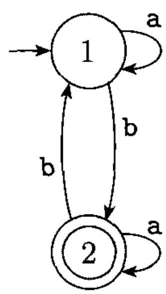
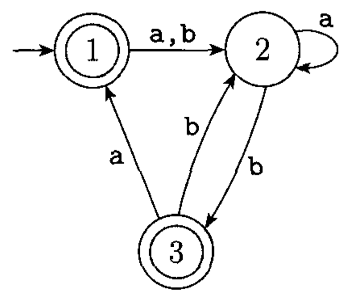
COMP 330 2022, Assignment 2
Due Tuesday, February 15th 23:59

[20%]

1.21 Use the procedure described in Lemma 1.60 to convert the following finite automata to regular expressions.



(a)



(b)

[10%]

1.47 Let $\Sigma = \{1, \#\}$ and let

$$Y = \{w \mid w = x_1\#x_2\#\cdots\#x_k \text{ for } k \geq 0, \text{ each } x_i \in 1^*, \text{ and } x_i \neq x_j \text{ for } i \neq j\}.$$

Prove that Y is not regular.

IN EITHER 1.47 (above) or 1.53 (below), AT YOUR CHOOSING, YOU MUST USE THE MYHILL-NERODE THEOREM TO PROVE NON-REGULARITY, while in the other you must use the pumping lemma.

[10%]

1.53 Let $\Sigma = \{0, 1, +, =\}$ and

$$ADD = \{x{=}y{+}z \mid x, y, z \text{ are binary integers, and } x \text{ is the sum of } y \text{ and } z\}.$$

Show that ADD is not regular.

[10%]

1.54 Consider the language $F = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$.

a. Show that F is not regular.

b. Show that F acts like a regular language in the pumping lemma. In other words, give a pumping length p and demonstrate that F satisfies the three conditions of the pumping lemma for this value of p.

[10%] **1.60** Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. For each $k \geq 1$, let C_k be the language consisting of all strings that contain an $\mathtt{a}$ exactly k places from the right-hand end. Thus $C_k = \Sigma^*\mathtt{a}\Sigma^{k-1}$. Describe an NFA with $k+1$ states that recognizes C_k, both in terms of a state diagram and a formal description.

[10%] **1.61** Consider the languages C_k defined in Problem 1.60. Prove that for each k, no DFA can recognize C_k with fewer than 2^k states.

1.64 Let N be an NFA with k states that recognizes some language A.

[12%]

a. Show that, if A is nonempty, A contains some string of length at most k.

b. Show that, by giving an example, that part (a) is not necessarily true if you replace both A's by $\overline{A}$.

c. Show that, if $\overline{A}$ is nonempty, $\overline{A}$ contains some string of length at most 2^k.

d. Show that the bound given in part (b) is nearly tight; that is, for each k, demonstrate an NFA recognizing a language A_k where $\overline{A_k}$ is nonempty and where $\overline{A_k}$'s shortest member strings are of length exponential in k. Come as close to the bound in (b) as you can.

[8%] **1.998** Consider the (decimal) languages defined below. For each one, either give a regular expression for its elements or prove the language is non-regular:

In all examples, a number cannot start with a 0 (unless it *is* 0 itself) and the empty string is *NOT* a number

a) **L_a** $=\{$ w | as an integer w is a multiple of 50$\}$.
b) **L_b** $=\{$ w | as an integer w is a multiple of 40$\}$.
c) **L_c** $=\{$ w | as an integer w is a power of 10$\}$.
d) **L_d** $=\{$ w | as an integer w is a multiple of 6$\}$.
e) **L_e** $=\{$ w | as an integer w is s. t. the sum of its digits is a multiple of 10$\}$.
f) **L_f** $=\{$ w | as an integer w is a power of 2$\}$.
g) **L_g** $=\{$ w | w is a rational number$\}$.
(with $\sum = \{ 0 , 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9 , - , / \}$)
Examples of such strings are -76403/3300, or 100/100 but not 1/0 or -0/0.
h) **L_h** $=\{$ w | w is a relatively prime rational number$\}$.
(with $\sum = \{ 0 , 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9 , - , / \}$)
Examples of such strings are -76403/1117, or 100/101 but not 5/10 or 2/4.

[10%] **1.999** Exhibit and explain an algorithm to find the shortest regular expression R_{min} equivalent to a given regular expression R.
(Shortest means least number of symbols and operators. The alphabet is {0,1}.)