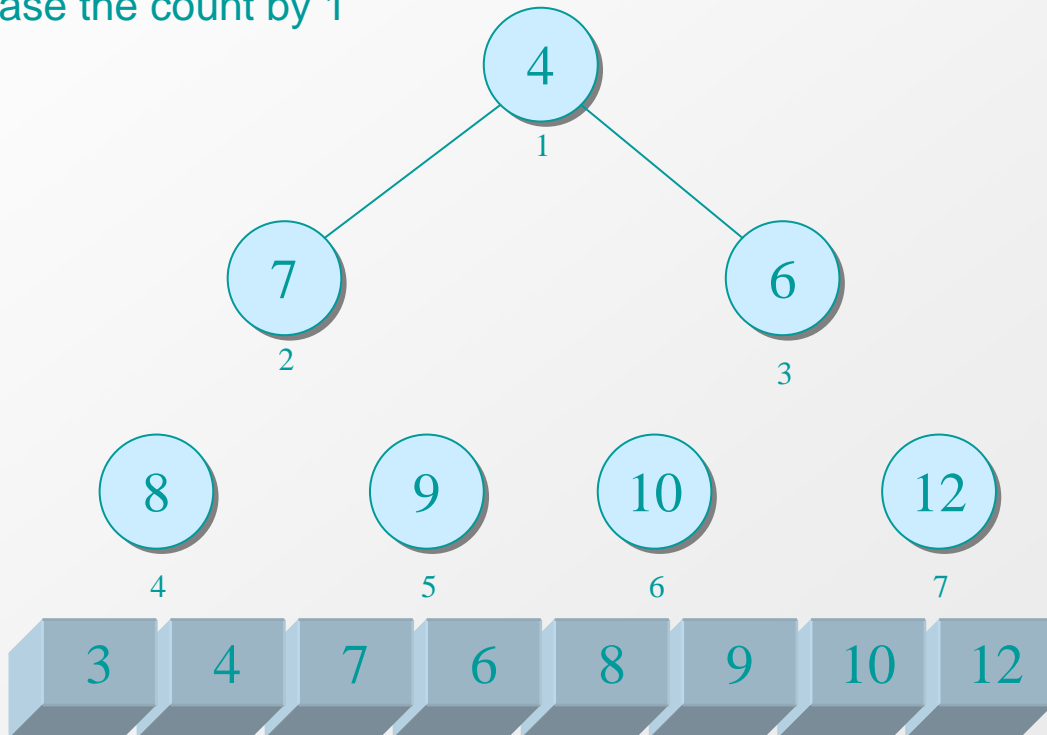
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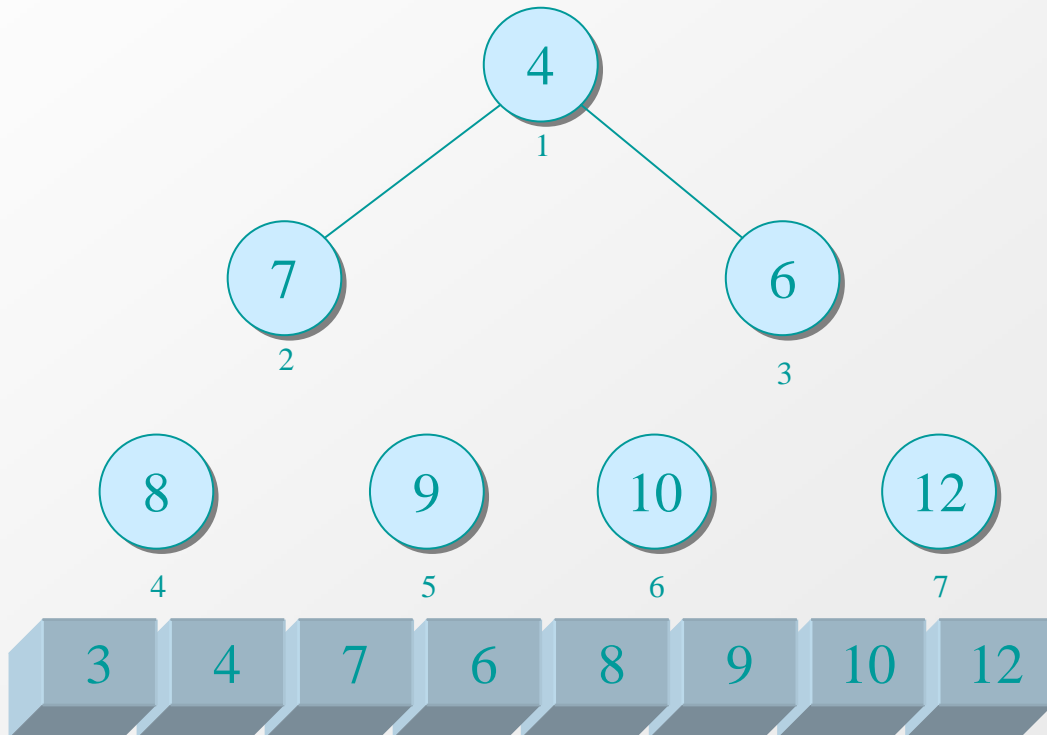
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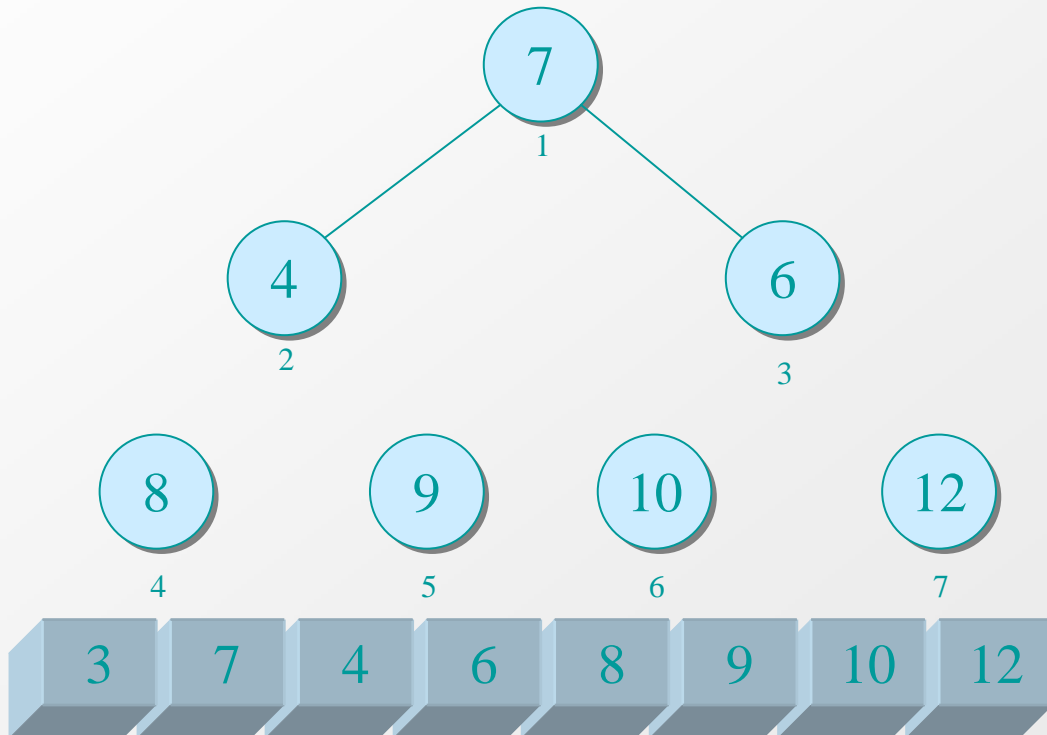
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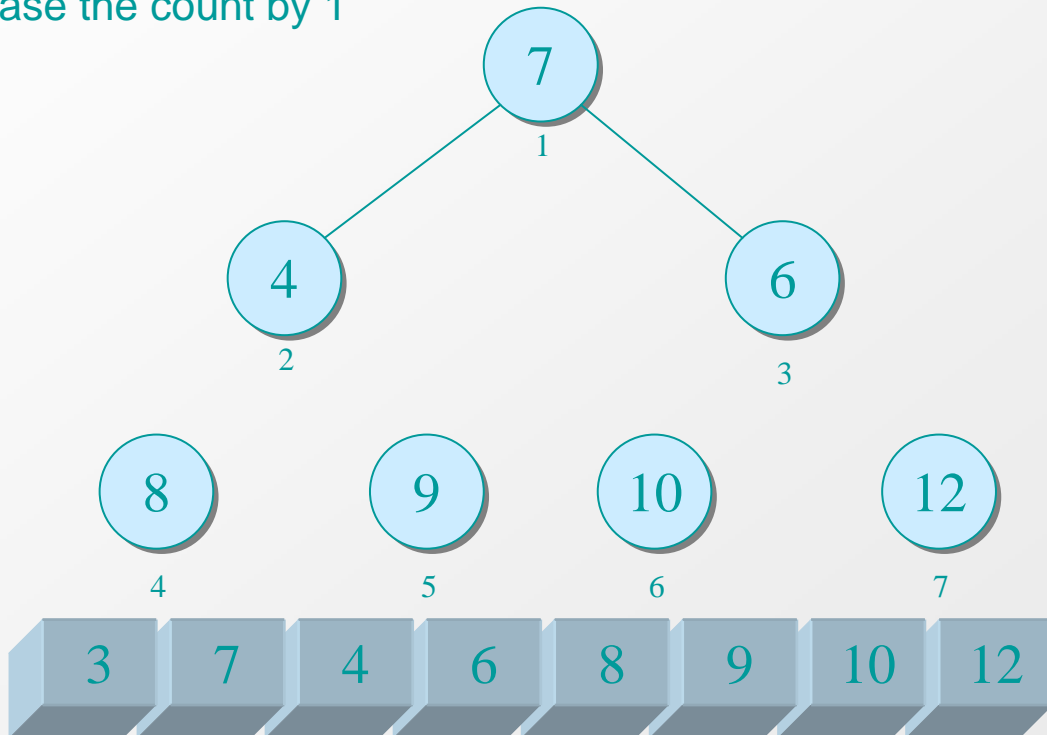
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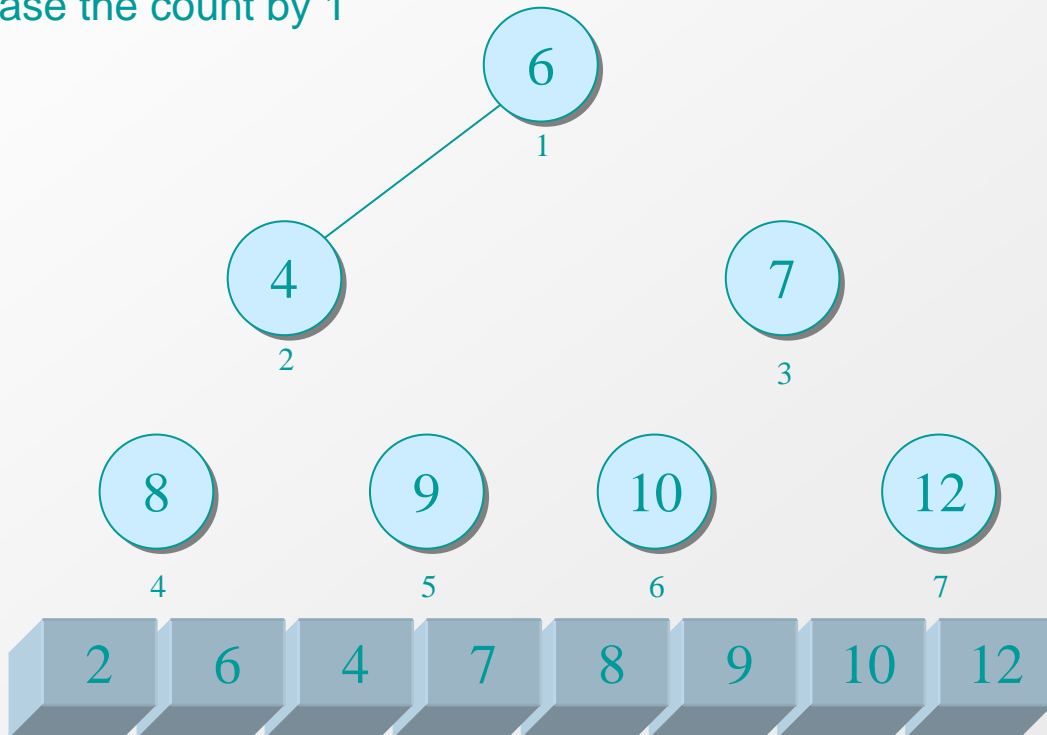
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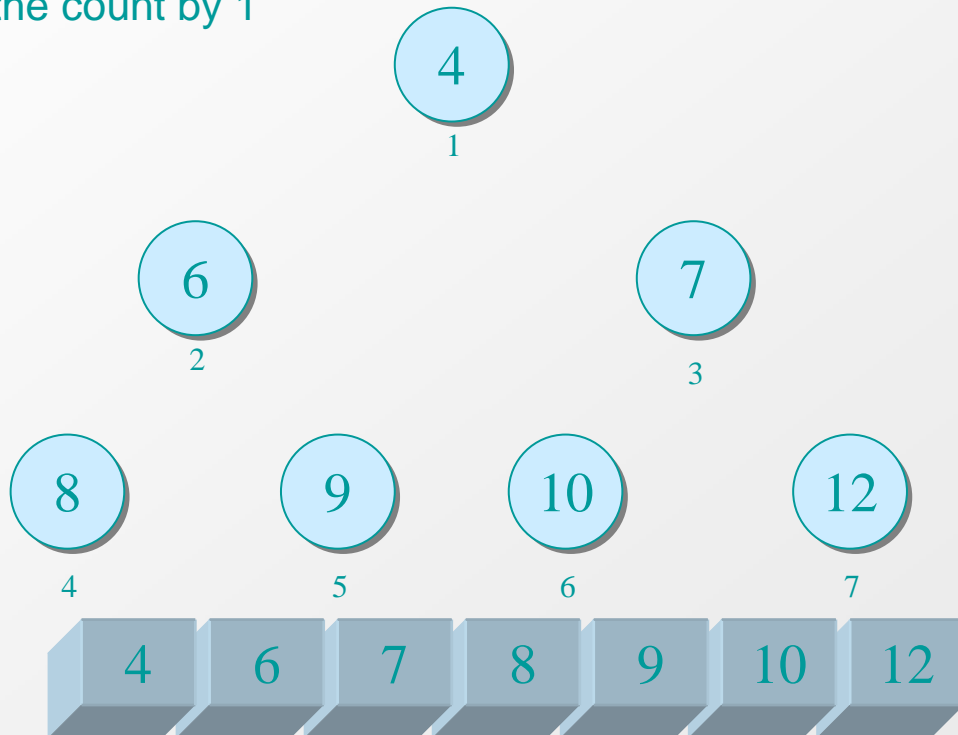
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Heap Sort

```
sub pushdown {  
    my ($heap, $i) = @_;  
    my $size = $heap->[0];  
    while($i <= $size/2) {  
        my $child = $i * 2;  
        if ($child < $size and $heap->[$child]) {  
            $child++;  
        }  
        if ($heap->[$i] >= $heap->[$child]) { last }  
        ($heap->[$i], $heap->[$child])  
            = ($heap->[$child], $heap->[$i]);  
        $i = $child;  
    }  
}
```

Building the Heap

- To build a heap
 - ◆ First, unshift the array's size into its first element



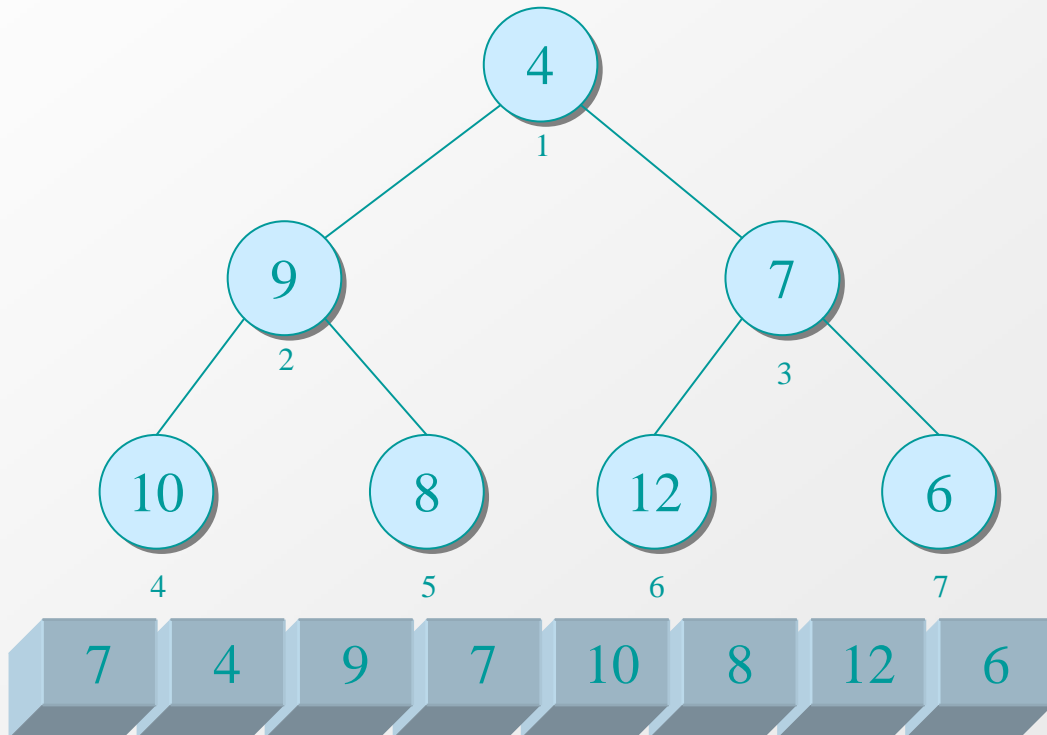
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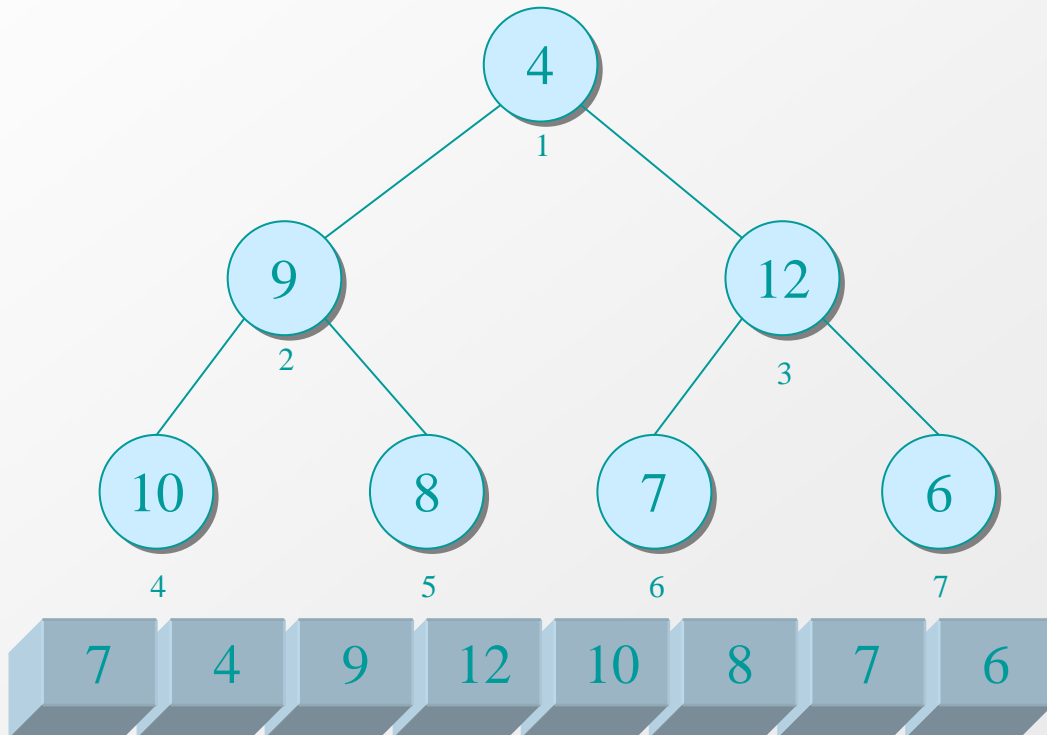
Building the Heap

- To build a heap
 - ◆ Call `pushdown()` on the last parent node



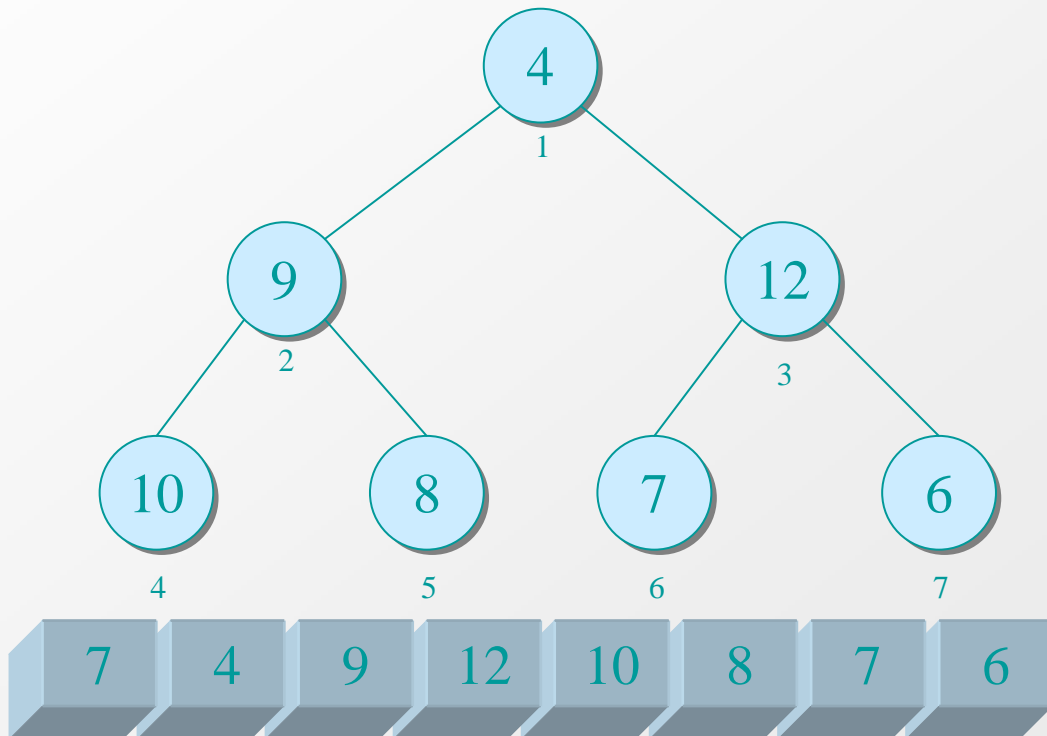
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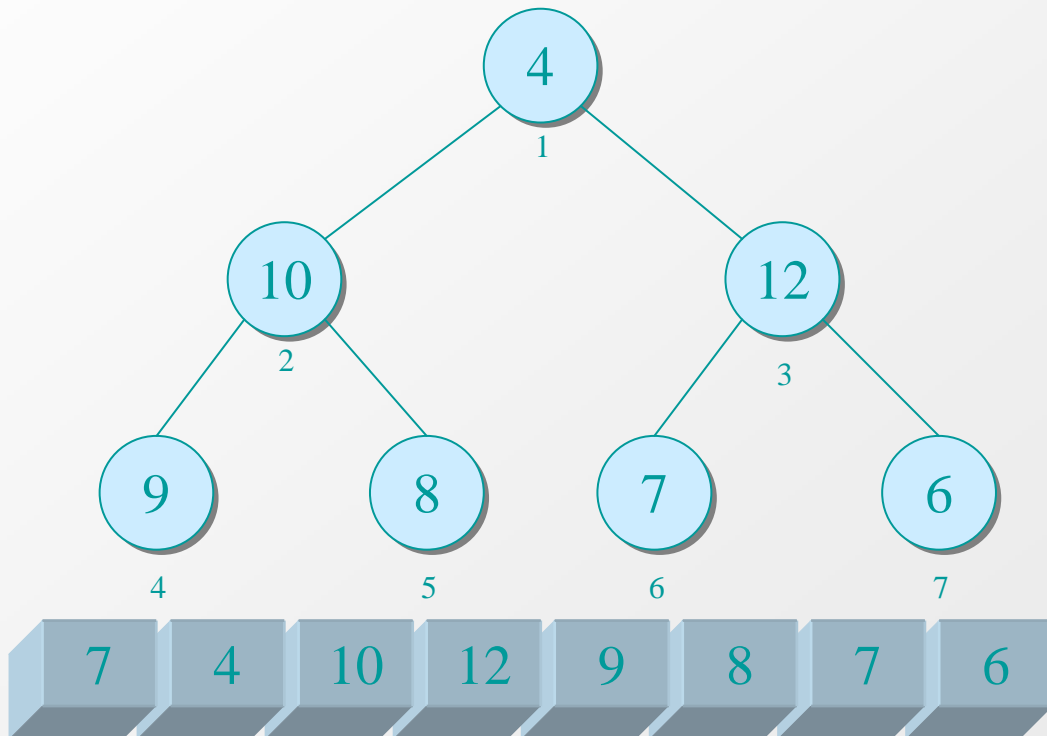
Building the Heap

- To build a heap
 - ◆ Call `pushdown()` on the next to the last parent node



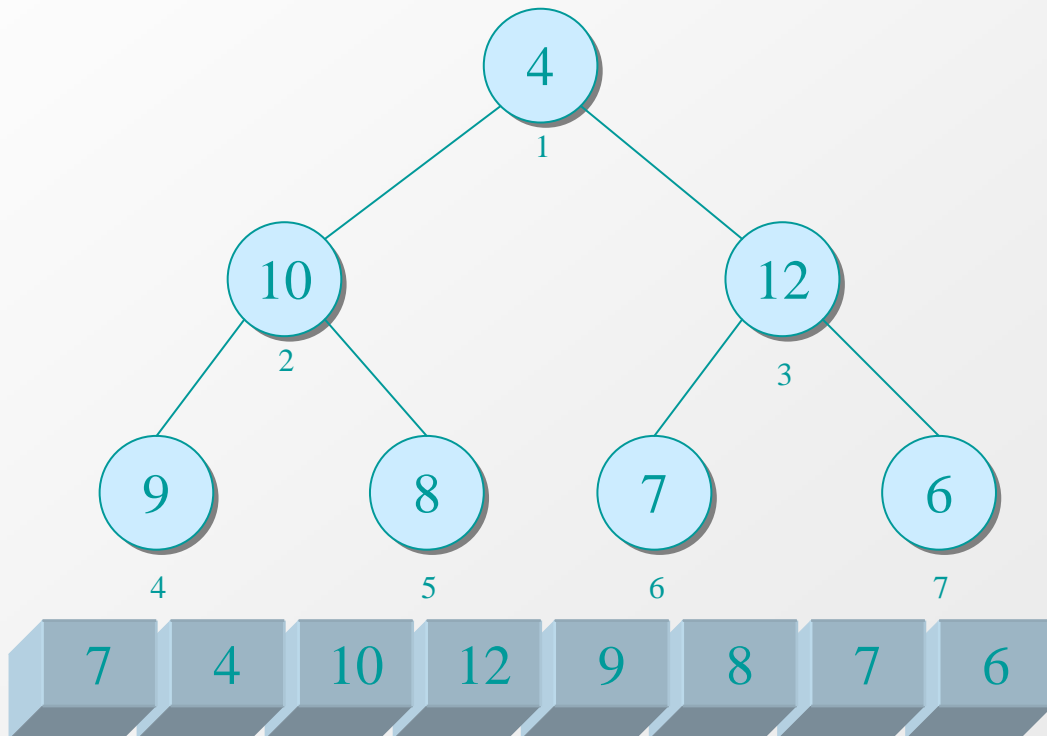
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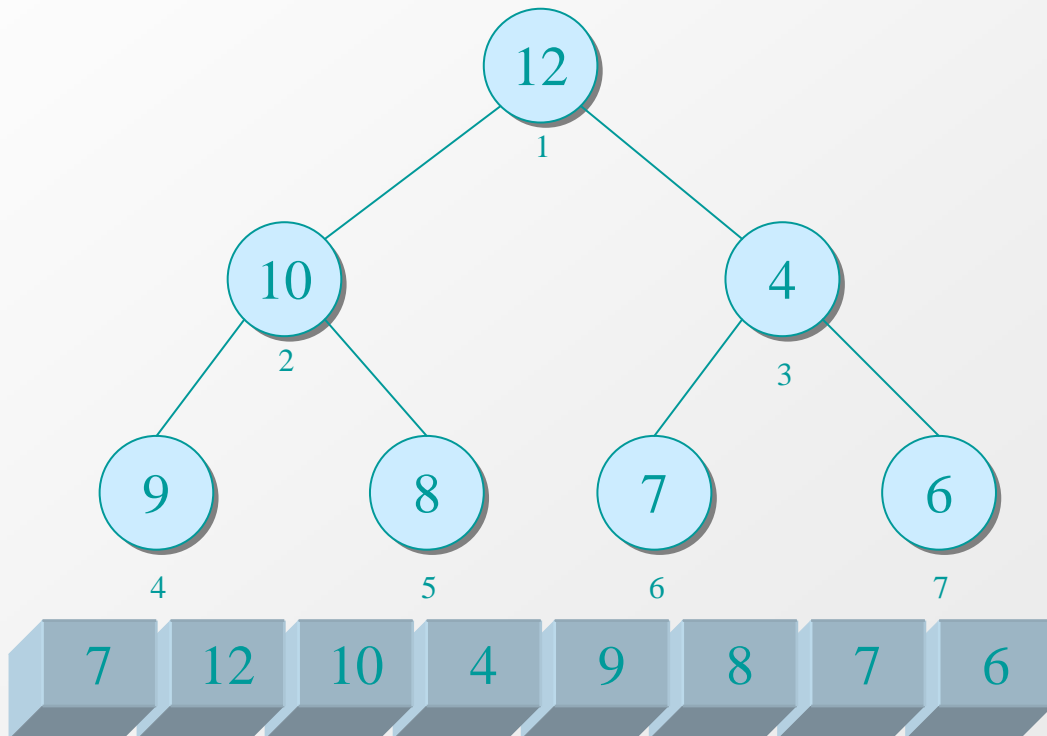
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- To build a heap
 - ◆ Call `pushdown()` on the head parent node



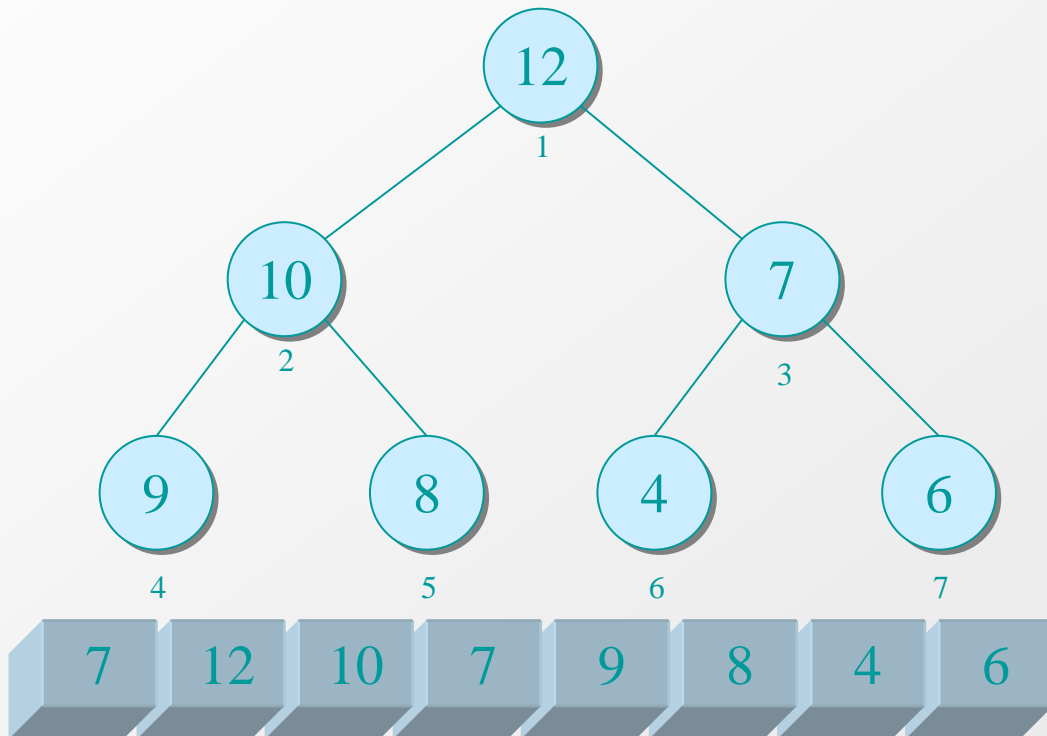
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Building the Heap

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Heap Sort

```
sub heap_sort {  
    my $heap = \@_  
    unshift @$heap, scalar @_  
    for (my $i = int($heap->[0] / 2); $i >= 1; $i--) {  
        pushdown($heap, $i);  
    }  
    for (my $i = $heap->[0]; $i >= 2; $i--) {  
        ($heap->[1], $heap->[$i]) = ($heap->[$i], $heap->[1]);  
        $heap->[0]--;  
        pushdown($heap, 1);  
    }  
    shift $heap;  
}
```

Heap Sort

- This appears to be as much time, and a lot more work than, say, an insertion sort
- The heap sort works in $N \cdot \log_2 N$
- $\log_2 N$ is roughly the height of the heap, so $N \cdot h$
- The height grows slowly: for $N = 1000$, h is 10
- So for larger arrays a heap sort is faster!

Other Algorithms

- The binary heap is an advanced data structure, there are others
 - ◆ Linked Lists, Circular Linked Lists, Doubly-Linked Lists
 - ◆ General heaps
 - ◆ Check out the `Heaps` module on CPAN
- Other Algorithms
 - ◆ Sets, Matrices
 - ◆ Graphs
 - ◆ Strings
 - ◆ Statistics, Numerical Analysis
 - ◆ Number Theory, Cryptography

Homework Ten

- Rewrite the `pushdown()` routine to use recursion rather than a while loop to push an element down the heap. Will this help or hurt the running time of the heap sort or make little difference?