A3 Q4

Saturday, October 9, 2021 7:05 PM

4.

a. LU factorization with a $B^{n \times p}$ can be solved in the same way we would solve Ax=b. First we use GEPP ($\frac{2n^3}{3}$ $flops + \frac{1}{2}n^2$ $comparisons$) to reduce the matrix A into it's upper triangular form, then extracting our L, U, and P vectors, as we would with regular LU factorization.

The cost of the forward and backward substitution processes is the same, where we do *n* divisions to find the value of our variable in each equation, and then for every calculated variable, we do a multiplication and subtraction to substitute in the appropriate value and move to the other side of the equation. We can see that for the first three terms, we have 1 operation, then 3 operations (1 known variable and 1 unknown), then 5 operations. Thus, we can define the amount of operations at each step as $2n-1$, so the total cost of all n operations is $n(2n-1)=2n^2-n$, or just n^2. Since we do forward and backward substitution, that comes out to a cost of $2n^2$

Since each corresponding column in matrix "X" and matrix "B" represent a new set of linear equations, this means that we will have to repeat the forward and backward substitution process *p* times, once for every column in matrix B. This means that the cost will be *p* times the forward and backward substitution process, since there is no need to recompute the L, U, and P matrixes.

The final cost then is:

$$\frac{2n^3}{3} + 2pn^2 \; flops + \frac{1}{2}n^2 \; comparisons$$

b.

i. From the printout, we can see that $\frac{\|X_C - X_T\|_F}{\|X_T\|_F}$ is smaller than $\epsilon\|A\|_F\|A^{-1}\|_F$, which confirms that GEPP is numerically stable, as seen in the 7th page of class notes on Linear Equations

ii. From the printout, we can see that $\frac{\|B - AX_C\|_F}{\|A\|_F\|X_C\|_F}$ is smaller than ϵ, which means that the residual is smaller than the epsilon and our result is numerically stable. We would expect this as the difference between two floating point values should be at most the machine epsilon.