More Efficient Optimization of Long-term Water Supply Portfolios

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ABSTRACT

The use of temporary transfers, such as options and spot market leases, has grown as utilities attempt to meet increases in demand while reducing dependence on the expansion of costly infrastructure capacity (e.g. reservoirs). Earlier work has been done to construct optimal portfolios comprised of firm capacity, transfers, and the decision rules that determine the timing and volume of such transfers. However, such work has only focused on the short term (e.g. one year scenarios). Long term modeling efforts allow for the investigation of multi-year portfolios, but can greatly increase the computational burden.

This work utilizes a coupled hydrologic-economic model to stochastically simulate the long-term performance of a city's water supply portfolio. The model is linked with an optimization search algorithm that is designed to handle the high-frequency, low-amplitude noise inherent in stochastic simulations. This noise is a detriment to the accuracy and precision of the optimized solution, and has traditionally been controlled through investing greater computational effort in the simulation. However, given the complexity of some simulations and the computational effort required to manage this noise, a variance reduction technique known as the control variate method is applied within the simulation/optimization approach in order to more efficiently arrive at an accurate optimum. Random variation in model output (i.e. noise) is moderated using knowledge of random variations in stochastic input variables (e.g. reservoir inflows, demand), thereby reducing the computational burden by approximately 50%. Using these gains in efficiency, different scenarios and option contract structures are evaluated for their ability to reduce costs and adapt to growth in demand while continuing to meet reliability goals.

INTRODUCTION

Increased scarcity of water is leading many utilities to consider the integrated management of multiple water supply alternatives, as rising costs, environmental regulations, and public opposition provide growing disincentives to large-scale water supply projects (NRC 2001). Among these water supply alternatives are temporary water transfers, many of the uses and advantages of which having been documented (Michelsen and Young 1993; Lund and Israel 1995a; Characklis, Griffin et al. 1999; Knapp, Weinberg et al. 2003; Brown 2006). Managing a range of water supply alternatives can be analogous to the approach power utilities use to manage a “portfolio” of production assets, delivering energy to consumers through a mix of infrastructure and market-based transfers. In both cases, diversifying the range of alternatives provides utilities with the flexibility to reduce the firm, or baseload, capacity that must be maintained in order to meet demand, an advantage that generally translates to lower costs. Some work has been done in simulating and optimizing water supply portfolios over short-term planning horizons (e.g. one-year) (Smith and Marin 1993; Lund and Israel 1995b; Watkins and McKinney 1997; Wilchfort and Lund 1997; Characklis, Kirsch et al. 2006). While these short-term studies provide insights into portfolio development, utilities are likely to be interested in arrangements designed to accommodate longer planning horizons, and the composition and computational approaches involved in formulating multi-year portfolios remains largely unexplored. Most of the earlier work has involved linear (LP) or dynamic (DP) programming, but simulation-based Monte Carlo (MC) methods can be used as a means of representing the uncertainty inherent in many water resource systems, as well as the probabilistic nature of the decision making (e.g. when to purchase leases or exercise options). MC methods can be particularly useful if the objective is characterized in terms of an expected value (e.g., expected cost, reliability), but the computational burden associated with such methods has traditionally been cited as a deterrent to their widespread application. In recent years, however, improved search-based optimization techniques, increases in computing power, and the greater degree of realism often afforded by simulations have made MC a more attractive option.

As MC models increase in size and complexity, concerns over the computational burden continue to be a primary concern. This is particularly true when the MC model is linked to a search-based optimization algorithm. This study builds on previous work (Characklis, Kirsch et al. 2006) by expanding the portfolio development problem through consideration of a multi-year planning horizon. In doing so, a new methodology is applied, one designed to improve the efficiency of portfolio optimization, as it explores the composition of several different multi-year portfolios. The goal is minimizing a portfolio's expected costs, but the inherent variance in this expected value (i.e. the objective function) results in a "noisy" optimization surface (see Figure 1a), one that is difficult for gradient-based search methods to navigate without becoming stuck in local minima. Variance in the objective function (i.e. noise) can be reduced by increasing the number of realizations on which the expected values are based. This improves the accuracy and precision of the search algorithm, but can add significantly to the computational burden.

This work applies a variance reduction technique known as the control variate (CV) method to this class of water resource planning problems (Lavenberg and Welch 1981; Avramidis and Wilson 1996). The CV method can reduce the noise inherent in optimization surfaces based on expected values by using knowledge of how random variations in a simulation's inputs affect its outcome, and using this information to reduce the variance in simulation output. The method has been used in other areas, most notably the pricing of stock options (Boyle 1977; Johnson and Shanno 1987; Broadie and Glasserman 1996), but always with a standalone MC simulation. In this case, it is applied within the simulation-optimization framework with the intent of characterizing its potential for improving the efficiency of search-based optimization.

This work develops water supply portfolios composed of permanent water rights, leases, and options and evaluates their expected costs and ability to meet water supply reliability goals. These portfolios can be attractive, as leases and options provide a city with an ability to acquire water on more of an "as needed" basis, thereby reducing the volume of firm capacity (i.e. permanent rights) a city must maintain to meet demand (Young 1986; Lund 1993; Lund and Israel 1995a; Characklis, Griffin et al. 1999). Earlier work (Characklis, Kirsch et al. 2006) combined an MC approach with an implicit filtering search algorithm (Kelley 1999) to identify portfolios which minimized expected costs while meeting constraints related to supply reliability and cost variability. The one year planning horizon considered in this study, however, is likely to be substantially shorter than that often used by utilities, but optimizing portfolios over multi-year periods can be much more computationally intensive. This highlights the need for the CV method, and the results of this work demonstrate the ability of the CV method to improve optimization efficiency. Multi-year scenarios also allow for an exploration of long-term option contracts, those which provide the purchaser with flexibility over the duration of the agreement in terms of the volume of options purchased and exercised. These contracts may be especially attractive given that they provide security while freeing the utility from the cost and inconvenience of renegotiating every year.

These results suggest that the CV method may be useful in simulation-based optimization problems across a broad range of water resource contexts. Similarly, results related to the composition of long-term water supply portfolios, including multi-year option contracts, may provide insights valuable in the formulation of water supply strategies.

METHODOLOGY

The control variate method is first applied to the one-year hydrologic economic model developed in earlier work (Characklis, Kirsch et al. 2006), then applied to an expanded multi-year scenario. While a detailed description of the one-year model can be obtained in the earlier paper, an abbreviated description is appropriate here. This description will be followed by an introduction to the control variate method, which is applied to the one-year model to determine its efficacy on a well-understood system. The expanded, multi-year model is then described, followed by the application of the CV method to this more complex long-term scenario.

The study region is the U.S. side of the Lower Rio Grande Valley (LRGV) in the southern tip of Texas. The primary water sources in the region are the Falcon and Amistad reservoirs, with a combined storage capacity of 5.8 million acre-feet (ac-ft), in which storage is strictly divided between the U.S. and Mexico according to the Treaty of 1944. Approximately 1600 users own rights to the U.S. portion of this storage, with the vast majority of water consumed by the agricultural sector. Municipal consumption has risen, however, from 7% to 13% of the regional total between 1970 and 2002, and this growth is expected to continue. The water supply portfolio is developed using water consumption patterns for a city of 120,000 (roughly corresponding to Brownsville, TX) that uses an average of 21,000 ac ft per year. Historic data for reservoir inflows, losses, and storage volume are used within the model, as is price information from the regional water market.

The original model evaluates the expected cost of the city's water supply portfolio over a single calendar year (discretized to monthly time steps). The municipality has access to surface water allocated via a system of marketable water rights, and several different transfer types exist within the market. The agricultural cycle is roughly annual, and since transferred water invariably originates from irrigators, many decisions that affect the availability and price of water on the market are made just before the beginning of the year. As a result, transfers of permanent rights and purchases of options are assumed to take place on or before December 31, with spot market leases and exercised options available to augment supply over the course of the following year. Thus, the city's portfolio is composed of three different types of assets:

(i) permanent rights, which are owned in perpetuity and provide allocations of new inflows to the reservoir system as they occur throughout the year;

(ii) one-year option contracts, all or part of which can be exercised on May 31 (or otherwise lapse);

(iii) spot-market leases which can be purchased during any monthly interval, but whose price varies stochastically in a manner linked to reservoir levels.

It is important to note that transfers of permanent rights require lengthy regulatory approvals, so unlike temporary transfers involving leases and options which can be approved in a matter of days, changes in the volume of permanent rights are assumed to be made only at annual intervals. Within the model, decision rules governing whether or not to exercise options or purchase leases, as well as the amount to be exercised/leased, are based on the ratio of expected supply (S_E) to expected demand (D_E) over the remainder of the year. This S_E/D_E ratio, based on historical data on reservoir inflows and the city's demand, reflects uncertain information that is available to utilities well in advance of a "failure", which is defined as any monthly interval in which demand exceeds supply. This sort of anticipatory approach to augmenting supply is likely to be more representative of utility decision-making than a situation in which choices regarding temporary transfers are made only when a failure is imminent (and the volume of water required is known).

The S_E/D_E ratio is recomputed for each monthly time step and compared against a threshold value, α, such that if the ratio drops below α, then spot leases are purchased or options exercised (the latter only in May). While α governs the timing of lease/exercise decisions, another threshold value, β, determines the quantity of water acquired via leases or options, such that the value of S_E is increased through leasing and/or exercising until the ratio of S_E/D_E is equal to β. Optimal values for α and β (i.e. those that result in a minimum cost portfolio) are not known *a priori* and therefore represent decision variables. Different α/β pairs could be assigned to each month or maintained constant throughout the year (or at any other intermediate time step). In this case, two sets are defined: α_1, β_1 for lease/option decisions made from January-May and α_2, β_2 for the period June-December.

The expected portfolio cost in any year is determined by identifying the number of permanent rights (N_R) and options (N_O) held at the beginning of the year, as well as optimal values for α and β, which control acquisitions via leases and options. Repeated model simulations involving stochastic inputs for reservoir inflows and demands then generate expected values for the volume of options exercised (N_X) and leases purchased (N_{L_i}). Given specification of initial conditions for reservoir storage (R_0) and the volume of water the city has in hand (expressed as a fraction of permanent rights owned, f_{R_0}), the expected portfolio cost can then be described in terms of the decision variables (N_R, N_O, α_1, β_1, α_2, β_2) such that,

$$E[Cost] = N_R\ p_R + N_O p_O + E[N_X]p_X + E\left[\sum_{t=0}^{11} N_{L_t} \hat{p}_{L_t}\right], \qquad [1]$$

where,

p_R = annualized price of permanent water right ($22.60/ac-ft)*;

p_O = option price ($5.30/ac-ft)*;

p_X = option exercise price ($15.00/ac-ft)*;

$\hat{p}_{L_i}$ = sampled lease price in month *i* ($/ac-ft);

N_X = number of options exercised (ac-ft);

N_{L_i} = number of leases purchased in month *i* (ac-ft).

The sum of the option price and exercise price are calculated to be risk neutral relative to the lease prices in the option exercise month using standard option-pricing theory (Black and Scholes 1973; Hull 1999), such that the sum of the option and exercise prices is equivalent to the expected lease price on the exercise date.

In addition to evaluating the expected cost, the simulation estimates two parameters used as constraints in the optimization. Water supply reliability is defined as the probability that the utility's supply exceeds demand in any given month (thus avoiding a "failure"). The second constraint limits the cost volatility of the portfolio using a concept called the Contingent Value at Risk (CVAR), a metric used in the power industry to measure the risk of high portfolio costs, and represents the average costs of the most expensive 5% of portfolios (i.e. if 1000 annual realizations are run, CVAR represents the average of the 50 most expensive years). In this case, a constraint involves the ratio of CVAR-to-expected cost to limit annual cost variability.

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* Parenthetical values represent output from an analysis of price data from LRGV. For more detail see Characklis, Kirsch et al. (2006).

The challenge of optimizing a solution surface (e.g. expected cost) characterized with high-frequency, low-amplitude noise is addressed using an approach called "implicit filtering", shown to be effective for searching surfaces that exhibit high-frequency, low-amplitude noise (Stoneking, Bilbro et al. 1991; Choi, Eslinger et al. 1999; Kelley 1999). Given an initial point ($\mathbf{x}_0$) of dimension N (where N is the number of decision variables), implicit filtering samples the solution surface at a set of points radiating from $\mathbf{x}_0$ in what is referred to as a stencil. Implicit filtering uses a stencil consisting of 2N points to implement a coordinate search along the positive and negative coordinate directions a distance h (stencil size) from $\mathbf{x}_0$ in search of another N-dimensional point ($\mathbf{x}$) with a lower objective function value (in the case of a minimization problem). Implicit filtering augments the coordinate search with a line search in which points are also sampled along a descent direction extending from $\mathbf{x}_0$ (and any subsequent $\mathbf{x}$). From these sampled points implicit filtering moves from $\mathbf{x}_0$ to the point ($\mathbf{x}$) that produces the lowest functional value. The process then repeats, and as the search progresses the stencil size decreases as $\mathbf{x}$ progresses towards the minimum. The search terminates when the desired tolerances for first order optimality conditions are met.

Despite the ability of implicit filtering to navigate noisy solution surfaces, the noise produced by an objective function ($f(\mathbf{x})$) is detrimental to the precision and accuracy of the optimized solution. This noise can be quantified using the standard error of the mean (*s.e.*), defined as,

$$s.e. = \frac{\sigma}{\sqrt{n}} \qquad [2]$$

where σ is the standard deviation of $f(\mathbf{x})$ and n the number of realizations. As the value of σ is intrinsic to $f(\mathbf{x})$, controlling the noise associated with a MC simulation typically means controlling the number of realizations performed. However, the square root in the denominator of [2] means that increasing *n* has decreasing marginal returns in terms of reducing standard error. An alternative approach to reducing noise would be to reduce the variance of the function, something that can be achieved through application of the control variate method.

Control Variate Method

The control variate (CV) method is a variance reduction technique that utilizes knowledge of how variation of stochastic input variables affects the value of the simulation output, in this case, the objective function $f(\mathbf{x})$. For example, a city may wish to estimate the expected cost of transfers through MC simulation. If there exists a known correlation between the cost of water transferred (simulation output) and the volume of water stored in the city's reservoir (stochastic input variable), the reservoir storage volume may be used as a control variate. Given a known mean reservoir storage volume, deviations from its mean can be used to account for deviations from the estimated mean cost of transfers, thereby reducing the variance in the transfer costs and improving the precision of cost estimation. A more in-depth discussion of the CV method exists in the literature (Lavenberg and Welch 1981; Avramidis and Wilson 1996), but a brief description is offered here.

If Z is a random input variable that is sufficiently correlated to model output, ($f(\mathbf{x})$), θ can be defined as the variance-reduced value of $f(\mathbf{x})$ such that

$$\theta = f(\mathrm{x}) + c \cdot (Z - \mathrm{E}[Z]), \qquad [3]$$

where c is a scaling factor and Z is the control variate. Taking the expected value of both sides of [3] produces

$$\mathrm{E}[\theta] = \mathrm{E}[f(\mathrm{x})], \qquad [4]$$

such that θ becomes an unbiased estimator of $f(\mathbf{x})$ when c is any real number. If the variance of both sides of [3] is calculated, the following is obtained:

$$\mathrm{Var}(\theta) = \mathrm{Var}(f(\mathrm{x})) - 2c\mathrm{Cov}(f(\mathrm{x}), Z) + c^2\mathrm{Var}(Z). \quad [5]$$

It can be shown that if

$$2c\mathrm{Cov}(f(\mathrm{x}), Z) > c^2\mathrm{Var}(Z), \quad [6]$$

then θ has a lower variance than $f(\mathbf{x})$. Further, it can be shown that the minimum variance occurs when

$$c^* = \mathrm{Cov}(f(\mathrm{x}), Z) / \mathrm{Var}(Z). \quad [7]$$

The reduction in variance then can be predicted with

$$\mathrm{Var}(\theta) = (1 - \rho^2)\mathrm{Var}(f(\mathrm{x})), \quad [8]$$

where ρ is the correlation coefficient between $f(\mathbf{x})$ and Z.

The control variate method can be extended to accommodate multiple control variates (Z_1, Z_2,…, Z_j), through the expansion of [3]:

$$\theta = f(\mathrm{x}) + c_1 \cdot (Z_1 - \mathrm{E}[Z_1]) + c_2 \cdot (Z_2 - \mathrm{E}[Z_2]) + ... + c_j \cdot (Z_j - E[Z_j]). \quad [9]$$

Similarly, the variance of θ is minimized through the choice of optimal values for c_1, c_2, …, c_j.

For the purpose of readability, references to the output variable $f(\mathbf{x})$ in this discussion will be replaced with expected cost (*Cost*), the output variable (or objective function) specific to this work. Likewise, the variance reduced output variable produced by the CV method, θ, will be replaced with $Cost_{\mathrm{var}}$, such that [3] could be rewritten as:

$$Cost_{\mathrm{var}} = Cost + c \cdot (Z - \mathrm{E}[Z]). \quad [10]$$

Application of CV Method

Selection of appropriate control variates is guided by the modeler's understanding of sources of variability in the objective function. In the example presented here, the objective function is the expected portfolio cost, *Cost*, and the source of the variability in the portfolio cost arises from the purchase of leases and exercise of options. More specifically, the variability can be identified as arising from variability in both the price and the quantity of transfers acquired, both of which are linked to variability in reservoir inflows and water demand.

This section will describe the selection of two control variates to be used in the single-year simulation. The majority of transfers occur at two decision points, the beginning of the year (t_0) and in May, the option exercise month (t_5). The lease price at t_0 is a function of a known distribution with a known expected value, obtained from water market data from the LRGV (Watermaster's Office 2004). Because each individual realization begins at t_0 with the same initial conditions, the quantity of leases purchased are unchanged from realization to realization. Thus, controlling for the variability in the lease price at t_0 accounts for all the cost variability that arises from leases purchased then, and the lease price at t_0 is designated as the first control variate, Z_{p_L}.

The second control variate accounts for portfolio cost variability arising from variability in the quantity of transfers acquired in t_5. Within the simulation, both the monthly rate of new reservoir inflows allocated to the city ($N_{R,i}$) and the city's monthly water demand (D_i) have known expected values, and the difference between the two is the monthly *net supply*. The second control variate, Z_{NS}, is thus defined as the net supply from the beginning of the year (t_0) to t_4, the month prior to the option exercise month, such that

$$Z_{NS} = \sum_{i=0}^{t_4} N_{R,i} - D_i \ . \qquad [11]$$

Therefore, below average values of Z_{NS} indicate above average lease purchasing or option exercising activity in t_4. Incorporating [11], the variance-reduced cost estimate for the one-year model ($Cost_{\mathrm{var}}$), can be represented as

$$Cost_{\mathrm{var}} = Cost + c_1 \cdot \left(Z_{p_L} - E\left[Z_{p_L}\right]\right) + c_2 \cdot \left(Z_{NS} - E\left[Z_{NS}\right]\right). \qquad [12]$$

The optimal values of c (c^*) in [12] are not known *a priori* and will change with different decision variables and initial conditions. Therefore, values for c_1^* and c_2^* are estimated for each new set of conditions using a pilot study, involving a very limited number of realizations that produce correlations between the control variates and *Cost*. Figure 2 illustrates how the optimization algorithm, the model, and the pilot study relate to each other. Without the CV method, the optimizer queries the model with an **x**, a vector describing all six decision variables, and the model returns *Cost*. With the CV method, the original simulation (labeled 'Main Model' in Fig. 2) immediately passes **x** to the pilot study, which performs a small number of realizations, calculates the c^* values and returns them to the main model. The main model then performs the simulation and applies [12] with the calculated c^* values before returning the variance-reduced cost estimate ($Cost_{\text{var}}$) to the optimizer. While the pilot study represents a computational investment, it is generally a small investment, and one that pays off in a decrease in the total number of realizations that are performed.

Expansion to Multi-Year Model

A multi-year simulation allows temporary transfers, particularly option contracts, to be evaluated on a time-scale that may be more useful for water supply planners. The expanded model is designed to accommodate any number of years, and in this work a 10-year planning horizon is used.

From an optimization standpoint, the greatest change made to the simulation is reflected in the objective function, which is now represented in a multi-year form

$$\sum_{k=1}^{10} Cost_k = \sum_{k=1}^{10}\left[N_{R_T,k}\,p_R + N_O\,p_{O,k} + E\left[N_{X,k}\right]p_X\right] + E\left[\sum_{k=1}^{10}\sum_{i=0}^{11} N_{L_t,k}\,\hat{p}_{L_t}\right] \qquad [13]$$

where k is the simulation year.

The operation of the hydrologic portion of the simulation remains unchanged from the single-year simulation with only a few exceptions. The multi-year simulation is set up to account for annual growth (r) by multiplying each demand value by the term $(1 + r)^{k-1}$. In addition, the reliability constraint is modified to accommodate multiple simulation years, such that the reliability for each year within the simulation period is required to meet a minimum value. The cost variability constraint in the multi-year scenario (when it is imposed) is also re-defined such that the average annual ratio of CVAR-to-expected cost must be less than a specified value.

In addition to exploring how a longer planning horizon alters optimal portfolio composition, the multi-year model presents opportunities for examining long-term option contracts, which can allow for inclusion of some interesting features. For example, an annual growth factor can be attached to the volume of options purchased each year. This growth factor can be calculated to increase in accordance with rising demand. In this case, model input is changed such that N_O refers to just the number of options purchased in the first year, with subsequent years' option purchases defined as

$$N_{O,k+1} = D_{E,k+1} - D_{E,k} + N_{O,k} \ . \qquad [14]$$

Application of Control Variate Method to Expanded Simulation

The expansion of the model to a multi-year simulation does not change the basic operation of the model, but it does require definitive changes in how the CV method is applied. In the single-year simulation, the objective of applying the CV method is to reduce the variance of the objective function, var($Cost$), whereas in the multi-year simulation the goal is the reduction of var($\sum_{k=1}^{10} Cost_k$). This is accomplished by applying the CV method to each simulation year separately and calculating a reduced-variance cost ($Cost_{\mathrm{var},k}$) for each $Cost_k$. In this case, each $Cost_{\mathrm{var},k}$ must achieve sufficient variance reduction that

$$\mathrm{var}(\sum_{k=1}^{10} Cost_{\mathrm{var},k}) < \mathrm{var}(\sum_{k=1}^{10} Cost_k). \qquad [15]$$

In the first year of the multi-year simulation, the control variates used to calculate $Cost_{\mathrm{var},1}$, the lease price in t_0 (Z_{p_L}) and the net supply of new water allocations from t_0 to t_4 (Z_{NS}), remain identical to those used in the single-year simulation. The lease price distribution, however, is dependent upon the reservoir level, but the expected reservoir level at t_0 of year $k + 1$ is dependent upon its observed value at year k, and thus the mean of Z_{p_L} cannot be calculated for years two through 10. Therefore, Z_{p_L} is excluded as a control variate from years two through 10.

The net supply control variate ($Z^{E}_{NS,k}$) is used in years $k > 1$ to account for the variability in the number of transfers that occur in the exercise month (t_5). However, the notation for the net supply control variate is changed to $Z^{E}_{NS,1}$ (where the superscript E and subscript 1 denote the early months (t_0 to t_4) and year one, respectively). The second control variate used in years two through 10 adapts the net supply used in $Z^{E}_{NS,k}$ to control for cost volatility arising from the variability of the number of leases purchased at the beginning of the year (t_0). To account for the variability in the quantity of leases purchased at t_0 of year k, one examines the net supply that accrues to the city in the latter portion of year k-1 (months t_5 to t_{11}),

$$Z^{L}_{NS,k} = \sum_{i=t_5}^{t_{11}} N_{R,i,k-1} - D_{i,k-1}\,. \qquad [16]$$

The CV method, as applied to the multi-year model, is

$$Cost_{\mathrm{var},1} = Cost_1 + c_{1,1}\cdot\left(Z_{p_L} - E\left[Z_{p_L}\right]\right) + c_{2,1}\cdot\left(Z^{E}_{NS,1} - E\left[Z^{E}_{NS,1}\right]\right), \qquad [17a]$$

and

$$Cost_{\mathrm{var},k} = Cost_k + c_{2,k} \cdot \left(Z^E_{NS,k} - E\left[Z^E_{NS,k}\right]\right) + c_{3,k} \cdot \left(Z^L_{NS,k} - E\left[Z^L_{NS,k}\right]\right) \quad [17b]$$

for k = 2, 3, …, 10.

A summary of control variates is included in Table 1.

RESULTS

Application of CV Method to Single-Year Model

Figures 1a and 1b illustrate the "smoothing" of the solution surface that occurs when the CV method is applied to the one-year model. Figure 1a plots the expected annual cost (*Cost*), while Figure 1b plots the variance-reduced cost ($Cost_{\mathrm{var}}$). Each expected value plotted in both figures was created using 125 realizations, but in Figure 1b, 25 of these are dedicated to the pilot study in order to calculate c^* values. These values are then applied to the 100 realizations performed in the main model to calculate the variance reduced estimate of expected cost ($Cost_{\mathrm{var}}$).

The smoothing effect of the CV method improves the accuracy and efficiency of the optimization of the model through noise reduction. Noise can be measured as standard error [2], but here, the definition is slightly modified to be the average standard error ($\overline{s.e.}$) as a percentage of the average portfolio cost ($\overline{Cost}$),

$$Noise = \overline{s.e.} / \overline{Cost} \cdot 100\% . \quad [18]$$

Figure 3 provides a comparison between the average optimal values of portfolio cost produced with and without the CV method using the same three levels of computational effort (125, 150, and 300 total realizations) and ordered according to the average noise level. For the runs utilizing the CV method, the pilot study accounted for 25, 25, and 50 of the total number of realizations, respectively. In stochastic optimization, search algorithms rarely converge to a unique solution, but rather to a relatively circumscribed region. As such, in Figure 3 each data point reflects the average optimized value of 100 optimization runs with the error bars representing the 25th and 75th percentiles, a range that varies from 1% to 4% of the mean, depending on the number of realizations.

The computational effort required to achieve optimality conditions can be measured either through the amount of work invested in the simulation (e.g. the number of realizations performed) or through the number of times the optimizer calls the simulation. In this case the number of calls to the simulation is not significantly affected when the CV method is applied. What is observed is that application of the CV method greatly reduces the number of realizations required to achieve the same level of accuracy and precision (using the same number of calls to the simulation) as when the CV method is not used. Results obtained both with and without the CV method, show that as more computational time (i.e. realizations) is invested and noise is reduced the average optimized value is lowered. With the CV method, runs using 125 and 150 realizations result in roughly the same level of noise and same average optimized portfolio cost as the case in which twice as many realizations (300) are used without the CV method. Also notice that range of optimal values (as reflected in the error bars) shrinks noticeably as the noise is reduced, evidence that reducing noise yields an optimized solution that is improved both in terms of lower portfolio costs and greater precision. Finally, to provide a basis of comparison, two limiting test cases (one with and one without the CV method) estimate a lower bound on the gains possible through noise reduction by using 10,000 realizations.

Applying the CV method to the one-year model, Figure 4 shows optimal portfolios arising from three strategies:

- (A) Permanent rights only. This represents a "typical" case in which the city maintains a sufficient volume of rights (i.e. firm capacity) to meet its reliability objectives.
- (B) Rights and options. Demand is met through a combination of permanent rights and a long-term (10 year) option contract, which commits the city to purchase a constant volume of options each year (the number exercised will vary with conditions).
- (C) Rights, options, and leases. The city can choose from all three water supply assets to construct its portfolio.

All strategies begin with an initial reservoir volume (R_0) of 800,000 ac-ft and a volume of water carried over from the previous year that is equal to 0.3 (f_{R_0}) of the volume of its permanent rights. As a city is unlikely to adopt a portfolio in which the firm capacity cannot at least meet demand in a typical year, all portfolios maintain a minimum 30,000 ac-ft of permanent rights, using leases and options as a hedge against extended dry periods. Further, all portfolios must meet a minimum reliability goal of 99%. As can be seen, the use of transfers significantly reduces the amount of firm capacity that must be maintained to meet reliability goals. As transfers supplant firm capacity, the expected portfolio costs are reduced, as well.

Multi-Year Portfolio Scenarios

The CV method is applied to the 10-year model to produce optimal long-term portfolios for the same three strategies as for the one-year model, plus one additional strategy (Figure 5). The initial conditions remain unchanged, but a 2% annual rate of demand growth is assumed. Strategy A reflects the situation in which firm capacity is maintained, and the city requires nearly 50,000 ac-ft of permanent rights to meet its reliability goals at a cost of $11.2 million. Allowing options to be purchased in conjunction with owning permanent rights (strategy B), the city is able to reduce excess firm capacity and continue to meet reliability requirements, reducing its expected 10-year portfolio cost by 14%, down to $9.7 million. Further savings are realized in strategy C when leases are considered. In this case, leases and options are both utilized to ensure reliability during dry periods as the expected portfolio cost drops to $8.6 million, but there is greater variability in portfolio cost as described by the CVAR values. The CVAR-to-portfolio cost metric rises from 1.06 in strategy B to 1.29 in strategy C, which highlights how options act to reduce cost variability by limiting exposure to spot market volatility. Strategy D employs rights, options, and leases, just as in (C), but with a cost variability constraint which limits the CVAR to expected cost ratio to less than 1.1, thereby demonstrating the tradeoff in cost that occurs as portfolio cost variability is restricted. Strategy C involves no variability constraints and a CVAR-to-expected cost ratio of 1.29, whereas strategy D, limited to a CVAR-to-expected cost ratio of 1.1, costs $600,000 more across the 10-year period.

The efficacy of the CV method is tested on the ten-year model just as with the one-year model. The average optimized multi-year portfolio costs were evaluated both with and without the CV method using 125, 150, and 250 total realizations in both cases, with values generated using the CV method consuming 25 of these realizations in the pilot study (Figure 6). Similarly to the one-year case, the ten-year optimization results demonstrate that the use of the CV method reduces the number of realizations required to reach a given solution by roughly 50%. However, one notable difference is that the 10-year model's average costs exhibit less noise than those of the one-year model, as a sum of ten years' costs tends to moderate the relative noise arising from extreme events observed in a single year.

The effectiveness of the CV method is not constant across the solution surface, which becomes apparent when the expected portfolio cost is mapped across a range of permanent rights and options holding α/β values constant (Figure 7). The application of the CV method over the domain considered yields variance reduction ranging from 0% to over 70%.

When surveying the landscape in Figure 7, it is clear that application of the CV method significantly reduces variance across broad sections of the potential range of solutions. Nonetheless, two areas of low variance reduction are observed. The first is when the city maintains a large volume of permanent rights well beyond (> 45,000 ac-ft) its average annual demand (~ 21,000 ac-ft), such that the city can reliably meet demand with few transfers. A low volume of transfers corresponds to little variability in expected portfolio costs, therefore the CV method's ability to reduce variance is low. Variance reduction is also low when the number of permanent rights and options is very low, a situation in which the city must rely largely on spot leases. While it seems paradoxical that regions involving both more and fewer transfers should both translate to low variance reduction, this region of heavy spot lease activity exhibits some of the greatest levels of noise, which largely arise from variability in the lease price. However, in the 10-year model the lease price variability is controlled for by the CV method only at t_0 of the first year. The other control variates ($Z^{E}_{NS,k}$ and $Z^{L}_{NS,k}$) only account for variability in the quantity of transfers executed throughout the simulation period. While regions of low variance reduction exist, the minimal cost portfolios developed in the next section are largely located in the broad swath of greatest variance reduction. Model constraints (and likely real-world portfolio strategies) generally preclude portfolios that either have an excess capacity of permanent rights or a large reliance on spot leases, meaning that variance reduction techniques may be particularly useful in identifying the types of portfolios that will be of greatest interest to utilities.

Long-Term Option Contracts

At this point, results have revolved around static 10-year portfolios, in which the number of permanent rights and options purchased remain constant throughout, but given an annual 2% growth in demand, it may be possible to develop multi-year arrangements that accommodate this growth more effectively. To this end, several more creative long-term option contracts are investigated. Figure 8 details the expected portfolio compositions and costs over time of three scenarios involving three different long-term option contracts, with all scenarios based on strategy B (permanent rights and options). The first panel describes the year-by-year evolution of the portfolio described by strategy B (from Figure 5), where the city has a long-term contract for a constant number of options. As one would expect, the average number of exercised options increases as demand increases. Alternatively, the second panel (B2) reflects a multi-year option contract in which the contract involves scaling the number of options purchased each year to correspond with demand growth according to [14]. This type of contract allows for fewer options to be purchased in the earlier years, and reduces the expected cost of the contract by 13%, or $378,000 over its life. The third panel (B3) structures the option purchase so that at the beginning of each year, the city either purchases a base number of options, or an enhanced number that is 30% greater than the base (roughly the same percentage increase as between years one and 10 of B2). The level of options purchased is decided by the state of the city's water supply entering the new year as measured by f_{R_0}. The base volume of options is purchased if $f_{R_0} > 0.2$, otherwise the city purchases the additional 30%, thus, the option purchase is responsive to the state of the city's water supply at the beginning of each year. The contract described in B3 results in a reduction of the contract's expected cost of 23% relative to the static case (B), and a total savings of $658,000.

The contingent clauses described in strategies B1 and B2 should not be difficult to write into a long-term contract, and both provide the city with long-term security and greater flexibility. The option contract in B2 is structured with the prediction of 2% annual growth in demand, however, if that growth in demand does not materialize, B3 is not committed to purchasing the enhanced quantity of options, providing additional flexibility that may be valuable.

CONCLUSIONS

When using a Monte-Carlo based simulation, there is a trade-off between sample error and the investment of computational effort. When the simulation is combined with a search algorithm, this sample error, or "noise", hinders the optimization process. Given the complexity and size of the simulations currently being used in water resource modeling, the ability to manage this noise may often be beneficial. The control variate method exhibits an ability to reduce the noise of both the one- and ten-year models such that only half the number of realizations is required to match the accuracy and precision of the optimal portfolios produced without the CV method. Use of more efficient simulation-optimization methods has growing application in water resource planning. In this case, it is applied toward the development of multi-year water supply portfolios, an application that also allows for an investigation of several types of long-term option contract structures. These long-term contracts that are structured to meet the individual needs of the utility, through such things as accounting for expected growth in demand or year-by-year flexibility, also appear to show promise. As municipal demands grow, this approach to stochastic simulation and optimization of long-term water supply portfolios is likely to find further application.

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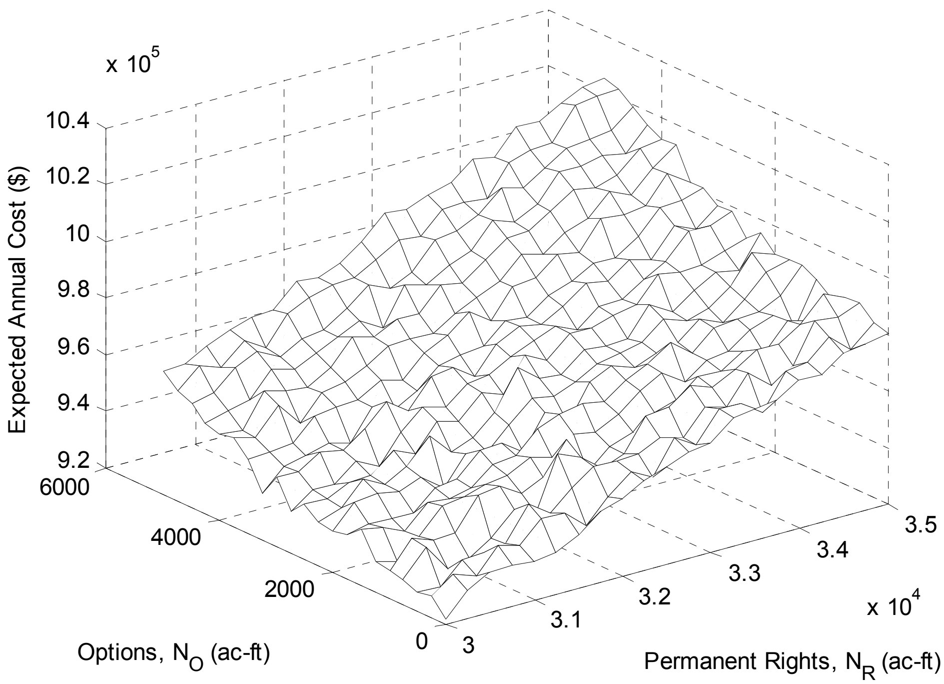

Figure 1a. Outcomes of a range of portfolio configurations, using 125 total realizations and no CV method.

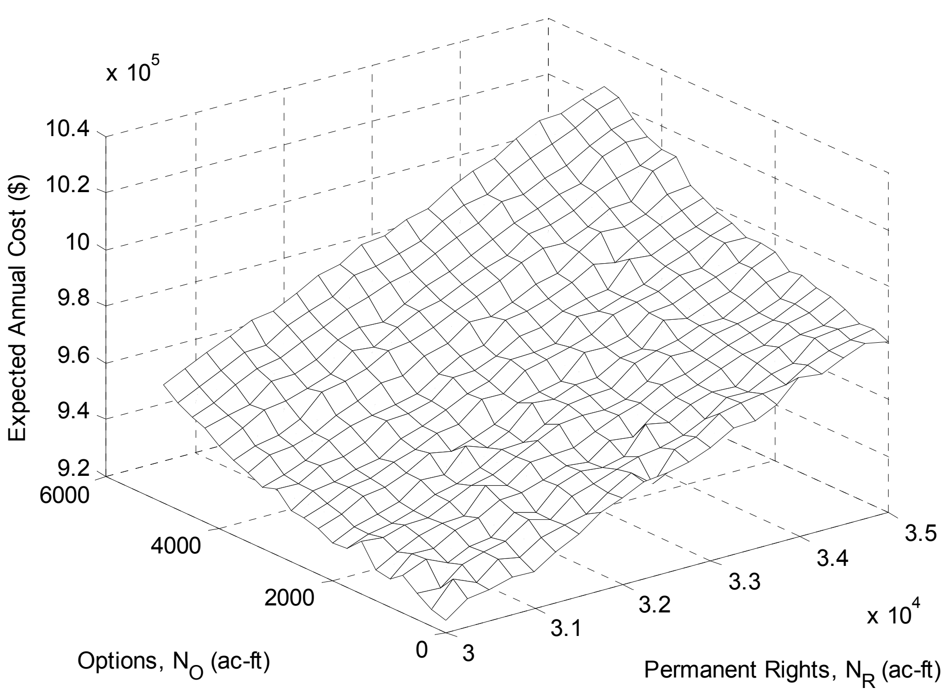

Figure 1b. Outcomes of a range of portfolio configurations, using 125 total realizations and applying the CV method

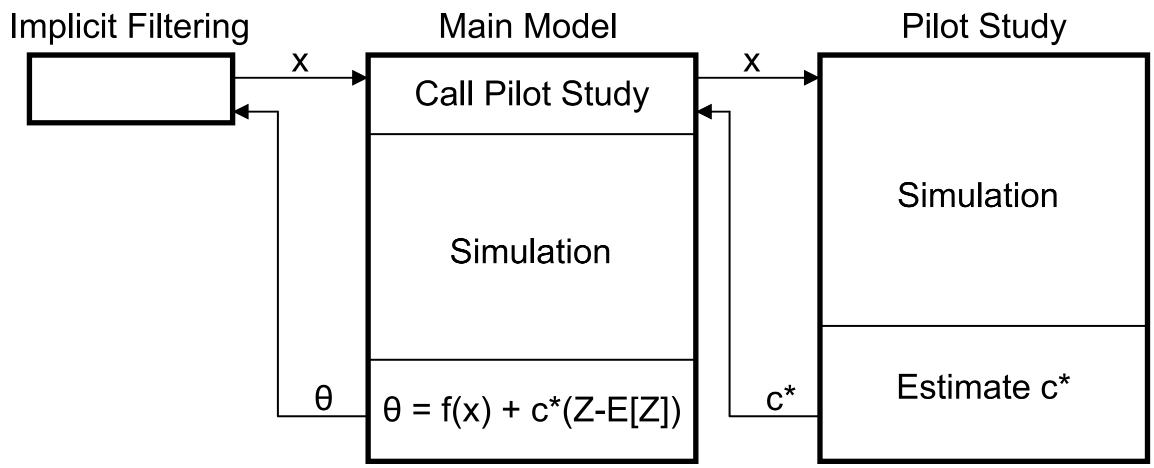

Figure 2. Schematic of optimization algorithm, model, and pilot study.

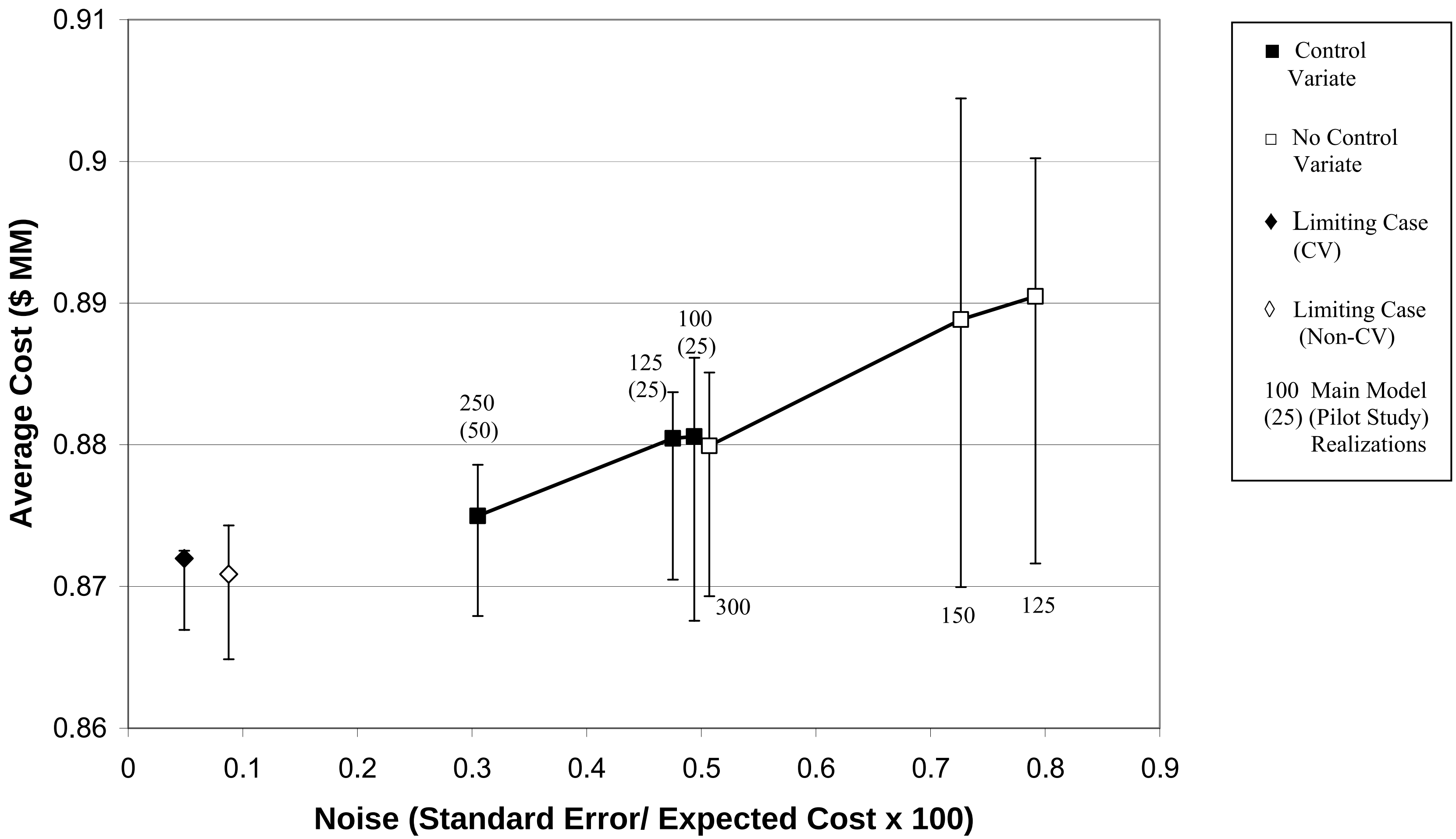

Figure 3. Average optimized results of the one-year model produced when controlling for noise, with and without the CV method. The scenario allowed the use of permanent rights, options, and leases, and initial conditions set to R_0 = 800,000 ac-ft and f_{R_0} =0.1. Note: Error bars represent 25th and 75th percentiles.

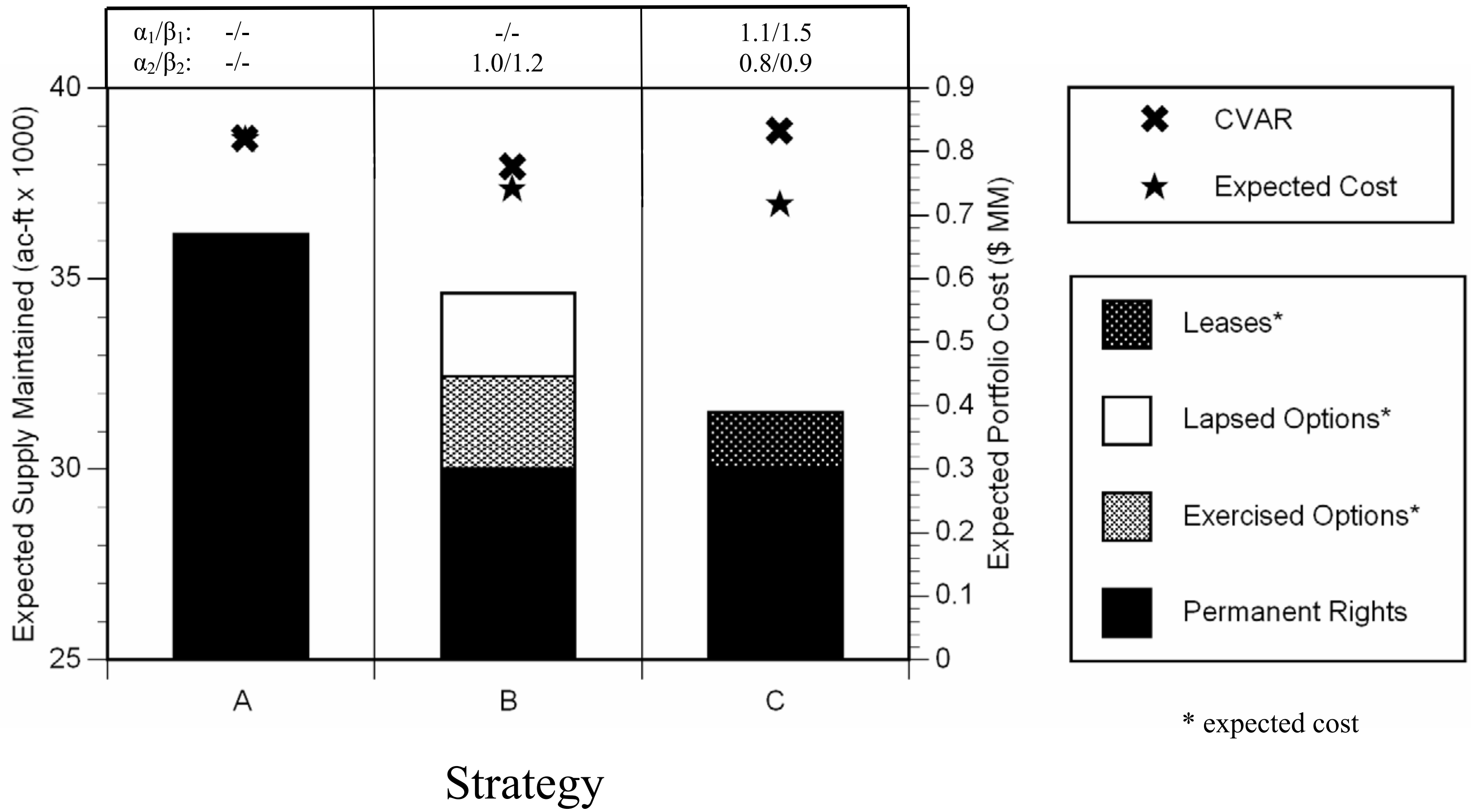

Figure 4. Optimized one-year portfolio results.

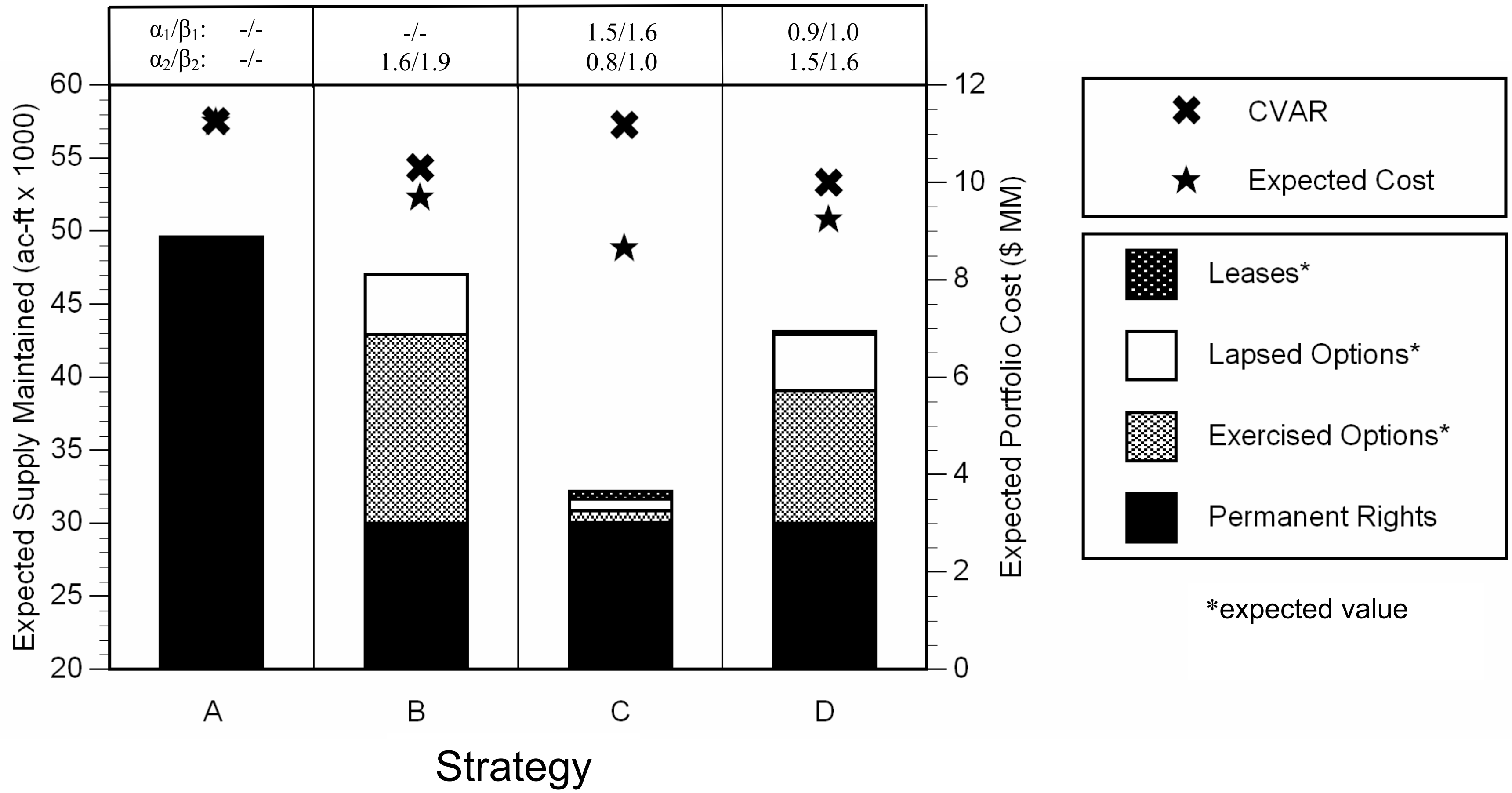

Figure 5. Optimized 10-year portfolio results.

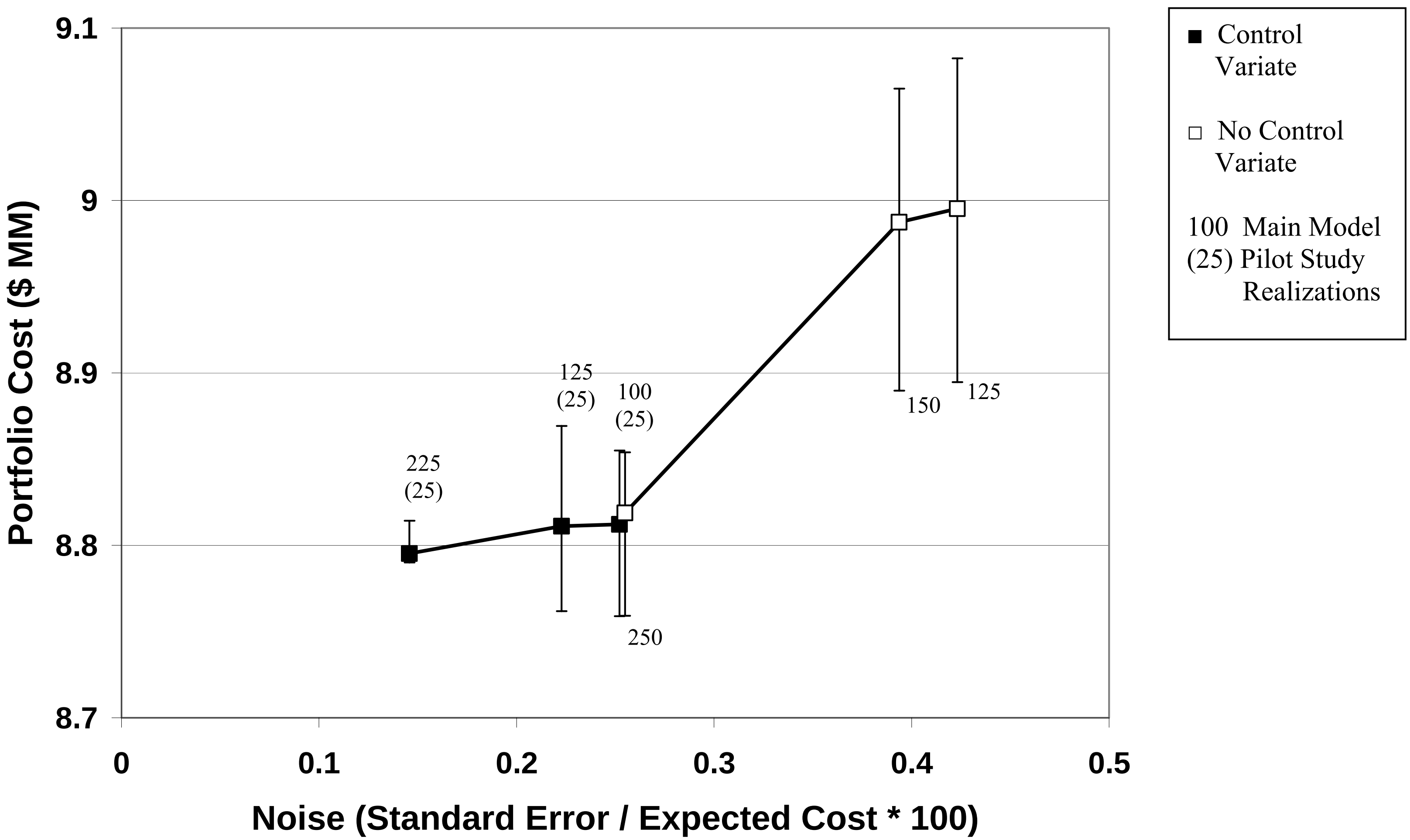

Figure 6. Average optimized results of the 10-year model obtained controlling for noise, with and without the CV method. The scenario allowed the use of permanent rights, options, and leases, and initial conditions set to R_0 = 800,000 ac-ft and f_{R_0} =0.1. Note: Error bars represent 25th and 75th percentiles.

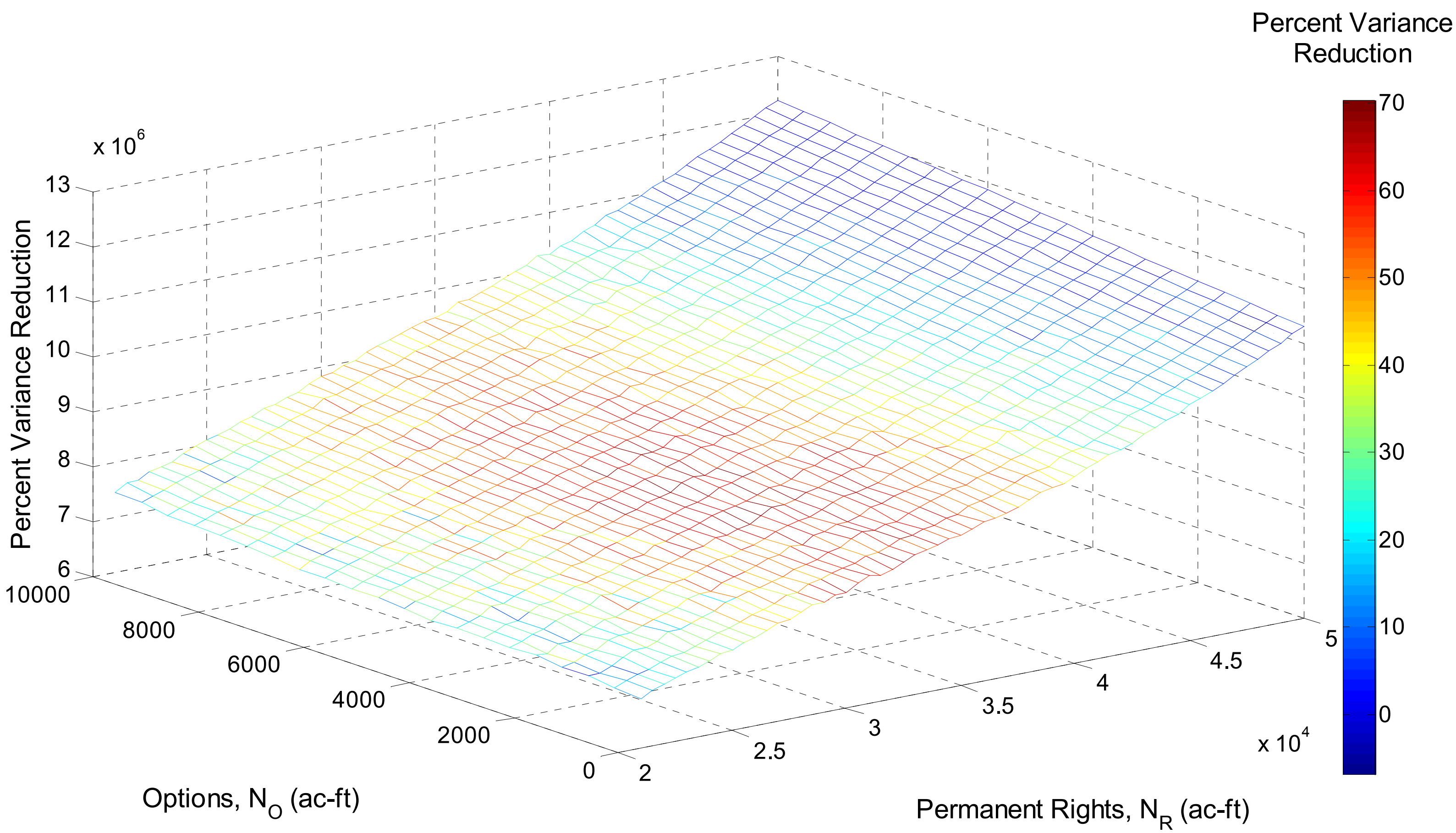

Figure 7. Map of variance reduction achieved across a range of permanent rights and options. (α_1= 1.3; β_1= 1.5; α_2= 1.0; β_2= 1.15

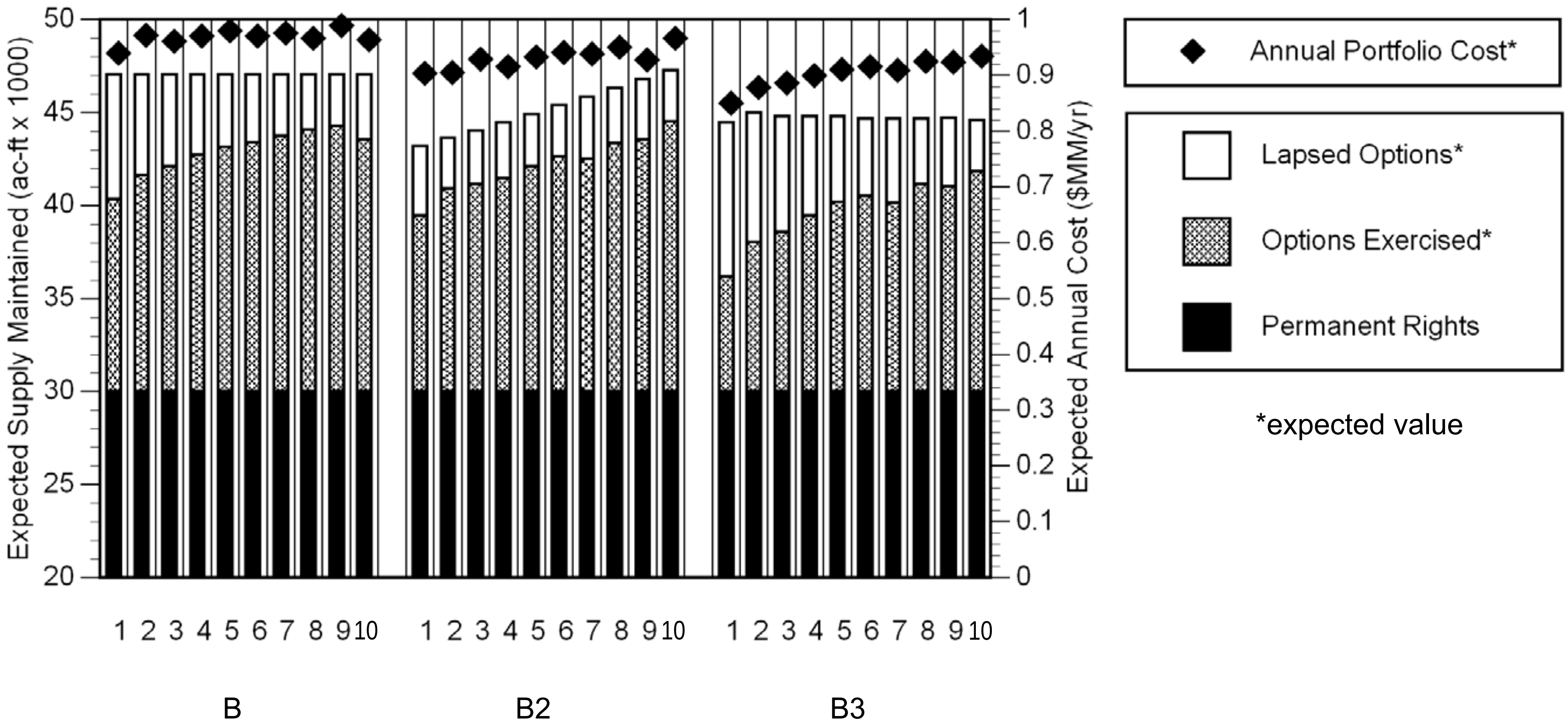

Figure 8. Expected option activity over 10 years, given (B) static option purchase, (B2) option purchases that grow with expected demand growth, and (B3) two levels of option purchases, dependent upon water supply conditions.

Table 1. Summary of control variates used in multi-year scenarios

Control Variate	Applied in Year(s)	Uses…	… to Account for Volatility Arising from…	…Which is Influenced by…
Z_{p_L}	1	The lease price at t_0	Price variability of leases purchased at the beginning of the year (t_0)	Reservoir storage in t_0.
$Z^{E}_{NS,k}$	1 – 10	The rate of new water allocation (net supply) from t_0 to t_4 in year k	Variability in leases purchased and/or options exercised in t_5 in year k	Volume of water available to city in t_5.
$Z^{L}_{NS,k}$	2 – 10	The net supply from months t_5 to t_{11} in year k-1	Variability in quantity of leases purchased in t_0 of year k	Volume of water available to city in t_0.