

Examples

January 7, 2026

```
[16]: include("notebook_init.jl")
```

This notebook generates the figures in the paper *Complexity of Projected Gradient Methods for Strongly Convex Optimization with Hölder Continuous Gradient Terms* by X. Chen, C. T. Kelley, and L. Wang

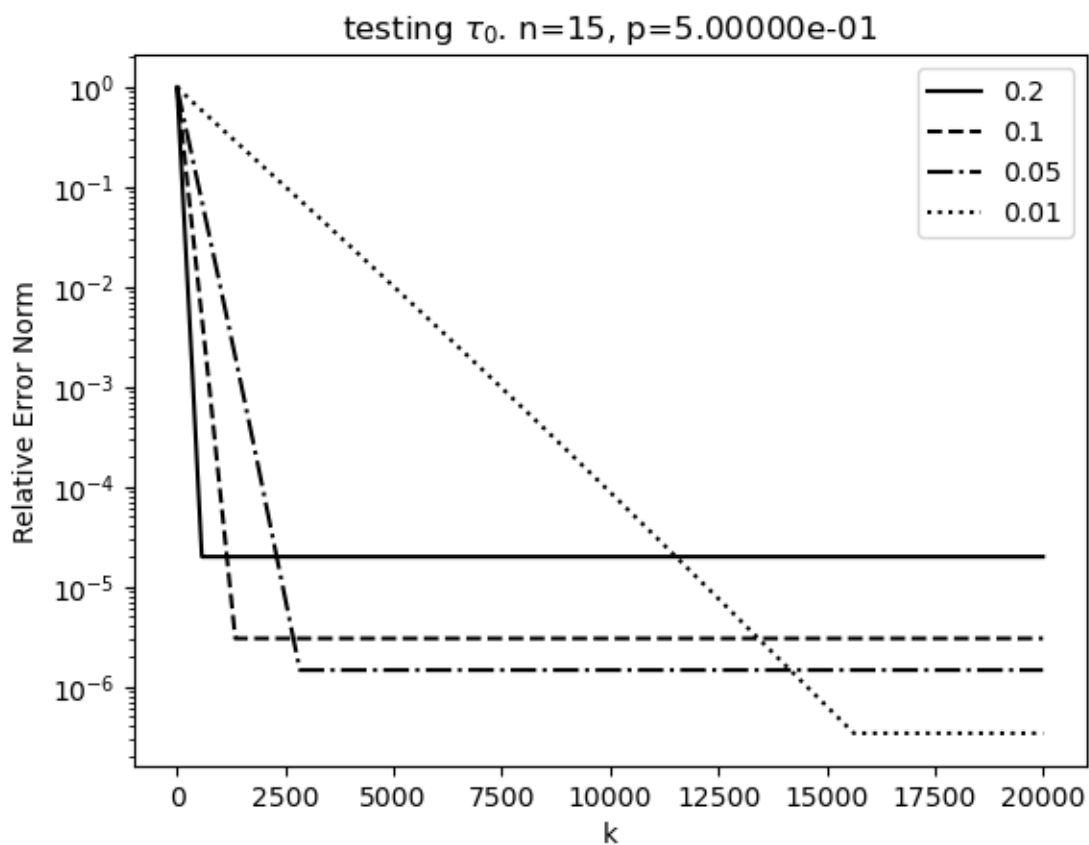
Example 1

Generate Figures 1(a), 1(b), 2(a), and 2(b). These figures are for Algorithm 1. Figures 1(a) and 2(a) compare various stepsizes $\tau = \tau_0 h^2$ which are consistent with the CFL condition. Figures 1(b) and 2(b) examine values of the exponent p .

The files for building the figures are in `/src/Figures`. The code is `Figures_Alg1.jl` and the functions `Figure1_2a` and `Figure1_2b`. The functions take the dimension as an argument.

```
[17]: Figure1_2a(15);
```

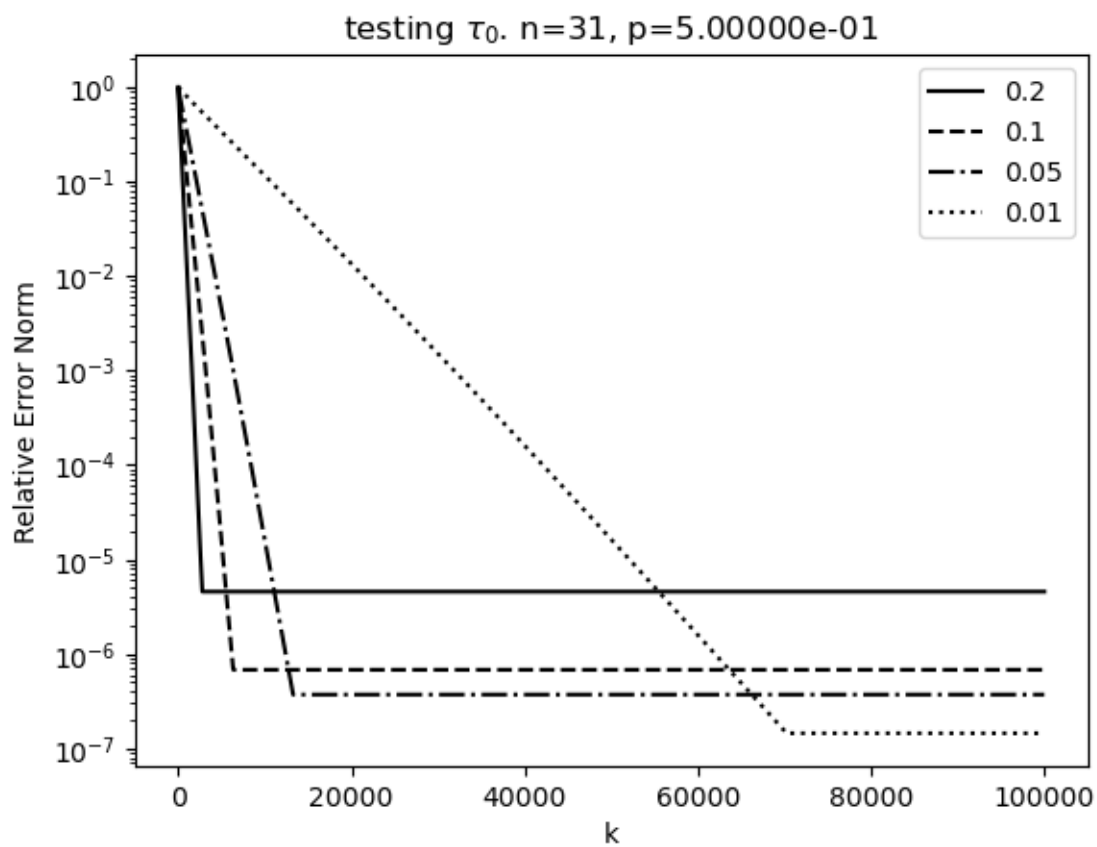
```
testing $\tau_0$. n=15, p=5.00000e-01
```



We run this again with a problem size of 31x31. This requires smaller stepsizes and the Lipschitz constant increases by a factor of four.

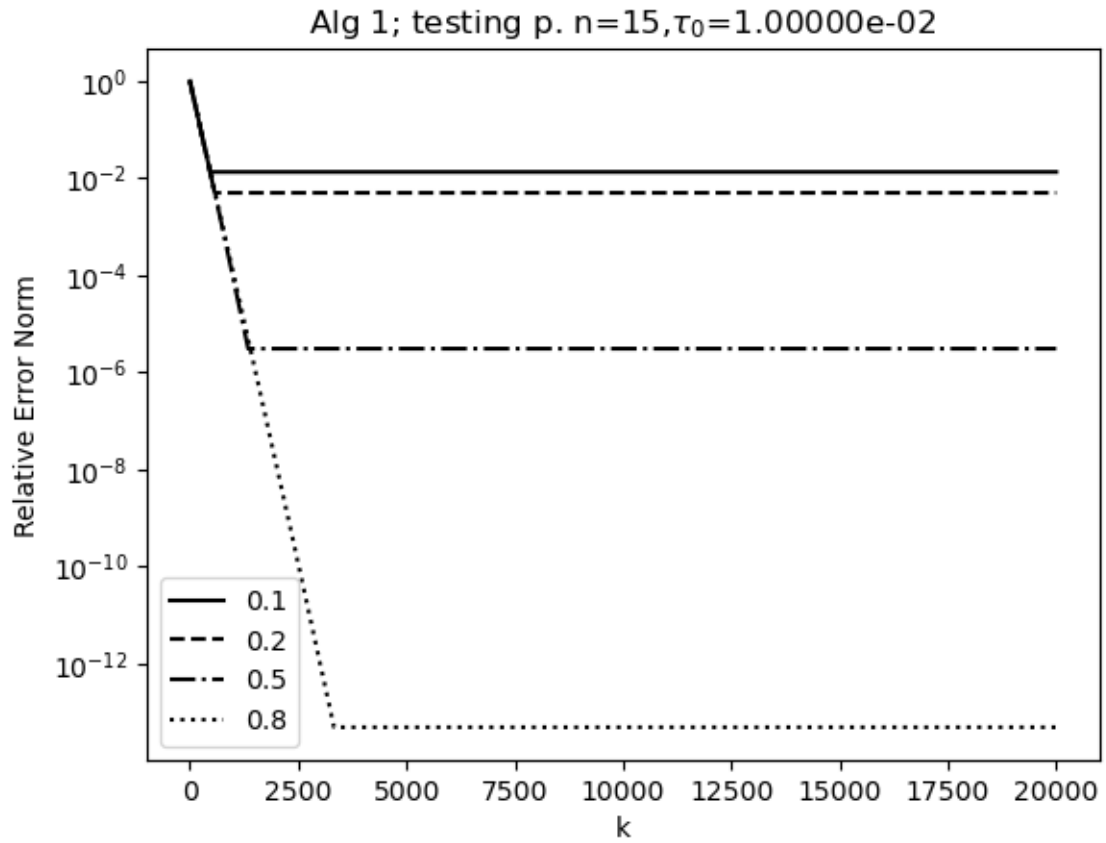
[18]: Figure1_2a(31);

testing τ_0 . $n=31$, $p=5.00000e-01$



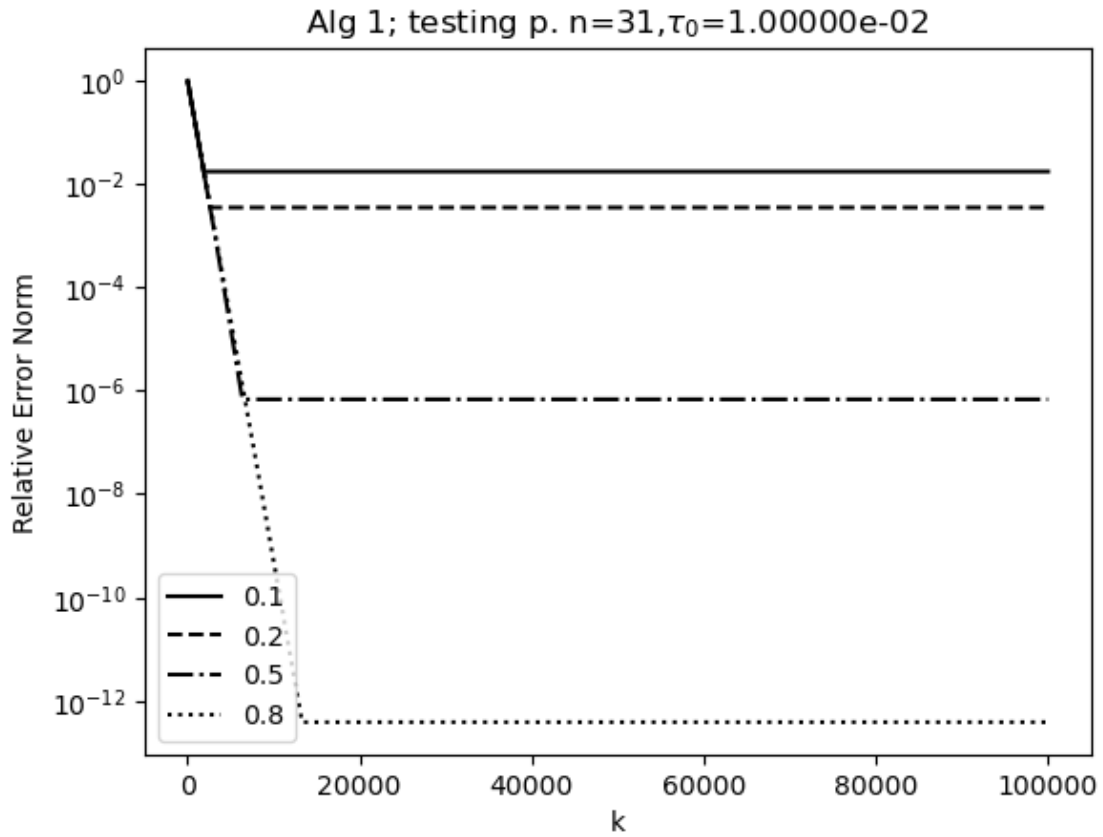
Now we compare the effects of changing the exponent p .

[19]: Figure1_2b(15);



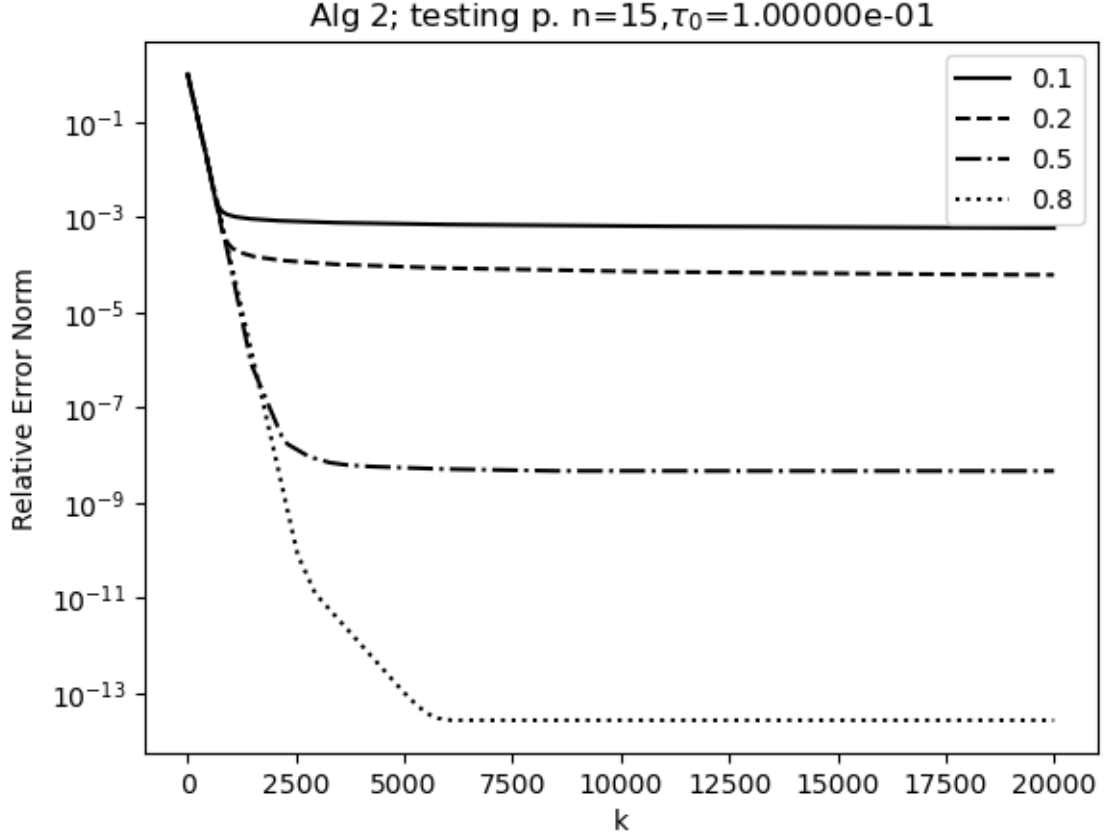
And finally repeat the computation for a 31x31 grid. This will complete the computations for Example 1 + Algorithm 1.

[20]: `Figure1_2b(31);`



The next example is Figure 3, which tests Algorithm 2. We start with $\tau_0 = 1$ and let the line search work.

[21]: `Figure3(15);`

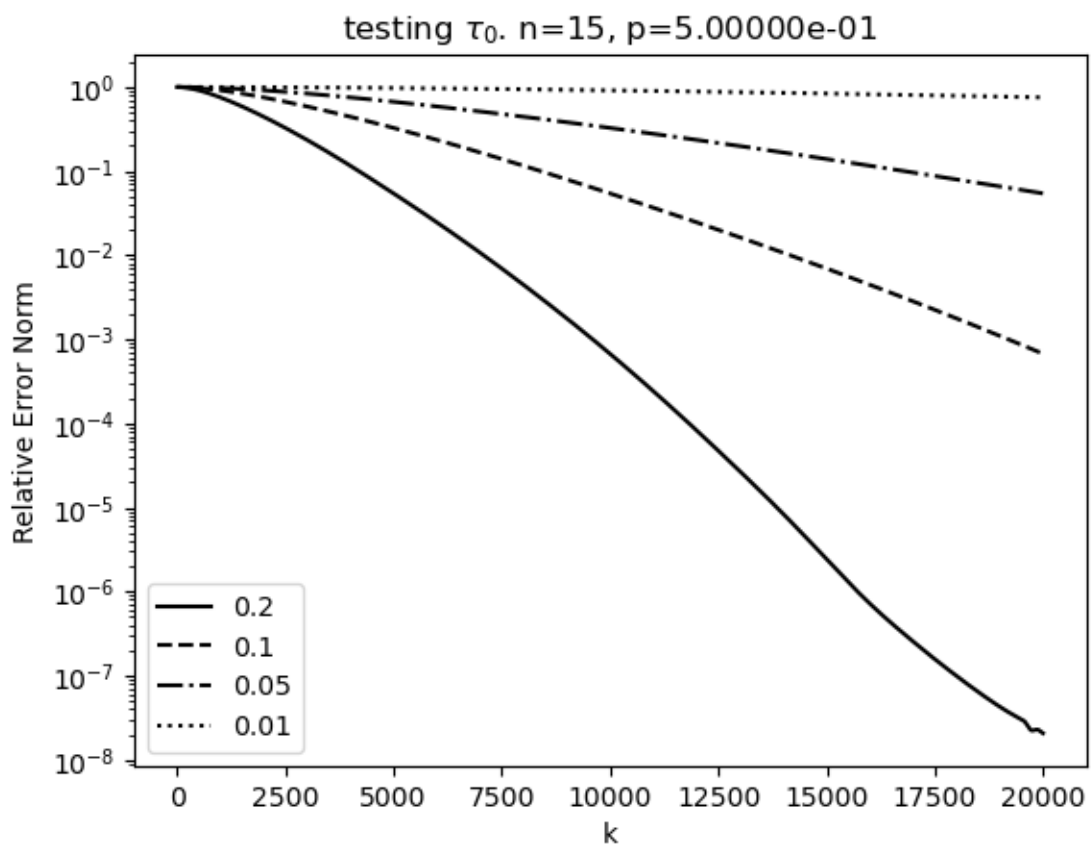


The advantage of the line search is that one does not have to manually adjust τ_0 .

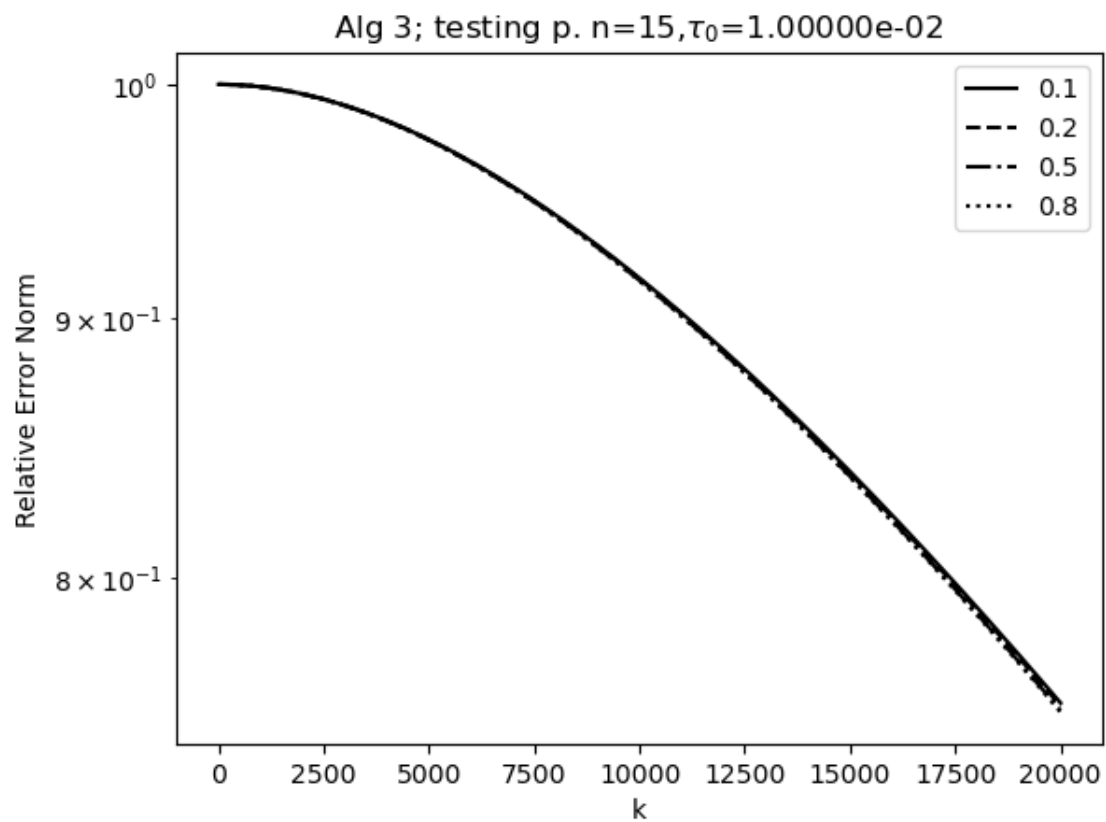
The results for Algorithm 3 are in Figures 4 and 5. We set $\nu = \tau_0 h^2$ in these examples and will need to modify that to use the estimate in Remark 4.2. In the first two figures 4(a) and 4(b) use the values of τ_0 we use in Figure 1.

[22]: `Figure4_5a(15);`

testing τ_0 . n=15, p=5.00000e-01



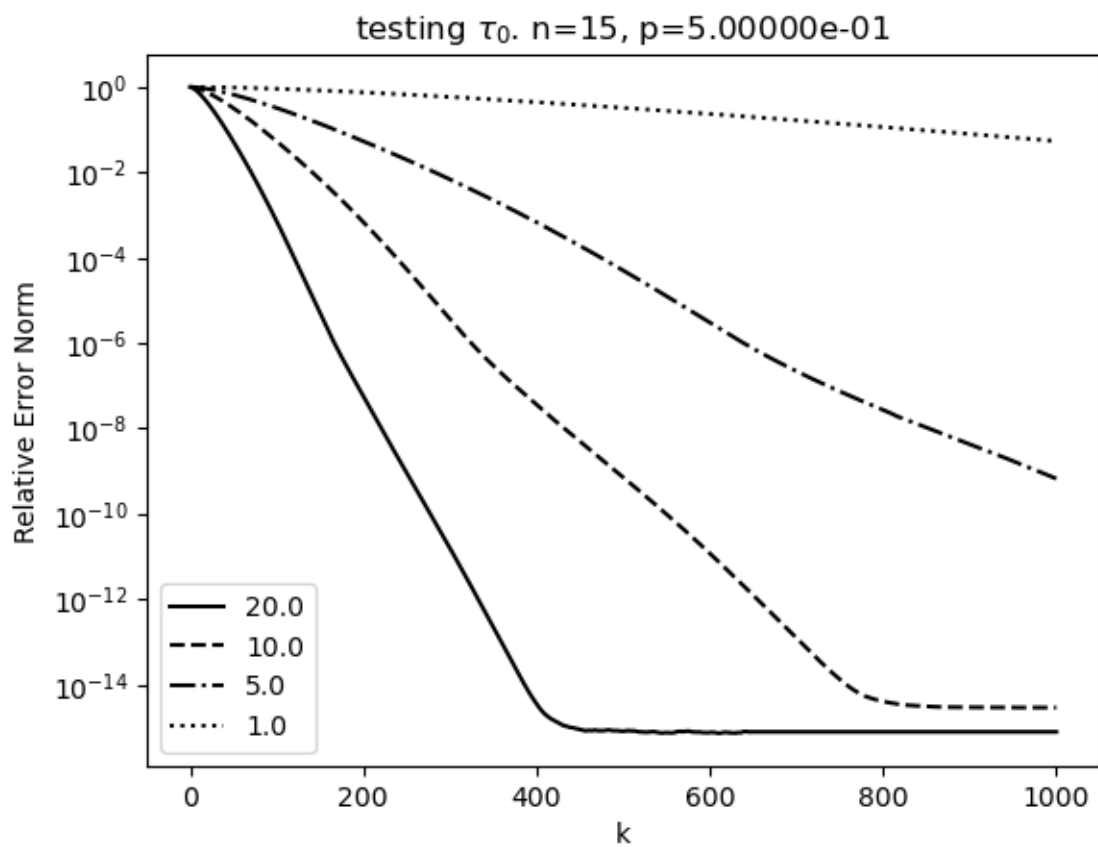
[23] : Figure4_5b(15);



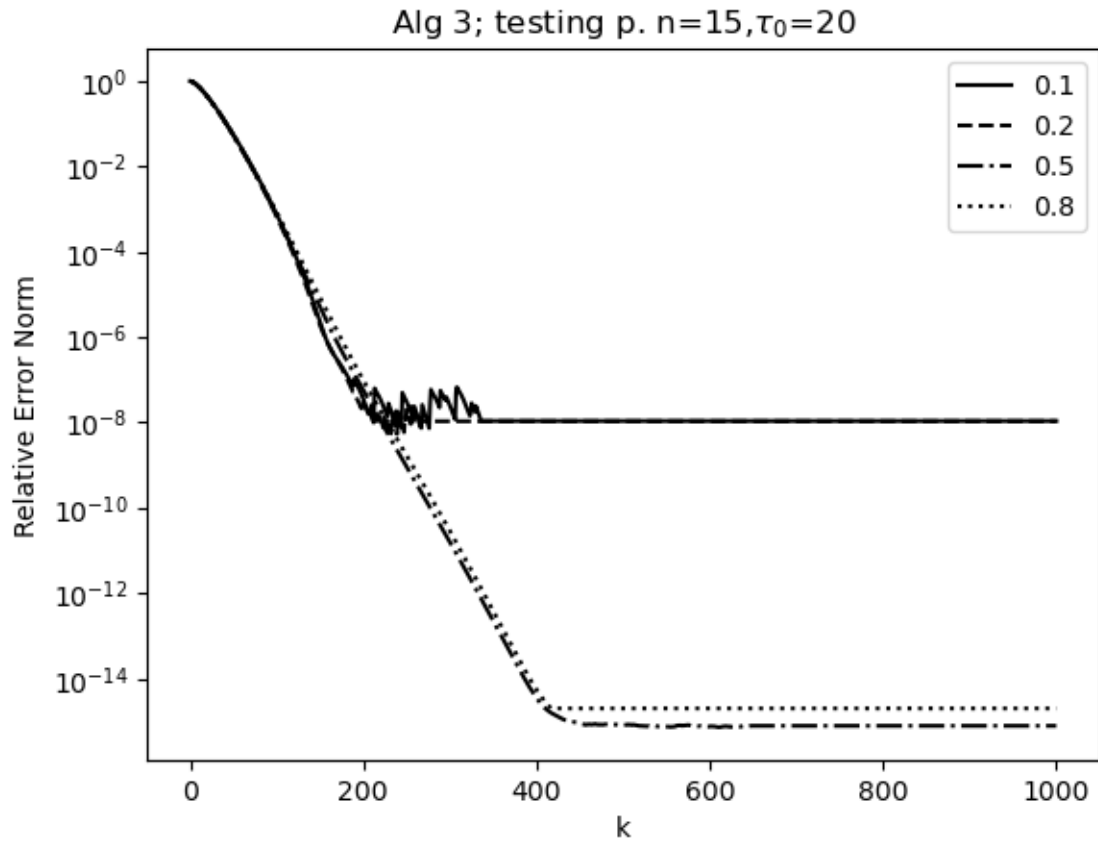
Now we use the larger values of τ_0 .

[24]: `Figure4_5a(15; maxit=1000, tauvec=[20.0, 10.0, 5.0, 1.0]);`

testing τ_0 . $n=15, p=5.00000e-01$

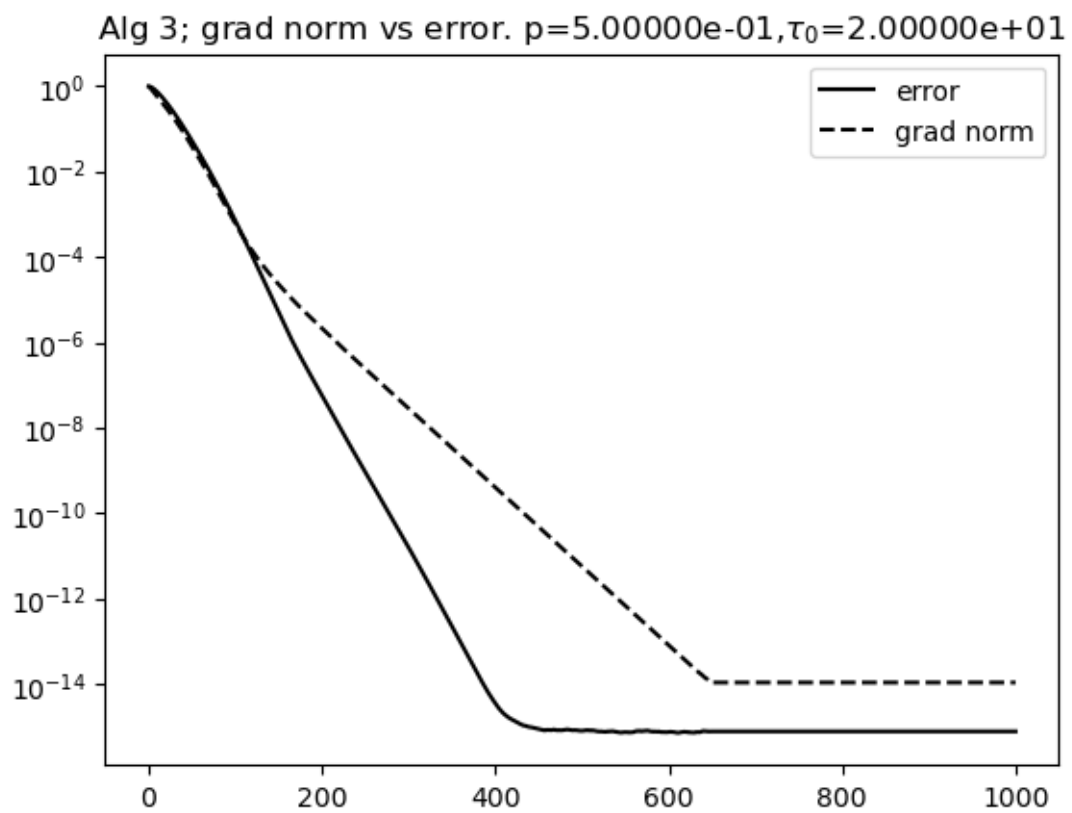


[25]: `Figure4_5b(15; maxit=1000, tau0=20);`

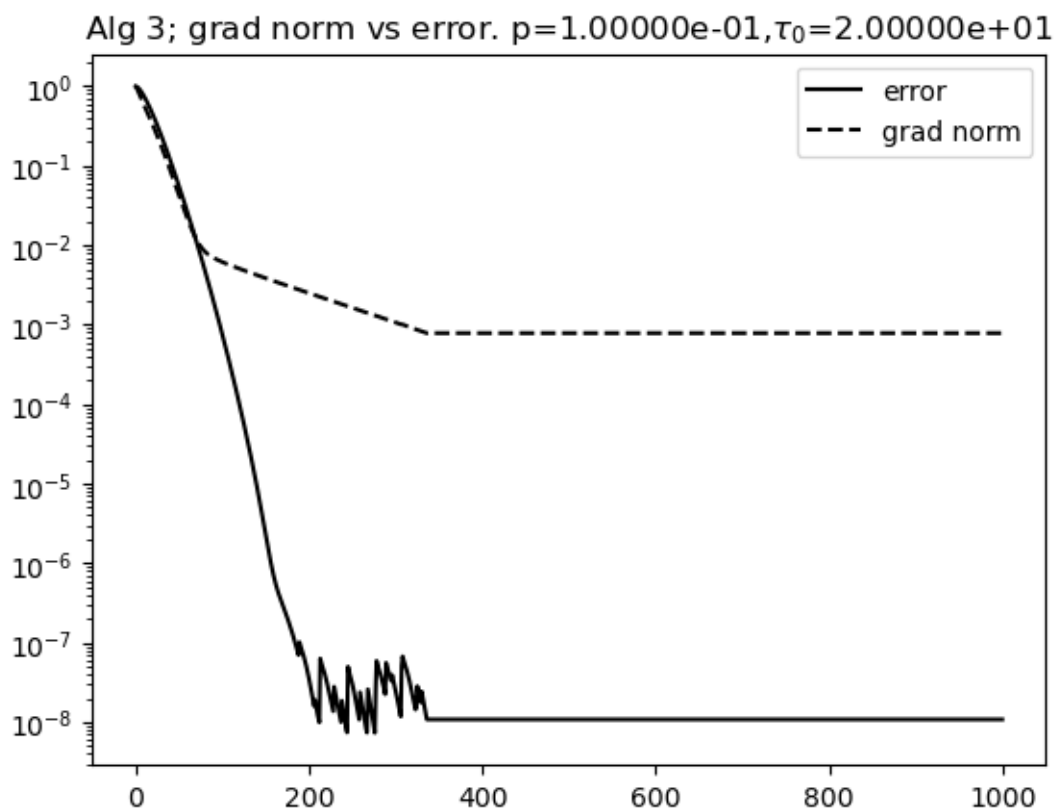


Figures 6ab compare the gradient norm to the error norm for $p=.5$ and $p=.01$.

[26]: `Figure6ab(15);`

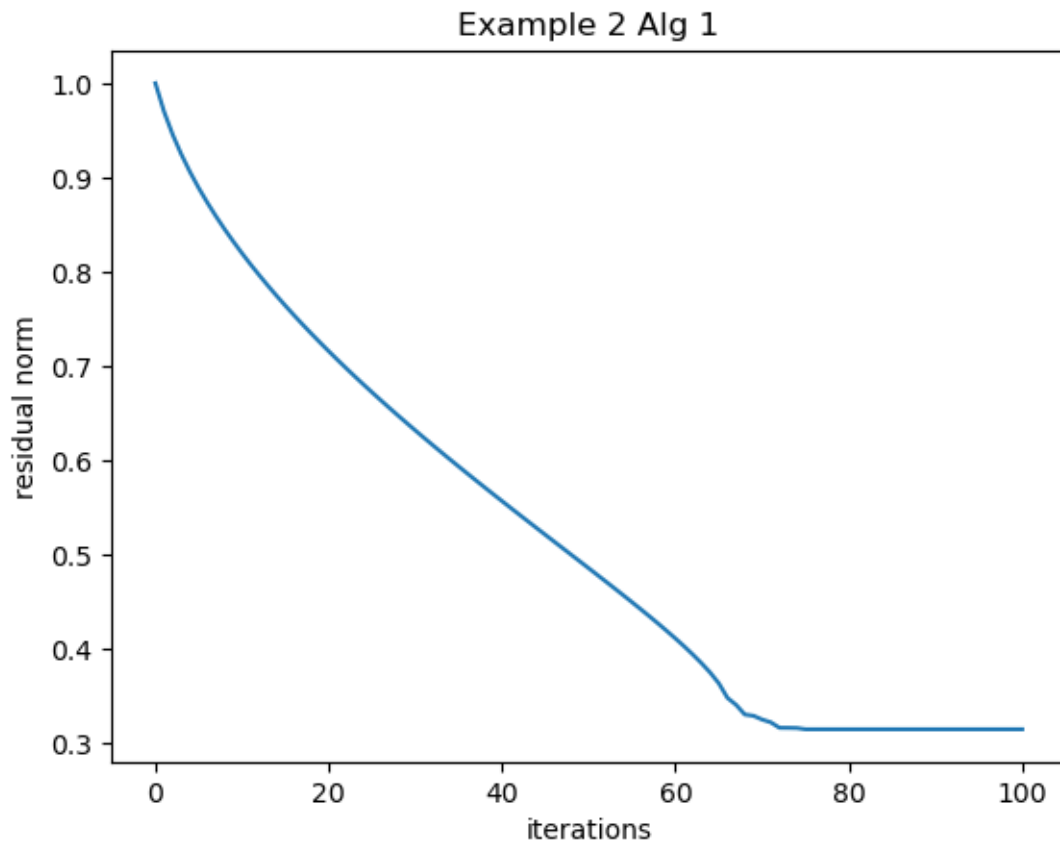


[27]: `Figure6ab(15; p=.1);`

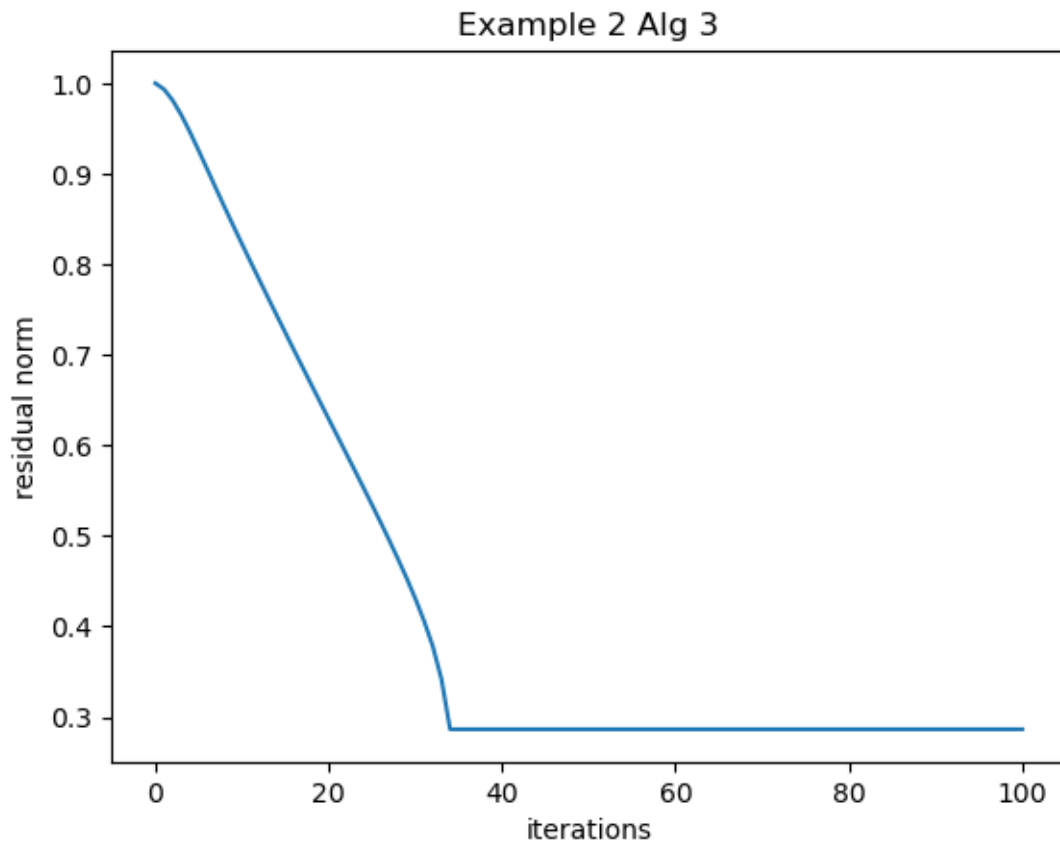


Here are the two figures for example 2

[28]: `Example2a();`



[29]: `Example2b();`



[]: