A3 Q2

Thursday, October 7, 2021 1:25 PM

2. Yes, it can be solved with real arithmetic operations, where we must break each complex number into a set of two variables that sum to the complex number. *As x* is also a complex number, it can be represented as $x = (x_r + x_c i)$. Thus, if we write out the product of some complex number represented *as (a+bi)* and x, *where a and b* are real numbers, we see:

$$(a+bi)(x_r + x_c i)$$
$$ax_r + ax_c i + bx_r i + bx_c i^2$$
$$ax_r + ax_c i + bx_r i - bx_c$$
$$ax_r - bx_c + ax_c i + bx_r i$$
$$(ax_r - bx_c) + (ax_c + bx_r)i$$

For two complex numbers to be equal, the real and imaginary parts must be equal. So if we introduce a complex number *c* in $(a+bi)x = c$, or $(a+bi)x = c_r + c_c i$, that means that:

$$ax_r - bx_c = c_r$$
$$ax_c + bx_r = c_c$$

We can represent this through matrices, with:

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix} \cdot \begin{bmatrix} x_r \\ x_c \end{bmatrix} = \begin{bmatrix} c_r \\ c_c \end{bmatrix}$$

And thus, given complex number C, we can solve for x_r and x_c as we would with any regular system of equations. For any variable like *x*, we can similarly decompose it into its real and complex components, solving for the variables. For instance:

$$(2+3i)x + (5+6i)y = (2+5i)$$
$$(4+10i)x + (7+8i)y = (6+9i)$$

$$\begin{bmatrix} 2 & -3 & 5 & -6 \\ 3 & 2 & 6 & 5 \\ 4 & -10 & 7 & -8 \\ 10 & 4 & 8 & 7 \end{bmatrix} \begin{bmatrix} x_r \\ x_c \\ y_r \\ y_c \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 6 \\ 9 \end{bmatrix}$$

As we can observe, for every variable *n*, we require an $A^{2n \times 2n}$ to calculate all the real and complex components of each variable. Using GEPP, we can substitute *2n* into the formula to get the cost of solving:

Original: $\frac{2}{3}n^3$ flops $+ \frac{1}{2}n^2$ comparisons

Complex: $\frac{2}{3}(2n)^3 + \frac{1}{2}(2n)^2 = \frac{16}{3}n^3$ flops and $2n^2$ comparisons