Examples

January 4, 2026

[13]:
```
include("notebook_init.jl")
```

This notebook generates the figures in the paper *Complexity of Projected Gradient Methods for Strongly Convex Optimization with H{"o}lder Continuous Gradient Terms* by X. Chen, C. T. Kelley, and L. Wang

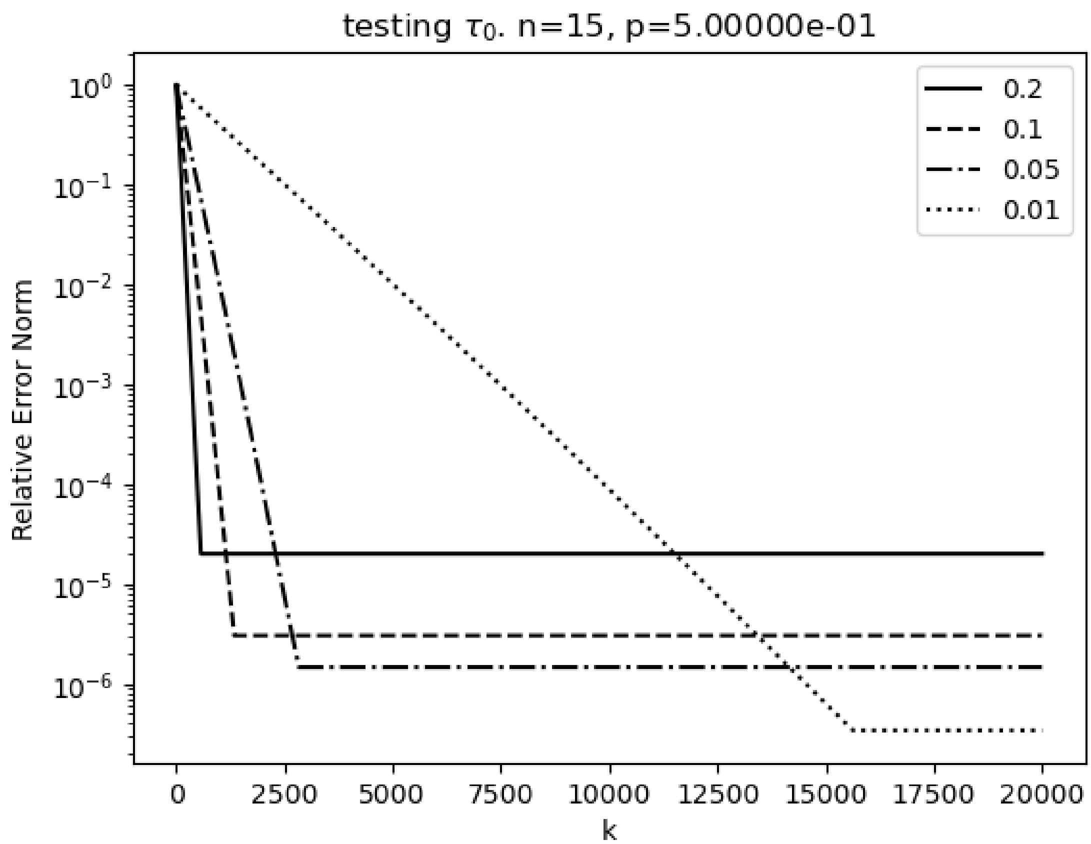
Example 1

Generate Figures 1(a), 1(b), 2(a), and 2(b). Theses figures are for Algorithm 1. Figures 1(a) and 2(a) compare various stepsizes $\tau = \tau_0 h^2$ which are consistent with the CFL condition. Figures 1(b) and 2(b) examine values of the exponent p.

The files for building the figures are in /**src**/**Figures**. The code is **Figures_Alg1.jl** and the functions **Figure1_2a** and **Figure1_2b**. The functions take the dimension as an argument.

[14]:
```
Figure1_2a(15);
```

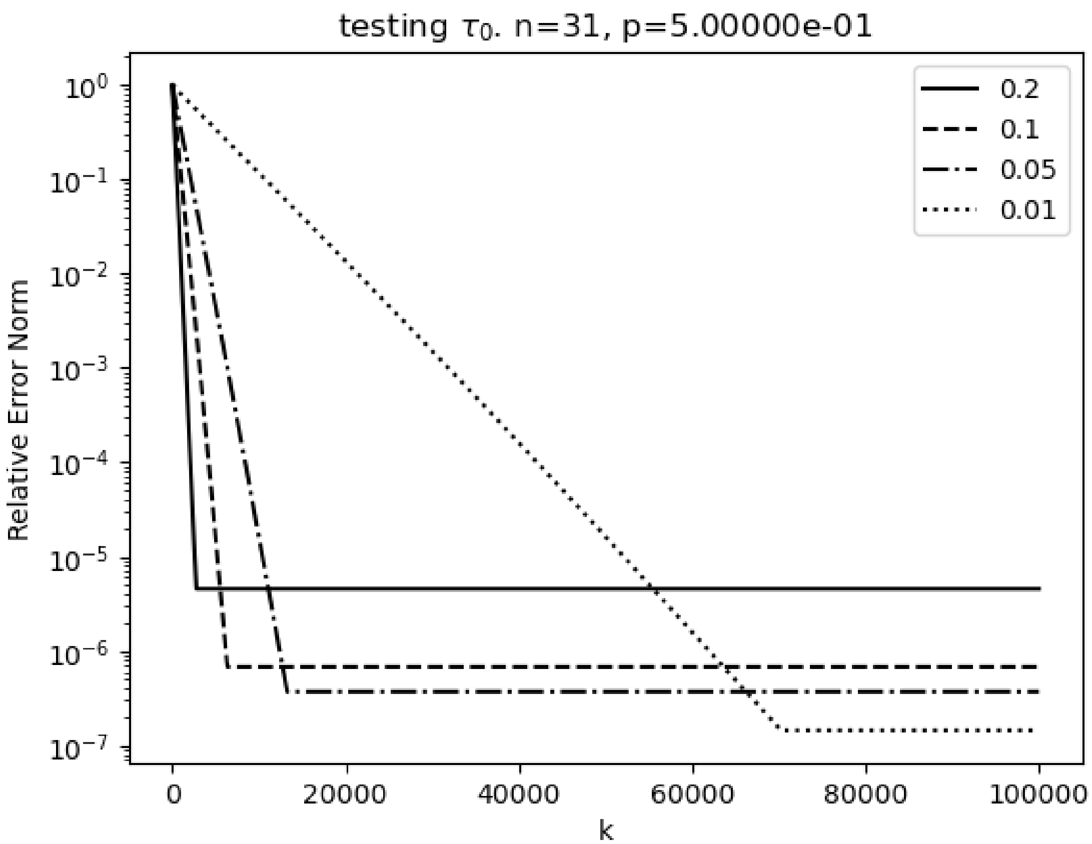
```
testing $\tau_0$. n=15, p=5.00000e-01
```



We run this again with a problem size of 31x31. This requiers smaller stepsizes ans the Lipschitz constant increases by a factor of four.

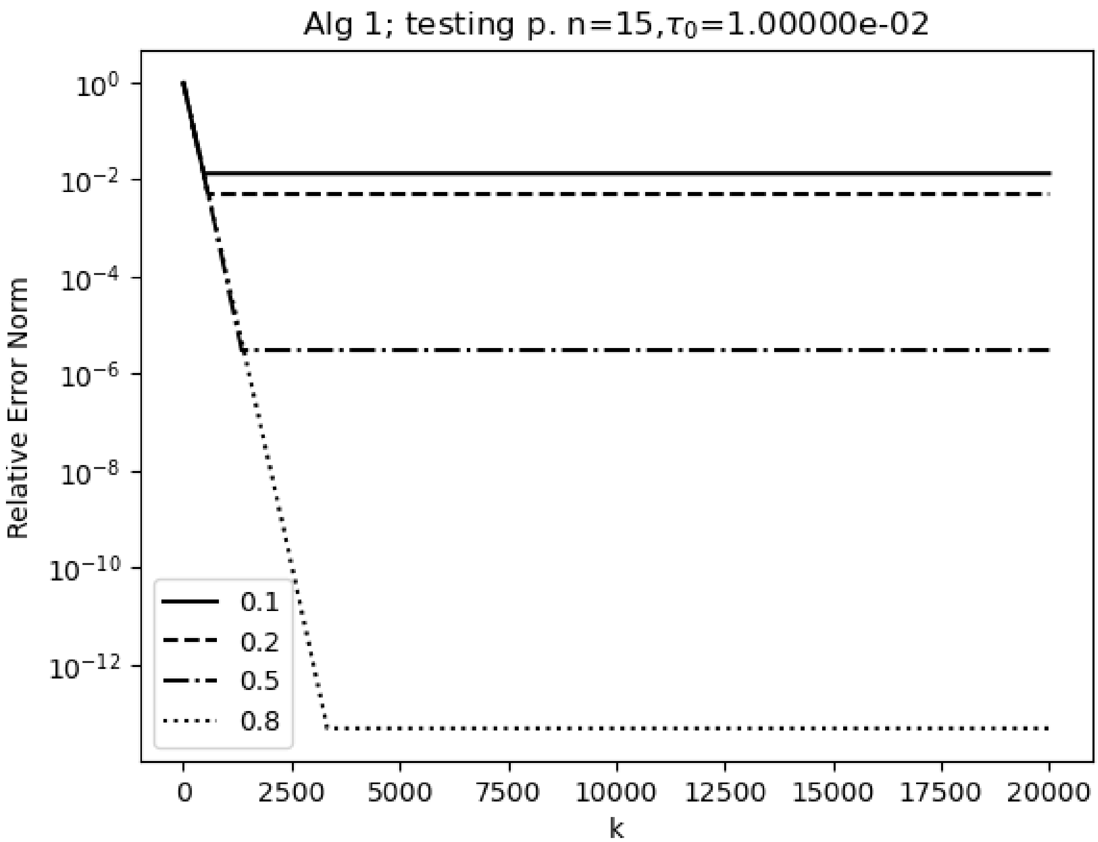
[15]:
```
Figure1_2a(31);
```

```
testing $\tau_0$. n=31, p=5.00000e-01
```



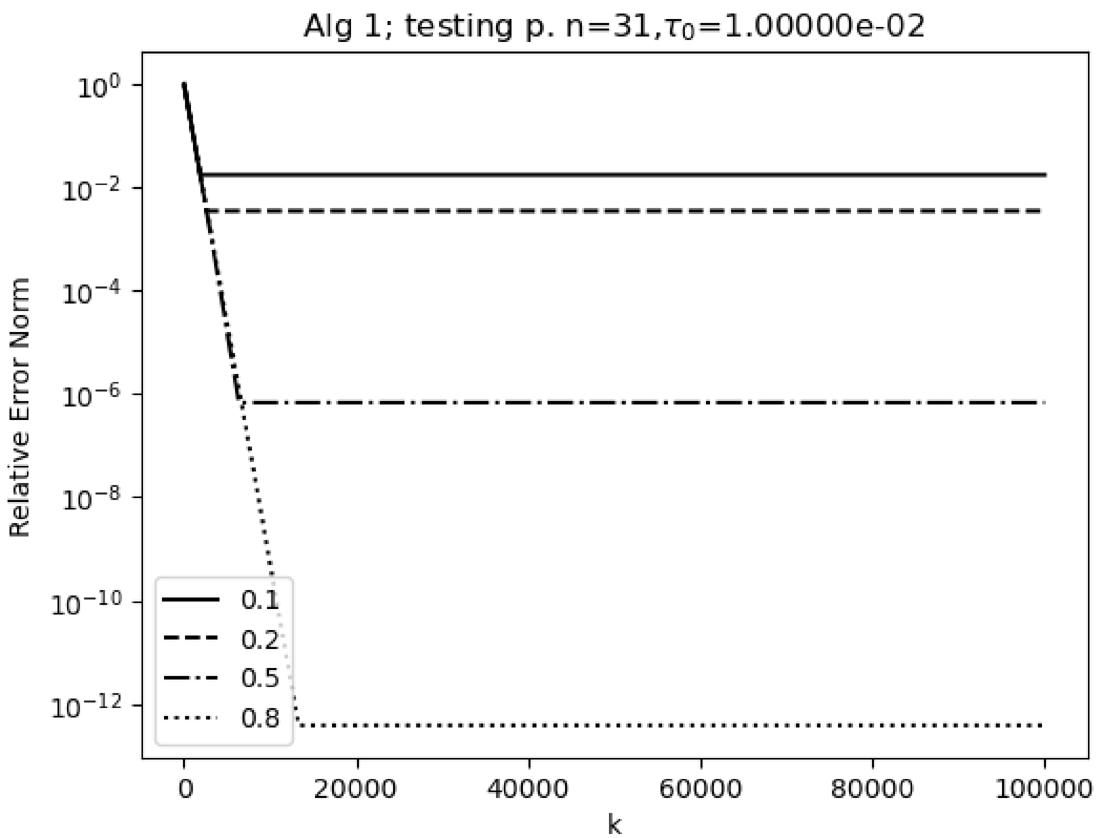
Now we compare the effects of changing the exponent p.

[16]:
```
Figure1_2b(15);
```



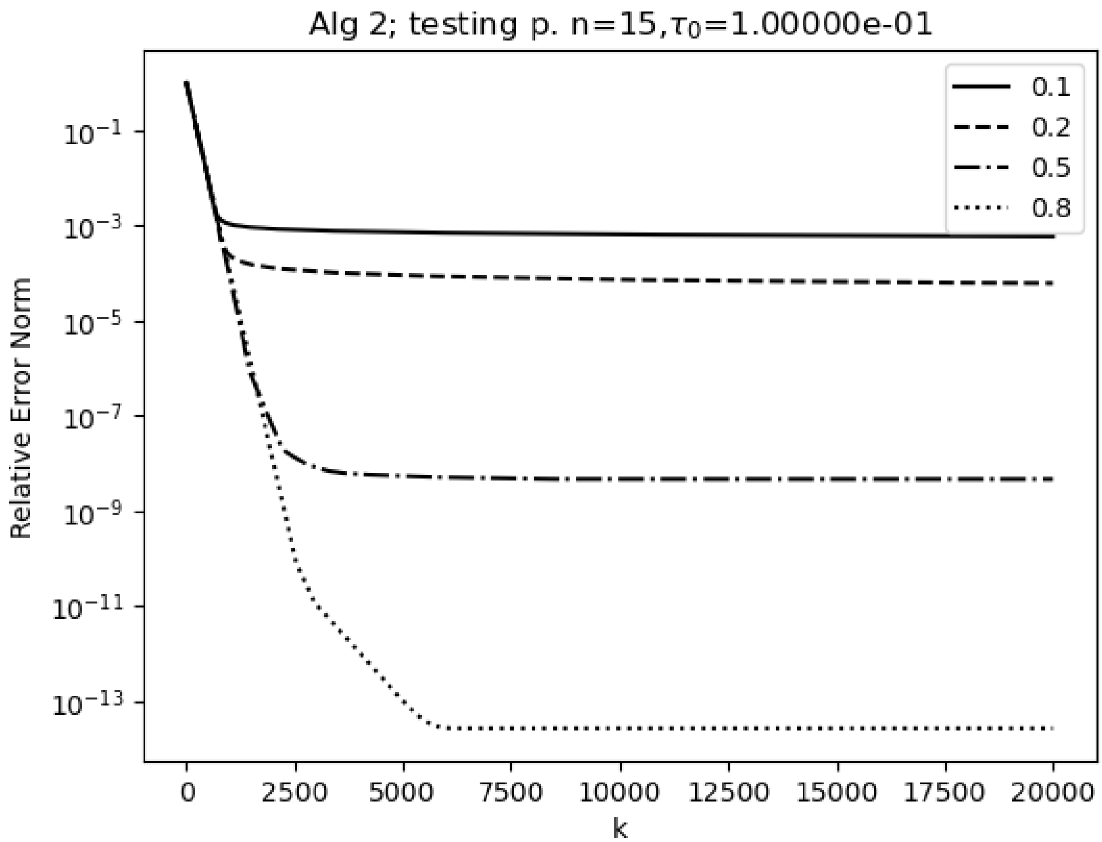
And finally repeat the computation for a 31x31 grid. This will complete the computations for Example 1 + Algorithm 1.

[17]:
```
Figure1_2b(31);
```



The next example is Figure 3, which tests Algorithm 2. We start with $\tau_0 = 1$ and let the line search work.

[18]:
```
Figure3(15);
```

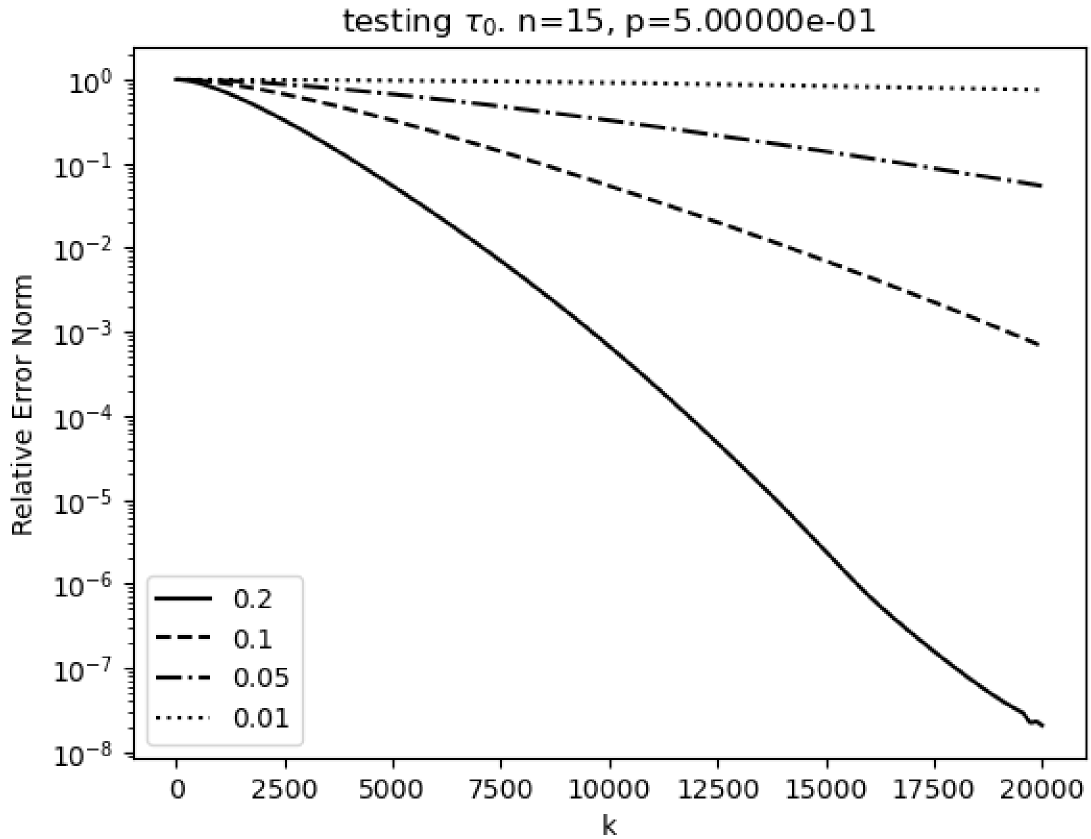


The advantage of the line search is that one does not have to manually adjust τ_0.

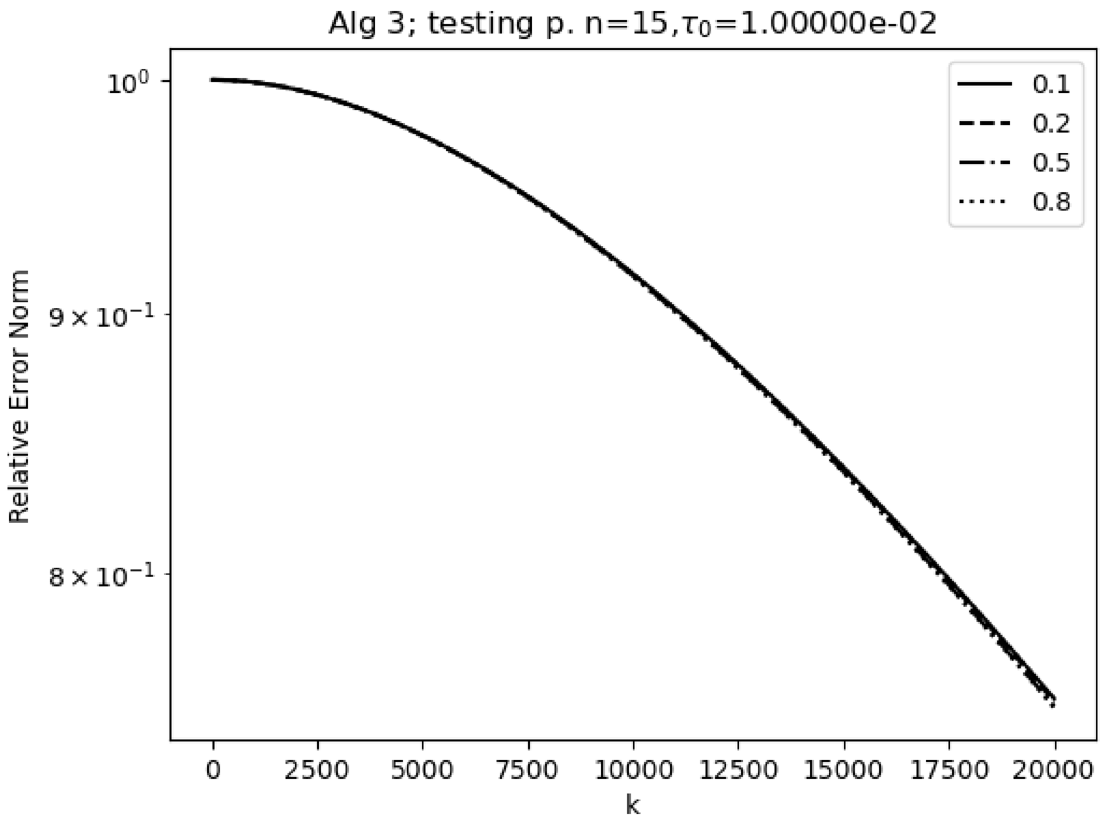
The results for Algoirthm 3 are in Figures 4 and 5. We set $\nu = \tau_0 h^2$ in these examples and will need to modify that to use the estimate in Remark 4.2. In the first two figures 4(a) and 4(b) use use the values of τ_0 we use in Figure 1.

[19]:
```
Figure4_5a(15);
```

```
testing $\tau_0$. n=15, p=5.00000e-01
```



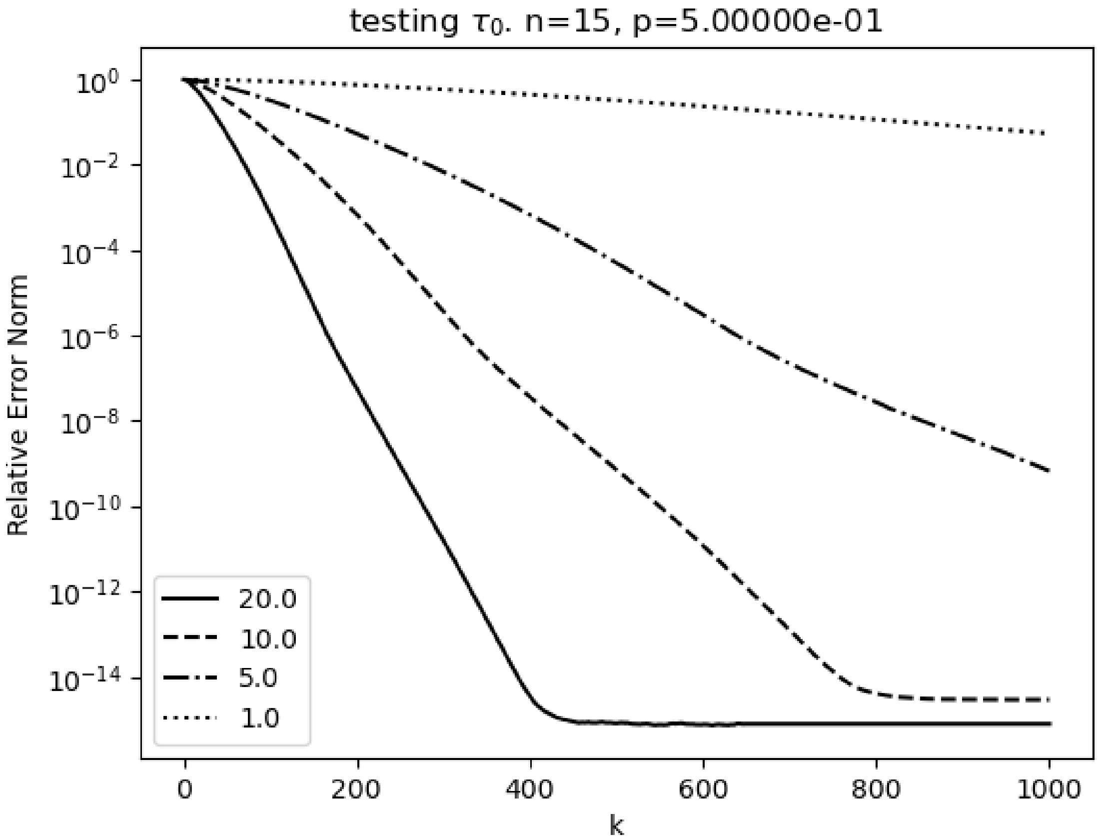
[20]:
```
Figure4_5b(15);
```



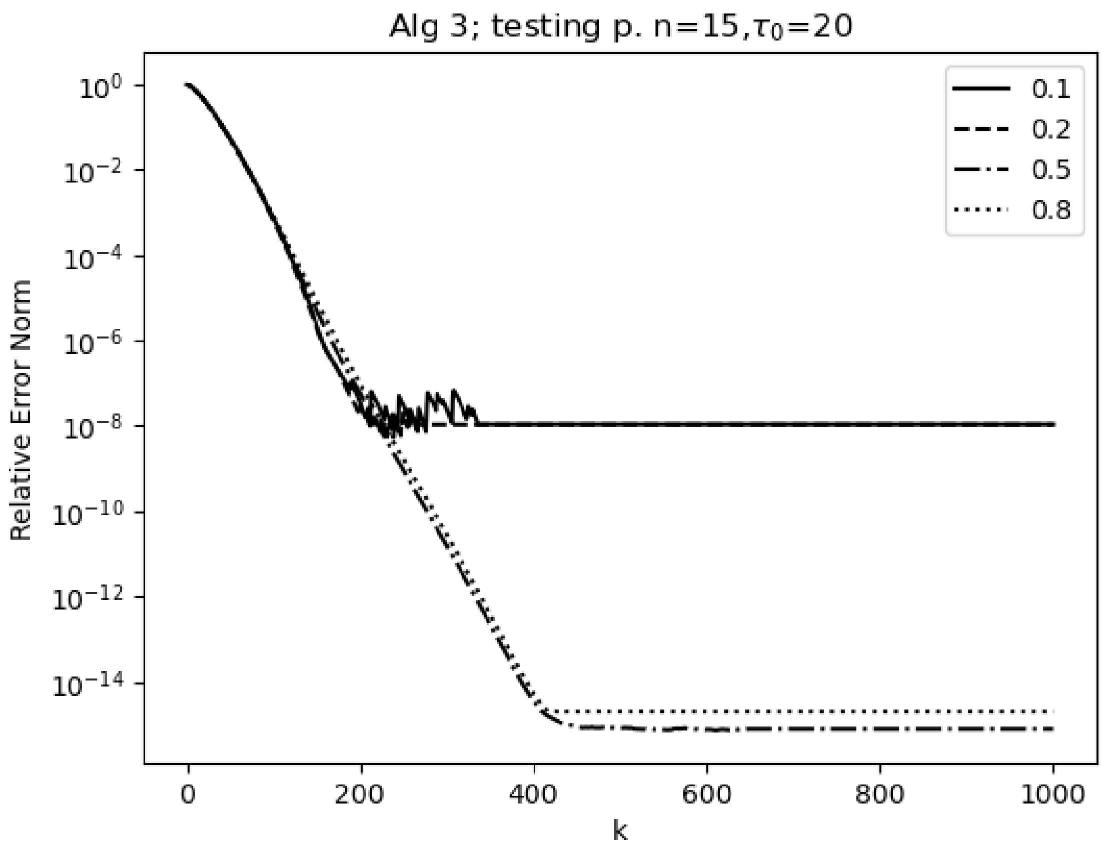
Now we use the larger values of τ_0.

[21]:
```
Figure4_5a(15; maxit=1000, tauvec=[20.0, 10.0, 5.0, 1.0]);
```

```
testing $\tau_0$. n=15, p=5.00000e-01
```

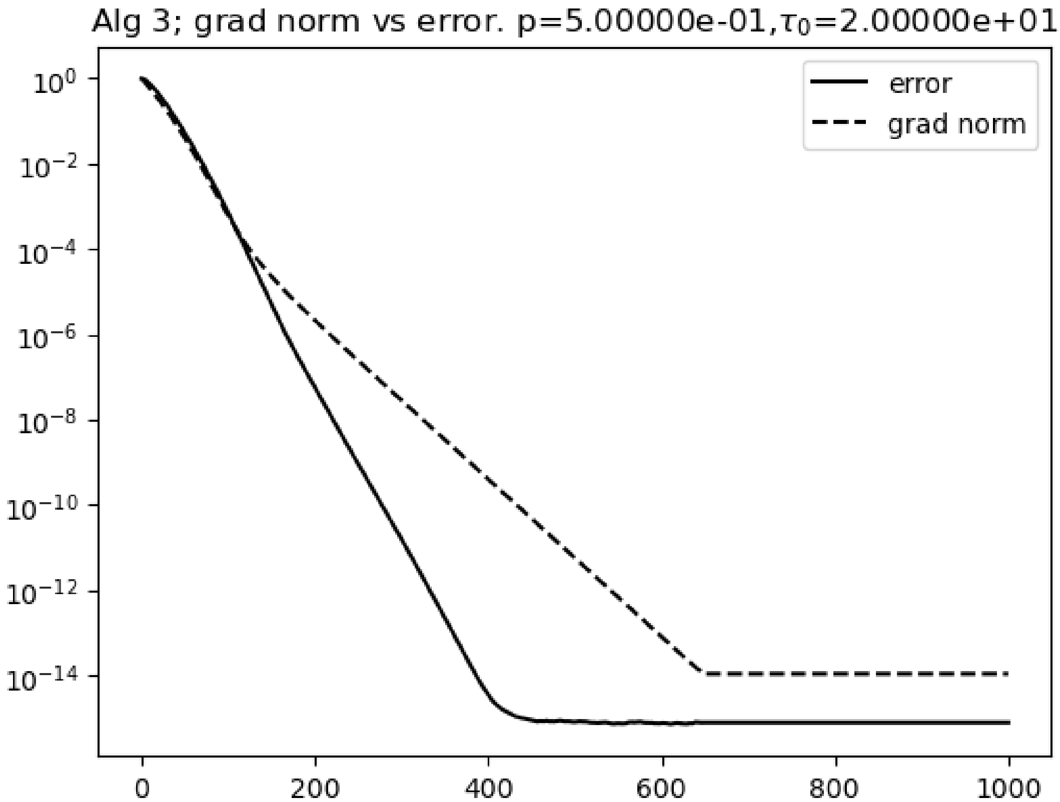



```
[22]: Figure4_5b(15; maxit=1000, tau0=20);
```



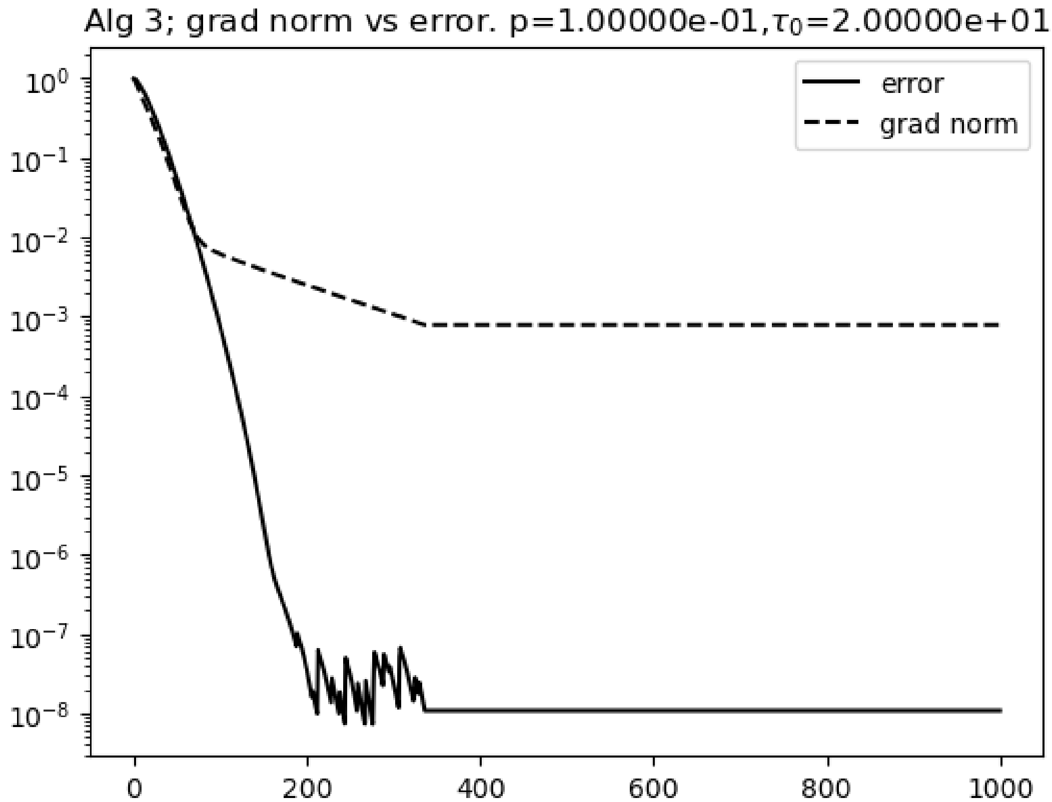
Figures 6ab compare the gradient norm to the error norm for p=.5 and p=.01.

[23]:
```
Figure6ab(15);
```



[24]:

```
Figure6ab(15; p=.1);
```



Alg 3; grad norm vs error. p=1.00000e-01,τ_0=2.00000e+01
error
grad norm
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
0
200
400
600
800
1000


```
[ ]:
```