PSTAT 174 HW #6

Chris Meade

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Problem 1

Given that $\sum_{i=1}^{5} \hat{\rho}_{\hat{W}}(i) = 0.005$, we may calcuate the the Protmanteau test statistic.

$$Q_W = n \sum_{j=1}^{h} \hat{\rho}^2_{\hat{W}}(j) = 50(0.005) = 0.25 \sim \chi^2_{7-1-1}$$

The the critical value for $\chi^2_{1-0.05,5}$ is \$1.145. Since $0.25 < 1.145$, we fail to reject the null hypothesis, which states that the residuals are white noise. Therefore we conclude that the residuals are compatible with the proposed model.

Problem 2

Portmanteau Test

$$Q_W = n \sum_{j=1}^{h} \hat{\rho}^2_{\hat{W}}(j) = 100[0.20^2 + 0.15^2 + 0.18^2 + 0.16^2 + 0.08^2 + 0.07^2 + 0.09^2] = 13.99 \sim \chi^2_{10-1-2}$$

Ljung-Box Test

$$\begin{aligned}\tilde{Q}_W = n(n+2)\frac{\sum_{j=1}^{h} \hat{\rho}^2_{\hat{W}}(j)}{n-j} &= 10200[0.20^2/99 + 0.15^2/98 + 0.18^2/97 + 0.16^2/96 + 0.08^2/95 + 0.07^2/94 + 0.09^2/93] \\ &= 14.697 \sim \chi^2_{10-1-2}\end{aligned}$$

Problem 3

$$\begin{aligned} AIC_{AR(2)} &= -2 \times \text{log-likelihood} + 2 \times \text{number of free parameters} \\ &= -2(-36.4) + 2(2) \\ &= 76.8 \end{aligned}$$

$$AIC_{MA(1)} = -2(-36.51) + 2(1) = 75.02$$

$$AIC_{ARMA(2,1)} = -2(-36.02) + 2(3) = 78.04$$

Since a smaller AIC is *better*, we would rank our time series $MA(1)$, $AR(1)$, $ARMA(2,1)$, from best to worst.

Problem 4

We assume, for contradiction, that the model $X_t - \phi_1 X_{t-1} = Z_t + \theta_1 Z_{t-1}$ is appropriate. This implies that the ACF of the residuals would be 0 at all lags $h > 0$. That is, $0 \in I_{\hat{\rho}_{\hat{W}}(h)}\ \forall h > 0$, where $I_{\hat{\rho}_{\hat{W}}(h)}$ represents the 95% confidence interval for $\hat{\rho}_{\hat{W}}(h)$.

Let $h = 1$. Then

$$
\begin{aligned}
I_{\hat{\rho}_{\hat{W}}(1)} &= \hat{\rho}_{\hat{W}}(1) \mp Z_{0.025} Sd_{\hat{\rho}_{\hat{W}}(1)}/\sqrt{n} \\
&= .50 \mp 1.96(0.08/\sqrt{(}26)) \\
&= (0.47,\ 0.53)
\end{aligned}
$$

But clearly $0 \notin I_{\hat{\rho}_{\hat{W}}(1)}$, so we arrive at a contradiction and conclude that the given model is not appropriate.