A3 Q3

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3. We know from class that when $Ax = b$, where A is some matrix containing real (or complex) numbers, x is the vector containing the unknowns, and b is the constant vector. We can then do some simple algebra to demonstrate that $x = A^{-1}b$. Using this, we suppose that in the equation $\alpha = c^T A^{-1} d$, $A^{-1}d$ can serve a similar purpose, where we say that some $x = A^{-1}d$, or $Ax = d$. Using LU decomposition, we can solve for x, which allows us to substitute out the $A^{-1}d$ in the original equation. To find *x,* the following is done:

We perform LU factorization with partial pivoting to extract the P, L, and U matrices

PA = LU

Ax = d

PAx = Pd

(LU)x = Pd

First we solve using forward substitution

Ly = Pd for y

yielding

LUx = Ly

Ux = y

Then we solve using backward substitution

Ux =y for x

Substituting the solved x back in, we get $\alpha = c^T x$, which we can compute without inverting anything.