Tutorial 9 - ART Stopping Criteria

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Overview

◆ StoppingCriterion

enum **CTL::ARTReconstructor::StoppingCriterion**

Enumeration for stopping criteria that can be used in **ARTReconstructor**. The enumeration values can be used as flags, i.e. they can be combined via logical OR operation to create arbitrary combinations of criteria to enable (for enabling see **setStoppingCriteria()**).

Enumerator	
NoStoppingCriterion	Using no stopping criterion at all. Note that reconstructions performed with this stopping criterion mode will not terminate.
MaximumNbIterations	Reconstruction will terminate after a defined total number of iterations has been performed. A single iteration includes processing all subsets associated with that particular iteration. The maximum number of iterations to check for when using this criterion can be set via **setMaxNbIterations()**.
MaximumTime	Reconstruction will terminate after a defined total time for reconstruction has passed. Note that stopping criteria are evaluated at the end of a full iteration (i.e. after all subsets have been processed). Therefore, the specified time may be exceeded by a more or less extensive amount, depending on the required duration for a single iteration. In other words, reconstruction stops after the first full iteration at which total time exceeds the threshold. The maximum reconstruction time to check for when using this criterion can be set via **setMaxNbIterations()**.
ProjectionErrorChange	Reconstruction will terminate when the relative change in the projection error from one iteration to the other falls below a defined threshold. Assume that ΔP_{old} is the projection error of the previous iteration and ΔP_{new} that of the current one, and given a threshold of ΔP_{min}, the stopping criterion is reached if $\frac{\lvert\Delta P_{new} - \Delta P_{old}\rvert}{\Delta P_{old}} < \Delta P_{min}$ The minimum relative change in projection error (i.e. threshold ΔP_{min}) to check for when using this criterion can be set via **setMinChangeInProjectionError()**.
VolumeDomainChange	Reconstruction will terminate when the relative change in the volume domain from one iteration to the other falls below a defined threshold. Assume that V_{old} is the volume estimate of the previous iteration and V_{new} that of the current one, and given a threshold of ΔV_{min}, the stopping criterion is reached if $\frac{\lVert V_{new} - V_{old}\rVert_{L2}}{\lVert V_{new}\rVert_{L2}} < \Delta V_{min}$ The minimum relative change in projection error (i.e. threshold ΔV_{min}) to check for when using this criterion can be set via **setMinChangeInVolumeDomain()**.
RelativeProjectionError	Reconstruction will terminate when the relative projection error in an iteration (w.r.t. the norm of the original projections) falls below a defined threshold. Assume that P_{sim} are the simulated projections of the current iteration and P_{orig} are the original ('measured') data, and given a threshold of ΔP_{min}, the stopping criterion is reached if $\frac{\lVert P_{sim} - P_{orig}\rVert_{L2}}{\lVert P_{orig}\rVert_{L2}} < \Delta P_{min}$ The minimum relative projection error (i.e. threshold ΔP_{min}) to check for when using this criterion can be set via **setMinRelativeProjectionError()**.
NormalEquationSatisfied	Reconstruction will terminate when the relative deviation from fulfilling the normal equation in an iteration falls below a defined threshold. Full details on the criterion can be found in **terminateByNormalEqTol()**. The threshold to check for when using this criterion can be set via **setNormalEqTolerance()**.
AllStoppingCriteria	Convenience enumeration value that contains all flags. Thus, enables all stopping criteria simultaneously if used.

The structure of the ART method

- We consider the linear system
$$Ax = b$$
- In presence of noise, no solution exists; thus, we solve
$$\min_x \|Ax - b\|^2$$
- which is equivalent to searching for the solution of the normal equation
$$A^{\mathrm{T}}(Ax - b) = 0$$
- a solution can be found by the Landweber iteration
$$x \to x - \omega A^{\mathrm{T}}(Ax - b)\ , \quad 0 < \omega < \omega_{\max}$$

Ordered Subsets (OS)

- If the system is subdivided in subsets A_i, $i = 0, ..., \texttt{nbSubsets} - 1$, then

$$x \to x - r\omega A_i^{\mathrm{T}}(A_i x - b) \ ,$$

$$r = \frac{\texttt{totalNbViews}}{\texttt{nbViewsPerSubset}}$$

- $\texttt{nbViewsPerSubset} = 1 \to$ SART type (maximum $\texttt{nbSubsets} \to$ max. speed)
- $\texttt{nbViewsPerSubset} = \texttt{totalNbViews} \to$ SIRT type (no OS $\to$ min. noise)
- Note that if you call `ARTReconstructor :: setRelaxation(float relax)` to set the relaxation manually, then

$$\texttt{relax} = \texttt{totalNbViews} \cdot \omega$$

Stopping Criteria

- for try out/play around with the `ARTReconstructor`
  - `MaximumNbIterations` (the default; set to 5)
    (1 iteration means all subsets have been processed)
  - `MaximumTime`
  - `VolumeDomainChange` or `ProjectionErrorChange`
    depends on the relaxation parameter and on the subset size

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- if noise level in known
  - `RelativeProjectionError`
    $\frac{\|Ax-b\|}{\|b\|} < \text{tol}$

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- if noise level in known
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    $\frac{\|Ax-b\|}{\|b\|} < \text{tol}$
- general criterion for quantitative analysis
  - `NormalEquationSatisfied`
    $\frac{\left\|A^T(Ax-b)\right\|}{\|A^Tb\|} < \text{tol}$
    For performance reasons, this criterion is only approximated in case of OS.