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Contents

Introduction

The behaviour of non-linear systems, at classical and quantum level, rises fundamental questions and is important in several applications. In classical mechanics, once the Hamiltonian systems become non-integrable, a sensitive dependence of the dynamics on the initial conditions manifests. This corresponds to chaotic dynamics and leads to several specific phenomena. In particular, the phase space is no longer filled by tori and depending on the energy, several trajectories will have an erratic behaviour in the available volume. An important question which is intensively debated in literature is related to the way in which the corresponding quantum system reflects this classical properties. The purpose of the present work is to explore the features of the energy levels statistics in relation with the departure from the integrable case and also as a function of excitation energy. We try to obtain a correlation between the global phase space structure in a given energy domain and the associated energy level distribution for the corresponding quantum system.

In **??** we review the basic concepts from classical Hamiltonian dynamics which are useful to the characterisation of chaotic dynamics.

In **??**, after a brief review of the principles of quantum mechanics, we discuss the nearest neighbour distributions and their relation to the non-integrability.

?? is devoted to a detailed description of the numerical procedures and to the analysis method of the level distributions performed with the aid of a `Python` program.

?? presents the main results of our investigations. We discuss the possible deviations from the Wigner distribution by proposing an effective level distribution which can interpolate between the Wigner and Poissonian one. We show that the behaviour of this distribution can reflect the energy variations in phase space. Moreover, it has peculiar behaviour at the variations of the parameter with the departure from integrability.

In the last chapter, the main conclusions of our work are summarised.

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Chapter 1

Conclusions

In this thesis we have studied the connection between the classical phase space structures and the spectra of the corresponding quantum system. The basic question was to explore how the relative partition between tori and chaotic trajectories in the classical phase space is reflected in the statistics of the energy levels of the quantum analogue.

Starting with a Hamiltonian that describes the dynamics of the nuclear surface in terms of quadrupole vibrations, we obtained the energy spectra through numerical diagonalisation. The Hamiltonian depends on parameters, which at classical level induce the transition from an integrable system to a non-integrable one. As a consequence, in agreement with the K.A.M. theorem, the phase space starts to be populated with chaotic trajectories which escape from the tori, filling a 3 dimensional volume. We observed that the relative weights of the tori volumes in the phase space depends in a non-trivial way with the energy and the control parameter for non-integrability. Starting from this observation we investigated systematically the properties of the energy spectra as a function of the same control parameter and in different energy ranges. Specifically, we focused on the nearest neighbour distribution and compared it with the predictions of the Poisson and Wigner distributions.

We observed that for all values of the control parameter, the distribution manifests deviations from the expected Wigner distribution characteristic to chaotic systems. We assumed that this effects are related to the significant influence of the tori at the classical level. Therefore, we proposed a distribution which is a linear superposition of the Poisson and Wigner distributions. Then, the coefficient factorising the Poisson distribution will be a measure of closeness to an integrable behaviour. Indeed, if this coefficient is equal to unity, the distribution becomes Poissonian, while when it goes to zero, it transforms Wigner one.

We noticed that when we rise the energy range for the analysis of the level distribution, this coefficient increases, reflecting the observed behaviour in classical phase space. In other words, the global structure of the classical phase space for an energy interval is reflected by the superposition fitting coefficient. More precisely, we observed that the value of this coefficient increases with the considered energy interval, while in the classical system, as we increase the energy, the volume of the regular trajectories confined on tori increases.

Furthermore, we obtained a non-trivial dependence of the superposition coefficient on the non-integrability parameter. While a greater value of this parameter is expected to make the system more chaotic, and consequently the level distribution gets closer to the Wigner one, from our analysis resulted a more intricate behaviour. The superposition coefficient decreases and increases periodically when the non-integrability parameter changes. This periodic shape shows that for specific values of the control parameter, the system will manifest a more robust integrable-like behaviour. Further work is required to better understand this aspect.