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ECE 6790

Homework 3

3.13

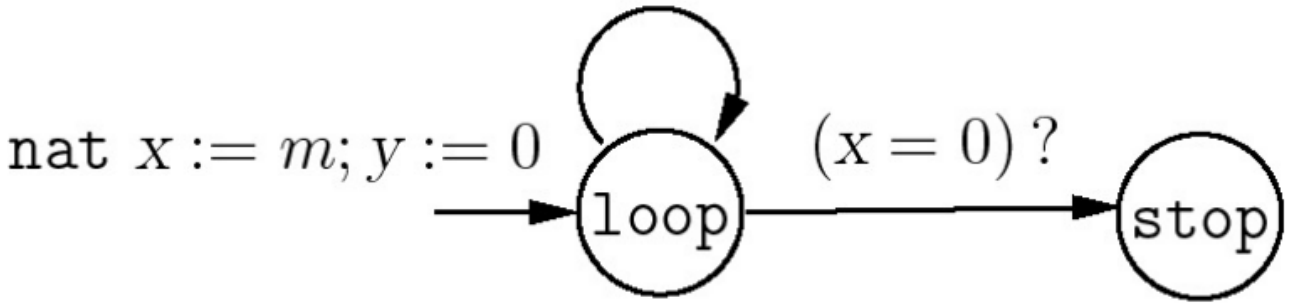
Consider the transition system $Mult(m,n)$ described in exercise 3.1. Describe this transition system symbolically using initialization and transition formulas.

3.1

Given two natural numbers *m* and *n*, consider the program Mult that multiplies the input numbers using two variables *x* and *y*, of type nat, as shown in the below figure. Describe the transition system *Mult*(m,n) that captures the behavior of this program on input numbers *m* and *n*, that is, describe the states, initial states, and transitions. Argue that when the value of the variable *x* is 0, the value of the variable *y* must equal the product of the input numbers *m* and *n*, that is, the following property is an invariant of this transition system:

```
(mode = stop) -> (y = m*n)
```

$$(x > 0) \rightarrow \{x := x - 1;\ y := y + n\}$$



Initial States = {x = m; y = 0;}

States = {loop, stop}

Transitions

```
if x > 0 then
    x := x - 1;
    y := y + n;
    states := loop;
```

```
else
    states := stop
```

When x = 0, y must be the product of *m* and *n* because that is the definition of multiplication. Taking a number (*n*) and adding it to itself a number of times (*m*). x is set to the number of times and y is counting the running total.

Initial $$ x = m; y = 0 $$ Transition $$ if (x > 0) then (x' = x - 1),(y' = y + n) $$

3.14

Consider the description of the component *Switch* given as an extended-state machine in figure 2.2. Give the initialization and reaction formulas corresponding to *Switch*. Obtain the transition formula for the corresponding transition system in as simplified a form as possible.

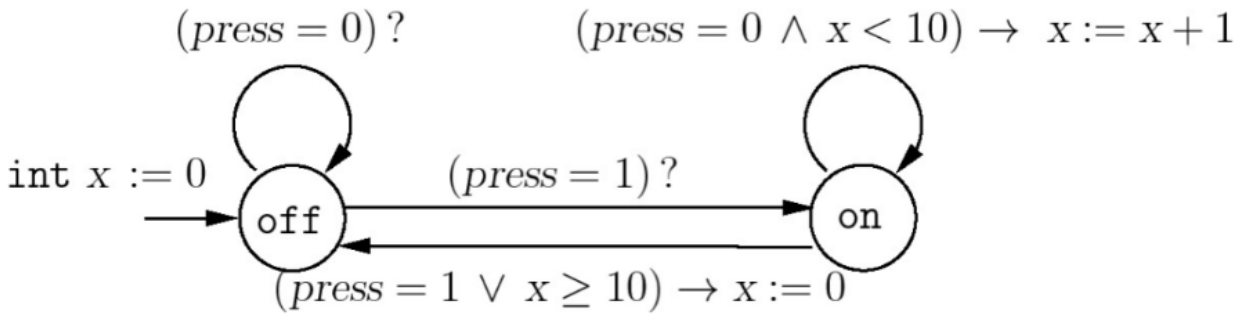


Figure 2.2: Description of `Switch` as an Extended-State Machine

Initial

```
x := 0
press := 0
mode := off
```

Reaction

```
if mode = off then
    if press = 1 then mode := on
else # mode = on
    if x >= 10 and press = 1 then x := 1 and mode := off
    else x := x + 1
endif
```

3.16

Consider the symbolic image computation for a transition system with two real-valued variables x ad y and transition description given by the formula $x' = x + 1$ or $y' = x$. Suppose the region A is described by the formula $0 \le x \le 4, y \le 7$. Compute the formula describing the post-image of A.

A' {$0 \le x' - 1 \le 4, 0 \le y \le 7$} A' {$1 \le x' \le 5, 0 \le y' \le 7$}

3.17

Consider a transition system *T* with two integer variables x and y. The transitions of the system corresponding to executing the statement:

$$ if (x < y) then x:= x + y else y := y + 1 $$

Write the transition formula over the variables $x, y, x', and y$ that captures the transition relation of the system. Consider a region A of the above transition system described by the formula $0 < x < 5$. Compute the formula describing the post-image of A.

$$ if (x < y) then x' := x + y else y' := y + 1 $$

A {$ 0 < x < 5 $} where $x = 2; y = 3$ A' {$ 0 < x' - y < 5$} A' {$ 0 < x' - 3 < 5$} A' {$ 3 < x' < 8$} A' {$ 3 < x < 8$}

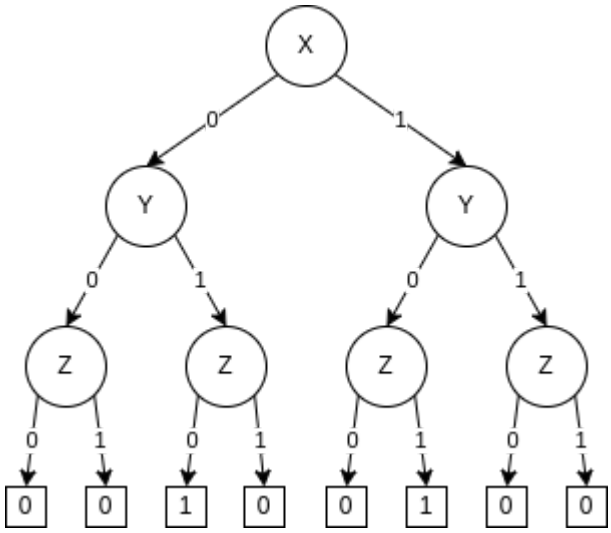
3.20

Consider the Boolean formula $$(x or y) and (~x or z) and (~y or ~z)$$

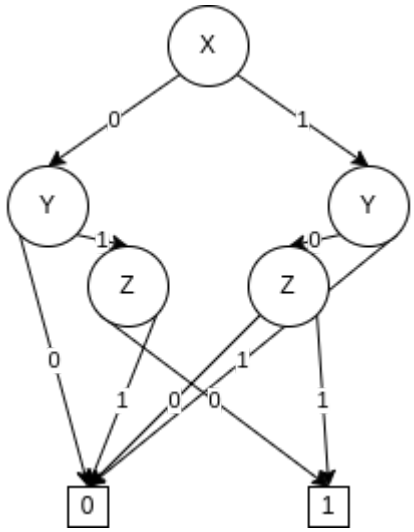
Draw the ROBDD for this formula with respect to the variable ordering $x < y < z$.

Note: The textbook has an example

Initial OBDD

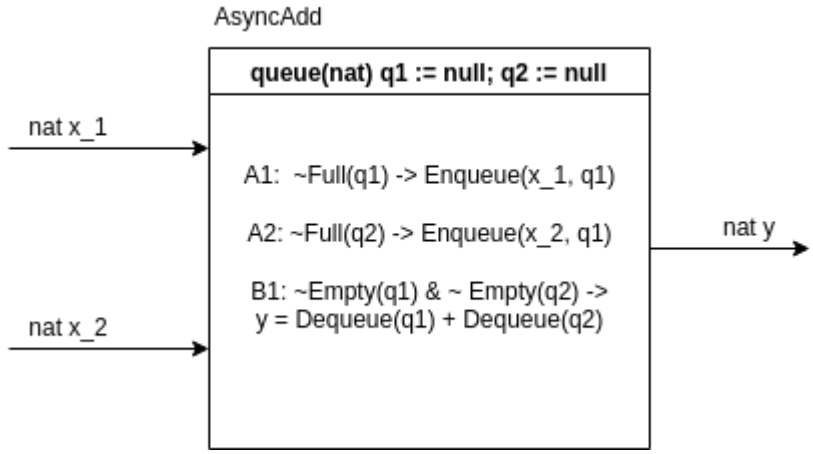


Final ROBDD



4.1

We want to design an asynchronous adder process *AsyncAdd* with input channels x_1 and x_2 and an output channel y, all of type nat. If the *i*th input message arriving on the channel x_1 is v and the *i*th input message arriving on channel x_2 is w, then the *i*th value output by the process *AsyncAdd* on its output channel should be $v + w$. Describe all the components of the process *AsyncAdd*



4.2

We want to design an asynchronous process *Split* that is the dual of *Merge*. The process *Split* has one input channel *in* and two output channels out_1 and out_2. The messages received on the input channel should be routed to one of the output channels in a nondeterministic manner so that all possible splittings of the input stream are feasible executions. Describe all the components of the desired process *Split*.