University of Burgundy

MSFT+SFM

Practical2

4 pts vs 2+1 pts algorithm

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1 Objective

The objective of this practical work is to reinforce the knowledge of homography estimation using four points' and two points' algorithm. The algorithms are tested given 50 points and also with data added white noise. RANSAC algorithm is also used to estimate the homography matrix for comparison.

2 Introduction

In the subject Structure from Motion, when we are estimating the homography. We know that at least 8 points are needed. Since there are 9 components in the homography matrix and 8 are unknown after normalization. Each point provides one equation so that for 8 unknown parameters, 8 points are needed.

2.1 4 pts algorithm

We take a hypothesis that all the points are belonging to one plane. In this case, 8 points can be reduced to 4. The homography is existing such as:

$$H = R - \frac{tn^T}{d} \tag{1}$$

We use p and p' to represent the points on two image planes, so that

$$p' \times Hp = 0 \tag{2}$$

2.2 2 pts algorithm

The scenario for 4 points algorithm is that all the points are on the same plane. If we take the hypothesis that this plane is vertical. The 4 points can then be reduces to 2 points. The homography can be computed like this:

$$H = R_y + [t_x, t_y, t_z]^T [n_x, 0, n_z] \tag{3}$$

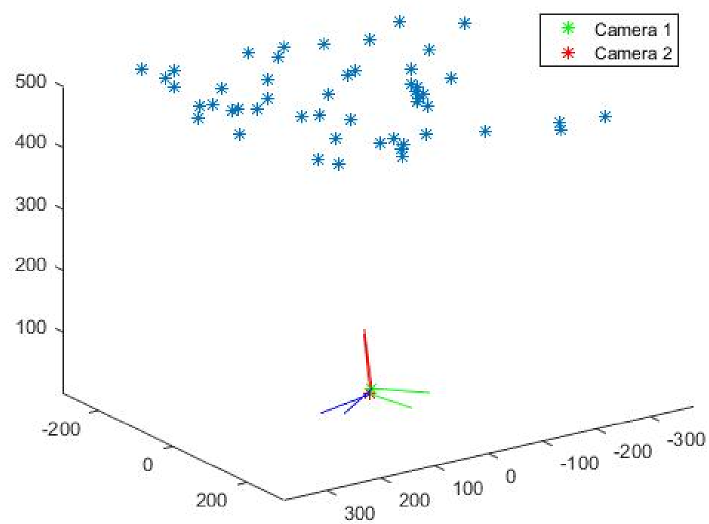

Figure 1: Random 50 points distribution on a plane & cameras' position

3 Tests and results

We are given 50 points which are randomly distributed in a plane of equation $N^T X_w + d = 0$ in the world frame (O_w, X_w, Y_w, Z_w). The camera is calibrated with the rotation R_i and T_i of the world coordinate ($X_w = R_i X_{ci} + T_i$). We can consider there are two cameras taking two images or there's one camera taking pictures twice from two views. The points are shown in figure 1.

3.1 A test with different position of the second camera

For the first test, we changed the position of the second camera (rotation and translation regarding to the world frame). From Figure 2 we can see that the second camera position is different from that in Figure 1.

3.1.1 4 pts algorithm

d is obtained by $d = (N' \times T1 - zposition)$(see more details in the code) to compute homography. Here, d is the distance between the the camera origin and the plane. Then, the theoretical hompgraphy is computed:

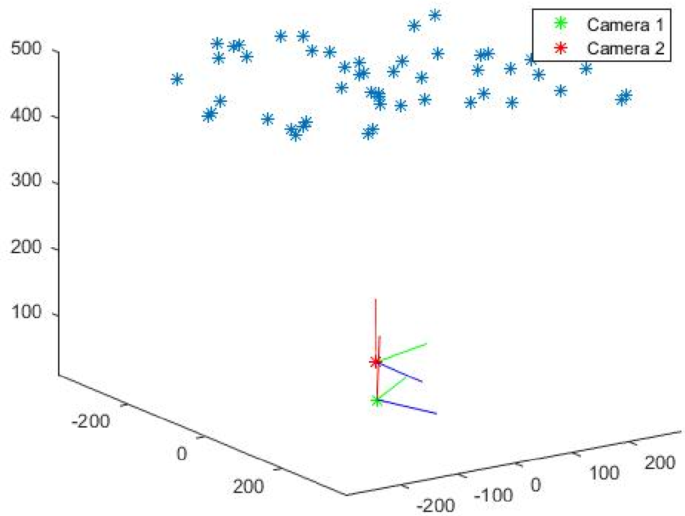

Figure 2: Different position of the second camera

$$H = \begin{matrix} 1.0628 & -0.4267 & 0.0257 \\ 0.4247 & 1.0628 & 0.0488 \\ -0.0318 & -0.0428 & 1.0000 \end{matrix}$$

For validation, we also compute HP1($HP1 = H \times p$) which is the coordinate of the points in image 2 computing from points in image 1. We got HP1 = [0.4799,-0.1990,1.0000]'. It is matching the ground truth.

Then we use the points to compute the homography:

$$\text{H4pt} = \begin{matrix} 1.0628 & -0.4267 & 0.0257 \\ 0.4247 & 1.0628 & 0.0488 \\ -0.0318 & -0.0428 & 1.0000 \end{matrix}$$

We can see that the results are the same.

3.1.2 2 pts algorithm

For 2 pts algorithm, we suppose that the roll and pitch angles are known. Z axis corresponds to the vertical direction. Using roll and pitch angles, we can obtain the points on virtual images. Homography matrix is computed by virtual image points:

$$\text{trueH} = \begin{matrix} 1.0596 & -0.4281 & -0.0169 \\ 0.4281 & 1.0596 & 0.0168 \\ 0 & 0 & 1.0000 \end{matrix}$$

Then the estimated homography matrix which is computed from the points:

$$\text{estimatedH} = \begin{matrix} 1.0596 & -0.4281 & -0.0169 \\ 0.4281 & 1.0596 & 0.0168 \\ 0 & 0 & 1.0000 \end{matrix}$$

The results are the same.

3.2 A test with image points adding white noise

For this test, we continue using the camera position as the previous test. But white noise are added to the second image points. Matlab function rand is used to add noise in 0-1 pixel standard since this function returns a single uniformly distributed random number in the interval (0,1).

3.2.1 4 pts algorithm

Firstly, we compute the hompgraphy using theoretical method. We got the same result as the previous test which is reasonable.

$$\text{H} = \begin{matrix} 1.0628 & -0.4267 & 0.0257 \\ 0.4247 & 1.0628 & 0.0488 \\ -0.0318 & -0.0428 & 1.0000 \end{matrix}$$

Then we use the noisy points to estimate the homography, we got

$$\text{estimatedH} = \begin{matrix} 1.0611 & -0.4253 & 0.0269 \\ 0.4243 & 1.0613 & 0.0496 \\ -0.0313 & -0.0437 & 1.0000 \end{matrix}$$

By comparison, we can see that the hompgraphy computed by points is approaching the true one but still has little difference. Because the inputing points have noise.

$$\text{diffH} = \begin{matrix} 0.0027 & 0.0023 & 0.0425 \\ -0.0033 & 0.0029 & 0.0319 \\ -0.0313 & -0.0437 & 0 \end{matrix}$$

3.2.2 2 pts algorithm

Then we test 2 pts algorithm by noisy image points. Firstly the true homography is computed theoretically.

$$\text{trueH} = \begin{matrix} 1.0596 & -0.4281 & -0.0169 \\ 0.4281 & 1.0596 & 0.0168 \\ 0 & 0 & 1.0000 \end{matrix}$$

And the homography conputed by noisy points:

$$\text{estimatedH} = \begin{matrix} 1.0584 & -0.4276 & -0.0156 \\ 0.4276 & 1.0584 & 0.0177 \\ 0 & 0 & 1.0000 \end{matrix}$$

When the image points have white noise, the estimation of the homography is not accurate. The difference is:

$$\text{diffH} = \begin{matrix} 0.0013 & -0.0005 & -0.0013 \\ 0.0005 & 0.0013 & -0.0009 \\ 0 & 0 & 0 \end{matrix}$$

3.2.3 RANSAC algorithm

We redo the computation of homography using RANSAC algorithm. We take the noisy points from image plane, the result is:

$$\text{ransac4H} = \begin{matrix} 1.0624 & -0.4262 & 0.0271 \\ 0.4247 & 1.0614 & 0.0493 \\ -0.0317 & -0.0454 & 1.0000 \end{matrix}$$

We take the points from virtual plane, the estimated homography is:

$$\text{ransac2H} = \begin{matrix} 1.0569 & -0.4268 & -0.0158 \\ 0.4295 & 1.0581 & 0.0178 \\ -0.0001 & 0.0013 & 1.0000 \end{matrix}$$

By comparing with the results from 4 pts and 2 pts algorithm, we notice that the results obtained from RANSAC algorithm are approching the ground truth and the difference has minimized. Most outliers are removed. But it doesn't get the exact same results.

3.3 A test with noise on IMU information

We firstly add noise with 0-2 degrees to the IMU information and then add the noise with 0-1 pixel to the image points. After computing using the same method, we got the homography.

3.3.1 4 pt algorithm

The ground truth:

$$H = \begin{matrix} 1.0628 & -0.4267 & 0.0257 \\ 0.4247 & 1.0628 & 0.0488 \\ -0.0318 & -0.0428 & 1.0000 \end{matrix}$$

The estimation from image points:

$$H = \begin{matrix} 1.0616 & -0.4261 & 0.0268 \\ 0.4238 & 1.0620 & 0.0497 \\ -0.0319 & -0.0428 & 1.0000 \end{matrix}$$

The estimation using RANSAC:

$$H = \begin{matrix} 1.0611 & -0.4317 & 0.0290 \\ 0.4236 & 1.0651 & 0.0492 \\ -0.0286 & -0.0383 & 1.0000 \end{matrix}$$

3.3.2 2 pts algorithm

The ground truth:

$$H = \begin{matrix} 1.0596 & -0.4281 & -0.0169 \\ 0.4281 & 1.0596 & 0.0168 \\ 0 & 0 & 1.0000 \end{matrix}$$

The estimation of homography using image points:

$$H = \begin{matrix} 1.0587 & -0.4275 & -0.0156 \\ 0.4275 & 1.0587 & 0.0179 \\ 0 & 0 & 1.0000 \end{matrix}$$

The estimation using RANSAC:

$$H = \begin{array}{ccc} 1.0580 & -0.4272 & -0.0156 \\ 0.4276 & 1.0586 & 0.0190 \\ -0.0015 & 0.0025 & 1.0000 \end{array}$$

We notice that as we adding more noise to more components, the error is accelerating.

4 Conclusion

In this practical work, we test the 4 pts and 2 pts algorithms for the scenario of points on the plane and on the vertical plane respectively. Then the algorithms are tested with noisy points and noisy IMU information. The estimated results are approaching the ground truth.