

| BEE core - Order Representation | |
|---|--|
| Declaring Variables | (Booleans and Integers) |
| (1) <code>new_bool(X)</code> | declare Boolean X |
| (2) <code>new_int(I, c₁, c₂)</code> | declare (order) I , $c_1 \leq I \leq c_2$ |
| (2b) <code>new_int(I, D)</code> | declare (order) I , <code>member(I, D)</code> |
| (3) <code>bool2int(X, I)</code> | $(X \Leftrightarrow I = 1) \wedge (\neg X \Leftrightarrow I = 0)$ |
| (3b) <code>int_order2bool_array(I, Xs, c_{min})</code> | $\bigwedge_{0 \leq i < Xs } Xs[i] \Leftrightarrow (I \geq i + c_{\min})$ |
| Boolean (reified) statements | $op \in \{\text{or, and, xor, iff}\}$ |
| (4) <code>bool_eq(X₁, X₂)</code> or <code>bool_eq(X₁, -X₂)</code> | $X_1 = X_2$ or $X_1 = -X_2$ |
| (5) <code>bool_array_eq_reif([X₁, ..., X_n], X)</code> | $(\bigwedge_{i < j} X_i = X_j) \Leftrightarrow X$ |
| (6) <code>bool_array_op([X₁, ..., X_n])</code> | $X_1 \text{ op } X_2 \cdots \text{ op } X_n$ |
| (7) <code>bool_array_op_reif([X₁, ..., X_n], X)</code> | $X_1 \text{ op } X_2 \cdots \text{ op } X_n \Leftrightarrow X$ |
| (8) <code>bool_op_reif(X₁, X₂, X)</code> | $X_1 \text{ op } X_2 \Leftrightarrow X$ |
| (9) <code>bool_ite(X₁, X₂, X₃)</code> | $X_1 ? X_2 : X_3$ |
| (10) <code>bool_ite_reif(X₁, X₂, X₃, X)</code> | $(X_1 ? X_2 : X_3) \Leftrightarrow X$ |
| Integer relations (reified) | $rel \in \{\text{leq, geq, eq, lt, gt, neq}\}$ |
| and arithmetic | $op \in \{\text{plus, times, div, mod, max, min}\}$, $op' \in \{\text{plus, times, max, min}\}$ |
| (11) <code>int_rel(I₁, I₂)</code> | $I_1 \text{ rel } I_2$ |
| (12) <code>int_rel_reif(I₁, I₂, X)</code> | $I_1 \text{ rel } I_2 \Leftrightarrow X$ |
| (13) <code>int_array_allDiff([I₁, ..., I_n])</code> | $\bigwedge_{i < j} I_i \neq I_j$ |
| (14) <code>int_array_allDiff_reif([I₁, ..., I_n], X)</code> | $(\bigwedge_{i < j} I_i \neq I_j) \Leftrightarrow X$ |
| (15) <code>int_array_allDiffCond([I₁, ..., I_n], [X₁, ..., X_n])</code> | $\bigwedge_{i < j} (X_i \wedge X_j \Rightarrow I_i \neq I_j)$ |
| (16) <code>int_abs(I₁, I)</code> | $ I_1 = I$ |
| (17) <code>int_op(I₁, I₂, I)</code> | $I_1 \text{ op } I_2 = I$ |
| (18) <code>int_array_op'([I₁, ..., I_n], I)</code> | $I_1 \text{ op}' \cdots \text{ op}' I_n = I$ |
| Cardinality | $rel \in \{\text{leq, geq, eq, lt, gt}\}$ |
| (19) <code>bool_array_sum_rel([X₁, ..., X_n], I)</code> | $(\sum X_i) \text{ rel } I$ |
| (20) <code>bool_array_pb_rel([c₁, ..., c_n], [X₁, ..., X_n], I)</code> | $(\sum c_i * X_i) \text{ rel } I$ |
| (21) <code>int_array_sum_rel([I₁, ..., I_n], I)</code> | $(\sum I_i) \text{ rel } I$ |
| (22) <code>int_array_lin_rel([c₁, ..., c_n], [I₁, ..., I_n], I)</code> | $(\sum c_i * I_i) \text{ rel } I$ |
| Cardinality Module K | |
| only support non-negative integers | |
| (23) <code>bool_array_sum_modK([X₁, ..., X_n], c, I)</code> | $((\sum X_i) \bmod c) = I$ |
| (24) <code>bool_array_sum_divK([X₁, ..., X_n], c, I)</code> | $((\sum X_i) / c) = I$ |
| (25) <code>bool_array_sum_divModK([X₁, ..., X_n], c, I_d, I_m)</code> | $((\sum X_i) \bmod c) = I_d$ $((\sum X_i) / c) = I_m$ |
| (26) <code>int_array_sum_modK([I₁, ..., I_n], c, I)</code> | $((\sum I_i) \bmod c) = I$ |

| | |
|--|--|
| (27) <code>int_array_sum_divK</code> ($[I_1, \dots, I_n], c, I$) | $((\Sigma I_i) / c) = I$ |
| (28) <code>int_array_sum_divModK</code> ($[I_1, \dots, I_n], c, I_d, I_m$) | $((\Sigma I_i) / c) = I_d$ $((\Sigma I_i) \bmod c) = I_m$ |
| Boolean arrays relations (reified) | |
| <i>if the arrays are of different length then pad shorter with zeros</i> | |
| (29) <code>bool_arrays_rel</code> (Xs_1, Xs_2) | $Xs_1 \text{ rel } Xs_2$ |
| (30) <code>bool_arrays_rel_reif</code> (Xs_1, Xs_2, X) | $Xs_1 \text{ rel } Xs_2 \Leftrightarrow X$ |
| (31) <code>bool_arrays_lex</code> (Xs_1, Xs_2) | $Xs_1 \preceq Xs_2$ |
| (32) <code>bool_arrays_lexLt</code> (Xs_1, Xs_2) | $Xs_1 \prec Xs_2$ |
| (33) <code>bool_arrays_lex_reif</code> (Xs_1, Xs_2, X) | $Xs_1 \preceq Xs_2 \Leftrightarrow X$ |
| (34) <code>bool_arrays_lexLt_reif</code> (Xs_1, Xs_2, X) | $Xs_1 \prec Xs_2 \Leftrightarrow X$ |
| Integer arrays relations (reified) | |
| <i>rel</i> $\in \{\text{eq, neq}\}$ | |
| (35) <code>int_arrays_rel</code> (Is_1, Is_2) | $Is_1 \text{ rel } Is_2$ |
| (36) <code>int_arrays_rel_reif</code> (Is_1, Is_2, X) | $Is_1 \text{ rel } Is_2 \Leftrightarrow X$ |
| (37) <code>int_arrays_lex</code> (Is_1, Is_2) | $Is_1 \preceq Is_2$ |
| (38) <code>int_arrays_lexLt</code> (Is_1, Is_2) | $Is_1 \prec Is_2$ |
| (39) <code>int_arrays_lex_implied</code> (Is_1, Is_2, X) | $X \Rightarrow Is_1 \preceq Is_2$ |
| (40) <code>int_arrays_lexLt_implied</code> (Is_1, Is_2, X) | $X \Rightarrow Is_1 \prec Is_2$ |
| (41) <code>int_arrays_lex_reif</code> (Is_1, Is_2, X) | $Is_1 \preceq Is_2 \Leftrightarrow X$ |
| (42) <code>int_arrays_lexLt_reif</code> (Is_1, Is_2, X) | $Is_1 \prec Is_2 \Leftrightarrow X$ |

Table 1: Syntax of BEE Constraints.

| BEE core - Direct Representation | |
|---|--|
| Declaring Variables | |
| (1) <code>new_direct</code> (I, c_1, c_2) | declare (direct) $I, c_1 \leq I \leq c_2$ |
| (2) <code>int_direct2bool_array</code> (I, Xs, c_{\min}) | $\bigwedge_{0 \leq i < Xs } Xs[i] \Leftrightarrow (I = i + c_{\min})$ |
| Integer relations (reified) | |
| <i>rel</i> $\in \{\text{leq, geq, eq, lt, gt, neq}\}$ | |
| (3) <code>direct_rel</code> (I_1, I_2) | $I_1 \text{ rel } I_2$ |
| (4) <code>direct_array_allDiff</code> ($[I_1, \dots, I_n]$) | $\bigwedge_{i < j} I_i \neq I_j$ |

Table 2: Syntax of Direct BEE Constraints.

| BEE core - Mix Representation | |
|--|---|
| BEE integers can have multi bit-level representations, and can be defined using one of the following: <code>new_int</code> (order), <code>new_direct</code> (direct), <code>new_int_dual</code> (order & direct). | |
| While most BEE constraints are applied only on the Order encoding of the integer (Table 1), few applied only on the Direct encoding (Table 2), and other few applied on both (e.g. <code>int_array_allDiff</code>). | |
| When a constraint is applied on an integer and the integer doesn't have (all) the requested representation(s) BEE will add the required representation to the integer and add a channeling constraint automatically. | |
| Declaring Mix Variables | |
| (1) <code>new_int_dual(I, c₁, c₂)</code> | declare (order & direct) $I, c_1 \leq I \leq c_2$ |
| Channeling Variables | |
| (2) <code>channel_int2direct(I)</code> | Adds Direct representation to an Order integer. |
| (3) <code>channel_direct2int(I)</code> | Adds Order representation to a Direct integer. |

Table 3: Syntax of Mix BEE Constraints.