

Examples

January 7, 2026

```
[16]: include("notebook_init.jl")
```

This notebook generates the figures in the paper *Complexity of Projected Gradient Methods for Strongly Convex Optimization with Hölder Continuous Gradient Terms* by X. Chen, C. T. Kelley, and L. Wang

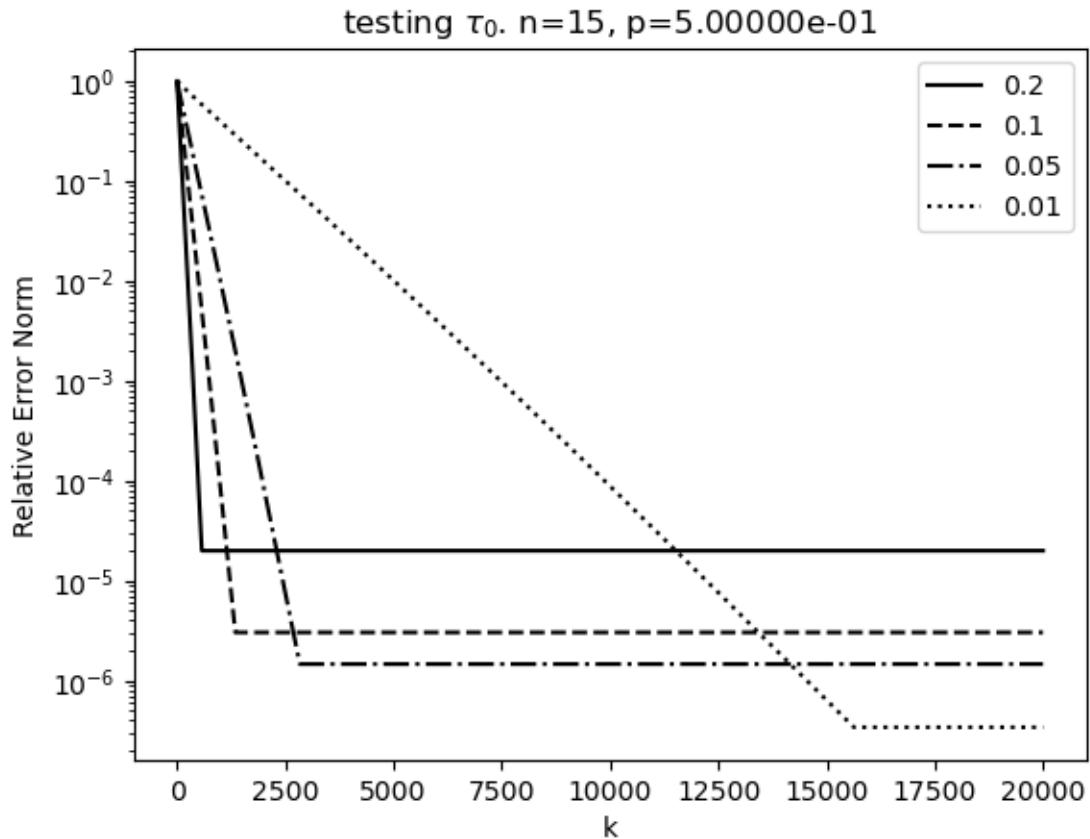
Example 1

Generate Figures 1(a), 1(b), 2(a), and 2(b). Theses figures are for Algorithm 1. Figures 1(a) and 2(a) compare various stepsizes $\tau = \tau_0 h^2$ which are consistent with the CFL condition. Figures 1(b) and 2(b) examine values of the exponent p .

The files for building the figures are in `/src/Figures`. The code is **Figures_Alg1.jl** and the functions **Figure1_2a** and **Figure1_2b**. The functions take the dimension as an argument.

```
[17]: Figure1_2a(15);
```

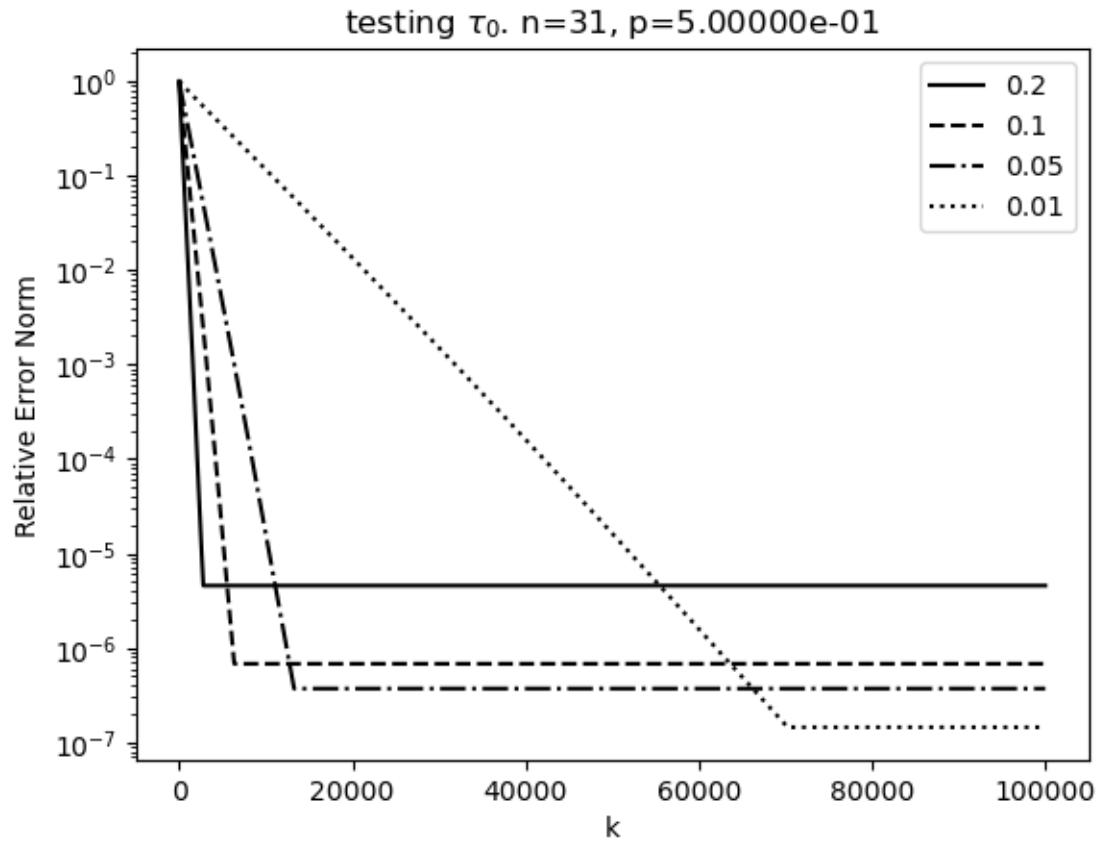
```
testing $\tau_0$. n=15, p=5.00000e-01
```



We run this again with a problem size of 31x31. This requiers smaller stepsizes ans the Lipschitz constant increases by a factor of four.

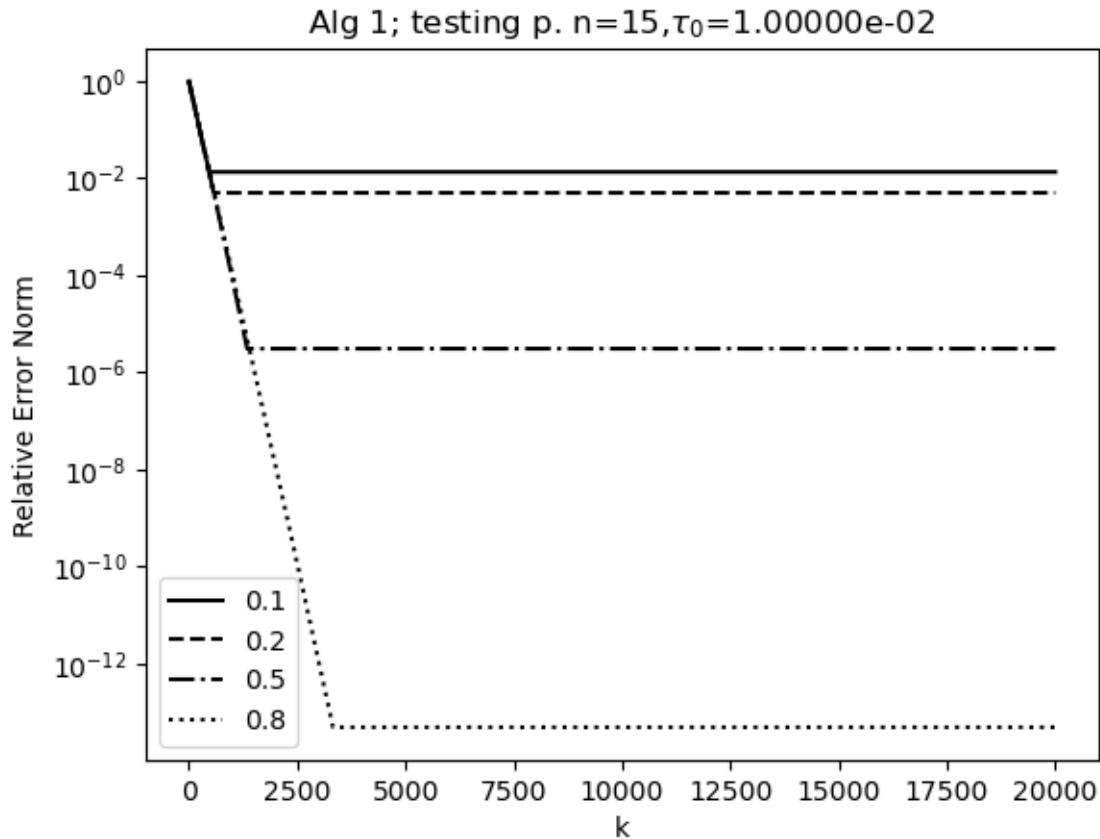
[18]: Figure1_2a(31);

```
testing $\tau_0$. n=31, p=5.00000e-01
```



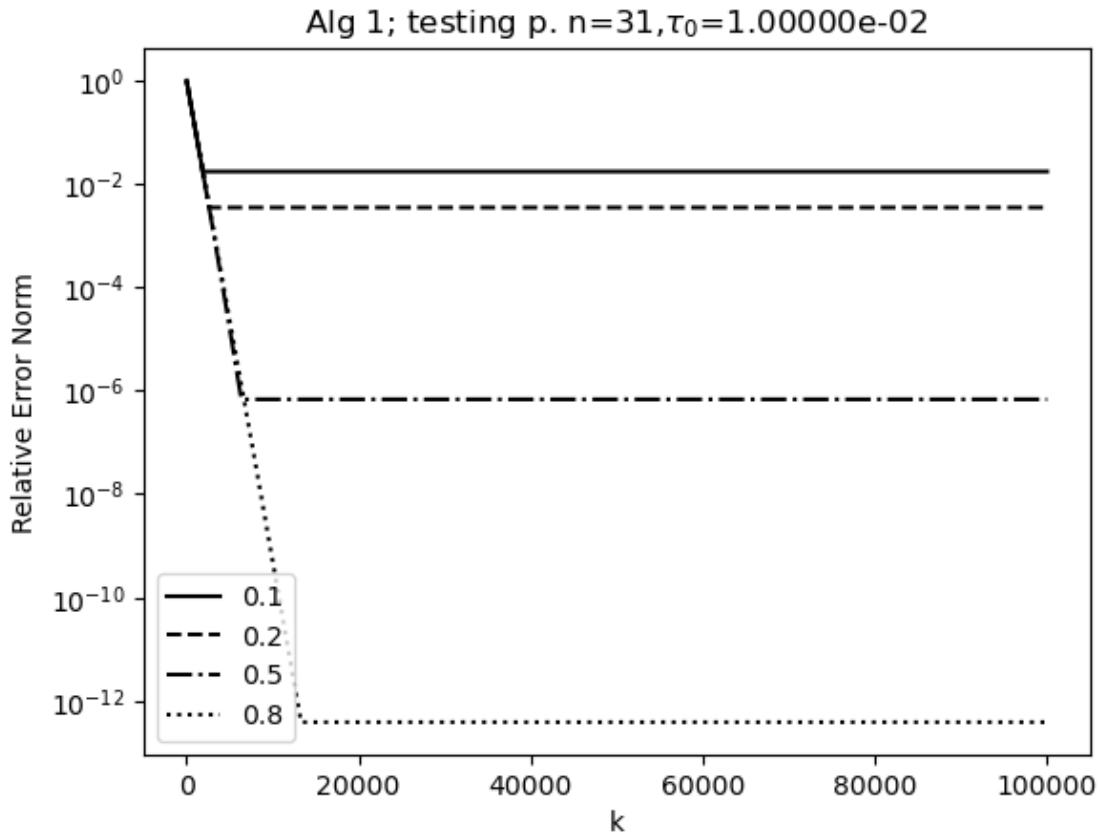
Now we compare the effects of changing the exponent p .

[19]: Figure1_2b(15);



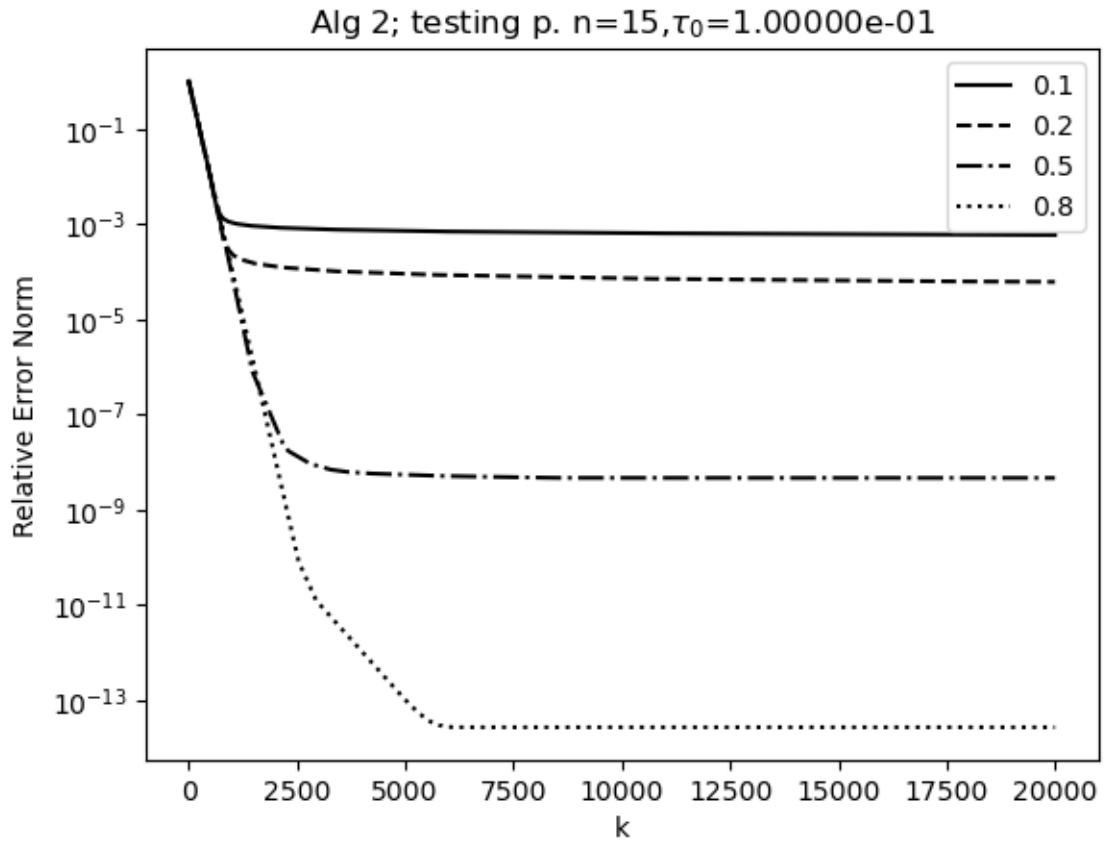
And finally repeat the computation for a 31x31 grid. This will complete the computations for Example 1 + Algorithm 1.

[20] : Figure1_2b(31);



The next example is Figure 3, which tests Algorithm 2. We start with $\tau_0 = 1$ and let the line search work.

[21]: Figure3(15);

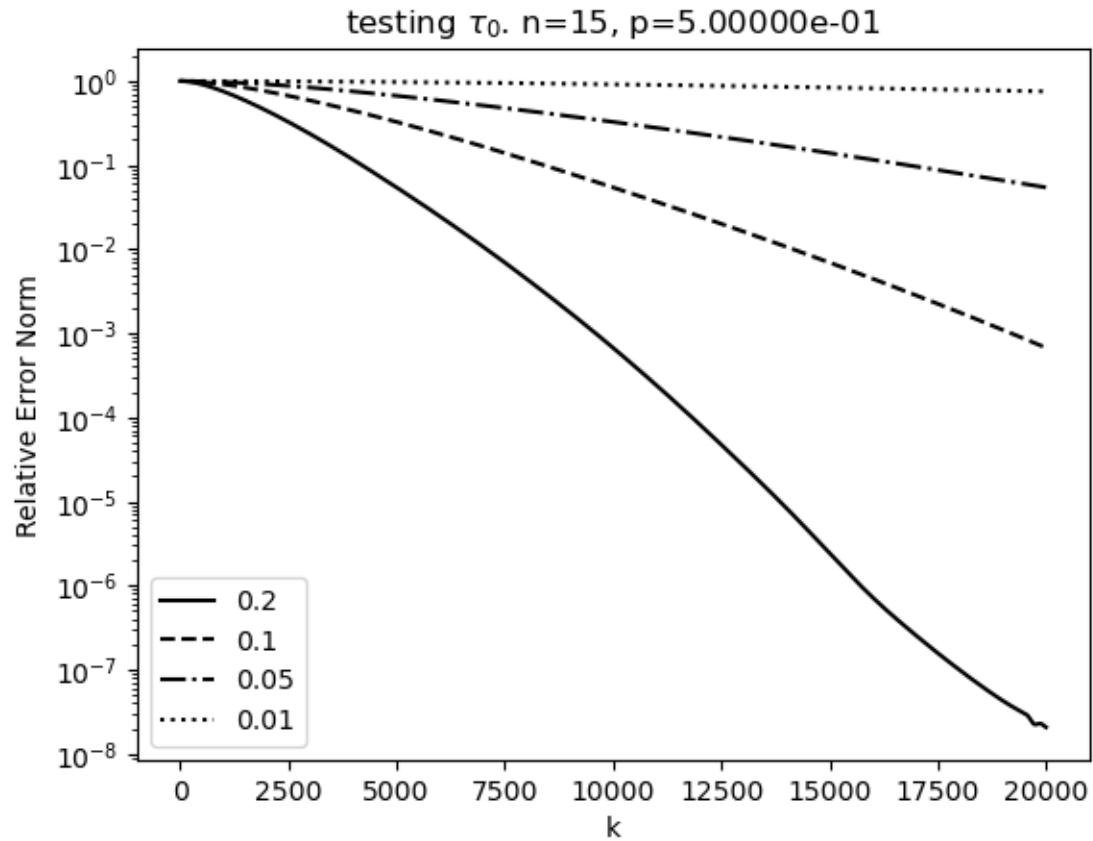


The advantage of the line search is that one does not have to manually adjust τ_0 .

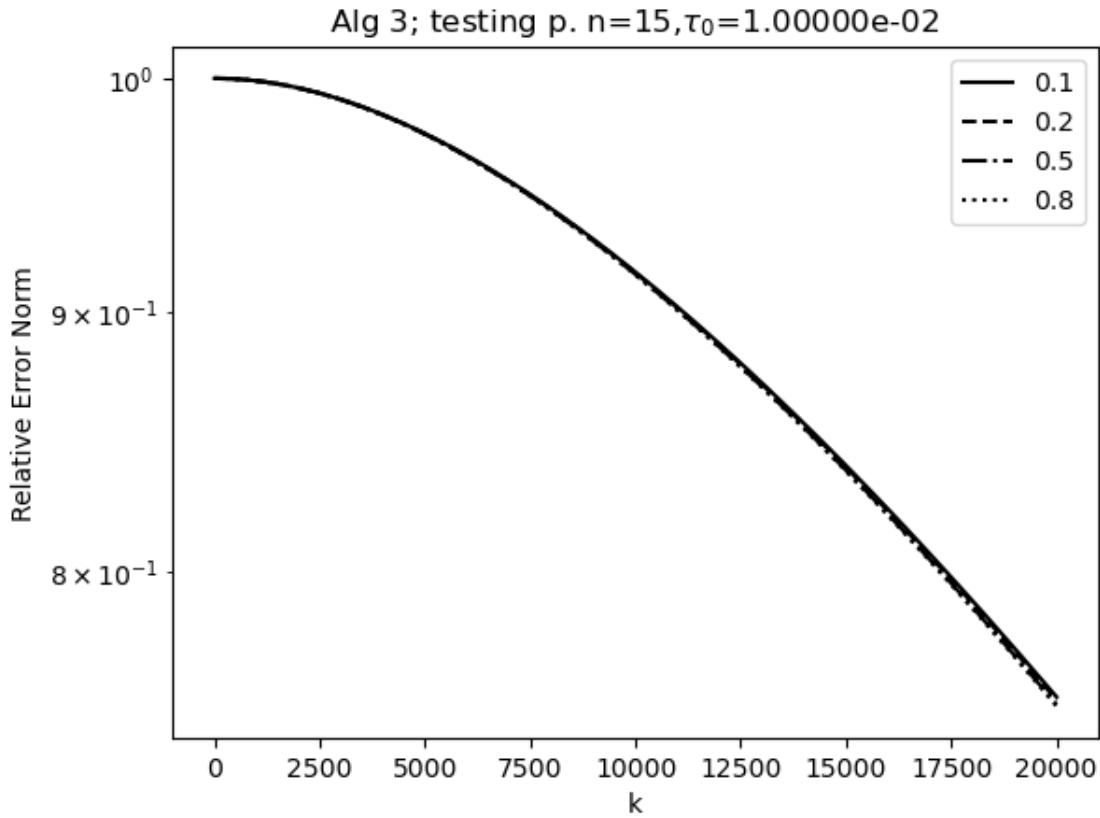
The results for Algoirthm 3 are in Figures 4 and 5. We set $\nu = \tau_0 h^2$ in these examples and will need to modify that to use the estimate in Remark 4.2. In the first two figures 4(a) and 4(b) use the values of τ_0 we use in Figure 1.

[22]: Figure4_5a(15);

```
testing $\tau_0$. n=15, p=5.00000e-01
```



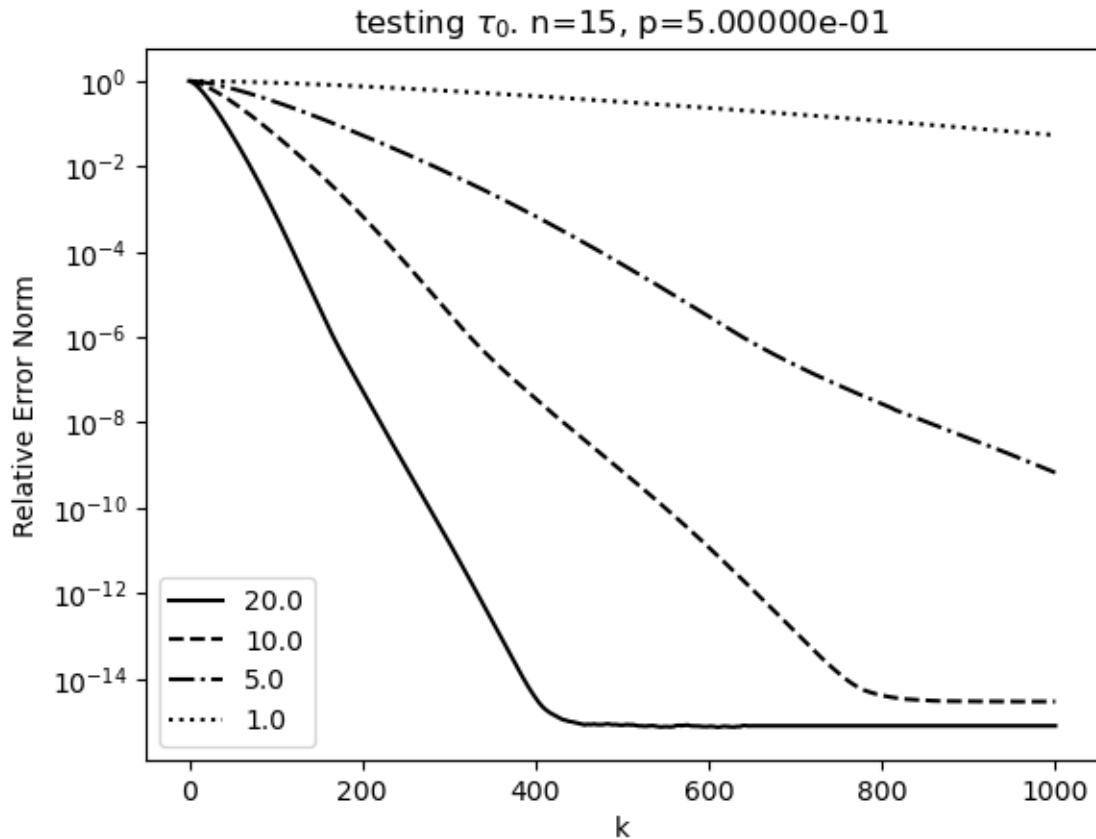
[23] : Figure4_5b(15);



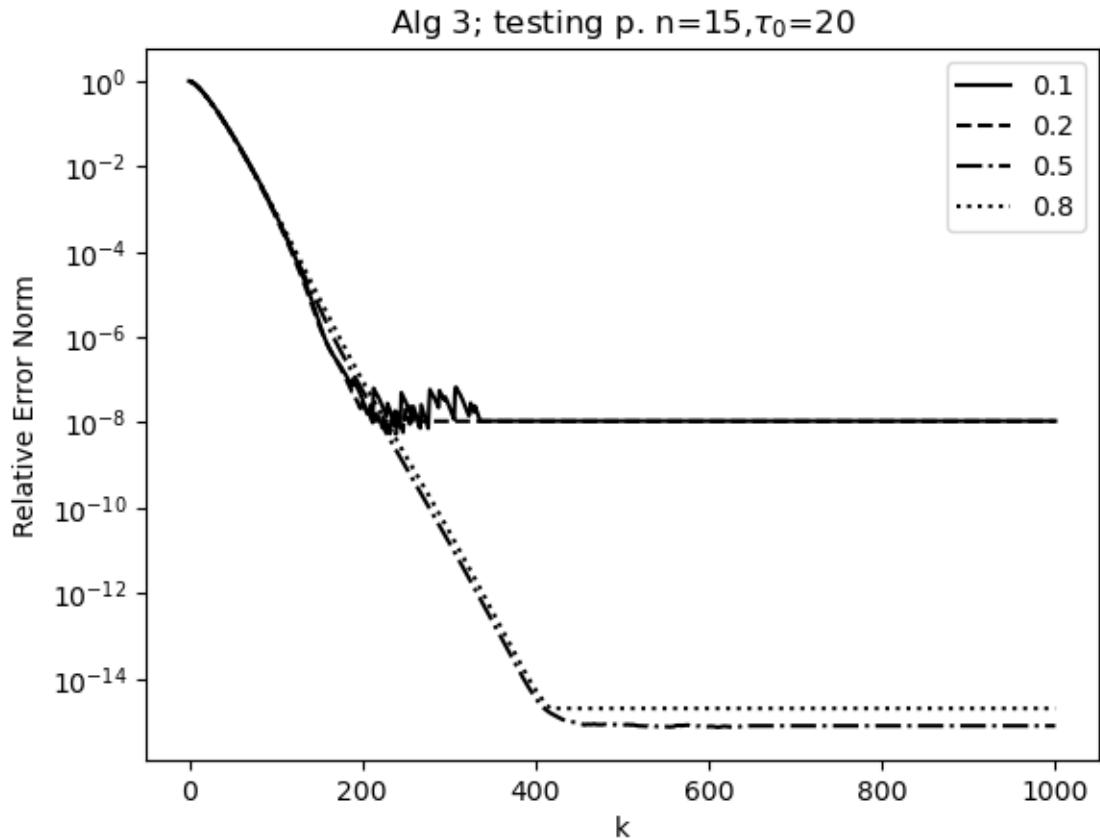
Now we use the larger values of τ_0 .

[24]: `Figure4_5a(15; maxit=1000, tauvec=[20.0, 10.0, 5.0, 1.0]);`

testing \$\tau_0\$. n=15, p=5.00000e-01



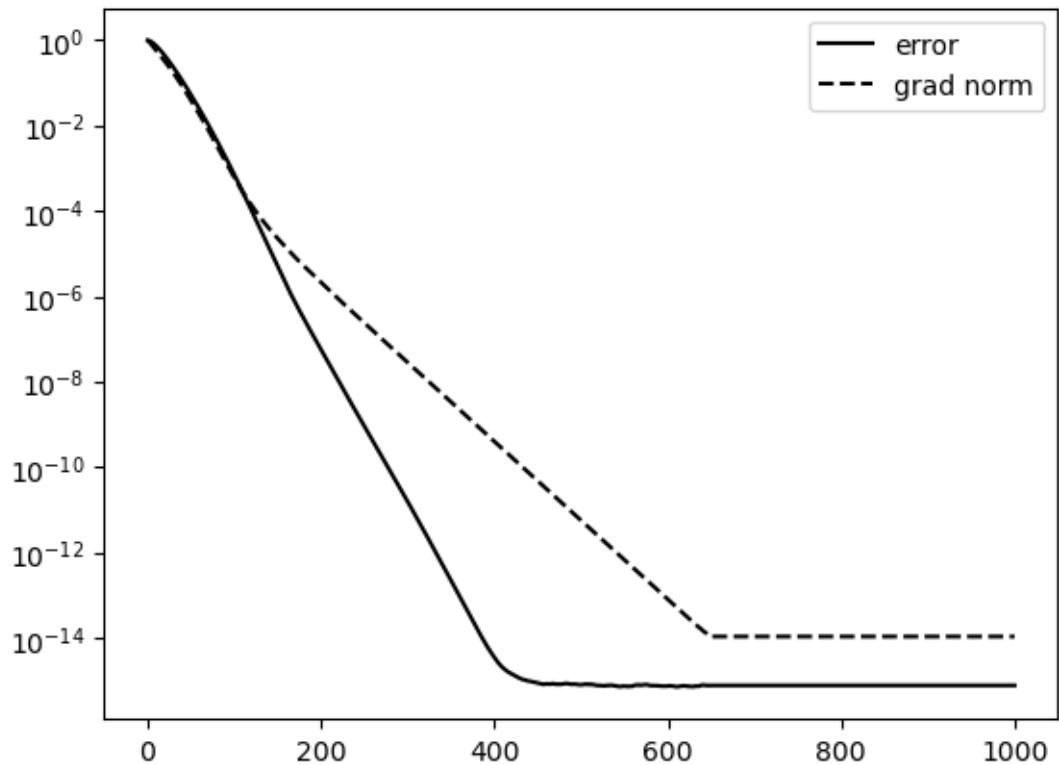
```
[25]: Figure4_5b(15; maxit=1000, tau0=20);
```



Figures 6ab compare the gradient norm to the error norm for $p=.5$ and $p=.01$.

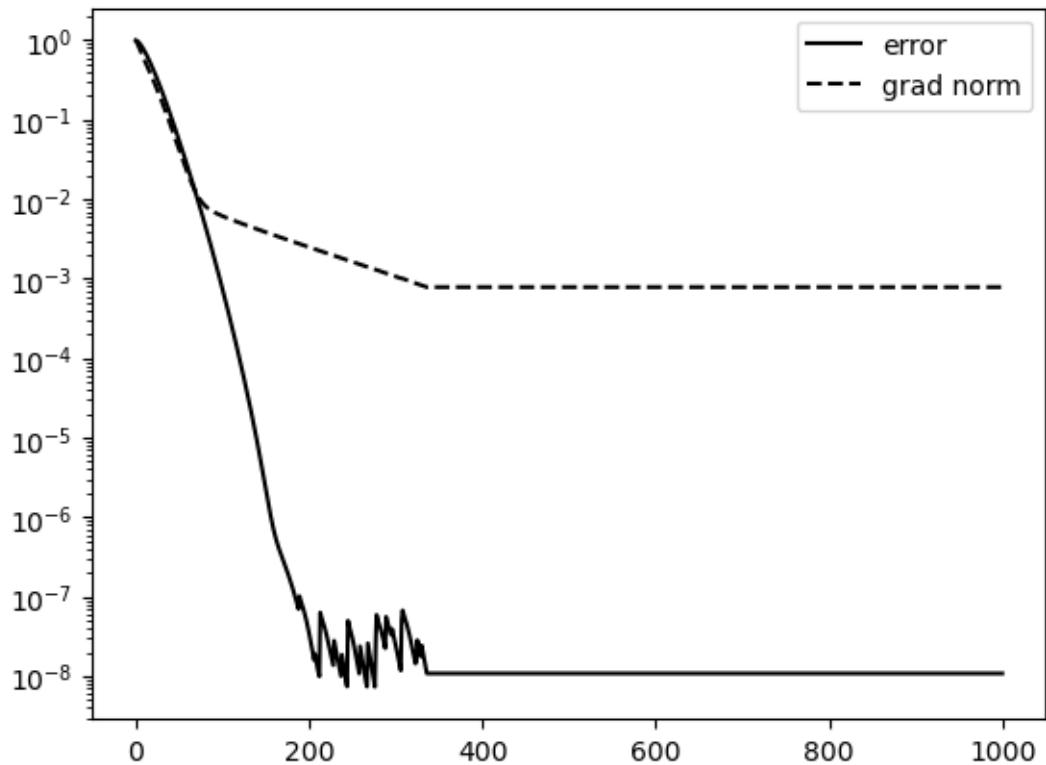
[26] : Figure6ab(15);

Alg 3; grad norm vs error. $p=5.00000e-01, \tau_0=2.00000e+01$



[27]: Figure6ab(15; p=.1);

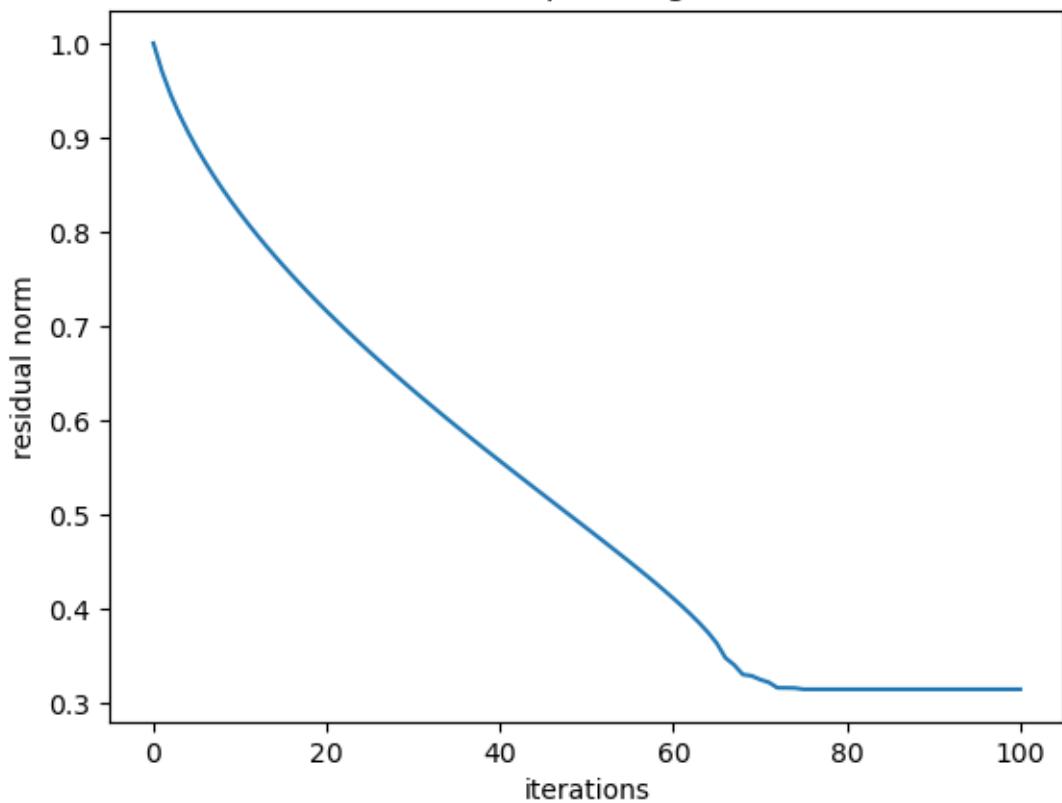
Alg 3; grad norm vs error. $p=1.00000e-01, \tau_0=2.00000e+01$



Here are the two figures for example 2

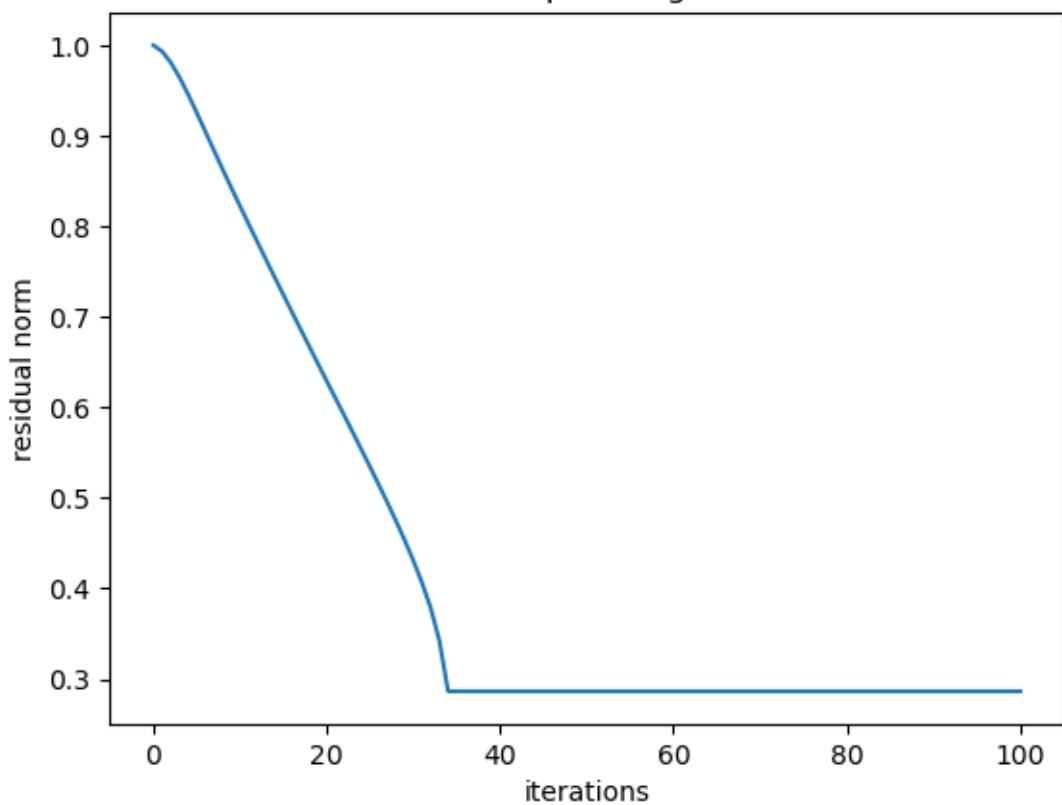
[28]: Example2a();

Example 2 Alg 1



```
[29]: Example2b();
```

Example 2 Alg 3



[]: