Structural Patterns of Tractable Sequentially-Optimal Planning

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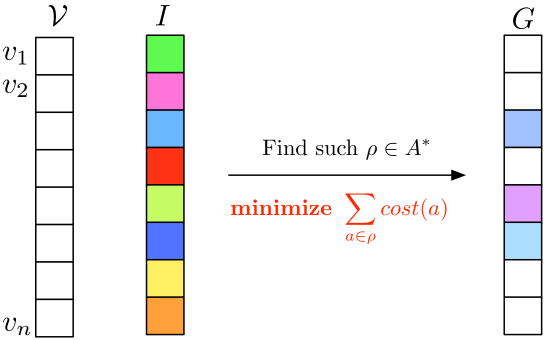
Sequentially-Optimal Classical Planning

Plan Cost = Total Cost of Plan's Actions



HOW TO SOLVE?

A^*/IDA^* WITH AN ADMISSIBLE HEURISTIC

1. Admissible heuristic $\leftarrow$ based on optimal cost of solving an over-approximating abstraction of the original problem Π
2. Homomorphism abstractions

   PRINCIPLE Systematic state merging

   DONE BY Projecting Π onto a subset of its state variables

   KNOWN AS Pattern Databases (PDBs)

ACHILLES HEEL OF PDBS: $|V_i| = O(\log |V|)$

- Each pattern V_i is required to be **small** to allow exhaustive search in the state space of $\Pi^{[V_i]}$
- Typically implies: Limited scalability

Generalizing PDBs to Structural Patterns

Select a (relatively small) set of subsets $V_1, \ldots, V_k$ of V such that, for $1 \leq i \leq k$,

1. $\Pi^{[V_i]}$ is an over-approximating abstraction of Π, and
2. the reachability analysis in $\Pi^{[V_i]}$ is tractable
   (not necessarily due to the size of but)
   due to the specific structure of $\Pi^{[V_i]}$
   - Possibly $|V_i| = \Theta(|V|)$!

Similarly to PDBs,

Disjoint?

YES

$$h(s) = \sum_{i=1}^{k} h^{[\mathcal{V}_i]}(s)$$

NO

$$h(s) = \max_{i=1}^{k} h^{[\mathcal{V}_i]}(s)$$

Structural Patterns: The Good, the Bad, and the Needed

What is Good?

- Generalization of PDBs to homomorphism abstractions of unlimited dimensionality

What is Bad?

- Structural patterns correspond to tractable fragments of optimal planning
- The palette of such known fragments is extremely limited

What is Needed?

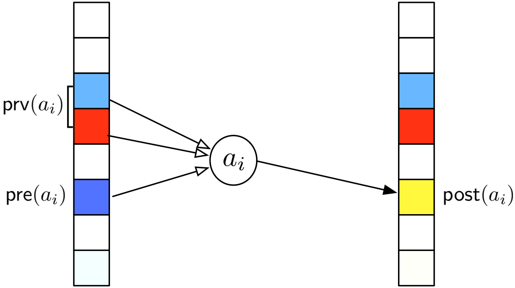
- Discovering new islands of tractability of optimal planning
- This is what we do!

Islands of Tractability

Our focus here: UB fragment of classical planning

1. All actions are **u**nary-effect
2. All state variables $\mathcal{V}$ are **b**inary-valued

$$A = \{a_1, \ldots, a_N\}$$



For all actions $a \in A$, we have $|\mathsf{post}(a)| = 1$

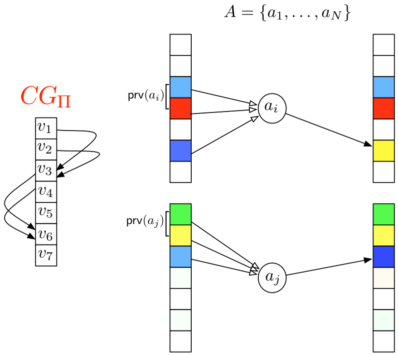
Islands of Tractability

Our focus here: UB fragment of classical planning

1. All actions are **u**nary-effect
2. All state variables $\mathcal{V}$ are **b**inary-valued

Tractable cases are characterized in terms of

1. Form of the problem's causal graph
2. $O(1)$ bounds on causal graph's indegree
3. $O(1)$ bounds on $|\mathrm{prv}(a)|$ (k-dependence)

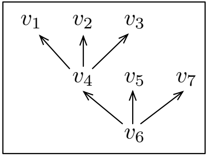


k-DEPENDENCE

Problem is k-dependent iff $|\mathsf{prv}(a)| \leq k$ for all actions $a \in A$

Focus on Acyclic Causal Graphs

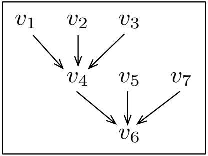
T



Tree

$\subset$

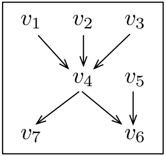
I



Inverted Tree

$\subset$

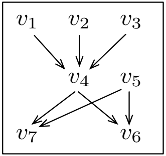
P



Polytree

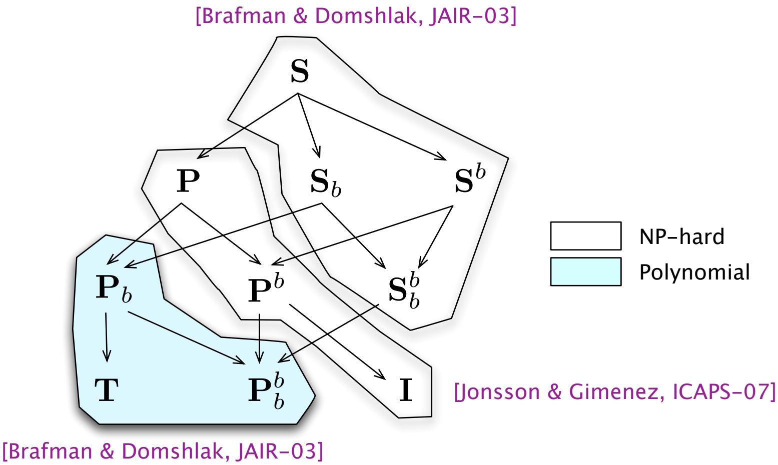
$\subset$

S



Directed–path
Singly Connected

Complexity Results for (Regular) UB Planning



	$k = 1$	$k = 2$	$k = 3$	$k = \Theta(n)$
$\mathbf{P}_b(k)$	—	—	—	P
$\mathbf{P}(k)$	P			NPC
$\mathbf{S}_b(k)$	NPC	—	—	NPC

Sequentially-optimal planning

	$k = 1$	$k = 2$	$k = 3$	$k = \Theta(n)$
$\mathbf{P}_b(k)$	—	—	—	P
$\mathbf{P}(k)$	P			NPC
$\mathbf{S}_b(k)$		NPC	—	NPC

Satisficing plan generation

General Principle

Compiling a problem Π into a *constraint optimization problem* $\mathsf{COP}_\Pi = (\mathcal{X}, \mathcal{F})$ over variables $\mathcal{X}$, functional components $\mathcal{F}$, and the global objective $\min \sum_{\varphi \in \mathcal{F}} \varphi(\mathcal{X})$ such that

(I) the compilation of a problem Π into COP_Π is polynomial in the description size of Π,

(II) the tree-width of the cost network of COP_Π is bounded by a constant,

(III) if Π is unsolvable then all the assignments to $\mathcal{X}$ evaluate the objective function to ∞, and otherwise, the optimum of the global objective is obtained on and only on the assignments to $\mathcal{X}$ that correspond to optimal solutions for Π,

(IV) given an optimal solution to COP_Π, an optimal plan for Π can be reconstructed from the former in polynomial time.

From General Principle to Concrete Algorithms

Reusing tractable constraint optimization

Having such a compilation scheme, we can

1. Solve COP_Π using a standard, poly-time algorithm for *constraint optimization over trees*, and
2. Reconstruct an optimal plan for Π

Same framework $\Leftrightarrow$ Different instances

Variables $\mathcal{X}$ and functional components $\mathcal{F} = \{\varphi_x \mid x \in \mathcal{X}\}$ **differ** from problem class to problem class

POLYTREE + BOUNDED INDEGREES ($\mathbf{P}_b$)

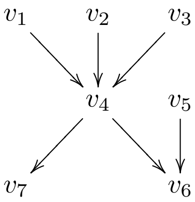
PROPERTY I [BD03]

For any solvable **P** problem Π over n state variables, any SO-plan ρ for Π, and any state variable v in Π, the number of value changes of v along ρ is $\leq n$.

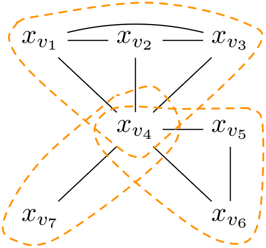
Polytree + Bounded Indegrees ($\mathbf{P}_b$)

Constraint Optimization Problem

- COP variables $\mathcal{X} \leftrightarrow$ State variables V
- Domain of $x_v \leftrightarrow$ Possible value changes of v along plan
- Function components $\leftrightarrow$ State variables' families



Causal graph of Π



Cost network of COP_{Π}

Polytree + 1-dependence ($\mathbf{P}(1)$)

Property II (Uniform-Cost Actions)

For any solvable problem Π, there exists an SO-plan ρ such that all the changes of each variable to a certain value are performed by the same type of action.

Property III (General Actions)

For any solvable problem Π, there exists an SO-plan ρ such that all the changes of each variable are done using at most three types of actions which are prevailed by at most two parents.

See the full version for the construction for $\mathbf{P}(1)$ with general actions.

Notation

Given an **S** problem $\Pi = \langle V, A, I, G \rangle$, for each variable $v \in V$, b_v denotes the initial value $I[v]$, and w_v denotes the opposite value of v.

$\sigma(v)$ denotes the **longest possible sequence of values obtainable by** v **along a cost-optimal plan** ρ, with $|\sigma(v)| = n + 1$, with b_v occupying all the odd positions of $\sigma(v)$, and w_v occupying all the even positions of $\sigma(v)$.

$\trianglerighteq^*[\sigma(v)]$ denotes the set of all **goal-valid prefixes of** $\sigma(v)$, that is, prefixes of $\sigma(v)$ that end up with $G[v]$ if the latter is specified.

P(1) with Uniform-Cost Actions

COP Variables

$$\mathcal{X} = \{x_v \mid v \in V\} \cup \{x_v^u \mid (\overrightarrow{u,v}) \in CG(\Pi)\}$$

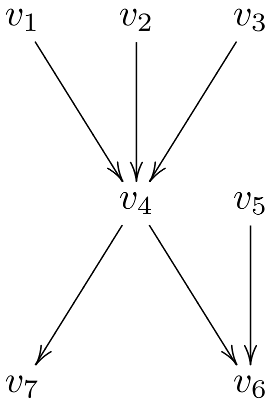
COP Variable Domains

$$Dom(x_v) = \trianglerighteq^*[\sigma(v)]$$
$$Dom(x_v^u) = \{[\delta_{\mathrm{w}}, \delta_{\mathrm{b}}, \eta] \mid \delta_{\mathrm{w}}, \delta_{\mathrm{b}} \in \{0,1\}, 0 \leq \eta \leq n\}$$

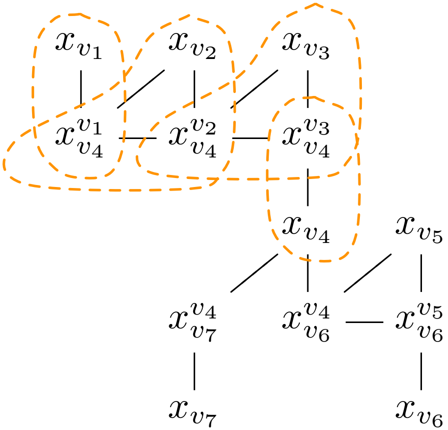
COP Functional Scopes ($\mathrm{pred}(v) = \{u_1, \ldots, u_k\}$)

$$Q_x = \begin{cases} \{x_v\}, & x = x_v, k = 0 \\ \{x_v, x_v^{u_k}\}, & x = x_v, k > 0 \\ \{x_v^{u_1}, x_{u_1}\}, & x = x_v^{u_1}, k > 0 \\ \{x_v^{u_j}, x_v^{u_{j-1}}, x_{u_j}\}, & x = x_v^{u_j}, 1 < j \leq k \end{cases}$$

POLYTREE + 1-DEPENDENCE ($\mathbf{P}(1)$)



Causal graph of Π



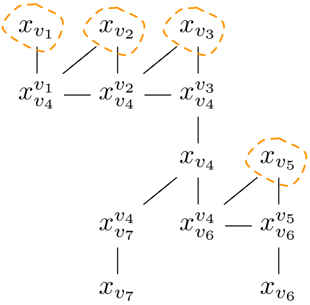
Cost network of COP_{Π}

COP Functionals (I)

$\mathbf{P}(1)$ with Uniform-Cost Actions

For each x_v with $\mathrm{pred}(v) = \emptyset$

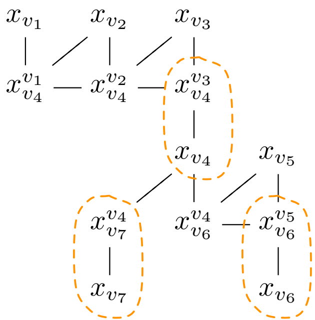
$$\varphi_{x_v}(\sigma_v) = \begin{cases} 0, & |\sigma_v| = 1, \\ 1, & |\sigma_v| = 2, a_{w_v} \in A_v, \\ |\sigma_v| - 1, & |\sigma_v| > 2, a_{w_v}, a_{b_v} \in A_v, \\ \infty, & \text{otherwise} \end{cases}$$



COP Functionals (II)
$\mathbf{P}(1)$ with Uniform-Cost Actions

For each x_v with $\mathsf{pred}(v) = \{u_1, \ldots, u_k\}$, $k > 0$

$$\varphi_{x_v}(\sigma_v, [\delta_w, \delta_b, \eta]) = \begin{cases} 0, & |\sigma_v| = 1, [\delta_w, \delta_b, \eta] = [0,0,0], \\ 1, & |\sigma_v| = 2, [\delta_w, \delta_b, \eta] = [1,0,1], \\ |\sigma_v| - 1, & |\sigma_v| > 2, [\delta_w, \delta_b, \eta] = [1,1,|\sigma_v| - 1], \\ \infty, & \text{otherwise} \end{cases}$$



Indicator Function

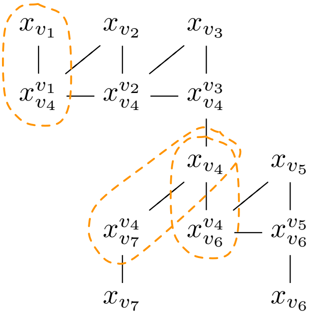
$$\varphi\left(\left[\delta_{\mathsf{w}}, \delta_{\mathsf{b}}, \eta\right], \sigma_u\right) = \begin{cases} 0, & \delta_{\mathsf{w}} = 0, \delta_{\mathsf{b}} = 0, \\ 0, & \delta_{\mathsf{w}} = 1, \delta_{\mathsf{b}} = 0, a_{\mathsf{w}_v|\mathsf{b}_u} \in A_v, \\ 0, & \delta_{\mathsf{w}} = 1, \delta_{\mathsf{b}} = 0, |\sigma_u| > 1, a_{\mathsf{w}_v|\mathsf{w}_u} \in A_v, \\ 0, & \delta_{\mathsf{w}} = 0, \delta_{\mathsf{b}} = 1, a_{\mathsf{b}_v|\mathsf{b}_u} \in A_v, \\ 0, & \delta_{\mathsf{w}} = 0, \delta_{\mathsf{b}} = 1, |\sigma_u| > 1, a_{\mathsf{b}_v|\mathsf{w}_u} \in A_v, \\ 0, & \delta_{\mathsf{w}} = 1, \delta_{\mathsf{b}} = 1, a_{\mathsf{w}_v|\mathsf{b}_u}, a_{\mathsf{b}_v|\mathsf{b}_u} \in A_v, \\ 0, & \delta_{\mathsf{w}} = 1, \delta_{\mathsf{b}} = 1, |\sigma_u| > 1, a_{\mathsf{w}_v|\mathsf{w}_u}, a_{\mathsf{b}_v|\mathsf{w}_u} \in A_v, \\ 0, & \delta_{\mathsf{w}} = 1, \delta_{\mathsf{b}} = 1, |\sigma_u| \geq \eta, a_{\mathsf{w}_v|\mathsf{b}_u}, a_{\mathsf{b}_v|\mathsf{w}_u} \in A_v, \\ 0, & \delta_{\mathsf{w}} = 1, \delta_{\mathsf{b}} = 1, |\sigma_u| > \eta, a_{\mathsf{w}_v|\mathsf{w}_u}, a_{\mathsf{b}_v|\mathsf{b}_u} \in A_v, \\ \infty, & \text{otherwise} \end{cases}$$

COP Functionals (III)

P(1) with Uniform-Cost Actions

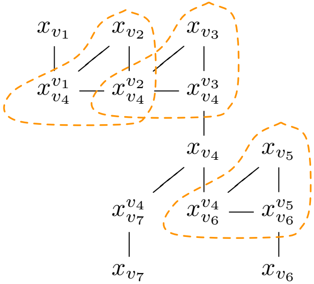
For $x_v^{u_1}$

$$\varphi_{x_v^{u_1}}\left(\left[\delta_{\mathsf{w}}, \delta_{\mathsf{b}}, \eta\right], \sigma_{u_1}\right) = \varphi\left(\left[\delta_{\mathsf{w}}, \delta_{\mathsf{b}}, \eta\right], \sigma_{u_1}\right)$$



For each $x_v^{u_j}$, $j > 1$

$$\varphi_{x_v^{u_j}}\left(\mathbf{[}\delta_\mathsf{w}, \delta_\mathsf{b}, \eta\mathbf{]}, [\![\delta'_\mathsf{w}, \delta'_\mathsf{b}, \eta']\!], \sigma_{u_j}\right) = \begin{cases} \varphi\left([\![\delta_\mathsf{w} - \delta'_\mathsf{w}, \delta_\mathsf{b} - \delta'_\mathsf{b}, \eta]\!], \sigma_{u_j}\right), & \eta = \eta' \\ \infty & \text{otherwise} \end{cases}$$



Future work

This is just the beginning ...

- Discovering new islands of tractability of optimal planning
  - New results for non-binary domains with *fork* and *inverted fork* causal graphs [KD07b]
  - Positive evidence of relevance to structural patterns *heuristics*
- Devising as efficient as possible algorithms for such islands
- Translating and/or abstracting the general planning problems into such islands