A5 Q1

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Vandermode

$$c_0 + c_1\cdot(-2) + c_2\cdot(-2)^2 + c_3\cdot(-2)^3 = 2$$
$$c_0 + c_1\cdot 0 + c_2\cdot 0^2 + c_3\cdot 0^3 = 4$$
$$c_0 + c_1\cdot 1 + c_2\cdot 1^2 + c_3\cdot 1^3 = 2$$
$$c_0 + c_1\cdot 3 + c_2\cdot 3^2 + c_3\cdot 3^3 = 7$$

$$\begin{bmatrix} 1 & -2 & 4 & -8 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 3 & 9 & 27 \end{bmatrix} \cdot \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 2 \\ 7 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 4 & -8 & 2 \\ 1 & 0 & 0 & 0 & 4 \\ 1 & 1 & 1 & 1 & 2 \\ 1 & 3 & 9 & 27 & 7 \end{array}\right] \begin{matrix} \\ -R_1 \\ -R_1 \\ -R_1 \end{matrix} \quad \left[\begin{array}{cccc|c} 1 & -2 & 4 & -8 & 2 \\ 0 & 2 & -4 & 8 & 2 \\ 0 & 3 & -3 & 9 & 0 \\ 0 & 5 & 5 & 35 & 5 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 4 & -8 & 2 \\ 0 & 5 & 5 & 35 & 5 \\ 0 & 3 & -3 & 9 & 0 \\ 0 & 2 & -4 & 8 & 2 \end{array}\right] \begin{matrix} \\ \\ -\frac{3}{5}R_2 \\ -\frac{2}{5}R_2 \end{matrix} \quad \left[\begin{array}{cccc|c} 1 & -2 & 4 & -8 & 2 \\ 0 & 5 & 5 & 35 & 5 \\ 0 & 0 & -6 & -12 & -3 \\ 0 & 0 & -6 & -6 & 0 \end{array}\right] -R_3 \quad \left[\begin{array}{cccc|c} 1 & -2 & 4 & -8 & 2 \\ 0 & 5 & 5 & 35 & 5 \\ 0 & 0 & -6 & -12 & -3 \\ 0 & 0 & 0 & 6 & 3 \end{array}\right]$$

$c_0 + 4 - 2 - 4 = 2 \qquad c_0 = 4$

$5c_1 - 2.5 + 17.5 = 5 \qquad 5c_1 = -10 \qquad c_1 = -2$

$-6c_2 - 12(0.5) = -3 \qquad -6c_2 - 6 = -3 \qquad -6c_2 = 3 \qquad c_2 = -0.5$

$6c_3 = 3 \qquad c_3 = 0.5$

$$S(x) = 4 - 2x - 0.5x^2 + 0.5x^3$$

Lagrange

$$p(x) = Q(x)\sum_{i=0}^{n} \frac{c_i}{x - x_i} \qquad (x+2)(x-0)(x-1)(x-3)$$

$$(x+2)(x-0)(x-1)(x-3)\cdot\left(-\frac{1}{15(x+2)} + \frac{2}{3x} - \frac{1}{3(x-1)} + \frac{7}{30(x-3)}\right)$$

$$-\frac{x^3-4x^2+3x}{15} + \frac{2(x^3-2x^2-5x+6)}{3} - \frac{x^3-x^2-6x}{3} + \frac{7(x^3+x^2-2x)}{30}$$

$$-\frac{2x^3-8x^2+6x}{30} + \frac{20x^3-40x^2-100x+120}{30} - \frac{10x^3-10x^2-60x}{30} + \frac{7x^3+7x^2-14x}{30}$$

$$\frac{15x^3-15x^2-60x+120}{30}$$

$$S(x) = \frac{x^3-x^2}{2} - 2x + 4$$

$i=0$: $\frac{2}{(-2-0)(-2-1)(-2-3)} \cdot \frac{1}{(x+2)} = \frac{2}{-2\cdot-3\cdot-5}\cdot\frac{1}{(x+2)} = \frac{2}{-30(x+2)} = -\frac{1}{15(x+2)}$

$i=1$: $\frac{4}{(0+2)(0-1)(0-3)} \cdot \frac{1}{(x-0)} = \frac{4}{2\cdot-1\cdot-3}\cdot\frac{1}{x} = \frac{4}{6}\cdot\frac{1}{x} = \frac{2}{3x}$

$i=2$: $\frac{2}{(1+2)(1-0)(1-3)} \cdot \frac{1}{(x-1)} = \frac{2}{3\cdot1\cdot-2}\cdot\frac{1}{(x-1)} = \frac{2}{-6}\cdot\frac{1}{(x-1)} = -\frac{1}{3(x-1)}$

$i=3$: $\frac{7}{(3+2)(3-0)(3-1)} \cdot \frac{1}{(x-3)} = \frac{7}{5\cdot3\cdot2}\cdot\frac{1}{(x-3)} = \frac{7}{30}\cdot\frac{1}{(x-3)} = \frac{7}{30(x-3)}$

Newton

$$\begin{bmatrix} 1 & & & \\ 1 & (0+2) & & \\ 1 & (1+2) & \begin{matrix}(1+2)\\(1-0)\end{matrix} & \\ 1 & (3+2) & \begin{matrix}(3+2)\\(3-0)\end{matrix} & \begin{matrix}(3+2)\\(3-0)\\(3-1)\end{matrix} \end{bmatrix} \cdot \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 2 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & & & \\ 1 & 2 & & \\ 1 & 3 & 3 & \\ 1 & 5 & 15 & 30 \end{bmatrix} \cdot \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 2 \\ 7 \end{bmatrix}$$

$a_0 = 2$

$2 + 2a_1 = 4 \qquad a_1 = 1$

$2 + 3 + 3a_2 = 2 \qquad a_2 = -1$

$2 + 5 - 15 + 30a_3 = 7 \qquad a_3 = 1/2$

$$p_n(x) = 2 + (x+2) - (x+2)(x-0) + 0.5(x+2)(x-0)(x-1)$$
$$2 + x + 2 - x^2 - 2x + 0.5x^3 + 0.5x^2 - x$$
$$0.5x^3 - 0.5x^2 - 2x + 4$$

$$p_n(x) = 0.5x^3 - 0.5x^2 - 2x + 4$$