A4 Q1

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1. We say $f(r) = 0$, where r is a root of $f(x)$. We can then represent $f(r)$ as a Taylor Series. As f, f', and f'' are continous, they are defined for some point x_0.
We also know that x_{n+1} approaches r, so we can write r as x_{n+1}, since it doesn't change the final calculation

$$f(x_{n+1}) = f(x_n) + f'(x_n)(x_{n+1} - x_n) + \frac{f''(\lambda)(x_{n+1} - x_n)^2}{2},$$ where λ is between r and x_n

Divide by $f'(x_n)$

$$\frac{f(x_{n+1})}{f'(x_n)} = \frac{f(x_n)}{f'(x_n)} + (x_{n+1} - x_n) + \frac{f''(\lambda)(x_{n+1} - x_n)^2}{2f'(x_n)}$$

We already know definition of $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ or $x_{n+1} - x_n = -\frac{f(x_n)}{f'(x_n)}$

$$\frac{f(x_{n+1})}{f'(x_n)} = \frac{f(x_n)}{f'(x_n)} - \frac{f(x_n)}{f'(x_n)} + \frac{f''(\lambda)}{2f'(x_n)} \cdot \left(-\frac{f(x_n)}{f'(x_n)}\right)^2$$

$$\frac{f(x_{n+1})}{f'(x_n)} = \frac{f''(\lambda)}{2f'(x_n)} \cdot \frac{f(x_n)^2}{f'(x_n)^2}$$

Multiply by $f'(x_n)$

$$f(x_{n+1}) = \frac{f''(\lambda)f(x_n)^2}{2f'(x_n)f'(x_n)}$$

$$\frac{f(x_{n+1})}{f(x_n)^2} = \frac{f''(\lambda)}{2f'(x_n)^2}$$

$$\frac{f''(\lambda)}{2f'(x_n)^2} = C$$

$$\lim_{n\to\infty} \frac{f(x_{n+1})}{f(x_n)^2} = \lim_{n\to\infty} (C) = C$$