$\underline{13.1}$ –

① $\vec{AB} = \langle 2,3 \rangle$
    $\vec{PQ} = \langle 6,9 \rangle$

② $\vec{OR} = \langle -2,7 \rangle$ $\|OR\| = \sqrt{4+49} = \sqrt{53}$

③ $\langle 14, 8 \rangle$

$\underline{13.2}$ –

② $\langle 8,12,-6 \rangle = 2 \langle 4,6,-3 \rangle$ same dir ✓

  $r(0) = \langle 3,-1,4 \rangle = s(-2)$ same pt. ✓

③ $r_1(t) = \langle 2-4t, 1, 1+t \rangle$

  $v_2(s) = \langle -4+2s, 1+s, 5-2s \rangle$

$\underline{y}$: $1 = 1+s$ =) $s = 0$

$\underline{x}$: $2-4t = -4+2s = -4$ $\Rightarrow t = -6/-4 = \frac{3}{2}$

$\underline{z}$: $1+\frac{3}{2} \neq s$ No intersection

13.3 —

① $\theta = \cos^{-1}\left(\dfrac{\langle 3,1,1\rangle \cdot \langle 2,-4,2\rangle}{\sqrt{9+1+1}\cdot\sqrt{4+16+4}}\right)$ $\sqrt{}$

$= \cos^{-1}\left(\dfrac{6-4+2}{\sqrt{11}\,\sqrt{24}}\right) = \cos^{-1}\left(\dfrac{4}{\sqrt{264}}\right)$

② $\cos\theta_{u,v} = \dfrac{u\cdot v}{\|u\|\|v\|} = \dfrac{c_v \cdot v + c_w \cdot v}{\sqrt{2}\ \|v\|}$

$= \dfrac{\|v\| + \|v\|\cos\theta_{v,w}}{\sqrt{2}\ \|v\|}$

$= \dfrac{1+\cos\theta_{v,w}}{\sqrt{2}}$

$\cos\theta_{u,v} = \dfrac{u\cdot w}{\|u\|\|w\|} = \dfrac{\|w\|\cos\theta_{v,w} + \|w\|}{\sqrt{2}\ \|w\|}$

$= \dfrac{1+\cos\theta_{v,w}}{\sqrt{2}} = \cos\theta_{v,w}$ ✓

③ No, f. $\underline{ex}$ $i \cdot k = j \cdot k = 0$

but $i \neq j$

13.4

① a) $3\, u \times w + \overset{6}{4\, \cancel{w \times w}}$

   $= 3 \langle 0, 3, 1 \rangle$

b) $\overset{0}{u \cancel{\times} u} - u \times v + v \times u - \overset{0}{v \cancel{\times} v}$

   $= 2\, v \times u = -2 \langle 1, 1, 0 \rangle$

② $i \times j = k,\quad j \times k = i,\quad k \times i = j$

③ $|u \cdot (v \times w)| = \begin{vmatrix} 2 & 2 & 1 \\ 1 & 0 & 3 \\ 0 & -4 & 0 \end{vmatrix}$

$= 4 \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 4(6-1) = \boxed{20}$

13.5

① $x + 3y + 2z = d$

$4 - 3 + 2 = d \implies d = 3$

$\boxed{x + 3y + 2z = 3}$

② $\vec{PQ} = \langle -1, 2, -3 \rangle$

$\vec{PR} = \langle 1, 2, -6 \rangle$

$PQ \times PR = \begin{vmatrix} i & j & k \\ -1 & 2 & -3 \\ 1 & 2 & -6 \end{vmatrix}$

$= i(-12 + 6) - j(6 + 3) + k(-2 - 2)$

$= -6i - 9j - 4k = \langle -6, -9, -4 \rangle$

$-6x - 9y - 4z = d$

Plug in: $(1,1,1) \implies -6 - 9 - 4 = d \implies \boxed{d = -19}$

$$\boxed{-6x - 9y - 4z = -19}$$

③ $r(t) = \langle 1, 1+2t, 4t \rangle$

$\quad 1 + 1 + 2t + 4t = 14$

$\quad\quad \Rightarrow 6t = 12 \quad \Rightarrow t = 2$

$\quad\quad r(2) = \boxed{(1, 5, 8)}$

14.1 –

① $\quad \sin t = 0 \quad \Rightarrow \quad t = K\pi$

$\quad\quad \cos t/2 = 0 \quad \Rightarrow \quad \frac{t}{2} = K\frac{\pi}{2}$

$\quad \underline{Yes}$, when t is a multiple of π

② $\quad \underline{center}: \quad (7, 0, 0)$

$\quad \underline{radius}: \quad 12$

① $\dfrac{dr_1}{dt} = \langle 2t, 3t^2, 1 \rangle$

$\dfrac{dr_2}{dt} = \langle 3e^{3t}, 2e^{2t}, e^t \rangle$

a. $\dfrac{dr_1}{dt} \cdot r_2 + r_1 \cdot \dfrac{dr_2}{dt}$

$= 2te^{3t} + 3t^2 e^{2t} + e^t + 3t^2 e^{3t} + 2t^3 e^{2t} + te^t$

$$\boxed{\; (2t + 3t^2)e^{3t} + (3t^2 + 2t^3)e^{2t} + (1+t)\,e^t \;}$$

b. $\langle\, t^3 e^t + 3t^2 e^t - 2te^{2t} - e^{2t},$

$\quad 3te^{3t} + e^{3t} - t^2 e^t - 2te^t,$

$\quad 2t^2 e^{2t} + 2te^{2t} - 3t^3 e^{3t} - 3t^2 e^{3t} \,\rangle$

② $\int_{-2}^{2} \langle t^2 + 4t, \; 4t^3 - t \rangle \; dt$

$= \langle 2t + 4, \; 12t^2 - 1 \rangle \Big|_{-2}^{2}$

$= \langle 8, \; 47 \rangle - \langle 0, \; 47 \rangle = \langle 8, 0 \rangle$

$\underline{14.3}$ -

① $\int_{1}^{4} \| r'(t) \| \; dt \;\; = \int_{1}^{4} \| \langle 2, \frac{1}{t}, 2t \rangle \| \; dt$

$= \int_{1}^{4} \sqrt{4 + \frac{1}{t^2} + 4t^2} \; dt \;\; = \int_{1}^{4} \sqrt{\frac{4t^4 + 4t^2 + 1}{t^2}} \; dt$

$= \int_{1}^{4} \sqrt{\frac{(2t + 1)^2}{t^2}} \; dt \;\; = \int_{1}^{4} \frac{2t + 1}{t} \; dt$

$$= \int_1^4 2 + \frac{1}{t} \, dt \;=\; 2t + \ln|t| \;\Big|_1^4$$

$$= \; 8 + \ln(8) - 2 - 0 = \boxed{6 + \ln(8)}$$

②
$$s(t) = \int_0^t \| \langle 2u, \; 4u, \; 3u^2 \rangle \| \, du$$

$$= \int_0^t \sqrt{4u^2 + 16u^2 + 9u^4} \; du$$

$$= \int_0^t u\sqrt{20 + 9u^2} \; du$$

$$= \frac{(9u^2 + 20)^{3/2}}{27} \; \Big|_0^t$$

$$= \frac{(9t^2 + 20)^{3/2}}{20} - \frac{(20)^{3/2}}{27}$$

③ a) $s = g(t) = \int_0^t \| \langle 3,4,2 \rangle \| \, du$

$$= \int_0^t \sqrt{9+16+4} \, du = \sqrt{29} \, t$$

$$g^{-1} = \frac{s}{\sqrt{29}}$$

$$r(g^{-1}(s)) = \left\langle \frac{3s}{\sqrt{29}} + 1, \ \frac{4s}{\sqrt{29}} - s, \ \frac{2s}{\sqrt{29}} \right\rangle$$

b) $s = g(t) = \int_0^t \| \langle e^u \cos u + e^u \sin u, \ -e^u \sin u + e^u \cos u, \ e^u \rangle \| \, du$

$$= \int_0^t e^u \sqrt{\cos^2 + 2\cos\sin + sh^2 + \sin^2 - 2\sin\cos + \cos^2 + 1} \, du$$

$$= \int_0^t e^u \sqrt{3} \, du = \sqrt{3}\, e^u \Big|_0^t = \sqrt{3}\, e^t - \sqrt{3}$$

$$s + \sqrt{3} = \sqrt{3}\, e^t \implies t = \ln\left(\frac{s+\sqrt{3}}{\sqrt{3}}\right) = g^{-1}(s)$$

$$r(g^{-1}(s)) = \frac{s+\sqrt{3}}{\sqrt{3}} \left\langle \sin\left(\ln\left(\frac{s+\sqrt{3}}{\sqrt{3}}\right)\right), \ \cos\left(\ln\left(\frac{s+\sqrt{3}}{\sqrt{3}}\right)\right), \ 1 \right\rangle$$