

Homework 8

Problem 2

A

How does the feedback work?

The feedback for $1 + x + x^4$ works that the output is saved into the left register and the previous value of the left register is XOR'ed with the output.

At count 4: the output is 1, the left register is 0. So the second-to-left register is given the value $1^0=1$, and the left register is given the output, 1.

At count 13: the output is 1, the left register is 1, so the second-to-left register is $1^1=0$, and the left register is given the output, 1.

At count 15: this is like 13 where the output is 1 and the left register is 1. So the second-to-left register is 0, and the left register is given the output, 1.

This takes 16 clock cycles to return to its original form.

Count	State	Output
0	1 0 0 0	0
1	0 1 0 0	0
2	0 0 1 0	0
3	0 0 0 1	1
4	1 1 0 0	0
5	0 1 1 0	0
6	0 0 1 1	1
7	1 1 0 1	1
8	1 0 1 0	0
9	0 1 0 1	1
10	1 1 1 0	0
11	0 1 1 1	1
12	1 1 1 1	1
13	1 0 1 1	1
14	1 0 0 1	1
15	1 0 0 0	0

B

The output for the starting register state [1,0,0,0] is

Count	State	Output
0	1 1 0 0	0
1	0 1 1 0	0
2	0 0 1 1	1
3	1 1 1 1	1
4	1 0 0 1	1
5	1 0 1 0	0
6	0 1 0 1	1
7	1 1 0 0	0
8	0 1 1 0	0
9	0 0 1 1	1
10	1 1 1 1	1
11	1 0 0 1	1
12	1 0 1 0	0
13	0 1 0 1	1
14	1 1 0 0	0
15	0 1 1 0	0

The state repeats after at count 7. Below are the 4 initial states that do traverse all states. Something interesting is most of the states are covered in two initial conditions, taking 7 cycles to repeat. Two of the states repeat every cycle, and don't go into any other states ([0,0,0,0], and [1,0,1,1])

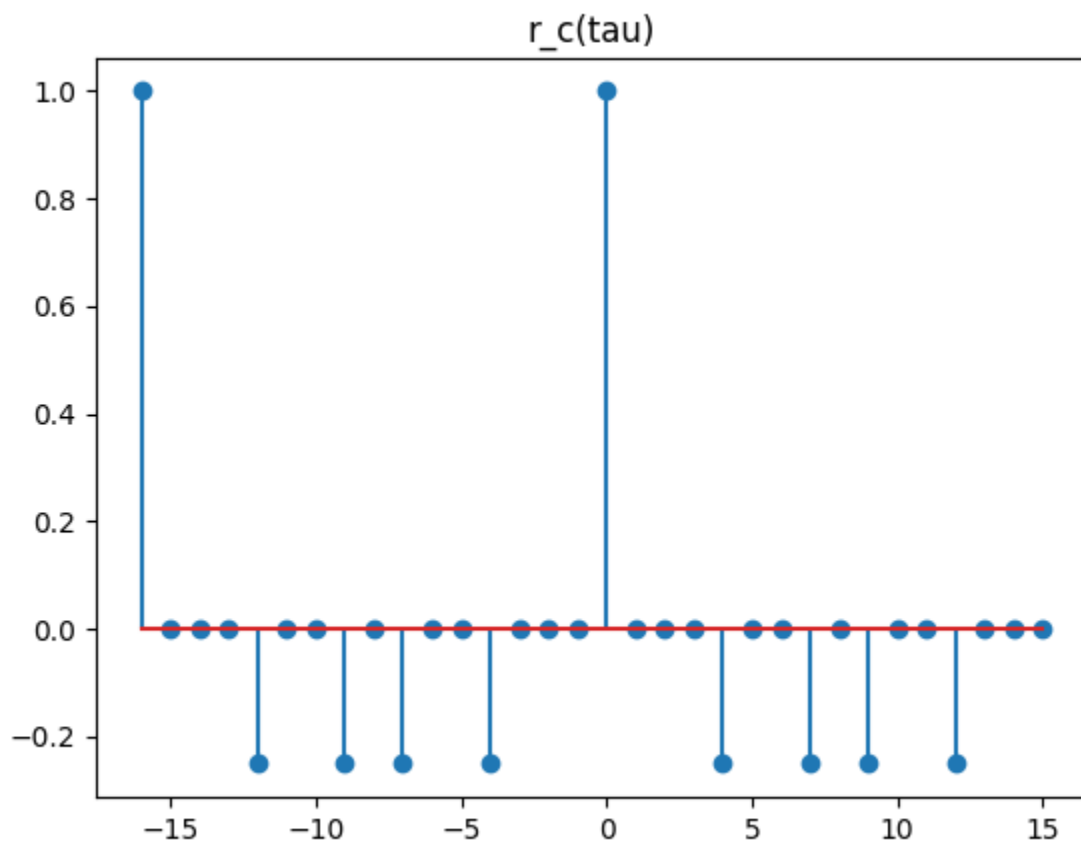
[1, 1, 0, 0] | Count | State | Output | |-----|-----|-----| | 0 | 1 1 0 0 | 0 | | 1 | 0 1 1 0 | 0 | | 2 | 0 0 1 1 | 1 | | 3 | 1 1 1 1 | 1 | | 4 | 1 0 0 1 | 1 | | 5 | 1 0 1 0 | 0 | | 6 | 0 1 0 1 | 1 | | 7 | 1 1 0 0 | 0 |

[0, 0, 0, 0] | Count | State | Output | |-----|-----|-----| | 0 | 0 0 0 0 | 0 | | 1 | 0 0 0 0 | 0 | | 2 | 0 0 0 0 | 0 | | 3 | 0 0 0 0 | 0 | | 4 | 0 0 0 0 | 0 | | 5 | 0 0 0 0 | 0 | | 6 | 0 0 0 0 | 0 | | 7 | 0 0 0 0 | 0 |

[0, 0, 0, 1] | Count | State | Output | |-----|-----|-----| | 0 | 0 0 0 1 | 1 | | 1 | 1 1 1 0 | 0 | | 2 | 0 1 1 1 | 1 | | 3 | 1 1 0 1 | 1 | | 4 | 1 0 0 0 | 0 | | 5 | 0 1 0 0 | 0 | | 6 | 0 0 1 0 | 0 | | 7 | 0 0 0 1 | 1 |

[1, 0, 1, 1] | Count | State | Output | |-----|-----|-----| | 0 | 1 0 1 1 | 1 | | 1 | 1 0 1 1 | 1 | | 2 | 1 0 1 1 | 1 | | 3 | 1 0 1 1 | 1 | | 4 | 1 0 1 1 | 1 | | 5 | 1 0 1 1 | 1 | | 6 | 1 0 1 1 | 1 | | 7 | 1 0 1 1 | 1 |

C



What do you observe about this autocorrelation function?

- I notice that there are only 4 values, $[-1/4, 0, 1/4, 1]$, and they seem to be repeating every mod N

D

I saw the pattern pretty early, the outputs of the long division match the 1's of the output in Table 4.11. For example, the first coefficient is x^3 and the first 1 in the output of 4.11 is in cycle 3.

$$\begin{array}{r}
 x^3 + x^6 + x^7 \\
 \hline
 1 + x^3 + x^4 \mid x^3 \\
 \quad -x^3 + x^6 + x^7 \\
 \quad \hline
 \quad 0 + x^6 + x^7 \\
 \quad \dots
 \end{array}$$

E

