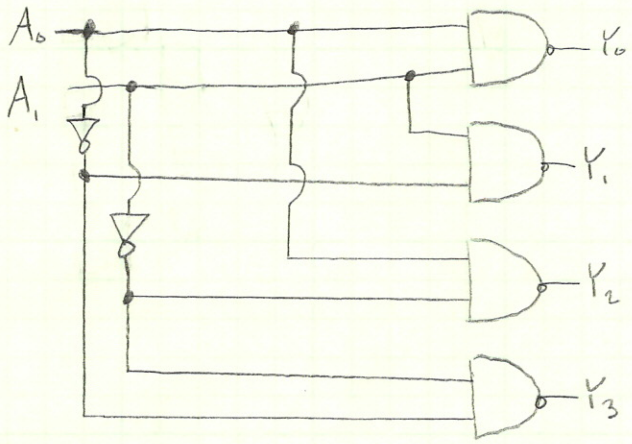
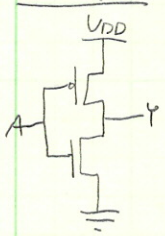
Problem 1

a)

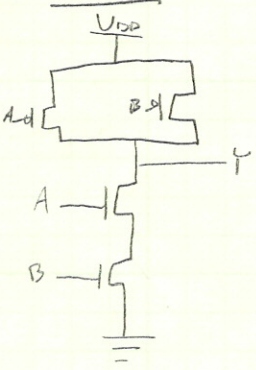
	A_1	A_0
Y_0	0	0
Y_1	0	1
Y_2	1	0
Y_3	1	1

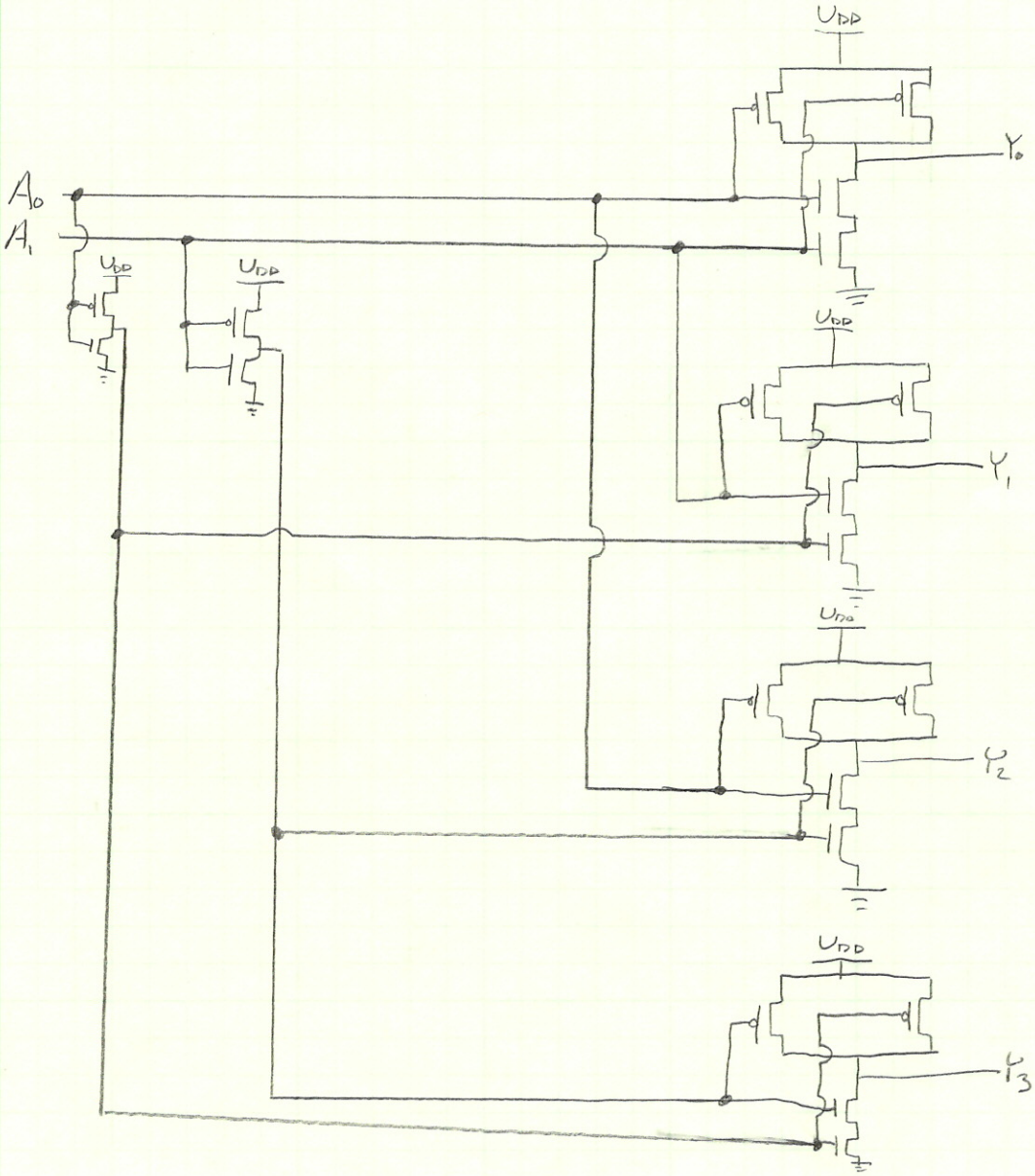


Inverter



NAND



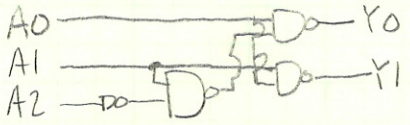


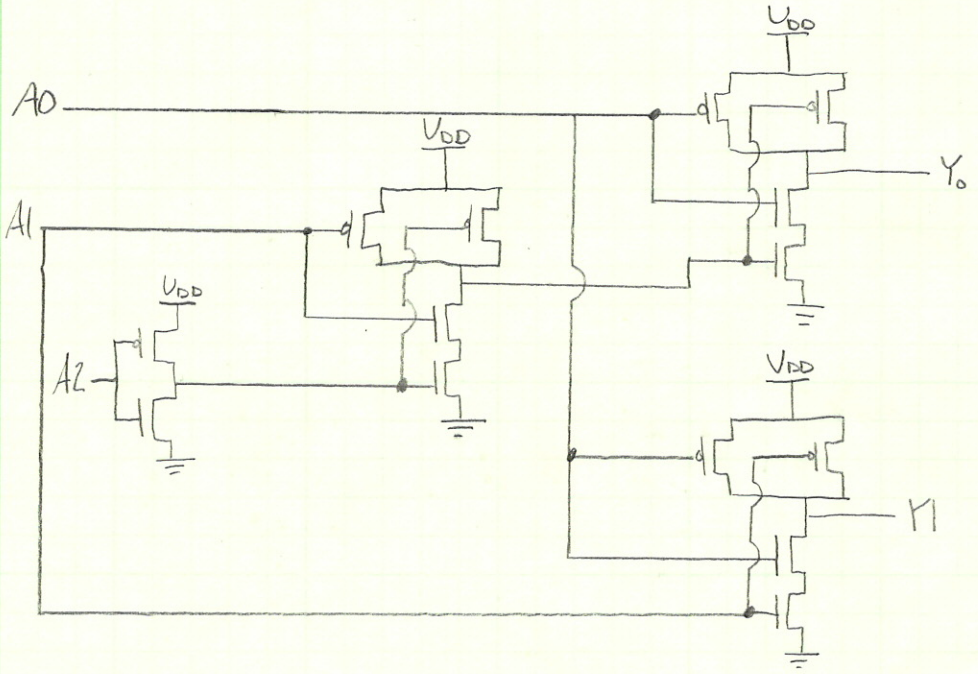
Problem 1

b)

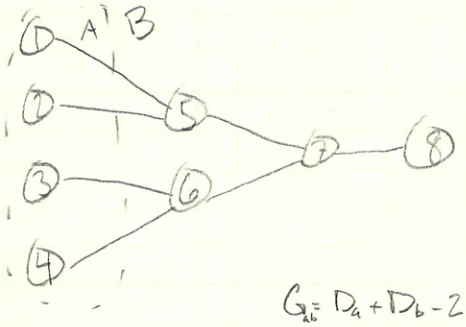
$$Y0 = \overline{A0} \cdot (A1 + \overline{A2}) = \overline{\overline{A0} \cdot (\overline{A1} \cdot A2)}$$

$$Y1 = \overline{A0} \cdot \overline{A1}$$





Problem 2



$D = E - I$

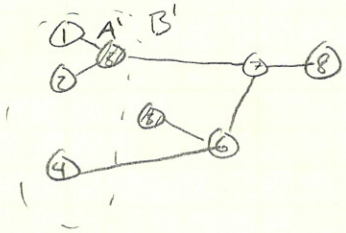
$G_{ab} = D_a + D_b - 2C_{ab}$

	I	E	D
1	0	1	1
2	0	1	1
3	0	1	1
4	0	1	1
5	1	2	1
6	1	2	1
7	3	0	-3
8	1	0	-1

$G_{15} = 1 + 1 - 2 \cdot 1 = 0$
$G_{16} = 1 + 1 - 2 \cdot 0 = 2$
$G_{17} = 1 + -3 - 2 \cdot 0 = -2$
$G_{18} = 1 + -1 - 2 \cdot 0 = 0$
$G_{25} = 1 + 1 - 2 \cdot 1 = 0$
$G_{26} = 1 + 1 - 2 \cdot 0 = 2$
$G_{27} = 1 + -3 - 2 \cdot 0 = -2$
$G_{28} = 1 + -1 - 2 \cdot 0 = 0$

$G_{35} = 1 + 1 - 0 = 2$ ←
$G_{36} = 1 + 1 - 2 = 0$
$G_{37} = 1 + -3 - 0 = -2$
$G_{38} = 1 + -1 - 0 = 0$
$G_{45} = 1 + 1 - 0 = 2$
$G_{46} = 1 + 1 - 2 = 0$
$G_{47} = 1 + -3 - 0 = -2$
$G_{48} = 1 + -1 - 0 = 0$

$g(3,5) = 2$



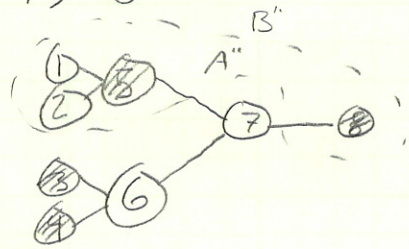
	I	E	D'
1	1	0	-1
2	1	0	-1
4	0	1	1
6	2	1	-1
7	2	1	-1
8	1	0	-1

$G'_{16} = -1 - 1 - 0 = -2$
$G'_{17} = -1 - 1 - 0 = -2$
$G'_{18} = -1 - 1 - 0 = -2$

$G'_{26} = -1 - 1 - 0 = -2$
$G'_{27} = -1 - 1 - 0 = -2$
$G'_{28} = -1 - 1 - 0 = -2$

$G'_{46} = 1 - 1 - 2 = -2$
$G'_{47} = 1 - 1 - 0 = 0$
$G'_{48} = 1 - 1 - 0 = 0$ ←

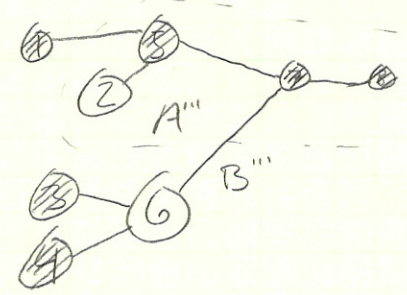
$g(4,8) = 0$



	I	E	D
1	1	0	-1
2	1	0	-1
6	3	0	-3
7	1	2	1

$G_{16} = -1 - 3 - 0 = -4$
$G_{17} = -1 + 1 - 0 = 0$ ←
$G_{26} = -1 - 3 - 0 = -4$
$G_{27} = -1 + 1 - 0 = 0$

$g(1,7) = 0$



	I	E	D
2	1	0	-1
6	2	1	-1

$G_{26} = -1 - 1 - 0 = -2$

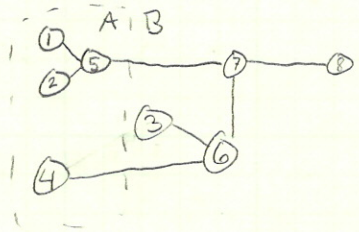
$g(2,6) = -2$

partial sum $G_1 = 2$ $G_2 = 2$ $G_3 = 0$

Problem 2

$D = E - I$

Pass 2



	I	E	D
1	1	0	-1
2	1	0	-1
3	1	0	-1
4	0	1	1
5	2	1	-1
6	2	1	-1
7	2	1	-1
8	1	0	-1

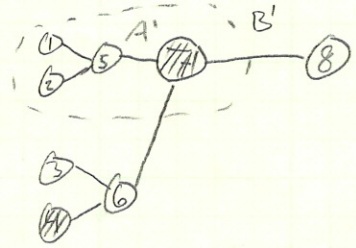
$G_{13} = -1 - 1 - 0 = -2$
$G_{16} = -1 - 1 - 0 = -2$
$G_{17} = -1 - 1 - 0 = -2$
$G_{18} = -1 - 1 - 0 = -2$

$G_{23} = -1 - 1 - 0 = -2$
$G_{26} = -1 - 1 - 2 = -4$
$G_{27} = -1 - 1 - 0 = -2$
$G_{28} = -1 - 1 - 0 = -2$

$G_{43} = 1 - 1 - 0 = 0$
$G_{45} = 1 - 1 - 0 = 0$
$G_{46} = 1 - 1 - 2 = -2$
$G_{47} = 1 - 1 - 0 = 0$

$G_{153} = -1 - 1 - 0 = -2$
$G_{56} = -1 - 1 - 0 = -2$
$G_{57} = -1 - 1 - 2 = -4$
$G_{58} = -1 - 1 - 0 = -2$

$g(4,7) = 0$



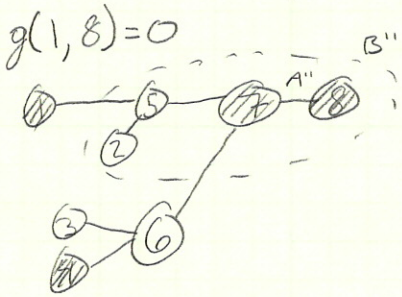
	I	E	D
1	1	0	-1
2	1	0	-1
3	1	0	-1
5	3	0	-3
6	2	1	-1
8	0	1	1

$G_{13} = -1 - 1 - 0 = -2$
$G_{16} = -1 - 1 - 0 = -2$
$G_{18} = -1 + 1 - 0 = 0$

$G_{23} = -1 - 1 - 0 = -2$
$G_{26} = -1 - 1 - 0 = -2$
$G_{28} = -1 + 1 - 0 = 0$

$G_{53} = -3 - 1 - 0 = -4$
$G_{56} = -3 - 1 - 0 = -4$
$G_{58} = -3 + 1 - 0 = -2$

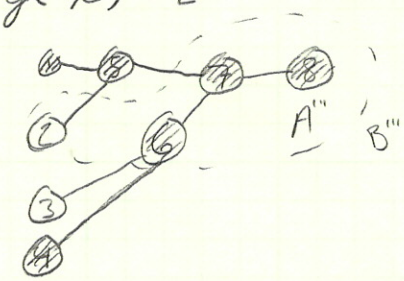
$g(1,8) = 0$



	I	E	D
2	1	0	-1
3	1	0	-1
5	2	1	-1
6	2	1	-1

$G_{23} = -1 - 1 - 0 = -2$
$G_{26} = -1 - 1 - 0 = -2$
$G_{53} = -1 - 1 - 0 = -2$
$G_{66} = -1 - 1 - 0 = -2$ ←

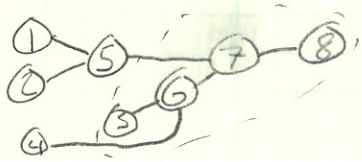
$g(5,6) = -2$



	I	E	D
2	0	1	1
3	0	1	1

$G_{23} = 1 + 1 - 0 = 2$

$g(2,3) = 2$



Problem 2

My guess of the optimal was $A=\{1,5,7,8\}$ $B=\{2,3,4,6\}$ cost = 2

The K-L algorithm gave me $A=\{2,3,4,5\}$ $B=\{1,6,7,8\}$ cost = 2

The K-L algorithm did generate <u>an</u> optimal solution.

Problem 3

a)

Cell	F	T	FS(i) (cross)	TE(i) (not cross)	g(i) = FS(i) - TE(i)
C4	A	B	$\|\{6\}\|$ = 1	$\|\{1,3,13\}\|$ = 3	-2
C7	B	A	$\|\{1,11\}\|$ = 2	$\|\{\}\|$ = 0	2

b)

Nets with C7: N4, N6, N8, N11, N13

Cells in those nets: ~~C7, C8, C4, C9, C1, C2, C6, C11, C12~~

C1, C2, C3, C4, C6, C8, C9, C12

Cells that change gain: C1, C2, C3, C4, C6, C8, C9, C12

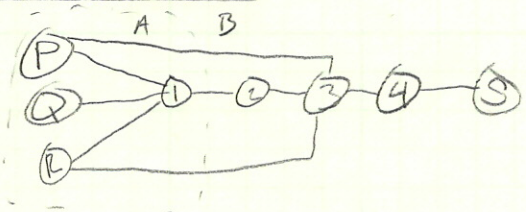
c) $r|U| - s_{max} \leq |A| \leq r|U| + s_{max}$ $\quad s_{max}=1 \quad |U|=12 \quad r=0.5 \quad A=7$

$0.5(12) - 1 \leq 1 \leq 0.5(12) + 1$ $\quad 5 \leq 7 \leq 7 \therefore$ Balanced

d)

Free Cells	Gain
C1	decrease - C7 joing partition
C2	decrease - C7 joing partition
C3	decrease - C7 joining partition
C4	decrease - C7 joining partition - on two nets
C6	decrease - C7 joing partition
C8	increase - C7 leaving partition
C9	increase - C7 leaving partition
C12	increase - C7 leaving partition

Problem 4 - Textbook 2.7



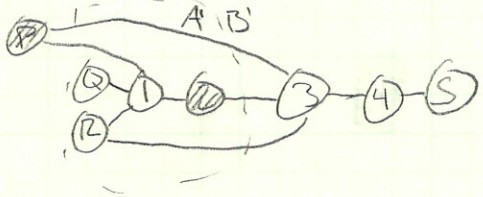
$A = \{P, Q, R, 1\}$

$B = \{2, 3, 4, S\}$

	I	E	D
P	1	1	0
Q	1	0	-1
R	1	1	0
1	3	1	-2
2	1	1	0
3	2	2	0
4	2	0	-2
S	1	0	-1

$G_{P2} = 0+0-0 = 0$
$G_{P3} = 0+0-2 = -2$
$G_{P4} = 0-2-0 = -2$
$G_{PS} = 0-1-0 = -1$
$G_{Q2} = -1+0-0 = -1$
$G_{Q3} = -1+0-0 = -1$
$G_{Q4} = -1-2-0 = -3$
$G_{QS} = -1-1-0 = -2$
$G_{R2} = 0+0-0 = 0$
$G_{R3} = 0+0-2 = -2$
$G_{R4} = 0-2-0 = -2$
$G_{RS} = 0-1-0 = -1$
$G_{12} = -2+0-2 = -4$
$G_{13} = -2+0-0 = -2$
$G_{14} = -2-2-0 = -4$
$G_{1S} = -2-1-0 = -3$

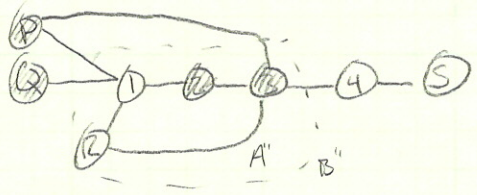
$g(P, 2) = 0$



	I	E	D
Q	1	0	-1
R	1	1	0
S	1	0	-1
1	3	1	-2
3	2	2	0
4	2	0	-2

$G_{Q3} = -1+0-0 = -1$ ←
$G_{Q4} = -1-2-0 = -3$
$G_{QS} = -1-1-0 = -2$
$G_{R3} = 0+0-2 = -2$
$G_{R4} = 0-2-0 = -2$
$G_{RS} = 0-1-0 = -1$
$G_{13} = -2+0-0 = -2$
$G_{14} = -2-2-0 = -4$
$G_{1S} = -2-1-0 = -3$

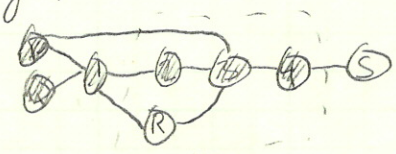
$g(Q, 3) = -1$



	I	E	D
R	2	0	-2
S	1	0	-1
1	2	2	0
4	1	1	0

$G_{R4} = -2+0-0 = -2$
$G_{RS} = -2-1-0 = -3$
$G_{14} = 0+0-0 = 0$ ←
$G_{1S} = 0-1-0 = -1$

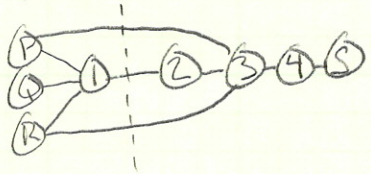
$g(1, 4) = 0$



	I	E	D
R	1	1	0
S	0	1	1

$G_{RS} = 0+1-0 = 1$

$g(R, S) = 1$



$A = \{P, Q, R, 1\}$
$B = \{2, 3, 4, S\}$

Problem 5 - Textbook 2.9

Unequal Size Blocks:

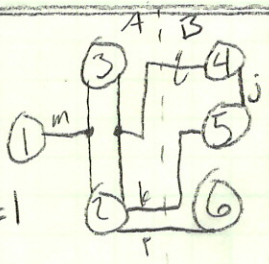
1) $|A| = Min(n_1, n_2) = n_1 \quad |B| = Max(n_1, n_2) = n_2$

2) Max number of swaps = n_1

After KL swaps, size of blocks remain unchanged:

$$|A| = n_1 \; ; \; |B| = n_2$$

Problem 6 - Textbook 2.14



$A(j)=0 \quad A(m)=3 \quad A(q)=2 \quad A(p)=1 \quad A(k)=1$

$B(j)=2 \quad B(m)=0 \quad B(q)=1 \quad B(p)=1 \quad B(k)=1$

$F = A$

$F = B$ - k_p t_{kp} values

$F(j)=0 \quad F(m)=3 \quad F(q)=2 \quad F(k)=1 \quad F(p)=1$

$T(j)=2 \quad T(m)=0 \quad T(q)=1 \quad T(k)=1 \quad T(p)=1$

$i=1$ F=A; T=B C_1 on N_m $\therefore$ $T(m)=0 \therefore g(1) = 0-1 = -1$

$i=2$ F=A; T=B C_2 on N_m/N_q $\therefore$ $T(m)=0 \therefore g(2) = 0-1 = -1$

$i=3$ F=A; T=B C_3 on N_m/N_q $\therefore$ $T(m)=0 \therefore g(3) = 0-1 = -1$

$i=4$ F=B; T=A C_4 on N_q/N_j $\therefore$ $F(j)=2 \therefore g(4) = 0$

$i=5$ F=B; T=A C_5 on N_j/N_k $\therefore$ $F(j)=2 \therefore g(5) = 0$

$i=6$ F=B; T=A C_6 on N_p $\therefore$ $F(p)=1 \therefore g(6) = 0+1 = 1$

Problem 7 – Textboook 2.15

I typed it into excel because it was easier to fix my mistakes :)

Cell	F	T	FS(i)	TE(i)	g(i)
1	A	B	0	1	-1
2	A	B	1	3	-2
3	A	B	0	2	-2
4	A	B	0	2	-2
5	A	B	0	2	-2
6	B	A	1	0	1

G1(1) = -1

Cell	m		q		k		p		j		Gain
	F	T	F	T	F	T	F	T	F	T	
2	2	1	3	0	2	0	1	1	-	-	-1
3	2	1	3	0	-	-	-	-	-	-	-1
4	-	-	3	0	-	-	-	-	2	0	-2
5	-	-	-	-	2	0	-	-	2	0	-2
6	-	-	-	-	-	-	1	1	-	-	1

G2(2) = -1

Cell	m		q		k		p		j		Gain
	F	T	F	T	F	T	F	T	F	T	
3	1	2	2	1	-	-	-	-	-	-	1
4	-	-	2	1	-	-	-	-	2	0	-1
5	-	-	-	-	1	1	-	-	2	0	0
6	-	-	-	-	-	-	2	0	-	-	-1

G3(3) = 1

Cell	m		q		k		p		j		Gain
	F	T	F	T	F	T	F	T	F	T	
4	-	-	1	2	-	-	-	-	2	0	0
5	-	-	-	-	1	1	-	-	2	0	0
6	-	-	-	-	-	-	2	0	-	-	-1

G4(4) = 0

Cell	m		q		k		p		j		Gain
	F	T	F	T	F	T	F	T	F	T	
5	-	-	-	-	1	1	-	-	1	1	2
6	-	-	-	-	-	-	2	0	-	-	-1

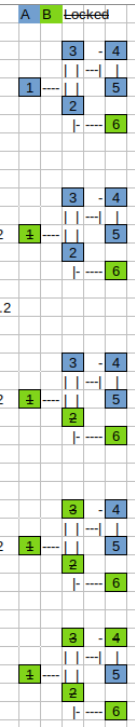
Cell	Weight
1	3
2	2
3	4
4	1
5	3
6	5

2.2<8<12.2

2.2<15<12.2

2.2<4<12.2

2.2<3<12.2



At this point, can't move 5 or 6 or would lose balance factor