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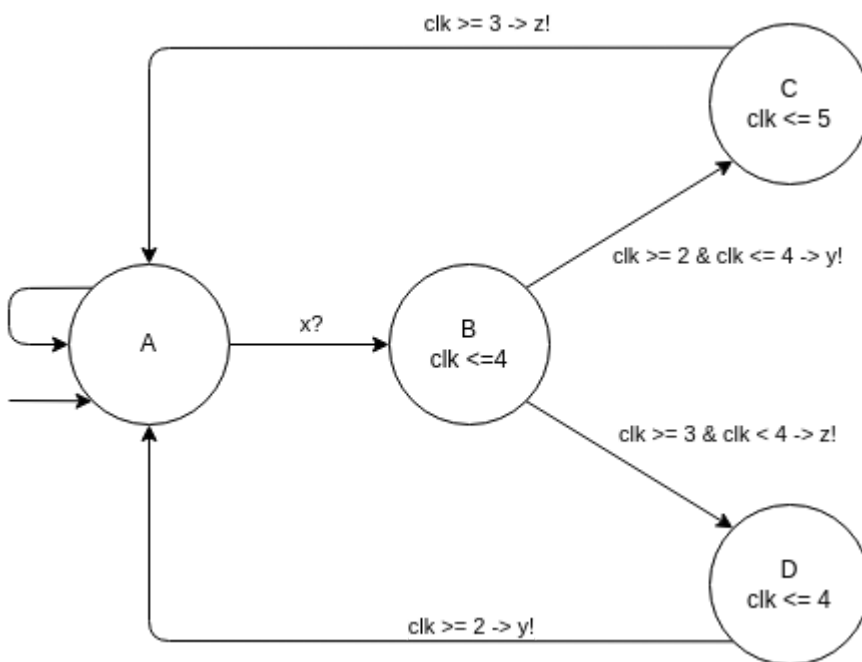
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Homework 7

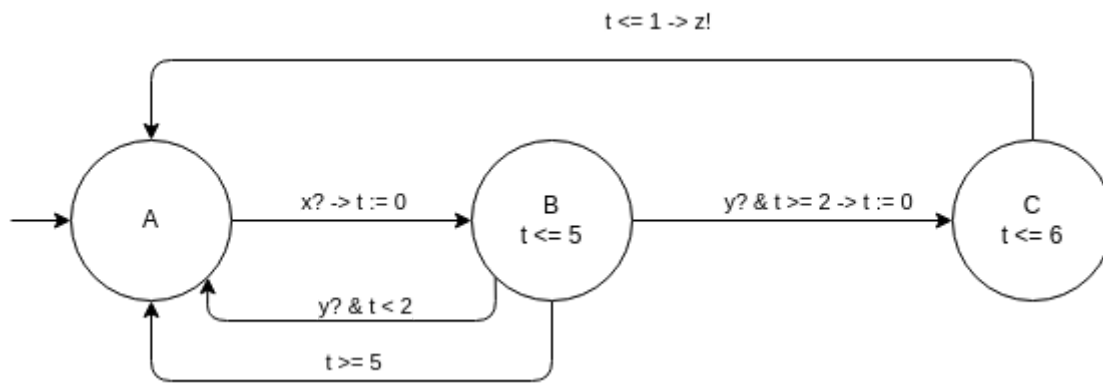
Exercise 7.1:

Consider a timed process with an input event x and two output events y and z . Whenever the process receives an input event on the channel x , it issues output events on the channels y and z such that (1) the time delay between $x?$ and $y!$ is between two and four units, (2) the time delay between $x?$ and $z!$ is between three and five units, and (3) while the process is waiting to issue its outputs, any additional input events are ignored. Design a timed state machine that exactly models this description.



Exercise 7.2:

Consider a timed process with two input events x and y and an output event z . Initially, the process is waiting to receive an input event $x?$. If this event occurs at time t , then the process waits to receive an input on the channel y . If the event $y?$ occurs before time $t+2$ or does not occur before time $t+5$, then the process simply returns to the initial state, and if the event $y?$ is received at some time t' between times $t+2$ and $t+5$, then the process issues an output event on z at some time between times $t'+1$ and $t'+6$ and returns to the initial state. Unexpected input events (e.g., the event y in the initial mode) are ignored. Design a timed state machine that exactly models this description.



Exercise 7.4:

Figure 7.6 shows the product extended-state machine that captures the behavior of the composition of two instances of the timed process `TimedBuf` shown in figure 7.5. Now consider the composition of two instances of the timed process `TimedBuf` connected in series as shown in figure 7.7. Draw the extended-state machine with four modes and two clocks that captures the behavior of this composite process.

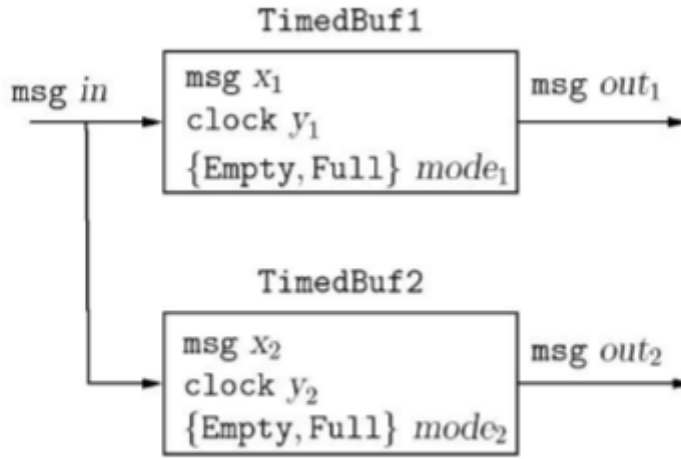


Figure 7.5: Composition of Two Instances of the Process TimedBuf

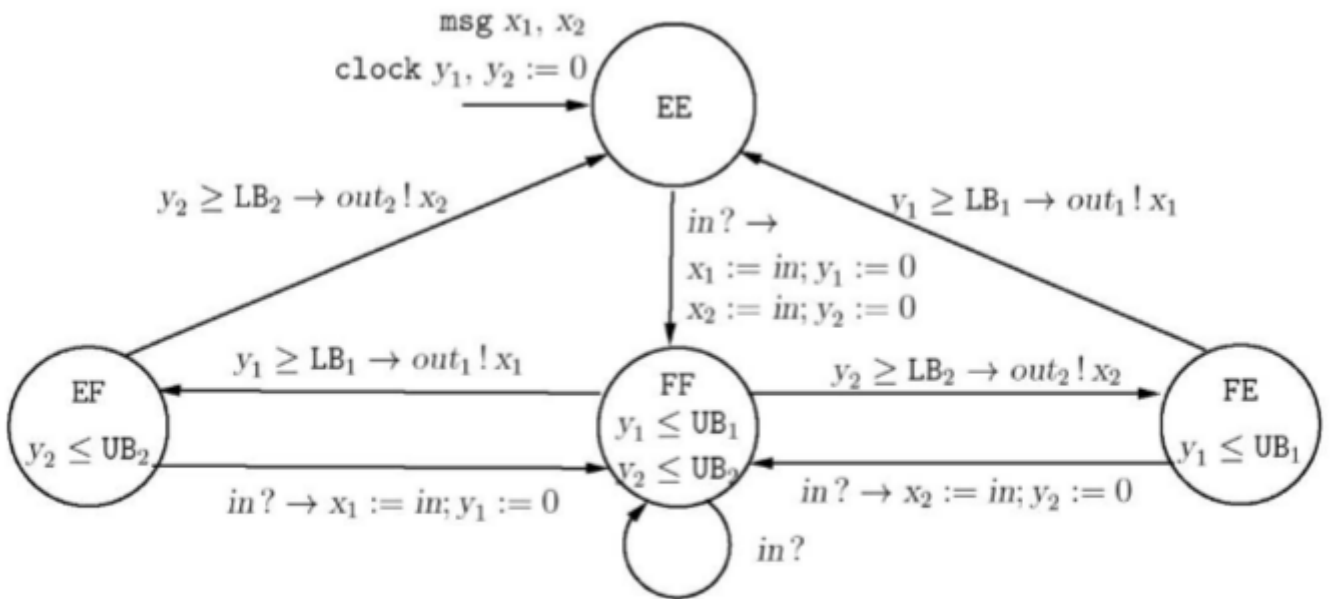


Figure 7.6: State Machine for Composition of Two TimedBuf Processes

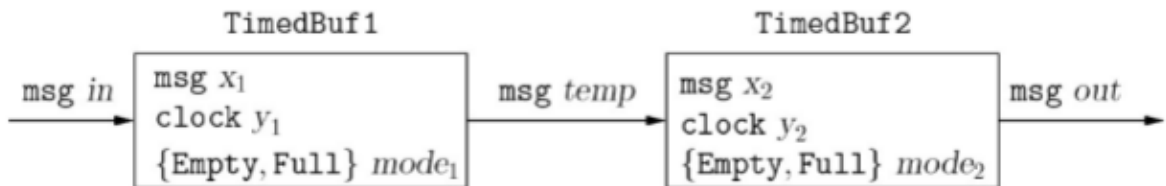
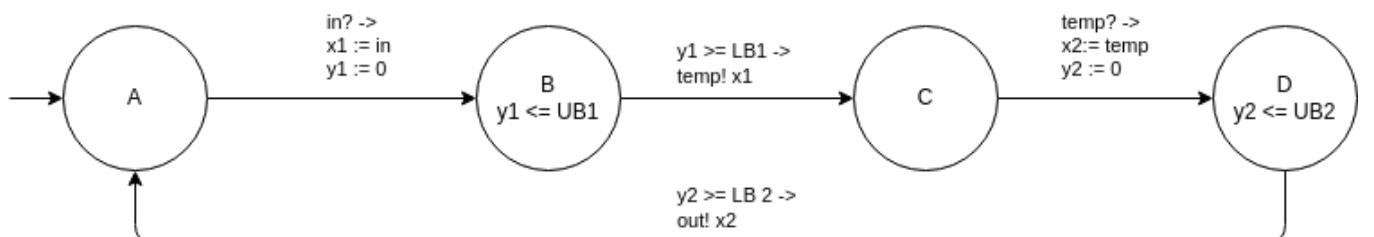


Figure 7.7: Composition of Two Instances of TimedBuf Processes in Series



Exercise 7.7:

For the timing-based mutual exclusion protocol of figure 7.9, consider the starvation-freedom requirement "if a process P enters the mode Test, then it will eventually enter the mode crit." Does the system satisfy the starvation-freedom requirement? If not, show a counter-example.

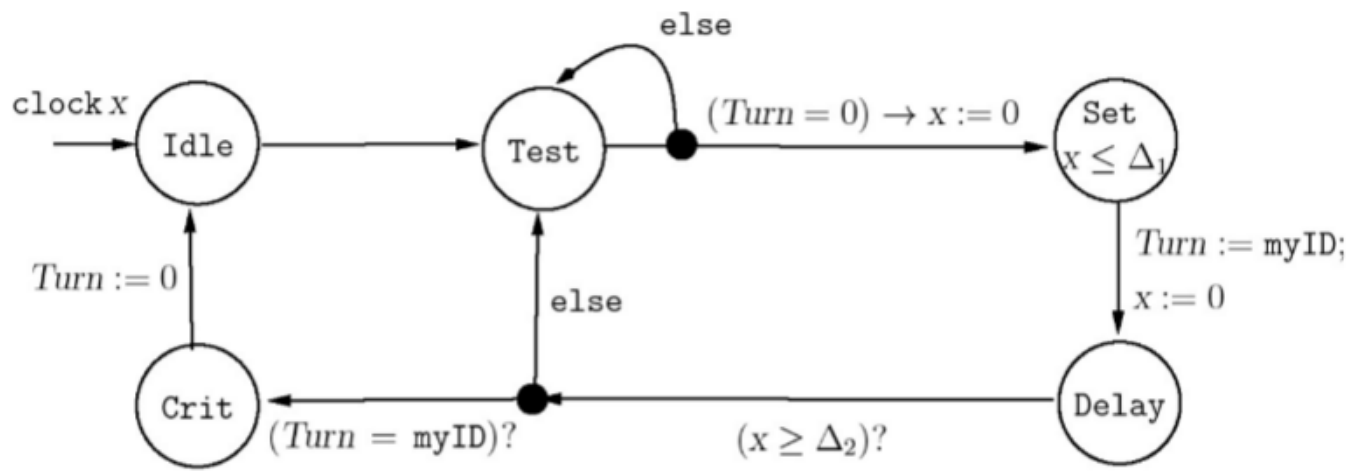


Figure 7.9: Timing-based Mutual Exclusion

Using discrete timing where $\Delta_1 = 2$ and $\Delta_2 = 1$

clock X	P1	P2	TURN
1	IDLE	IDLE	0
2	TEST	IDLE	0
3	SET	IDLE	0
0	DELAY	IDLE	1
1	DELAY	TEST	1
2	CRIT	TEST	1
3	IDLE	TEST	0
4	TEST	TEST	0
5	SET	TEST	0
6	DELAY	TEST	1
7	DELAY	TEST	1

Exercise 7.15:

Suppose a timed automaton has two clocks x_1 and x_2 . Before entering a mode A, suppose we know that $(3 \leq x_1 \leq 4)$ and $(1 \leq x_1 - x_2 \leq 6)$ and $(x_2 \geq 0)$:

1. Show the DBM corresponding to the given constraints.
 2. Is the DBM in part (1) canonical? If not, obtain an equivalent canonical form.
 3. Suppose the clock-invariant of mode A is ($x_2 \leq 5$). Compute the canonical DBM that captures the set of clock values that can be reached as the process waits in mode A.
 4. Consider a mode-switch out of mode A with guard ($x_1 \geq 7$) and update $x_1 := 0$. Compute the canonical DBM that captures the set of clock values that are possible after taking this transition.
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1.

$$x_1 - x_0 \leq 3$$

$$x_0 - x_1 \leq -4$$

$$x_1 - x_2 \leq 6$$

$$x_2 - x_1 \leq 1$$

$$x_2 - x_0 \leq 0$$

$$x_0 - x_2 \leq 0$$

	X0	X1	X2
X0	0	-4	0
X1	3	0	6
X2	0	1	0

2.

$$x_1 - x_0 \leq 3 \ \& \ x_2 - x_1 \leq 1 \ := \ x_2 - x_0 \leq 4 \text{ <---- Not more restrictive}$$

$$x_1 - x_0 \leq 3 \ \& \ x_0 - x_2 \leq 0 \ := \ x_1 - x_2 \leq 3$$

$$x_0 - x_1 \leq -4 \ \& \ x_1 - x_2 \leq 6 \ := \ x_0 - x_2 \leq 2 \text{ <---- Not more restrictive}$$

$$x_0 - x_1 \leq -4 \ \& \ x_2 - x_0 \leq 0 \ := \ x_2 - x_1 \leq -4$$

$$x_1 - x_2 \leq 6 \ \& \ x_2 - x_0 \leq 0 \ := \ x_1 - x_0 \leq 6 \text{ <---- Not more restrictive}$$

$$x_2 - x_1 \leq 1 \ \& \ x_0 - x_2 \leq 0 \ := \ x_0 - x_1 \leq 1 \text{ <---- Not more restrictive}$$

	X0	X1	X2
X0	0	-4	0
X1	3	0	3
X2	0	-4	0

3.

$$x_2 - x_0 \leq 5 \quad x_0 - x_2 \leq -5$$

	X0	X1	X2
X0	0	inf	-5
X1	inf	0	inf
X2	5	inf	0

	X0	X1	X2
X0	0	-4	-5
X1	3	0	3
X2	0	-4	0

4.

$$(7 \leq x_1 \leq 8) \text{ and } (8 \leq x_1 - x_2 \leq 13) \text{ and } (x_2 \geq 0) \rightarrow x := 0$$

$$x_1 - x_0 \leq 8$$

$$x_0 - x_1 \leq -7$$

$$x_1 - x_2 \leq 13$$

$$x_2 - x_1 \leq 8$$

$$x_2 - x_0 \leq 0$$

$$x_0 - x_2 \leq 0$$

	X0	X1	X2
X0	0	-7	0
X1	8	0	13
X2	0	8	0

$$x_1 := 0$$

	X0	X1	X2
X0	0	-7	0
X1	8	0	13
X2	0	8	0