PSTAT 174 HW #6

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Problem 1

Given that $\sum_{i=1}^{5} \hat{\rho}_{\hat{W}}(i) = 0.005$, we may calcuate the the Protmanteau test statistic.

$$Q_W = n \sum_{j=1}^{h} \hat{\rho}_{\hat{W}}^2(j) = 50(0.005) = 0.25 \sim \chi^2_{7-1-1}$$

The the critical value for $\chi^2_{1-0.05,5}$ is \$1.145. Since $0.25 < 1.145$, we fail to reject the null hypothesis, which states that the residuals are white noise. Therefore we conclude that the residuals are compatible with the proposed model.

Problem 2

Portmanteau Test

$$Q_W = n \sum_{j=1}^{h} \hat{\rho}_{\hat{W}}^2(j) = 100[0.20^2 + 0.15^2 + 0.18^2 + 0.16^2 + 0.08^2 + 0.07^2 + 0.09^2] = 13.99 \sim \chi^2_{10-1-2}$$

Ljung-Box Test

$$\begin{aligned}\tilde{Q}_W = n(n+2)\frac{\sum_{j=1}^{h} \hat{\rho}_{\hat{W}}^2(j)}{n-j} &= 10200[0.20^2/99 + 0.15^2/98 + 0.18^2/97 + 0.16^2/96 + 0.08^2/95 + 0.07^2/94 + 0.09^2/93] \\ &= 14.697 \sim \chi^2_{10-1-2}\end{aligned}$$

Problem 3

$$\begin{aligned} AIC_{AR(2)} &= -2 \times \text{log-likelihood} + 2 \times \text{number of free parameters} \\ &= -2(-36.4) + 2(2) \\ &= 76.8 \end{aligned}$$

$$AIC_{MA(1)} = -2(-36.51) + 2(1) = 75.02$$

$$AIC_{ARMA(2,1)} = -2(-36.02) + 2(3) = 78.04$$

Since a smaller AIC is *better*, we would rank our time series $MA(1)$, $AR(1)$, $ARMA(2,1)$, from best to worst.