PSTAT 174 HW #5

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Problem 1

Assume that the data are realized from an $AR(2)$ process. Then this process can be modelled as

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + Z_t$$

Let $\hat{\gamma}(0) = 1382.2$, $\hat{\gamma}(1) = 1114.4$, $\hat{\gamma}(2) = 591.73$, and $\hat{\gamma}(3) = 96.216$.

For an $AR(2)$ process, we have the following Yule Walker equations:

$$\begin{bmatrix} \hat{\gamma}(0) & \hat{\gamma}(1) \\ \hat{\gamma}(1) & \hat{\gamma}(0) \end{bmatrix} \begin{bmatrix} \hat{\phi}_1 \\ \hat{\phi}_2 \end{bmatrix} = \begin{bmatrix} \hat{\gamma}(1) \\ \hat{\gamma}(2) \end{bmatrix}$$

To find the estimates $\hat{\phi}_1$ and $\hat{\phi}_2$, we must invert the matrix.

$$\begin{bmatrix} \hat{\phi}_1 \\ \hat{\phi}_2 \end{bmatrix} = \begin{bmatrix} \hat{\gamma}(1) \\ \hat{\gamma}(2) \end{bmatrix} \begin{bmatrix} \hat{\gamma}(0) & \hat{\gamma}(1) \\ \hat{\gamma}(1) & \hat{\gamma}(0) \end{bmatrix}^{-1}$$

$$= \frac{1}{\hat{\gamma}(0)^2 - \hat{\gamma}(1)^2} \begin{bmatrix} \hat{\gamma}(0) & -\hat{\gamma}(1) \\ -\hat{\gamma}(1) & \hat{\gamma}(0) \end{bmatrix} \begin{bmatrix} \hat{\gamma}(1) \\ \hat{\gamma}(2) \end{bmatrix}$$

$$= \frac{1}{\hat{\gamma}(0)^2 - \hat{\gamma}(1)^2} \begin{bmatrix} \hat{\gamma}(0)\hat{\gamma}(1) & -\hat{\gamma}(1)\hat{\gamma}(2) \\ -\hat{\gamma}(1)^2 & \hat{\gamma}(0)\hat{\gamma}(2) \end{bmatrix}$$

We now substitute in the estimators for our autocovariances to find the values of $\hat{\phi}_1$ and $\hat{\phi}_2$.

$$\hat{\phi}_1 = \frac{\hat{\gamma}(0)\hat{\gamma}(1) - \hat{\gamma}(2)\hat{\gamma}(1)}{\hat{\gamma}(0)^2 - \hat{\gamma}(1)^2} = 1.3175$$

$$\hat{\phi}_2 = \frac{\hat{\gamma}(0)\hat{\gamma}(2) - \hat{\gamma}(1)^2}{\hat{\gamma}(0)^2 - \hat{\gamma}(1)^2} = -0.6342$$

Finally,

$$\begin{aligned} \hat{\sigma}^2 &= \hat{\gamma}(0) - \begin{bmatrix} \hat{\phi}_1 & \hat{\phi}_2 \end{bmatrix} \begin{bmatrix} \hat{\gamma}(1) \\ \hat{\gamma}(2) \end{bmatrix} \\ &= \hat{\gamma}(0) - \hat{\phi}_1\hat{\gamma}(1) - \hat{\phi}_2\hat{\gamma}(2) \\ &= 289.253 \end{aligned}$$

We may now construct confidence intervals for ϕ_1 and ϕ_2. Since the sample of data points is sufficiently large, we may write that

$$\begin{aligned}\hat{\phi}_1,\ \hat{\phi}_2 &\sim N(\begin{bmatrix}\phi_1 & \phi_2\end{bmatrix}, \frac{\sigma^2}{\hat{\gamma}(0)^2 - \hat{\gamma}(1)^2}\begin{bmatrix}\hat{\gamma}(0) & -\hat{\gamma}(1)\\ -\hat{\gamma}(1) & \hat{\gamma}(0)\end{bmatrix}/100)\\ &\sim N(\begin{bmatrix}\phi_1 & \phi_2\end{bmatrix}, 0.0063)\end{aligned}$$

This yields the following intervals:

$$\begin{aligned}I_{\phi_1} &= \hat{\phi}_1 \pm Z_{0.025}\sqrt{0.0063}\\ &= 1.3175 \pm 1.96\sqrt{0.0063} \quad = 1.3175 \pm 0.15557\end{aligned}$$

and

$$\begin{aligned}I_{\phi_2} &= \hat{\phi}_2 \pm Z_{0.025}\sqrt{0.0063}\\ &= -0.6342 \pm 1.96\sqrt{0.0063} \quad = -0.6342 \pm 0.15557\end{aligned}$$

Problem 2

We begin calculation of the partial autocorrelations $\hat{\phi}_{11}$, $\hat{\phi}_{22}$, and $\hat{\phi}_{33}$ by setting

$$\hat{\phi}_{11} = \hat{\rho}(1) = \frac{\hat{\gamma}(1)}{\hat{\gamma}(0)} = 0.806$$

By definition of the Durben-Levinson Algorithm,

$$\hat{\phi}_{22} = \frac{\hat{\rho}(2) - \hat{\phi}_{11}\hat{\rho}(1)}{1 - \hat{\phi}_{11}\hat{\rho}(1)} = \frac{0.428 - (0.806)^2}{1 - (0.806)^2} = -0.633$$

and

$$\hat{\phi}_{21} = \hat{\phi}_{11} - \hat{\phi}_{22}\hat{\phi}_{11} = 1.316$$

Finally we compute $\hat{\phi}_{33}$.

$$\hat{\phi}_{33} = \frac{\hat{\rho}(3) - \hat{\phi}_{21}\hat{\rho}(2) - \hat{\phi}_{22}\hat{\rho}(1)}{1 - \hat{\phi}_{21}\hat{\rho}(1) - \hat{\phi}_{22}\hat{\rho}(2)} = 0.081$$

To check if the value of $\hat{\phi}_{33}$ is compatible with the hypothesis that the data are generated from an $AR(2)$ process, we may assume that $\hat{\phi}_{33} \sim N(0, \frac{1}{100})$. We now construct a 95% confidence interval.

$$I_{\phi_{33}} = \hat{\phi}_{33} \mp Z_{0.025}\sqrt{1/100} = 0.081 \mp 1.96(0.1) = (-0.115, 0.277)$$

If these data were generated by an $AR(2)$ process, we would expect $\phi_{33} = 0$. Since $0 \in I_{\phi_{33}}$, our hypothesis is supported and we can conclude that the value of $\hat{\phi}_{33}$ is consistent with an $AR(2)$ model.

Problem 3

```r
# Get the current working directory
getwd()

# Set the working directory
setwd()

# Read data
read.table() or read.csv()

# Create Time-Series Object
ts()

# Plot at Time-Series Object
plot.ts()

# Simulate an ARMA Model
arima.sim()

# Add mean line to original time-series plot
abline(h=mean(ts, na.rm=T))

# Add trend line to original time-series plot
fit <- lm(ts ~ as.numeric(1:length(ts)))
abline(fit)

# Calculate and Plot Theoretical ACFs and PACFs
ARMAacf(..., pacf = FALSE)
ARMAacf(..., pacf = TRUE)
plot(ARMAacf())

# Calculate and Plot Same ACFs and PACFs
acf(..., plot = TRUE)
pacf(..., plot = TRUE)

# Check for Model Causality / Invertibility
polyroot()

# Perform Box-Cox Tranformation
library(MASS)
t = 1:length(timeseries)
fit = lm(wine ~ t)
bcTransform = boxcox(timeseries ~ t,plotit = TRUE)

# Perform differencing at lags 1 and 12
diff(ts, lag = 1)
diff(ts, lag = 12)

# Perform Yule-Walker Estimations and find variance of estimates
fit <- ar(ts, method="yule-walker")

# Forecast future observations of a model
predict(ts,)
```

```r
# Compare models using AICC:
AICc(fittedModel)
```