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ECE 6790

Homework 6

Exercise 5.9:

For each of the LTL formulas below, construct a Buchi automaton that accepts exactly those traces that satisfy the formula:

1. □⋄e V ⋄□f
2. □⋄e Λ □⋄f
3. □(e → e U f)

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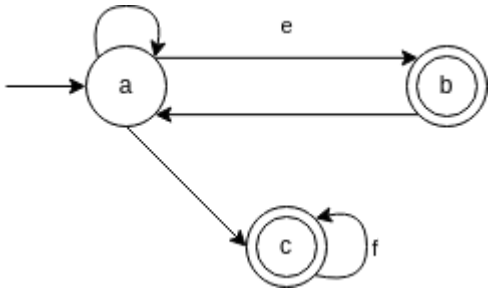
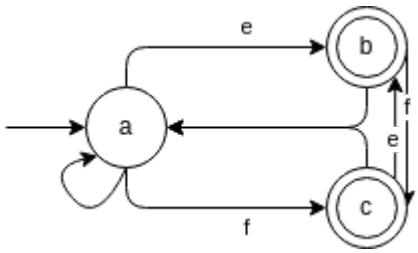
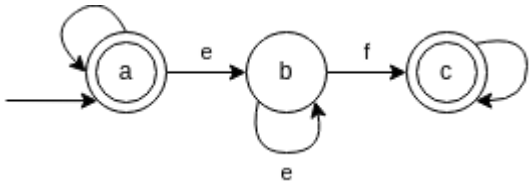
1. 

2. 

3. 

Exercise 5.10:

Write an LTL formula that exactly describes the set of traces that are accepted by the Buchi automaton shown in figure 5.9. Explain your answer.

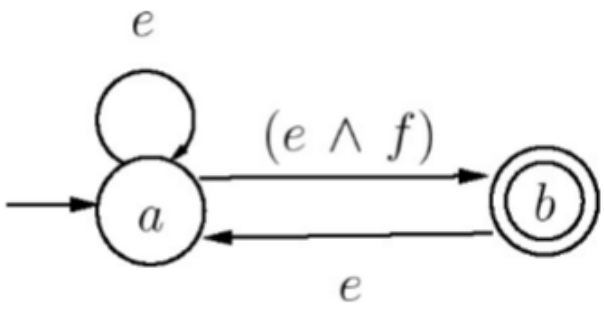


Figure 5.9: Exercise: From a Büchi Automaton to LTL

□e ∧ □⋄f

Exercise 5.15

φ = (e U Of) V ~e

You are strongly encouraged to read Exercise 5.14 and its solution on the next page first before attempting this problem. Answer the questions below instead of those in the problem description.

(a) Show all states included in Sub(φ).

(b) Tableau construction results in 16 states. Determine whether each of the 16 states listed below is a subset of Sub(φ). The sub-formula ψ below is ψ = (e U Of).

- q0 = {e,f,Of,Oψ,ψ,φ}
- q1 = {e,f,Of,ψ,φ}
- q2 = {e,f,Oψ,ψ,φ}
- q3 = {e,f}
- q4 = {e,Of,Oψ,ψ,φ}
- q5 = {e,Of,ψ,φ}
- q6 = {e,Oψ,ψ,φ}
- q7 = {e}
- q8 = {¬e,f,Of,Oψ,ψ,φ}
- q9 = {¬e,f,Of,ψ,φ}
- q10 = {¬e,f,Oψ,φ}
- q11 = {¬e,f,φ}
- q12 = {¬e,Of,Oψ,ψ,φ}
- q13 = {¬e,Of,ψ,φ}
- q14 = {¬e,Oψ,φ}
- q15 = {¬e,φ}

(c) You are not required to draw the entire Buchi automaton. Answer the questions below instead.

i. List all initial states.

ii. Draw all transitions and their next states out of the initial state q0.

iii. What states does the accepting set F_ψ include? List all such states.

(a) {φ, ψ, Oψ, ~e, e, Of}

(b)

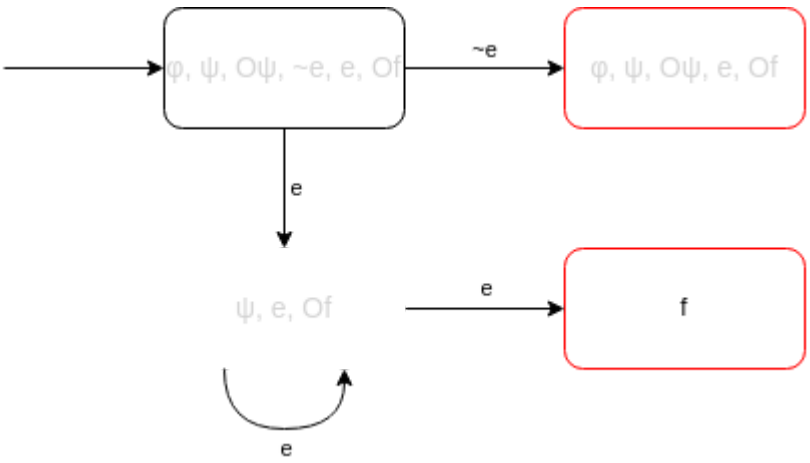
- q0 No
- q1 No
- q2 No
- q3 No
- q4 Yes
- q5 Yes
- q6 Yes
- q7 Yes
- q8 No
- q9 No
- q10 No
- q11 No
- q12 Yes
- q13 Yes
- q14 Yes
- q15 Yes

(c)

i.

- {φ, ψ, Oψ, ~e, e, Of}
- {φ, ψ, Oψ, ~e, e, f}
- {φ, ψ, Oψ, e, Of}

ii.



iii.

φ, ψ, Oψ, ~e, e, Of, f

Exercise 5.19:

Consider a transition system with two variables x and y of type nat. Suppose the transitions of the system are described by the conditional statement

```
    if ( x > y ) then x := x + 1 else y := x
```

First, describe the transition region as a formula *Trans* over the variables x, y, x', and y'. Consider the region A given by the formula 1 <= y <= 5. Compute the pre-image of the region A.

```
if (x > y) then
    x' := x + 1
    y' := y
else
    x' := x
    y' := y
```

The Pre-Image of A would be

```
(1 <= y <= 5 ∧ x > y) U (0 <= y <= 4 ∧ x <= y)
```
