COMP 330B 2022, Assignment 4 Due Tuesday, April 12th 2022 23:59

- [8%]
- **4.4** Let $A\varepsilon_{\mathsf{CFG}} = \{\langle G \rangle | G \text{ is a CFG that generates } \varepsilon \}$. Show that $A\varepsilon_{\mathsf{CFG}}$ is decidable.
- [10%]
- **4.13** Let $A = \{\langle R, S \rangle | R \text{ and } S \text{ are regular expressions and } L(R) \subseteq L(S) \}$. Show that A is decidable.
- [10%]
- **4.28** Let $C = \{\langle G, x \rangle | G \text{ is a CFG } x \text{ is a substring of some } y \in L(G) \}$. Show that C is decidable. (Hint: An elegant solution to this problem uses the decider for E_{CFG} .)
- [8%]
- 5.16 Let Γ = {0, 1, □} be the tape alphabet for all TMs in this problem. Define the busy beaver function BB: N→N as follows. For each value of k, consider all k-state TMs that halt when started with a blank tape. Let BB(k) be the maximum number of 1s that remain on the tape among all of these machines. Show that BB is not a computable function.
- [+5%]
- + Prove that BB(k) grows faster than any computable function f(k), i.e. $\lim_{k\to\infty} f(k)/BB(k)=0$.
- [10%]
- **5.19** In the *silly Post Correspondence Problem*, *SPCP*, in each pair the top string has the same length as the bottom string. Show that the *SPCP* is decidable.
- [10%]
- **5.20** Prove that there exists an undecidable subset of $\{1\}^*$.
- [10%]
- **5.21** Let $AMBIG_{CFG} = \{\langle G \rangle | G \text{ is an ambiguous CFG} \}$. Show that $AMBIG_{CFG}$ is undecidable. (Hint: Use a reduction from PCP. Given an instance

$$P = \left\{ \left[\frac{t_1}{b_1} \right], \left[\frac{t_2}{b_2} \right], \dots, \left[\frac{t_k}{b_k} \right] \right\},\,$$

of the Post Correspondence Problem, construct a CFG G with the rules

$$S \to T \mid B$$

$$T \to t_1 T \mathbf{a}_1 \mid \cdots \mid t_k T \mathbf{a}_k \mid t_1 \mathbf{a}_1 \mid \cdots \mid t_k \mathbf{a}_k$$

$$B \to b_1 B \mathbf{a}_1 \mid \cdots \mid b_k B \mathbf{a}_k \mid b_1 \mathbf{a}_1 \mid \cdots \mid b_k \mathbf{a}_k$$

where a_1, \ldots, a_k are new terminal symbols. Prove that this reduction works.)

$$f(x) = \begin{cases} 3x + 1 & \text{for odd } x \\ x/2 & \text{for even } x \end{cases}$$

for any natural number x. If you start with an integer x and iterate f, you obtain a sequence, x, f(x), f(f(x)), ... Stop if you ever hit 1. For example, if x = 17, you get the sequence 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1. Extensive computer tests have shown that every starting point between 1 and a large positive integer gives a sequence that ends in 1. But, the question of whether all positive starting points end up at 1 is unsolved; it is called the 3x + 1 problem.

Suppose that A_{TM} were decidable by a TM H. Use H to describe a TM that is guaranteed to state the answer to the 3x+1 problem.

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6.19 Recall the Post correspondence problem that we defined in Section 5.2 and its associated language PCP. Show that PCP is decidable relative to A_{TM} .

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[8%]

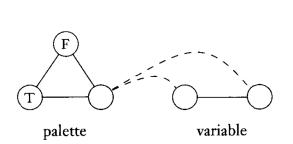
7.21 Let DOUBLE- $SAT = \{\langle \phi \rangle | \phi \text{ has at least two satisfying assignments} \}$. Show that DOUBLE-SAT is NP-complete.

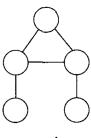
[10%]

7.27 A *coloring* of a graph is an assignment of colors to its nodes so that no two adjacent nodes are assigned the same color. Let

 $3COLOR = \{\langle G \rangle | \text{ the nodes of } G \text{ can be colored with three colors such that no two nodes joined by an edge have the same color} \}.$

Show that 3COLOR is NP-complete. (Hint: Use the following three subgraphs.)





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