Math 33A:

Today:

- **Topics**: Solving Systems of Linear Eqns.
  Matrix Algebra
  Linear Transformations

- **Office Hours**: 1-2pm, Zoom link on website
  & syllabus

Topic: Solving Systems of Linear Eqns.

& Gauss-Jordan elimination (Gaussian elimination, row reduction)

(1)
$$\begin{cases} x + 3y + 2z = 2 \\ 2x + 7y + 7z = -1 \\ 2x + 5y + 2z = 7 \end{cases}$$

$\downarrow$ Augmented Matrix

rows corresp. to eqns.

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{c|c} \underbrace{\begin{array}{ccc} 1 & 3 & 2 \\ 2 & 7 & 7 \\ 2 & 5 & 2 \end{array}}_{\text{coefficient matrix}} & \begin{array}{c} 2 \\ -1 \\ 7 \end{array} \end{array}\right] \end{array}$$

Gauss-Jordan Elimination:

1. swap two rows
2. Multiply / Divide a row by a scalar
3. Add a multiple of one row to another row

RREF: (Row Reduced Echelon Form)

1. First non-zero entry is a 1, called a leading 1
2. Every other entry in same col. as a leading 1 is 0
3. All leading 1's above another leading 1 are to the left

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 2 & 7 & 7 & -1 \\ 2 & 5 & 2 & 7 \end{array}\right] \xrightarrow{-2(\mathrm{I})} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 0 & 1 & 3 & -5 \\ 2 & 5 & 2 & 7 \end{array}\right] -2(\mathrm{I}) \longrightarrow$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 0 & 1 & 3 & -5 \\ 0 & -1 & -2 & 3 \end{array}\right] +(\mathrm{II}) \longrightarrow \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 0 & 1 & 3 & -5 \\ 0 & 0 & 1 & -2 \end{array}\right] \end{array}$$

① Read off matrix from here

$$\boxed{z = -2}$$

$$y + 3z = -5 \Rightarrow y - 6 = -5$$

$$\boxed{y = 1}$$

$$x + 3y + 2z = 2$$

$$x + 3 - 4 = 2 \Rightarrow x - 1 = 2$$

$$\boxed{x = 3}$$

② Reduce all the way to RREF

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 0 & 1 & 3 & -5 \\ 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{-3(\text{III})} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{-2(\text{III})}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 0 & 6 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{-3(\text{II})} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\boxed{\text{rank} = 3}$$

Def: The rank of a matrix is the number of leading 1's

- If my matrix has a row like $0 \ldots 0 \mid 0$ & not all my cols have a leading 1

  then I will have ∞ many solutions

- If there is a row like $0 \ldots 0 \mid$ non-zero

  Then I have no solutions

Topic: Matrix Algebra

Matrix Multiplication: "row-by-col"

$$\begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 2\cdot 3 + 1\cdot 2 & 2\cdot 1 + 1\cdot 1 \\ 3\cdot 3 + 1\cdot 2 & 3\cdot 1 + 1\cdot 1 \end{pmatrix}$$

2×2 $\quad$ 2×2

$$= \begin{pmatrix} 8 & 3 \\ 11 & 4 \end{pmatrix}$$

2×2 $\quad$ 2×3 $\quad\quad$ 2×3

$$\begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 2\cdot 1 + 1\cdot 0 & 2\cdot 0 + 1\cdot 1 & 2\cdot 1 + 1\cdot 1 \\ 3\cdot 1 + 1\cdot 0 & 3\cdot 0 + 1\cdot 1 & 3\cdot 1 + 1\cdot 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 1 & 3 \\ 3 & 1 & 4 \end{pmatrix}$$

Can only multiply matrices of different sizes when the

[number of cols of first matrix = number of rows of second matrix]

- You can add matrices of the same size

A, $m \times n$ matrix,

$n \times 1$

$$\begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$$

$A\vec{v}$ where $\vec{v}$ is a vector in $\mathbb{R}^n$,

$m \times n$ $n \times 1$ → vector in $\mathbb{R}^m$

3.1

$$\begin{pmatrix} 8 & 1 \\ 7 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 8 \cdot 2 + 1 \cdot (-2) \\ 7 \cdot 2 + 2 \cdot (-2) \end{pmatrix} = \begin{pmatrix} 16 - 2 \\ 14 - 4 \end{pmatrix}$$

$$= \begin{pmatrix} 14 \\ 10 \end{pmatrix}$$

Linear Transformations:

$T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is a linear transformation if:

- $T(\vec{x} + \vec{y}) = T(\vec{x}) + T(\vec{y})$
- $T(c\vec{x}) = cT(\vec{x})$ for any scalar c

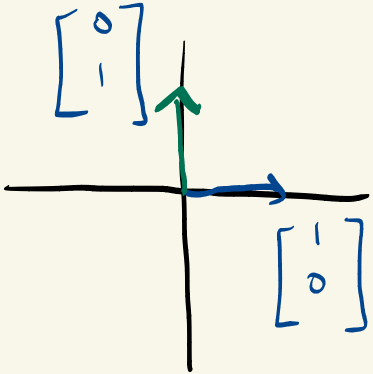
$\boxed{T(\vec{x}) = A\vec{x}}$, A $m \times n$ matrix

(5)

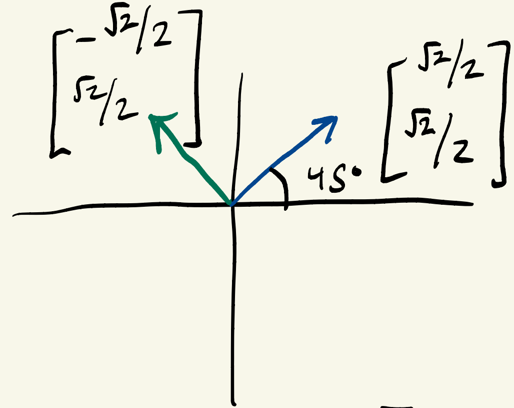
S.1: Find a 2×2 matrix s.t. is rotated a vector $\vec{v}$ by $45°$ counterclockwise

$\boxed{\text{Key Idea}}$: A linear transformation T is <u>completely determined</u> by where it sends the vectors $\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \dots, \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$

<u>(standard basis vectors)</u>



$$\xrightarrow[\text{CCW}]{\text{rotate } 45^\circ}$$



$$T(\vec{x}) = A\vec{x},$$

$$A = \left[T\begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad T\begin{bmatrix} 0 \\ 1 \end{bmatrix} \right]$$

$$= \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$