STRUCTURAL PATTERNS HEURISTICS: BASIC IDEA AND CONCRETE INSTANCE

Michael Katz   Carmel Domshlak
IE&M, Technion

Presentation in One Slide

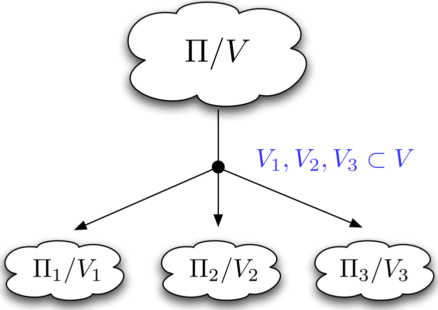
Main idea

Structurally-characterized tractable fragments of optimal planning can be operationalized in devising new admissible search heuristics.

- Focus on homomorphism abstractions
- Generalization of Pattern Database heuristics to Structural Patterns heuristics

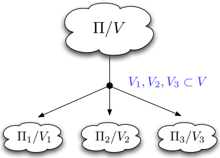
Select a (relatively small) set of subsets $V_1, \ldots, V_k$ of V such that, for $1 \leq i \leq k$,

1. $\Pi^{[V_i]}$ is an over-approximating abstraction of Π, and
2. the size of V_i is sufficiently small to perform reachability analysis in $\Pi^{[V_i]}$ by an (either explicit or symbolic) exhaustive search



Select a (relatively small) set of subsets $V_1, \ldots, V_k$ of V such that, for $1 \leq i \leq k$,

1. $\Pi^{[V_i]}$ is an over-approximating abstraction of Π, and
2. the size of V_i is sufficiently small to perform reachability analysis in $\Pi^{[V_i]}$ by an (either explicit or symbolic) exhaustive search



Disjoint?

YES

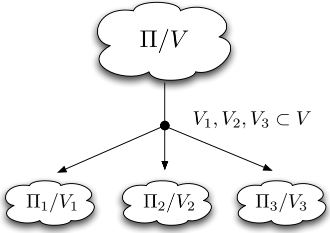
$$h(s) = \sum_{i=1}^{k} h^{[\mathcal{V}_i]}(s)$$

NO

$$h(s) = \max_{i=1}^{k} h^{[\mathcal{V}_i]}(s)$$

Disjoint Decomposition

Generalizing the standard "all-or-nothing" PDB disjoining



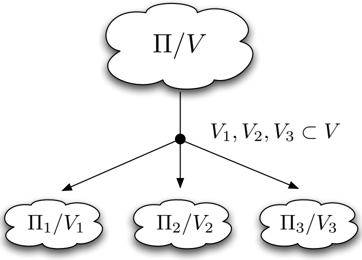
Each $a \in A$ satisfies $\mathcal{C}(a) \geq \sum_{i=1}^{m} \mathcal{C}_i(a^{[V_i]})$

Proposition

For any SAS$^+$ problem $\Pi = \langle V, A, I, G \rangle$, any set of V's subsets $\mathcal{V} = \{V_1, \ldots, V_m\}$, and any disjoint decomposition of Π over $\mathcal{V}$, we have $h^*(I) \geq \sum_{i=1}^{m} h_i^*(I^{[V_i]})$

Select a (relatively small) set of subsets $V_1, \ldots, V_k$ of V such that, for $1 \leq i \leq k$,

1. $\Pi^{[V_i]}$ is an over-approximating abstraction of Π, and
2. the size of V_i is sufficiently small to perform reachability analysis in $\Pi^{[V_i]}$ by an (either explicit or symbolic) **exhaustive search**



$$\boxed{|V_i| = O(\log |V|)}$$

Alternative Way to Go?

Select a (relatively small) set of subsets $V_1, \ldots, V_k$ of V such that, for $1 \leq i \leq k$,

1. $\Pi^{[V_i]}$ is an over-approximating abstraction of Π, and
2. the reachability analysis in $\Pi^{[V_i]}$ is tractable
   (not necessarily due to the size of but)
   due to the specific structure of $\Pi^{[V_i]}$

♠ Possibly $|V_i| = \Theta(|V|)$!

Structural Patterns: The Promise and the Skepticism

Promise

- Generalization of PDBs to homomorphism abstractions of unlimited dimensionality

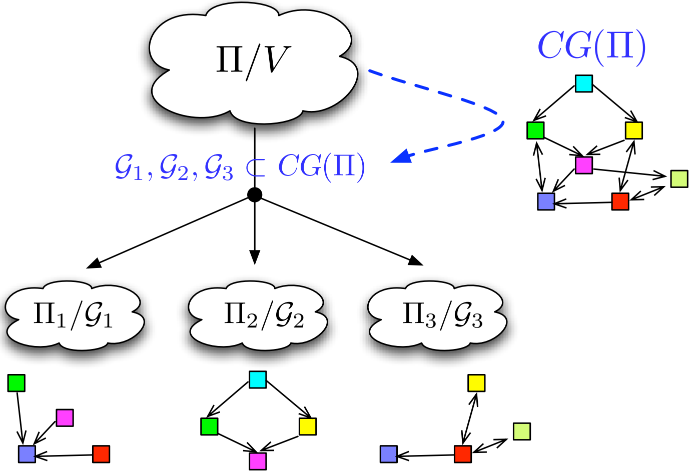
Structural Patterns: The Promise and the Skepticism

Promise

- Generalization of PDBs to homomorphism abstractions of unlimited dimensionality

Skepticism

- Structural patterns correspond to tractable fragments of optimal planning
- The palette of such known fragments is extremely limited



Π/V
$CG(\Pi)$
$\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3 \subseteq CG(\Pi)$
$\Pi_1/\mathcal{G}_1$
$\Pi_2/\mathcal{G}_2$
$\Pi_3/\mathcal{G}_3$

Disjoint CGSP Decomposition

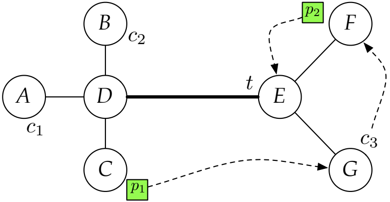
Definition

Let $\Pi = \langle V, A, I, G \rangle$ be a SAS$^+$ problem, and $\mathbf{G} = \{\mathcal{G}_1, \dots, \mathcal{G}_m\}$ be a set of subgraphs of the causal graph $CG(\Pi)$. A **disjoint CGSP decomposition of Π over $\mathbf{G}$** is a set of CGSPs $\{\Pi_{\mathcal{G}_1}, \dots, \Pi_{\mathcal{G}_m}\}$ such that each action $a \in A$ satisfies $\mathcal{C}(a) \geq \sum_{i=1}^{m} \sum_{a' \in A_{\mathcal{G}_i}(a)} \mathcal{C}_{\mathcal{G}_i}(a')$

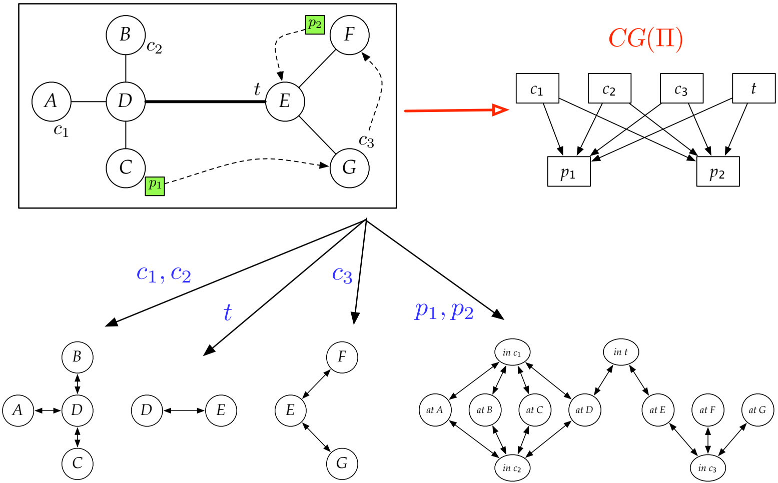
- *For the formal specification of $\Pi_{\mathcal{G}_i}$ see the paper.*

Proposition

For any SAS$^+$ problem $\Pi = \langle V, A, I, G \rangle$, any set of $CG(\Pi)$'s subgraphs $\mathbf{G} = \{\mathcal{G}_1, \dots, \mathcal{G}_m\}$, and any disjoint CGSP decomposition of Π over $\mathbf{G}$, we have $h^*(I) \geq \sum_{i=1}^{m} h_i^*(I_{\mathcal{G}_i})$



$$
\begin{aligned}
V &= \{p_1, p_2, c_1, c_2, c_3, t\} \\
Dom(p_1) &= Dom(p_2) = \{A, B, C, D, E, F, G, c_1, c_2, c_3, t\} \\
Dom(c_1) &= Dom(c_2) = \{A, B, C, D\} \\
Dom(c_3) &= \{E, F, G\} \\
Dom(t) &= \{D, E\} \\
I, G &\mapsto \text{see picture} \\
A &\mapsto \text{loads, unloads, single-segment movements}
\end{aligned}
$$



B
c_2
A
D
t
E
F
p_2
c_1
C
p_1
G
c_3
$CG(\Pi)$
c_1
c_2
c_3
t
p_1
p_2
c_1, c_2
t
c_3
p_1, p_2
in c_1
in t
at A
at B
at C
at D
at E
at F
at G
in c_2
in c_3

Disjoint Fork-Decomposition

Definition

Let $\Pi = \langle V, A, I, G \rangle$ be a SAS$^+$ problem. The **fork-decomposition**

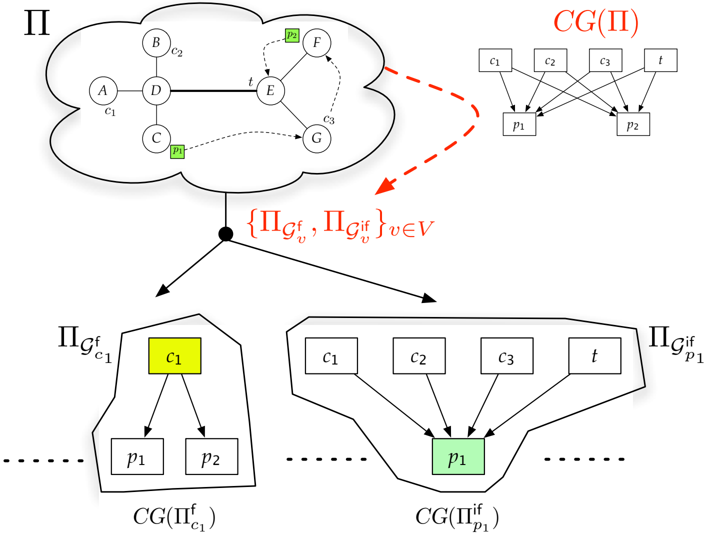
$$\{\Pi_{\mathcal{G}^{\mathrm{f}}_v}, \Pi_{\mathcal{G}^{\mathrm{if}}_v}\}_{v \in V}$$

is a disjoint CGSP decomposition of Π over subgraphs $\mathbf{G} = \{\mathcal{G}^{\mathrm{f}}_v, \mathcal{G}^{\mathrm{if}}_v\}_{v \in V}$ where, for $v \in V$,

$$V_{\mathcal{G}^{\mathrm{f}}_v} = \{v\} \cup \mathrm{succ}(v), \quad E_{\mathcal{G}^{\mathrm{f}}_v} = \bigcup_{u \in \mathrm{succ}(v)} \{(v, u)\}$$

$$V_{\mathcal{G}^{\mathrm{if}}_v} = \{v\} \cup \mathrm{pred}(v), \quad E_{\mathcal{G}^{\mathrm{if}}_v} = \bigcup_{u \in \mathrm{pred}(v)} \{(u, v)\}$$

Disjoint Fork-Decomposition



Optimal cost

Given uniform-cost actions, the optimal cost of solving the original problem is $h^* = 19$

Seminal admissible estimates

$h_{\max} = 8$ (Bonet and Geffner, '01)
$h^2 = 13$ (Haslum and Geffner, '00)

Fork-Decomposition Heuristics Value

$$h_{\mathbb{A}} = h^*_{\Pi^f_{c_1}} + h^*_{\Pi^f_{c_2}} + h^*_{\Pi^f_{c_3}} + h^*_{\Pi^f_{t}} + h^*_{\Pi^{if}_{p_1}} + h^*_{\Pi^f_{p_2}} = 15$$

Forks and Inverted Forks are Hard ...

- Even non-optimal planning for SAS$^+$ problems with fork and inverted fork causal graphs is NP-complete (Domshlak & Dinitz, '01).
- Even if the domain-transition graphs of all variables are strongly connected, optimal planning for forks and inverted forks remain NP-hard (Helmert, '03-04).

Shall we give up?

TRACTABLE CASES OF PLANNING WITH FORKS

PROPOSITION

Given a SAS$^+$ problem $\Pi = \langle V, A, I, G \rangle$ inducing a fork causal graph with a root $r \in V$, if

(I) $|Dom(r)| = 2$, or

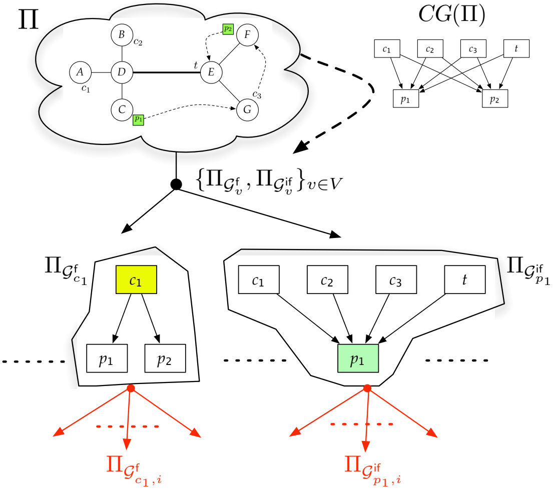
(II) for all $v \in V$, we have $|Dom(v)| = O(1)$,

then finding a cost-optimal plan for Π is poly-time.

PROPOSITION

Given a 1-dependent SAS$^+$ problem $\Pi = \langle V, A, I, G \rangle$ inducing an inverted fork causal graph with a root $r \in V$, if $|Dom(r)| = O(1)$, then finding a cost-optimal plan for Π is poly-time.

CGSPs Meet Domain Decompositions



$\phi_{c_1,i} : dom(c_1) \mapsto \{0, 1\}$

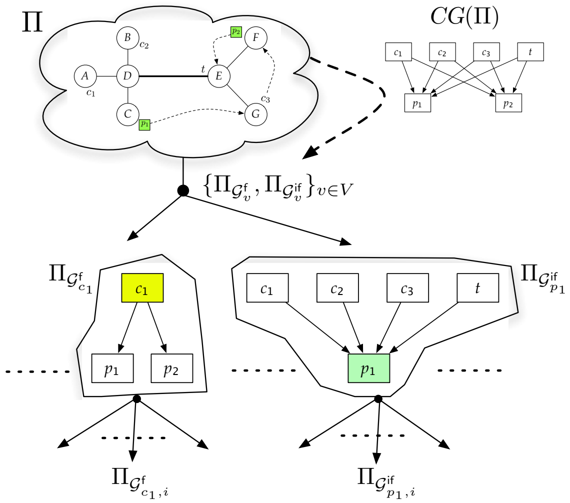
$\phi'_{p_1,i} : dom(p_1) \mapsto \{0, \ldots, k\}$

Disjoint Domain Decomposition

Basic Idea

- Disjoint decompositions can be applied at the level of variable domains
  1. Apply *various* domain abstractions on problem variables
  2. Normalize action costs in abstract problems to satisfy *admissibility-preserving action cost distribution*
  3. Sum-up optimal costs of solving abstract problems

♠ Domain decompositions and CGSPs can be naturally combined!



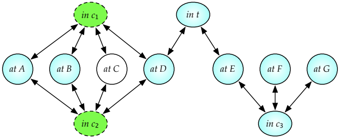
$\phi_{c_1,i} : dom(c_1) \mapsto \{0, 1\}$

$\phi'_{p_1,i} : dom(p_1) \mapsto \{0, 1, 2\}$

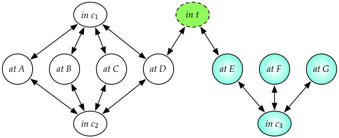
Back to our example

$$\forall l \in Dom(p_1): \quad \phi'_{p_1,i}(l) = \begin{cases} 0, d(I[p_1], l) < 2i-1 \\ 1, d(I[p_1], l) = 2i-1 \\ 2, d(I[p_1], l) > 2i-1 \end{cases}$$

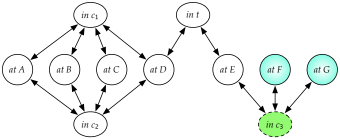
$\phi'_{p_1,1}$



$\phi'_{p_1,2}$



$\phi'_{p_1,3}$



Still Informative?

(Intractable) CGSPs

$h_{\max} = 8 \quad h^2 = 13 \quad h_{\curlywedge} = 15 \quad h^* = 19$

Still Informative? YES

(Intractable) CGSPs

$h_{\max} = 8 \quad h^2 = 13 \quad h_{\mathcal{M}} = 15 \quad h^* = 19$

(Tractable) CGSPs with Domain Abstractions

$h_{\max} = 8 \quad h^2 = 13 \quad h_{\mathcal{M}} = 16 \quad h^* = 19$

Hmm ... what?

Further abstraction gives a more precise estimate??

STILL INFORMATIVE? YES

(INTRACTABLE) CGSPS

$h_{\max} = 8 \quad h^2 = 13 \quad h_{\curlywedge} = 15 \quad h^* = 19$

(TRACTABLE) CGSPS WITH DOMAIN ABSTRACTIONS

$h_{\max} = 8 \quad h^2 = 13 \quad h_{\curlywedge} = 16 \quad h^* = 19$

HMM ... WHAT?

Further abstraction gives a more precise estimate??

HMM ... YES, THAT IS POSSIBLE!

- Domain abstraction may eliminate certain dependencies between the variables
- Less dependencies $\mapsto$ Less action representatives $\mapsto$ Less action cost erosion $\mapsto$ (Potentially) higher estimate

THIS IS JUST THE BEGINNING ...

1. Short-term
   - Implementation and empirical evaluation
   - *"In theory there is no difference between theory and practice. In practice there is."* (Yogi Berra)

2. Long-term
   - More tractability results for cost-optimal planning! (Katz & Domshlak, '07), ...
   - Optimization of structural patterns selection
   - Optimization of variable domains abstraction
   - Extensions to richer formalisms