University of Burgundy

Advanced Image Analysis

Project

Discrete 2D Wavelet Transform

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1 Introduction

The objective of this project is to grab the knowledge of wavelet transform and perform the image denoising as application. Two functions which are doing decomposition and reconstruction are programmed. The wavelet transform decomposition outputs an array of wavelet coefficients given an input image, the number of levels and a low-pass filter. On contrast, the reconstruction wavelet transform outputs a reconstructed image given the wavelet coefficients, the number of levels and the low-pass filter. The decomposition and reconstruction are inverse operations.

In this project, we are dealing with an image which is a 2D discrete signal. The main strategy is to use a one dimensional filter and convolve the rows and columns separately.

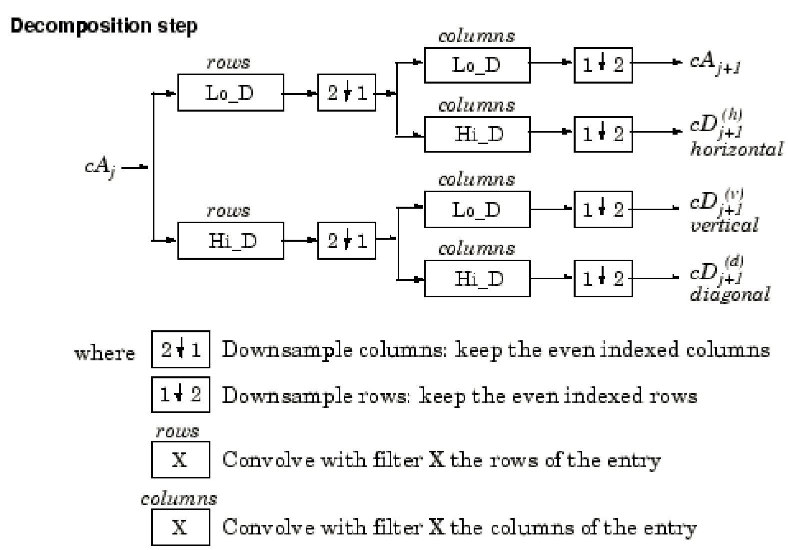


Figure 1: Decomposition step

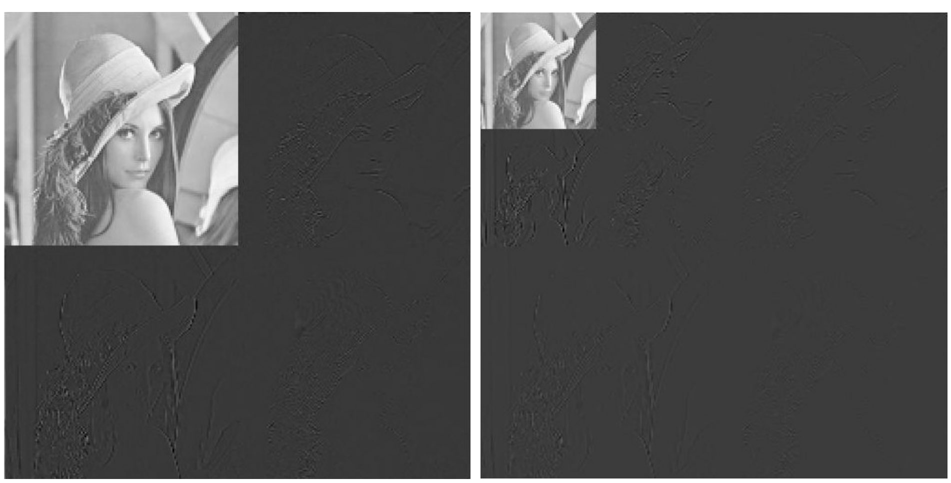
2 Image Decomposition

This kind of two-dimensional discrete wavelet transform leads to a decomposition of approximation coefficients at level j in four components: the approximation at level j+1, and the details in three orientations (horizontal, vertical, and diagonal). The decomposition step is showing in figure 1.

$$function[coef] = wt\_dec\_2d(img, h) \tag{1}$$

A function of decomposing the image with one level is programmed(1), where the input is an image NxN image and a low-pass filter. The output is an array of NxN coefficients.

Along the decomposition step, firstly given a low-pass filter, the high-pass filter is constructed. Then the pixels in rows of the image are convolved with the low and high pass filters respectively. Since there's less information after filtering, the signals are downsampled by only keeping the even indexed columns. At this moment, there are two parts which are the approximation and the detail. As we know, the approximation is remained after low-pass filter. The high-pass filter is removing the details. Then, the columns of the approximation and detail are convolved by low and high pass filters. The four parts are downsampled again by keeping the even indexed rows. So that the approximation, the horizontal detail, the vertical detail and the diagonal detail are obtained. The size of each component is a quarter of the original image.



(a) One-level decomposition. (b) Two-level decomposition.

Figure 2

$$function[coef] = wt\_dec\_2d\_mul\_lev(img, J, h) \tag{2}$$

A function which is doing multilevel decomposition is programmed based on the single level wavelet transform decomposition(2). Here, J is the number of the levels. Basically, this function is looping the previous one. The approximation component from the output is the input for next level.

3 Image Reconstruction

The image wavelet transform reconstruction is the inverse operation of the decomposition. Given the coefficients, an image can be reconstructed. The basic reconstruction step is showing in figure 3.

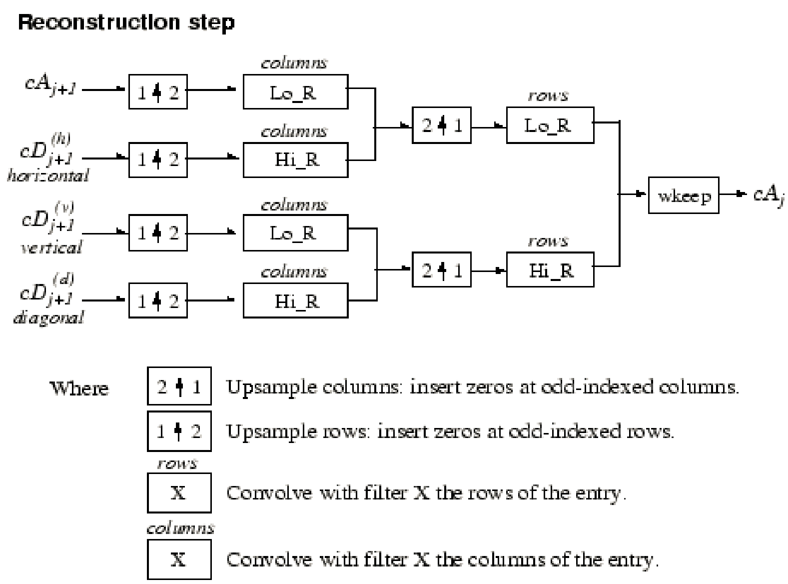


Figure 3: Reconstruction step

The four components are firstly upsampled by inserting zeros in odd indexed rows. After convolved with low and high pass filters, the approximation part is added from the approximation and horizontal detail of next level, as well as the detail part. Then the two parts are upsampled by inserting zeros in odd indexed columns. In the end, the approximation and the detail are added after convolving with low and high pass filters respectively to reconstruct the image. The image is four times as each component of coefficients.

(a) One-level reconstruction.

(b) Two-level reconstruction.

Figure 4

4 Conclusion

In this practical work, four functions about wavelet transform are programmed. They are one-level and multilevel decomposition and reconstruction. We learned that the wavelet transform captures both the frequency and the location information. The signal can be decomposed and reconstructed at any scale.